

# Isocurvature bounds on ALP DM

**Thomas Schwetz**

based on Enander, Pargner, TS, 1708.04466  
Feix, Frank, Pargner, Reischke, Schäfer, TS, 1903.06194  
Feix, Hagstotz, Pargner, Reischke, Schäfer, TS, in prep.

**LDMA'19 — Venice, Italy, 20-22 Nov 2019**



# Axion-like particles (ALPs)

- pNGB boson:  
spontaneous U(1) breaking scale  $f_a$  („PQ-scale“)  
expl. U(1) breaking at  $\Lambda$  by exotic non-pert. effects

$$V(a) \approx \Lambda^4 \left[ 1 - \cos \left( \frac{a}{f_a} \right) \right], \quad m_a^2 = \left. \frac{\partial^2 V}{\partial a^2} \right|_{\min} = \frac{\Lambda^4}{f_a^2}$$

- parametrize temperature dependence

$$m_a(T) = \min \left[ \frac{\Lambda^2}{f_a}, b \frac{\Lambda^2}{f_a} \left( \frac{\Lambda}{T} \right)^n \right]$$

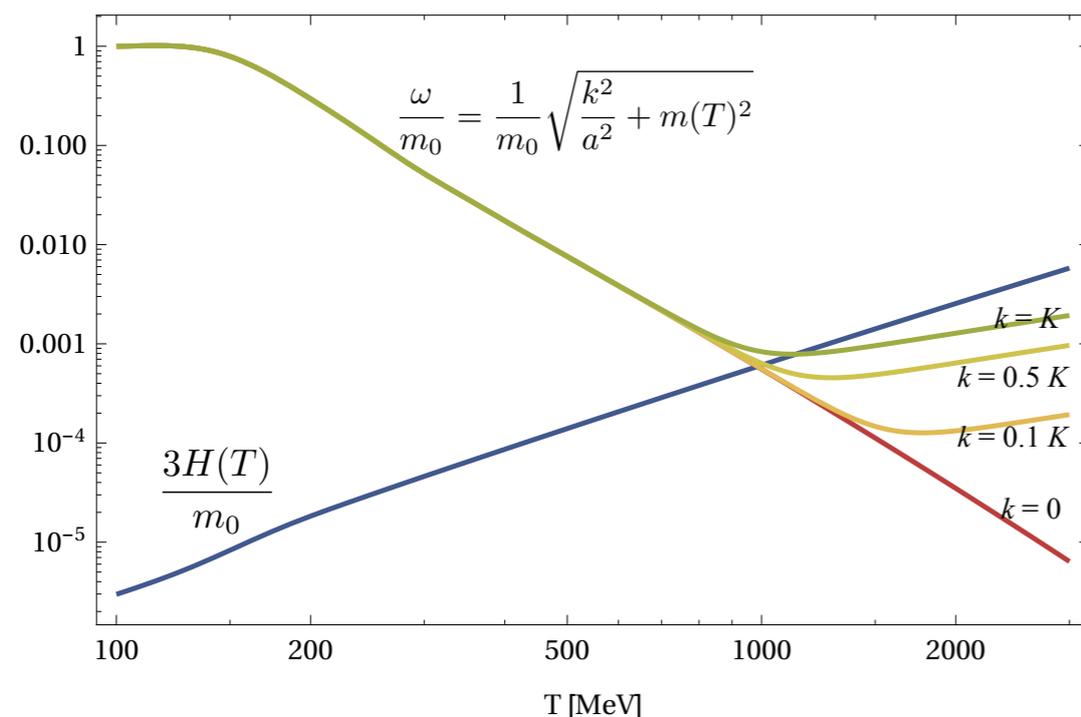
- QCD axion for:  $\Lambda = 75.5 \text{ MeV}, b = 10, n = 4$

# Cosmological evolution

small field values  $\rightarrow$  harmonic approx for potential:  $V(\theta, T) \approx \frac{1}{2}m^2(T)\theta^2$

$$\ddot{\theta}_k + 3H(T)\dot{\theta}_k + \omega^2\theta_k = 0 \quad \omega^2 = \frac{k^2}{a^2} + m(T)^2$$

- ▶ modes outside the horizon  $3H(T) > \omega$ :  
over-damped oscillator, field frozen:  $\theta_k = \text{const}$
- ▶ modes inside the horizon  $3H(T) < \omega$ :  
oscillator with frequency  $\omega$ , amplitude decays with expansion  
relativistic modes are red shifted relative to non-relativistic modes



- initial conditions depend crucially on PQ-symmetry breaking happening during or after inflation

# Adiabatic vs isocurvature fluctuations

- density contrast in component  $i = \text{DM}, b, \gamma$

$$\delta_i = \frac{\rho_i - \bar{\rho}_i}{\bar{\rho}_i}$$

- decompose DM fluctuations

$$\delta_{\text{DM}} = \delta_{\text{DM}}^{\text{ad}} + \delta_{\text{DM}}^{\text{iso}}$$

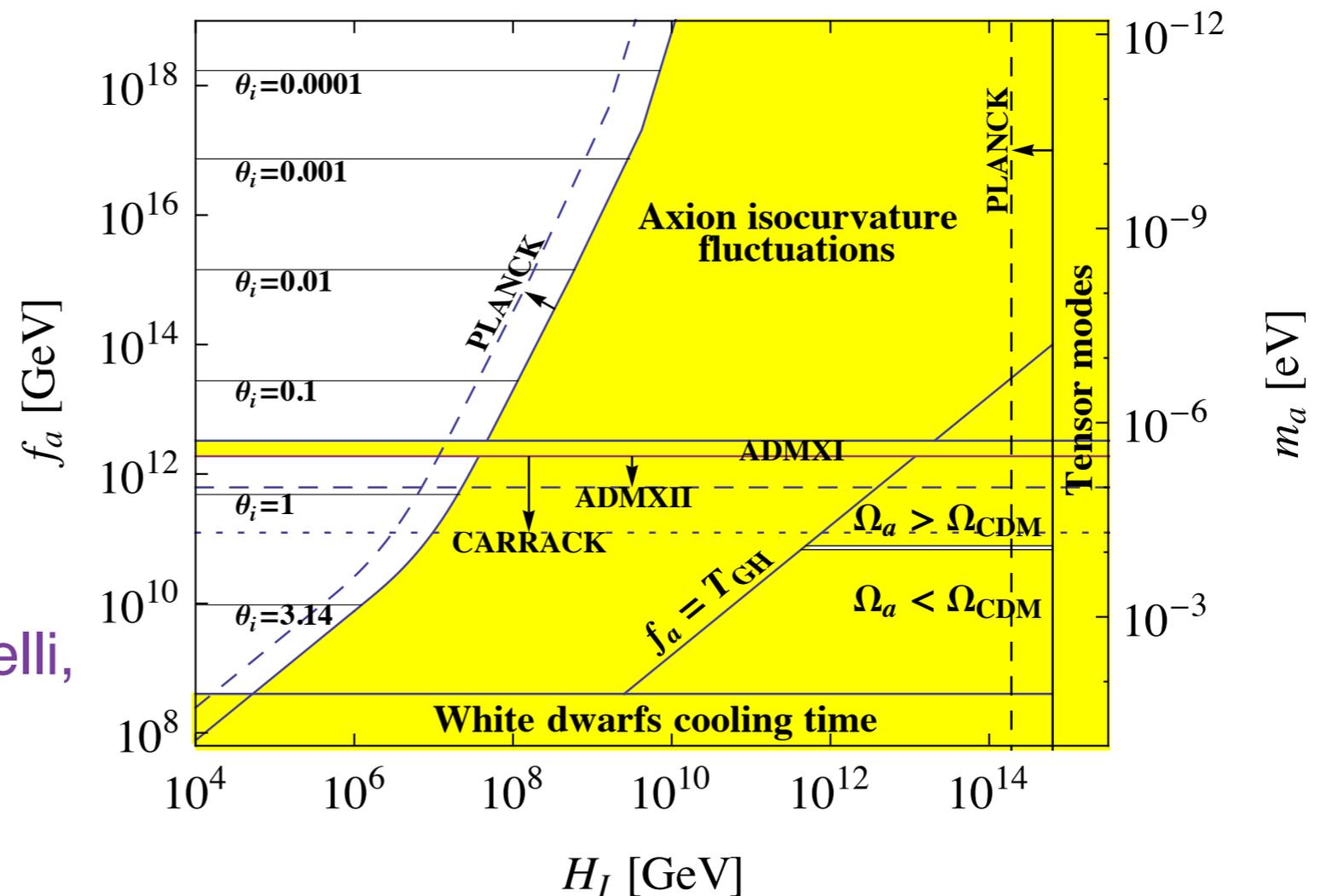
$$\delta_{\text{DM}}^{\text{ad}} = \frac{3}{4}\delta_\gamma \quad , \quad \delta_{\text{DM}}^{\text{iso}} = \delta_{\text{DM}} - \frac{3}{4}\delta_\gamma$$

- adiabatic and isocurvature fluctuations evolve differently
- constraints from CMB on isocurvature component  
( < few % at CMB scales)

# pre-inflationary case

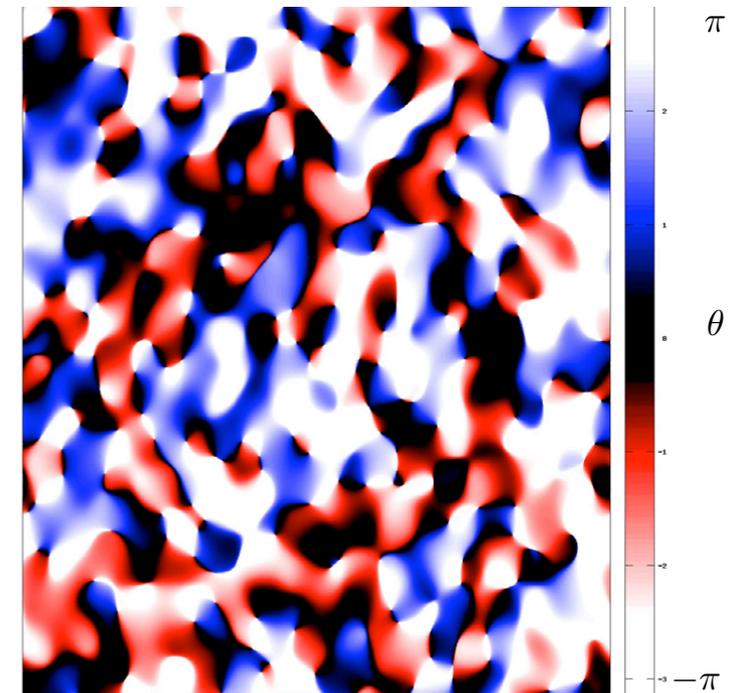
- PQ symmetry gets broken during inflation
- inflaton and axion are present simultaneously → generation of isocurvature fluctuations  
e.g. Turner Wilczek, 91; Lyth, 92

Gondolo, Visinelli,  
0903.4377



# This talk: post-inflationary scenario

- ALP field takes random values in causally disconnected regions
- random values of periodic field  $\rightarrow$  cosmic strings
- Kibble mechanism:  
`scaling` of string network: roughly one string per Hubble volume during expansion of Universe
- Before onset of field oscillations remains  
 $\sim$  one string per Hubble volume and  $\langle \theta(\vec{x})^2 \rangle = \pi^2/3$



J. Redondo

Klaer, Moore, 1708.07521; Gorghetto, Hardy, Villadoro, 1806.04677

# Initial condition

- axion field smooth on scales  $<$  horizon  
uncorrelated on scales  $>$  horizon
- assume power spectrum for axion field w Gaussian cut-off

$$\langle \theta_k \theta_{k'}^* \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P_\theta(k)$$

$$P_\theta^G(k) = \frac{8\pi^4}{3\sqrt{\pi}K^3} \exp\left(-\frac{k^2}{K^2}\right)$$

- normalization: fixed by flat distribution  $\langle \theta(\vec{x})^2 \rangle = \pi^2/3$
- cut-off: comoving horizon wave-number  
shortly before field starts oscillating:  $K = a_i H_i$

# Axion energy density

$$\rho(\vec{x}) = \frac{f_{\text{PQ}}^2}{2} \left[ \dot{\theta}^2 - \frac{1}{a^2} (\vec{\nabla} \theta)^2 + m^2 (T) \theta^2 \right]$$

$$\theta_k(a) = \theta_k f_k(a)$$

- go to Fourier space for field

$$\rho(\vec{x}) = \frac{1}{(2\pi)^6} \frac{f_{\text{PQ}}^2}{2} \int d^3 k d^3 k' \theta_k \theta_{k'}^* F(k, k') e^{-i\vec{x}(\vec{k}-\vec{k}')}$$

$$F(k, k') = \dot{f}_k \dot{f}_{k'} + \left( \frac{\vec{k} \cdot \vec{k}'}{a^2} + m^2 (T) \right) f_k f_{k'}$$

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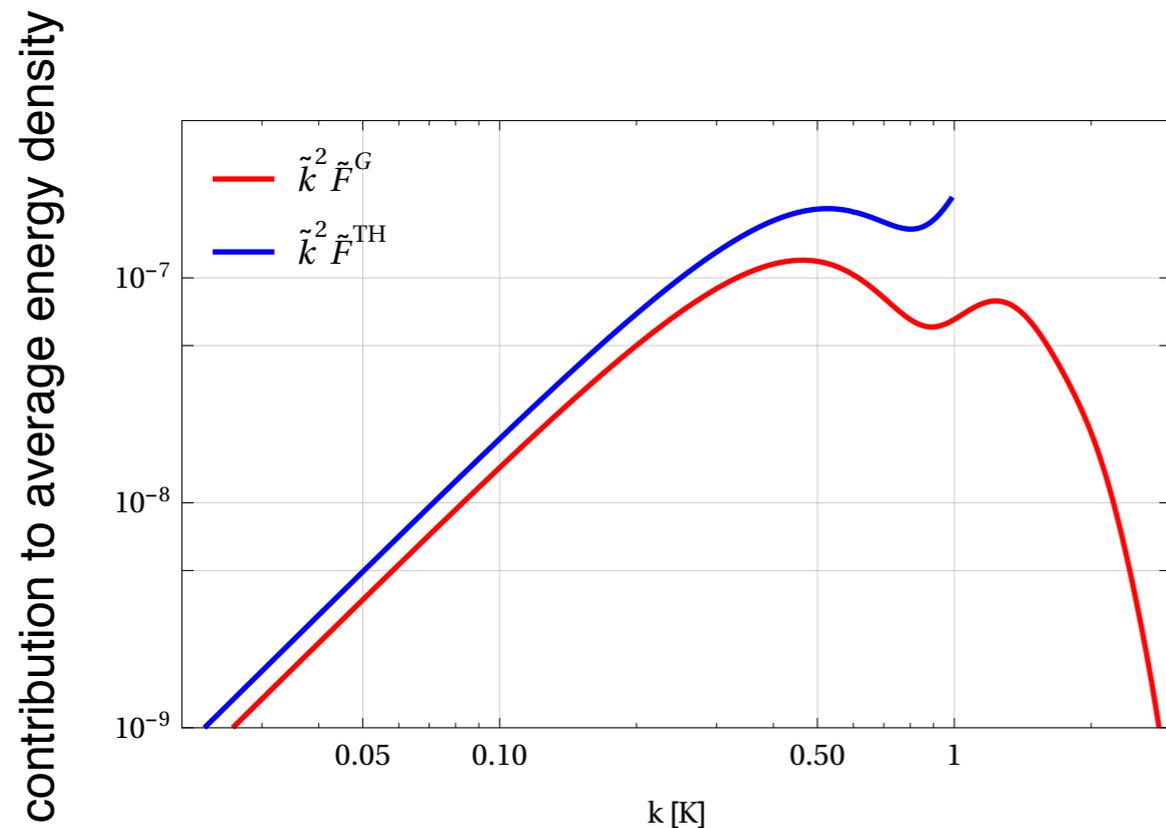
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- average energy density (**without string contribution**):

$$\bar{\rho} \equiv \langle \rho(\vec{x}) \rangle = \frac{1}{2\pi^2} \frac{f_{\text{PQ}}^2}{2} \int_0^\infty dk k^2 P_\theta(k) F(k, k)$$

$$F(k, k) = \dot{f}_k^2 + \omega_k^2 f_k^2$$

# Axion energy density



generalizes usual estimate:

$$\rho \sim f_a^2 m_0 m(T_{\text{osc}}) \left( \frac{a_{\text{osc}}}{a} \right)^3 \langle \theta^2 \rangle$$

- average energy density (without string contribution):

$$\bar{\rho} \equiv \langle \rho(\vec{x}) \rangle = \frac{1}{2\pi^2} \frac{f_{\text{PQ}}^2}{2} \int_0^\infty dk k^2 P_\theta(k) F(k, k)$$

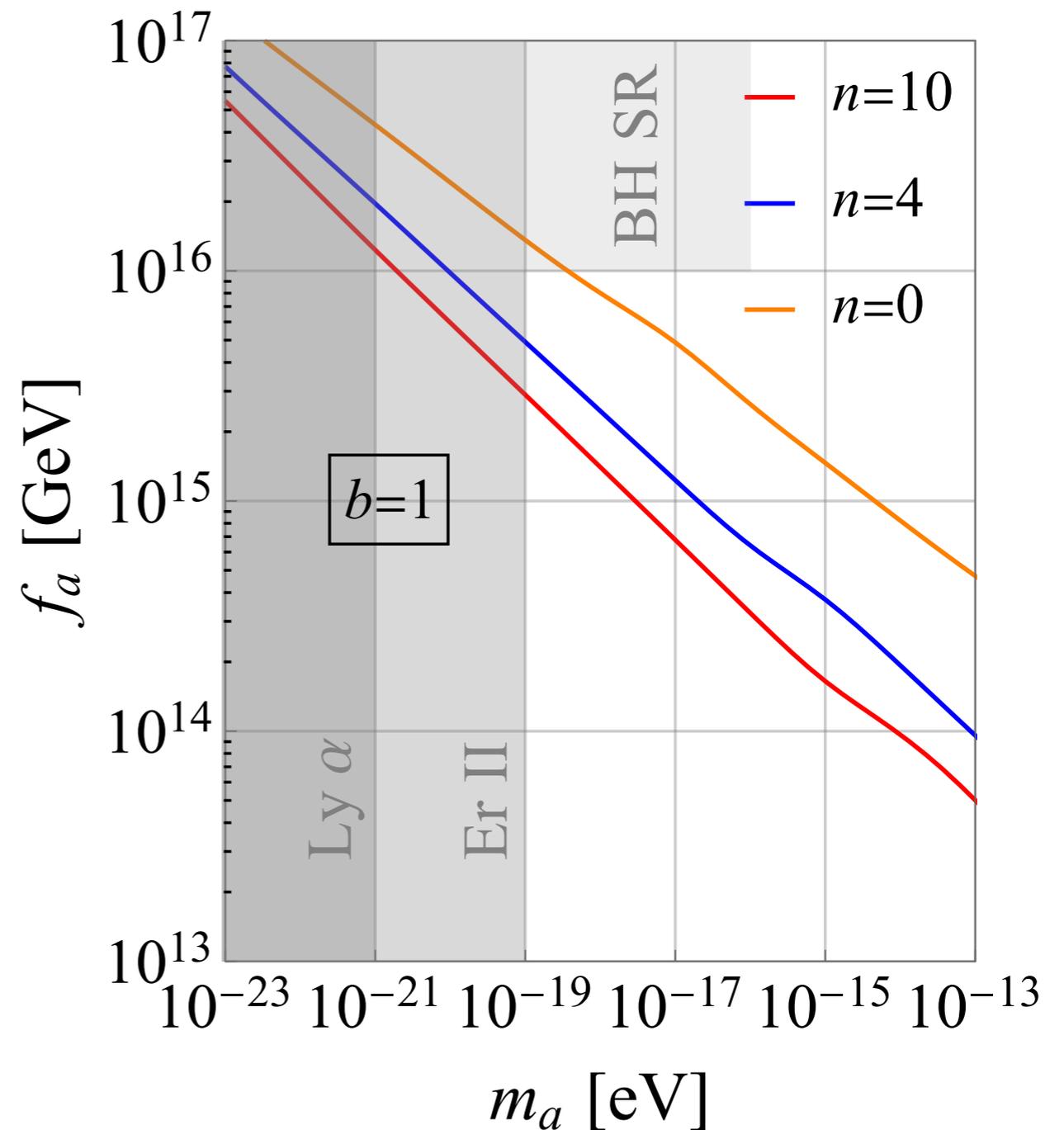
$$F(k, k) = \dot{f}_k^2 + \omega_k^2 f_k^2$$

# ALP DM relic density

use DM abundance to  
fix  $f_a$  as a function of ALP  
mass

$$m_a(T) = \min \left[ \frac{\Lambda^2}{f_a}, b \frac{\Lambda^2}{f_a} \left( \frac{\Lambda}{T} \right)^n \right]$$

$$m_0 = \frac{\Lambda^2}{f_a}$$



# Density power spectrum

Fourier transform of the density:

$$\rho_q = \frac{1}{(2\pi)^3} \frac{f_{\text{PQ}}^2}{2} \int d^3k \theta_k \theta_{k-q}^* F(k, k - q)$$

variance (use Wick's theorem):

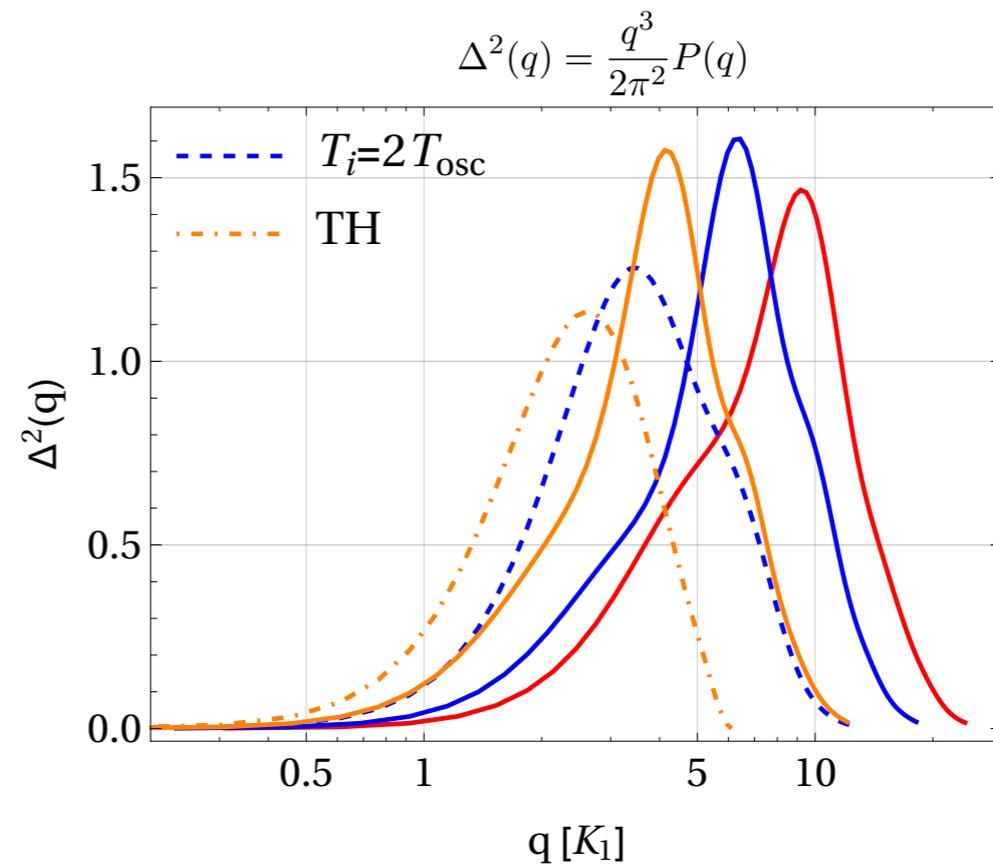
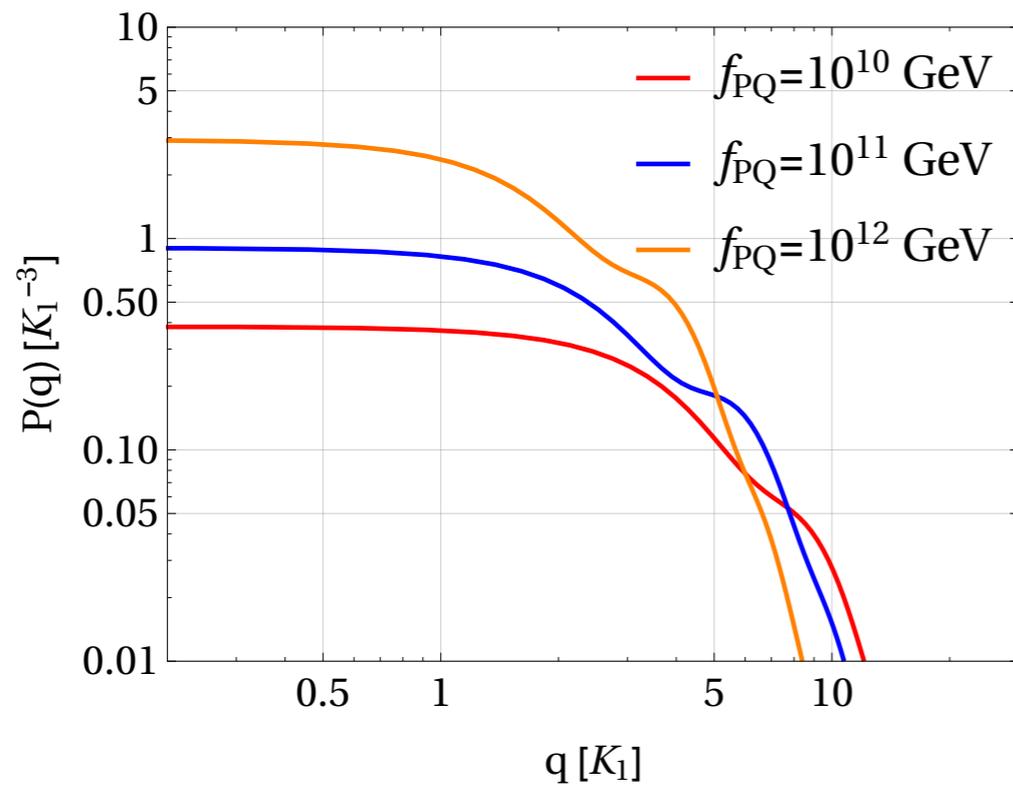
$$\begin{aligned} \langle |\rho_q|^2 \rangle &= \left[ \frac{1}{(2\pi)^3} \frac{f_{\text{PQ}}^2}{2} \right]^2 \int d^3k d^3k' \langle \theta_k \theta_{k-q}^* \theta_{k'}^* \theta_{k'-q} \rangle F(k, k - q) F^*(k', k' - q) \\ &= 2 \frac{V}{(2\pi)^3} \left( \frac{f_{\text{PQ}}^2}{2} \right)^2 \int d^3k P_\theta(|\vec{k}|) P_\theta(|\vec{k} - \vec{q}|) F(k, k - q)^2 \end{aligned}$$

this gives for the power spectrum

$$P(q) = \frac{1}{V} \frac{\langle |\rho_q|^2 \rangle}{\bar{\rho}^2} = 2(2\pi)^3 \frac{\int d^3k P_\theta(|\vec{k}|) P_\theta(|\vec{k} - \vec{q}|) F(k, k - q)^2}{\left[ \int d^3k P_\theta(k) F(k, k) \right]^2}$$

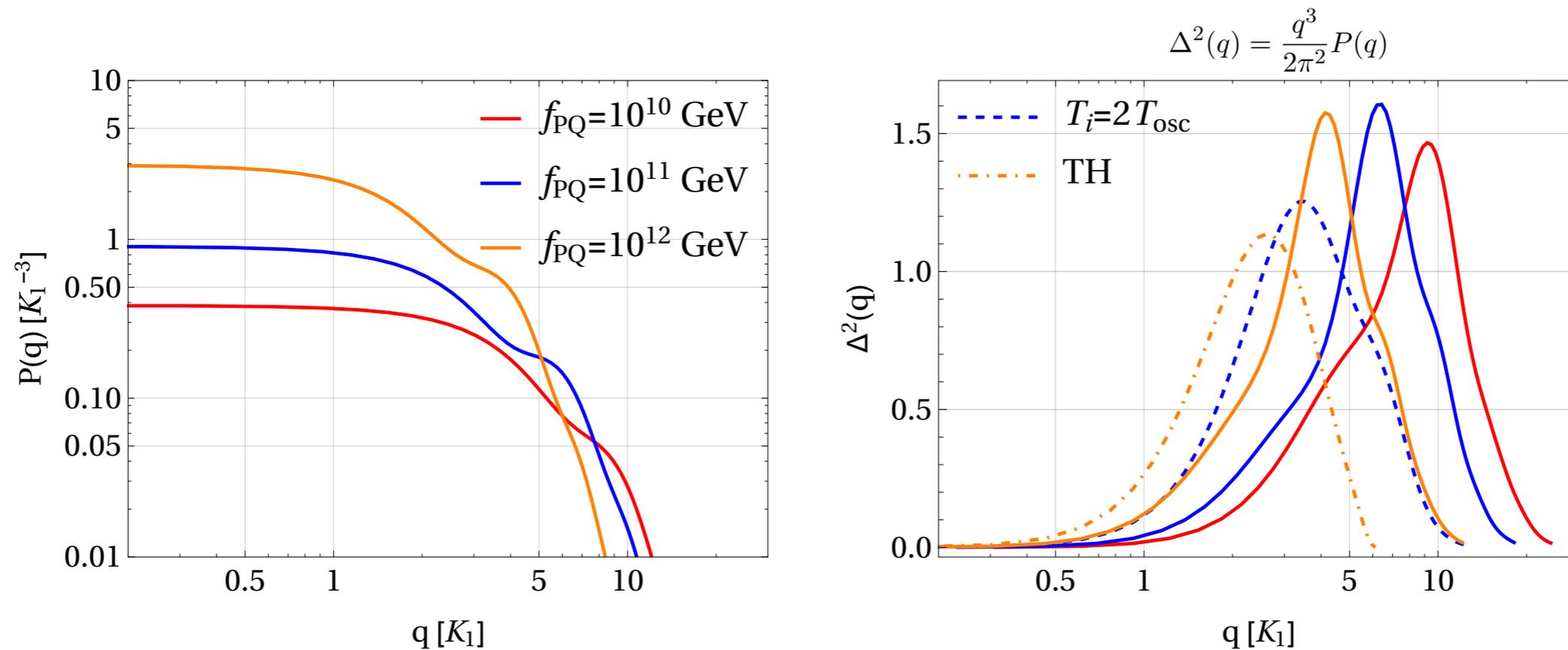
# Density power spectrum

Enander, Pargner, TS, 1708.04466



- density fluctuations of order one
- charact. size a few times smaller than horizon @  $T_{\text{osc}}$
- formation of grav. bound objects (mini cluster) Hogan, Rees, 1988

# Density power spectrum

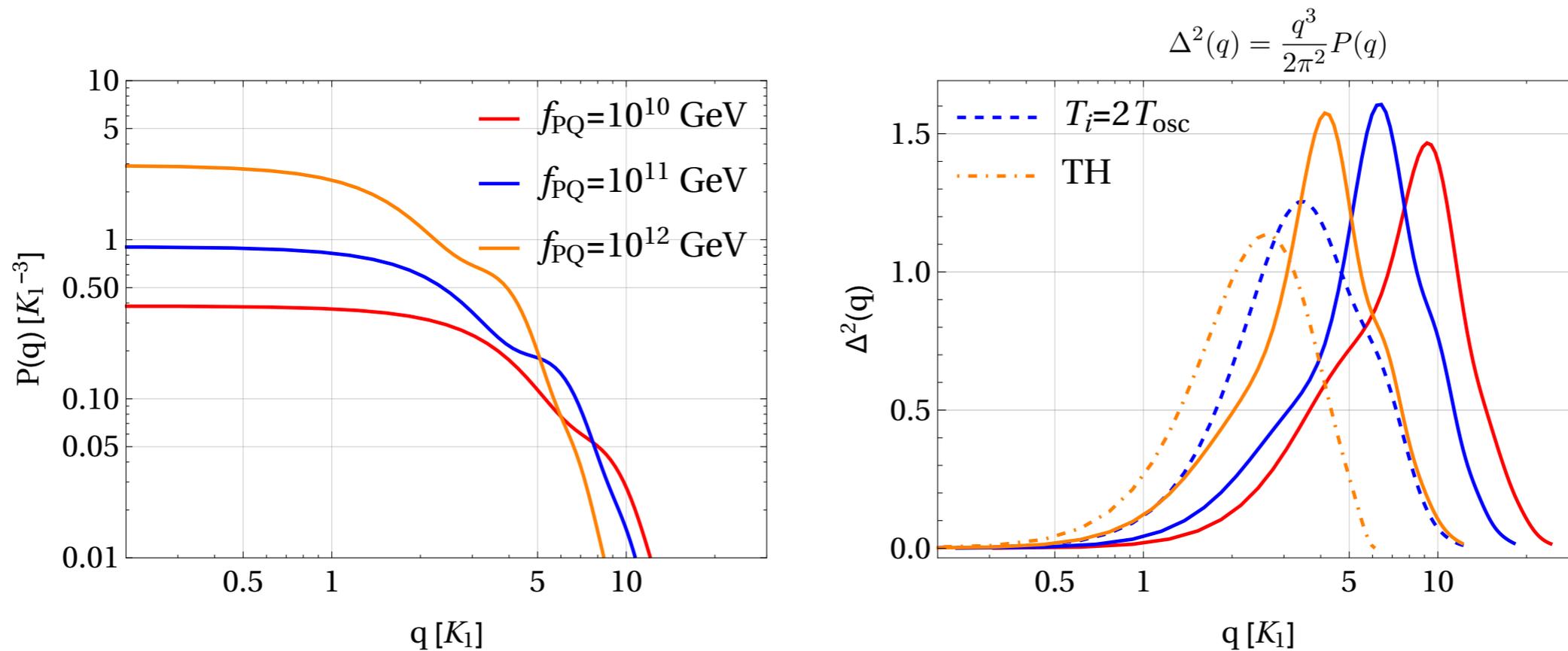


$$K \equiv a_{osc} H(T_{osc}) \quad \Delta^2(k) \approx C \left( \frac{k}{K} \right)^3 \quad k \ll K$$

$$P(k) \approx \frac{2\pi^2 C}{K^3}$$

- white noise power spectrum for small  $k$
- fluctuations linear for  $k \ll K$
- coeff.  $C \approx 0.04 - 0.3$  obtained by fitting to numerical results for  $P(k)$

# Density power spectrum

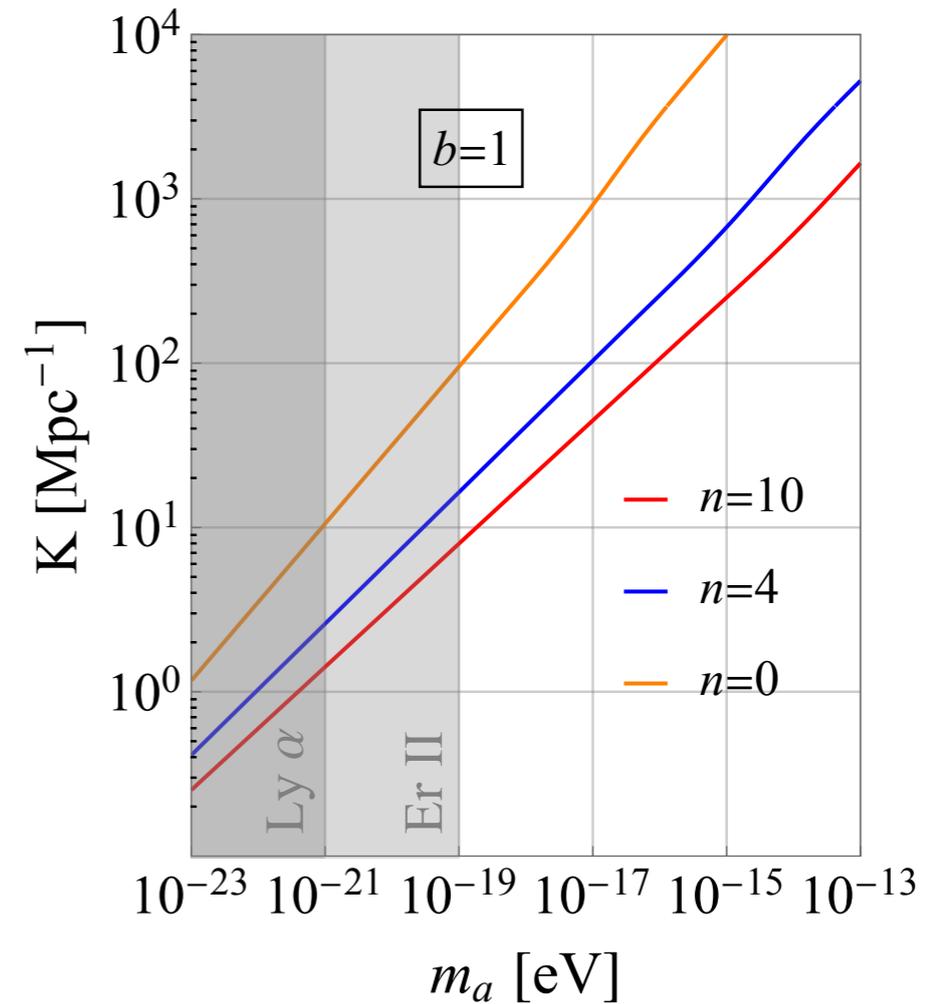
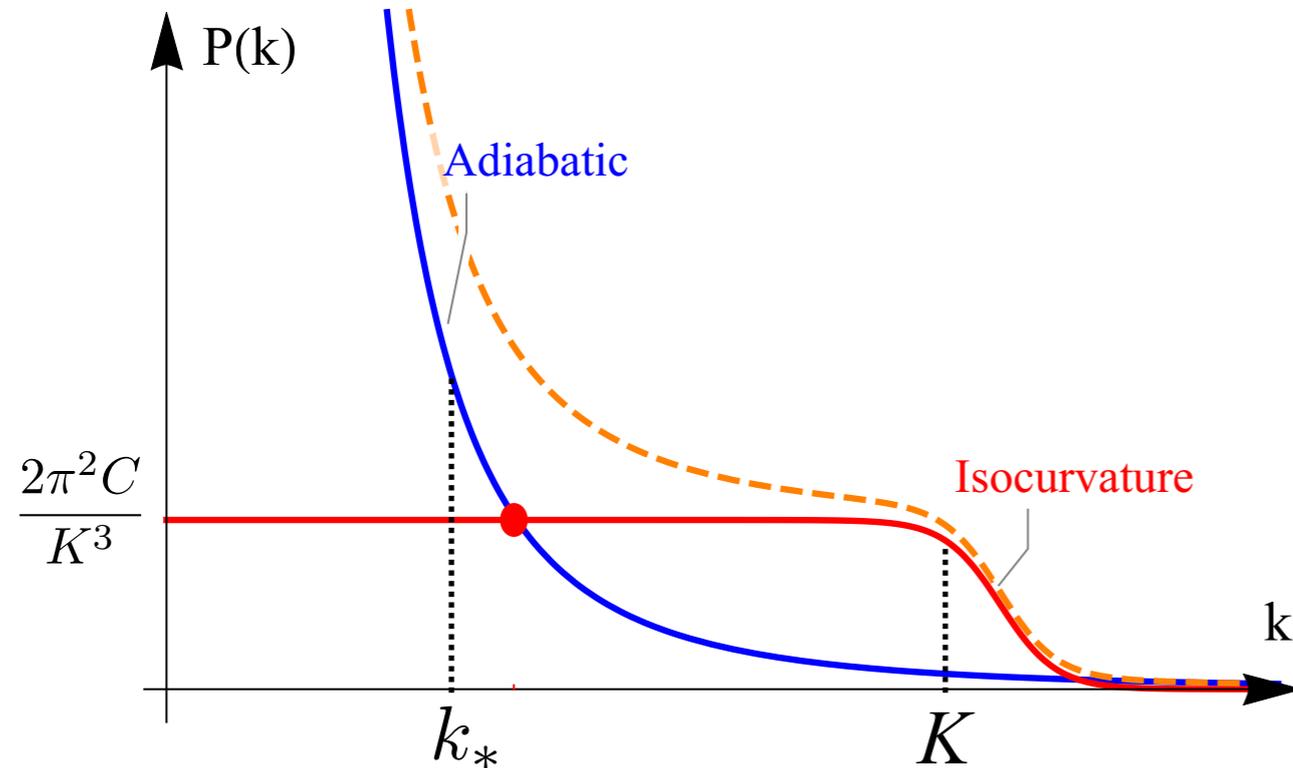


$$K \equiv a_{\text{osc}} H(T_{\text{osc}}) \quad \Delta^2(k) \approx C \left( \frac{k}{K} \right)^3 \quad k \ll K$$

$$P(k) \approx \frac{2\pi^2 C}{K^3}$$

- expect qualitative similar result including topological defects and the periodic potential
- allow for factor 5 uncertainty from comparison with simulations (incl. strings) [Vaquero, Redondo, Stadler, 1809.09241](#)

# Isocurvature fluctuations from post-inflationary ALP DM



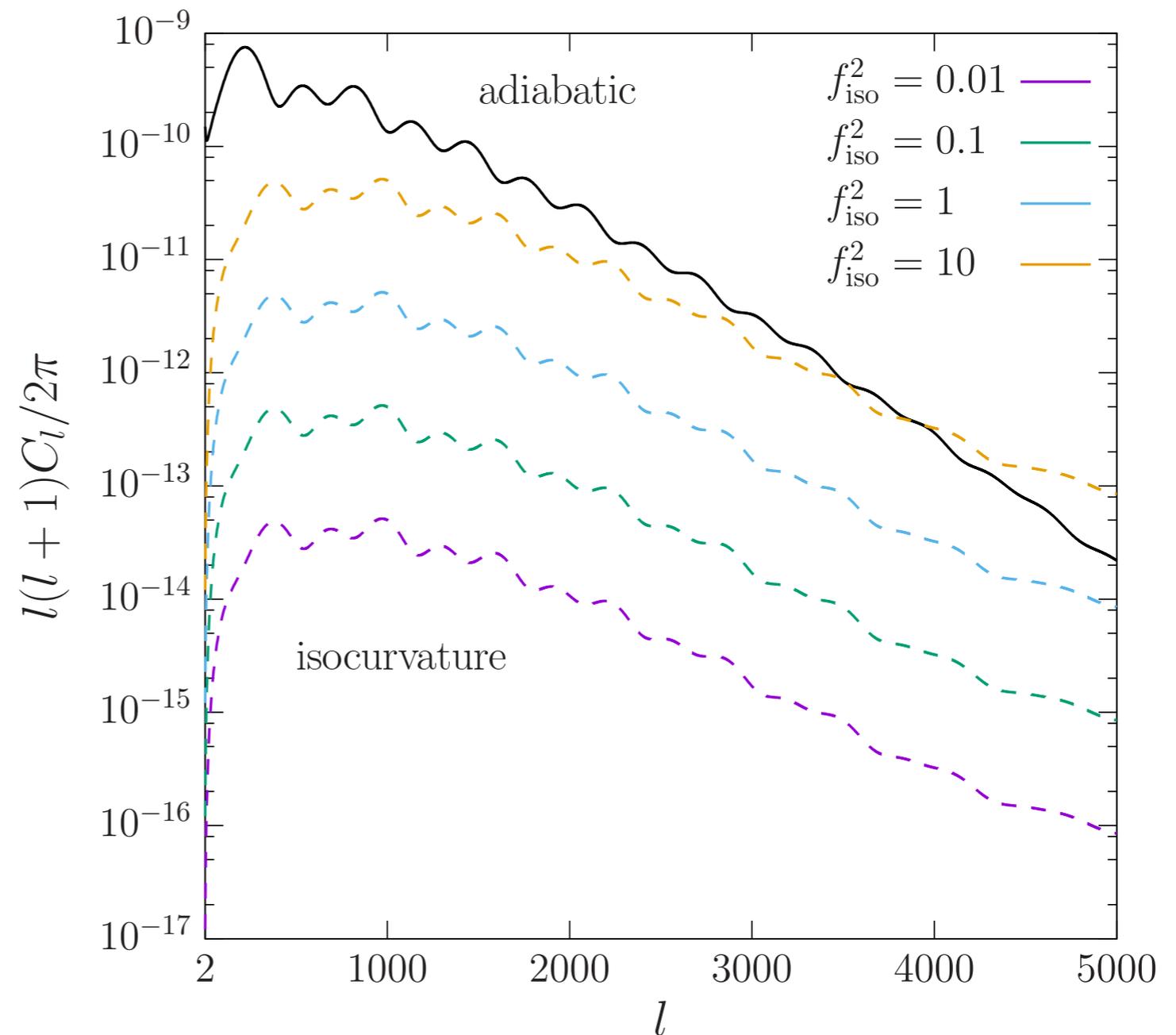
$$f_{\text{iso}}^2 \equiv \frac{P_{\text{iso}}(k)}{P_{\text{ad}}(k)} \Big|_{k_*}, \quad \Delta_{\text{ad}}^2(k) = \frac{k^3}{2\pi^2} P_{\text{ad}}(k) = A_s \left( \frac{k}{k_*} \right)^{n_s-1}$$

$$\Rightarrow f_{\text{iso}} \approx \sqrt{\frac{Ck_*^3}{A_s K^3}}$$

$$k_* = 0.05 \text{ Mpc}^{-1}$$

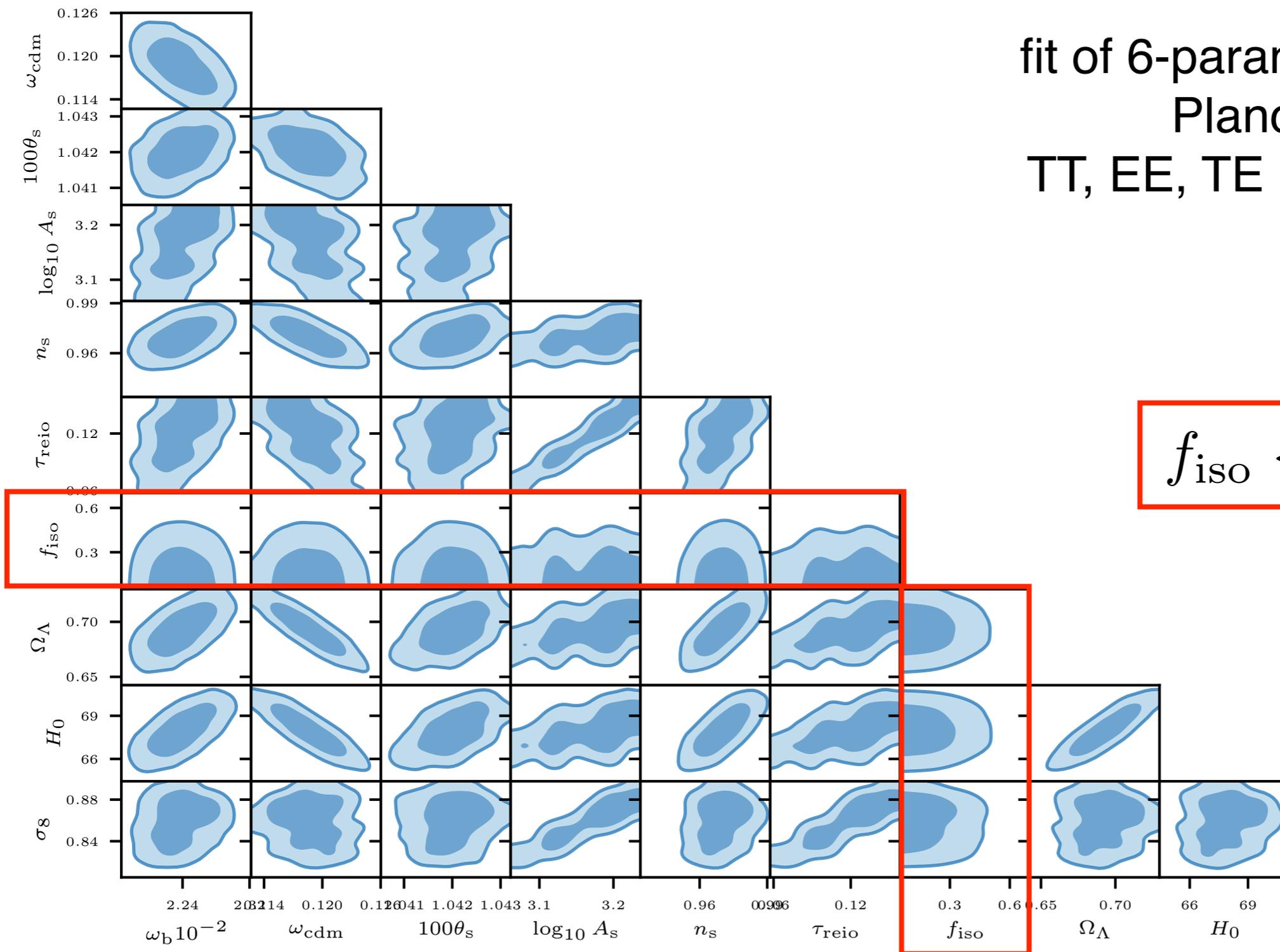
# White noise isocurvature fluctuations in the CMB

isocurvature component  
with white noise spectrum  
implemented in CLASS



Feix, Frank, Pargner, Reischke, Schäfer, TS, 1903.06194

# White noise isocurvature fluctuations in the CMB

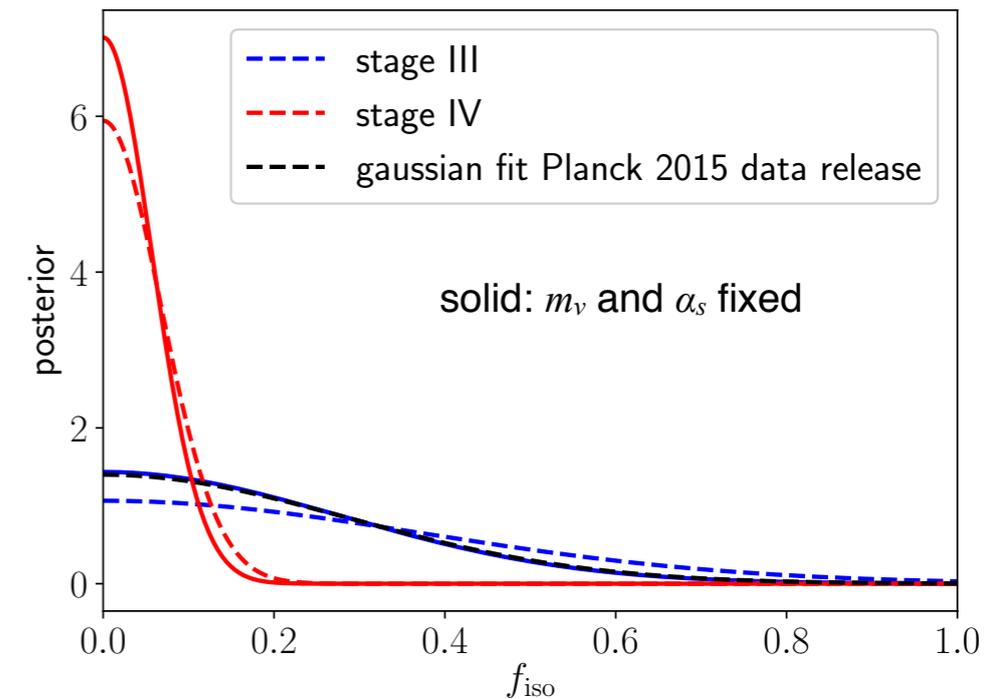


fit of 6-param LCDM + fiso  
Planck 2015  
TT, EE, TE power spectra

$$f_{\text{iso}} < 0.31 (2\sigma)$$

# Sensitivity forecasts

- Planck-like CMB experiment (s3)  
 $l = 30 - 2500$
- Stage IV CMB experiment (s4)  
 $l < 5000$  (temperature & polarization)
- 21cm observations with SKA:  
900 antennas (SKA1), 3600 antennas (SKA2)  
optimistic assumptions on foregrounds
- $\Lambda$ CDM +  $f_{\text{iso}}$  +  $\alpha_s$  +  $m_\nu$  +  $H_0$  +  $X_H$

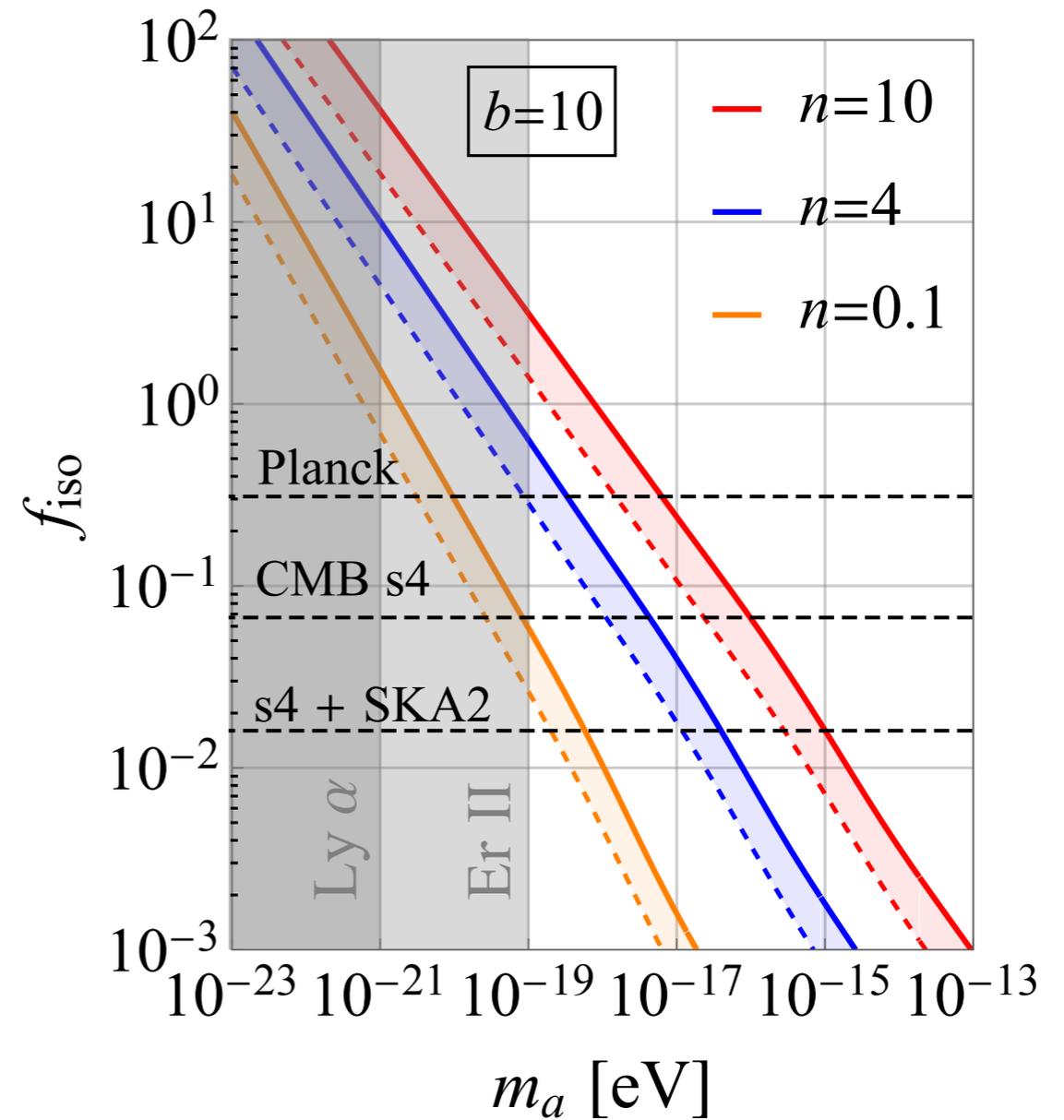
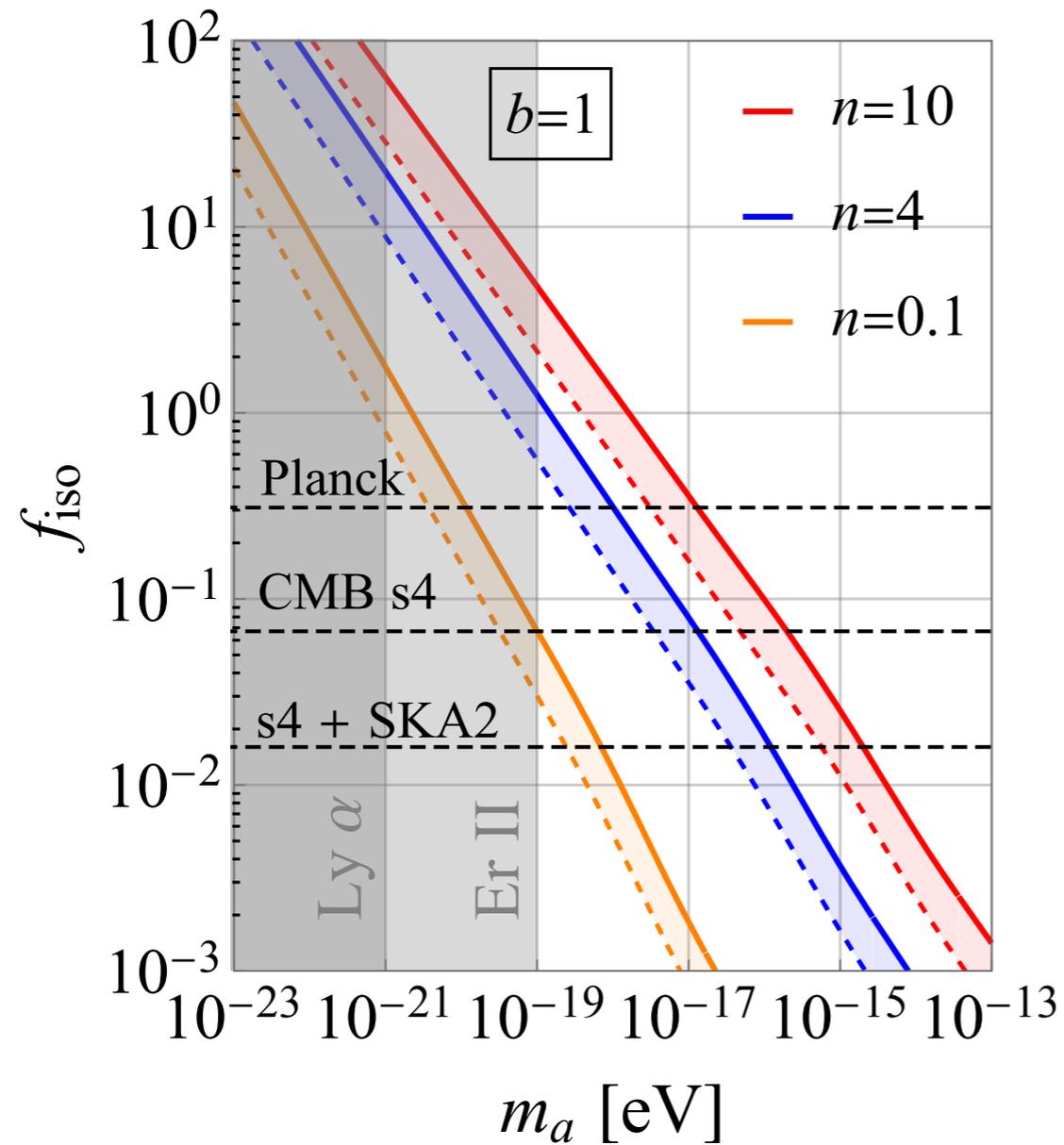


Experiment	$f_{\text{iso}}$	$\alpha_s$	$\sum m_\nu$ [eV]	$n_s$	$A_s$	$\Omega_b$	$\tau$	$h$	$\Omega_m$	$\bar{x}_H$
s3	0.38	0.0052	0.34	0.0034	0.021	0.0045	0.0045	0.032	0.038	-
s3+SKA1	0.20	0.0044	0.28	0.0031	0.021	0.0037	0.0043	0.027	0.031	0.082
s4	0.067	0.0018	0.050	0.0016	0.0080	0.00064	0.0017	0.0045	0.0053	-
s4 +SKA2	0.016	0.0017	0.042	0.0016	0.0080	0.00051	0.0017	0.0034	0.0040	0.012

1 $\sigma$  sens.

Feix, Frank, Pargner, Reischke, Schäfer, TS, 1903.06194

# Isocurvature component for DM ALPs



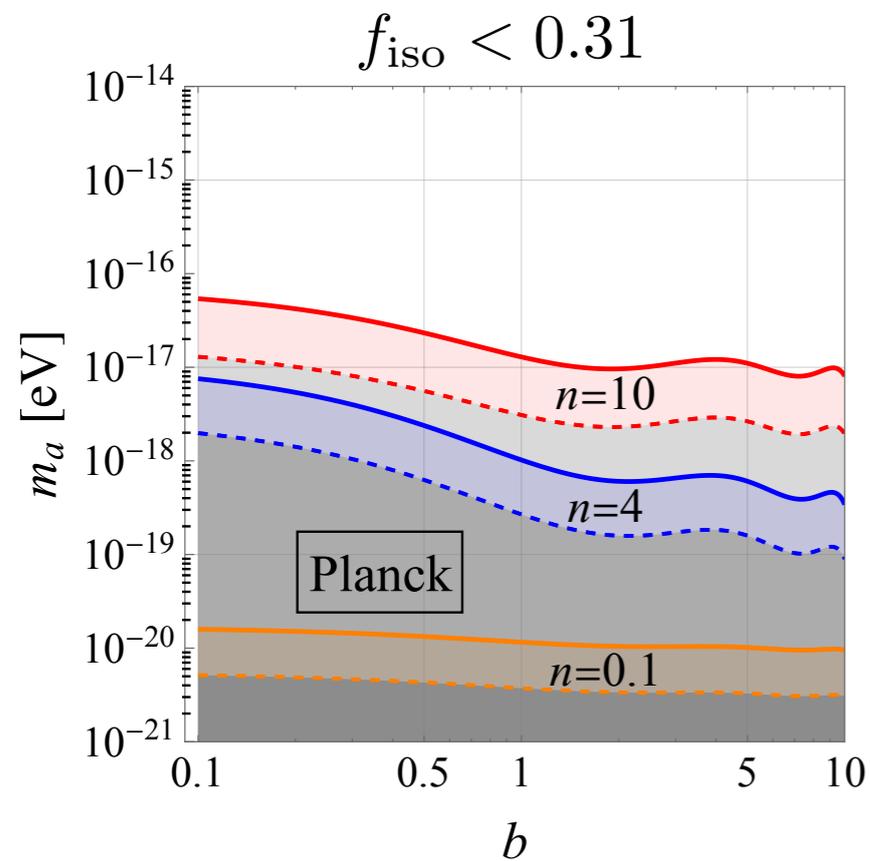
Feix et al., 1903.06194

$$f_{\text{iso}} \approx \sqrt{\frac{Ck_*^3}{A_s K^3}}$$

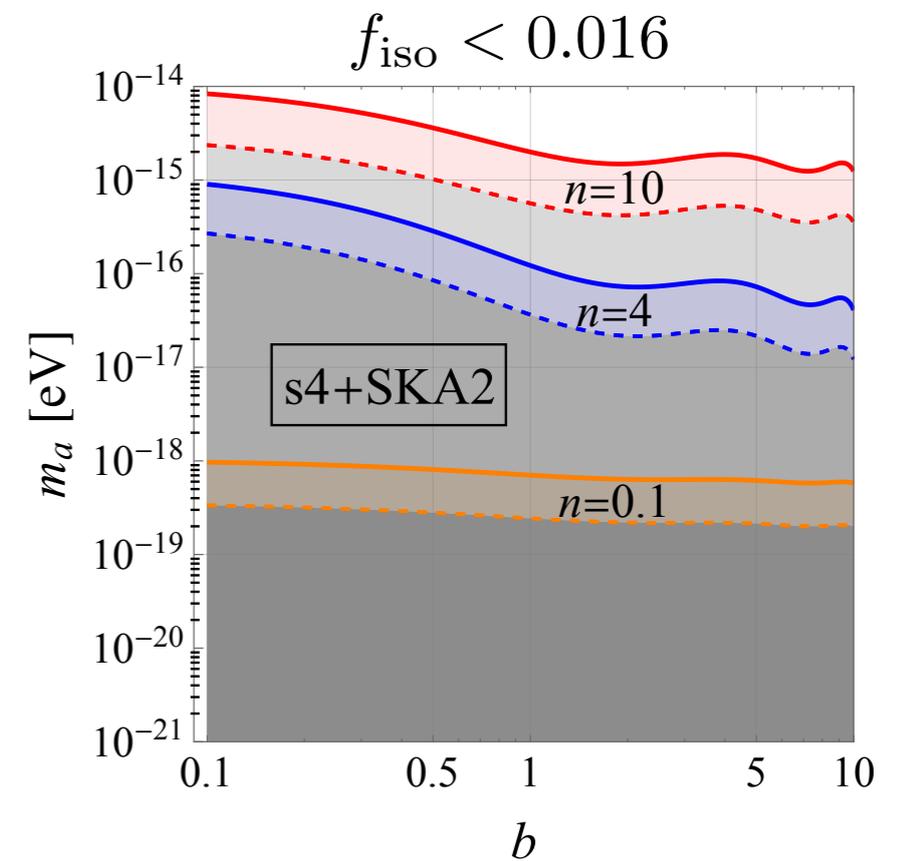
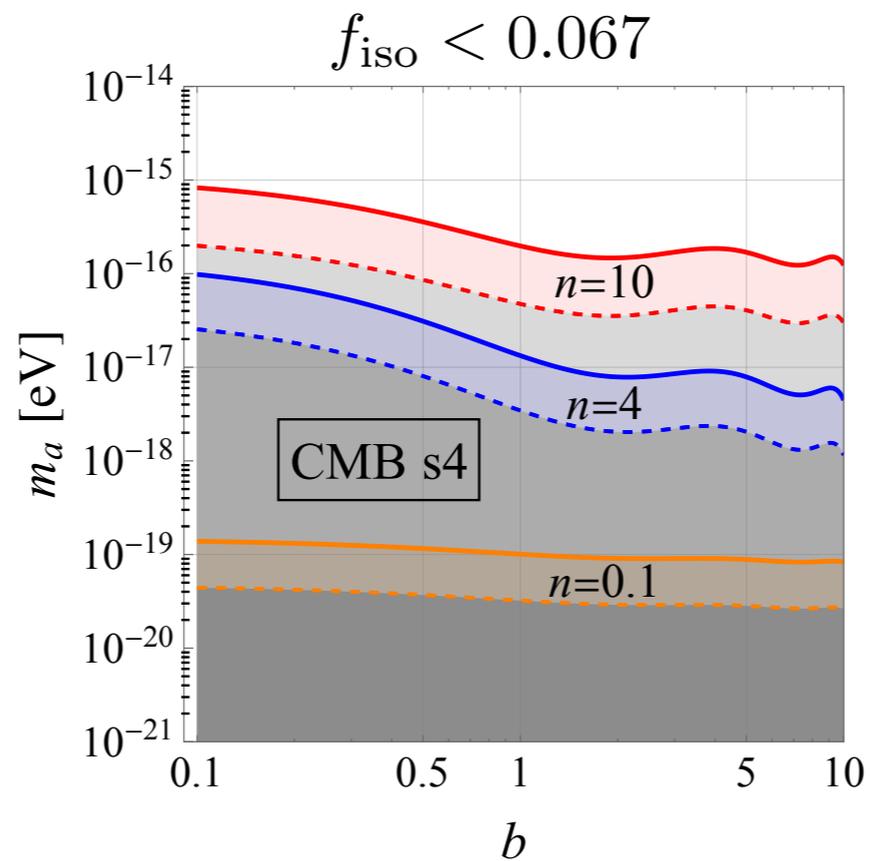
$$m_a(T) = \min \left[ \frac{\Lambda^2}{f_a}, b \frac{\Lambda^2}{f_a} \left( \frac{\Lambda}{T} \right)^n \right]$$

# Lower limits on ALP mass

Planck 2015



sensitivity forecasts



$$m_a(T) = \min \left[ \frac{\Lambda^2}{f_a}, b \frac{\Lambda^2}{f_a} \left( \frac{\Lambda}{T} \right)^n \right]$$

Feix et al., 1903.06194

# Crucial assumption: post-inflationary scenario

- PQ symmetry is broken only after end of inflation or it gets restored after inflation:

$$f_a < \max \left[ \frac{H_I}{2\pi}, \epsilon_{\text{eff}} E_I \right] \quad H_I^2 = \frac{8\pi}{3} \frac{E_I^4}{M_{Pl}^2}$$

e.g., Hertzberg, Tegmark, Wilczek, 0807.1726

- upper bound from non-observation of CMB tensor modes (in simple inflation models):

$$E_I < 1.7 \times 10^{16} \text{ GeV (95\% CL)}$$

$$\frac{H_I}{2\pi} < 10^{13} \text{ GeV} \quad \text{Planck, 1807.06211}$$

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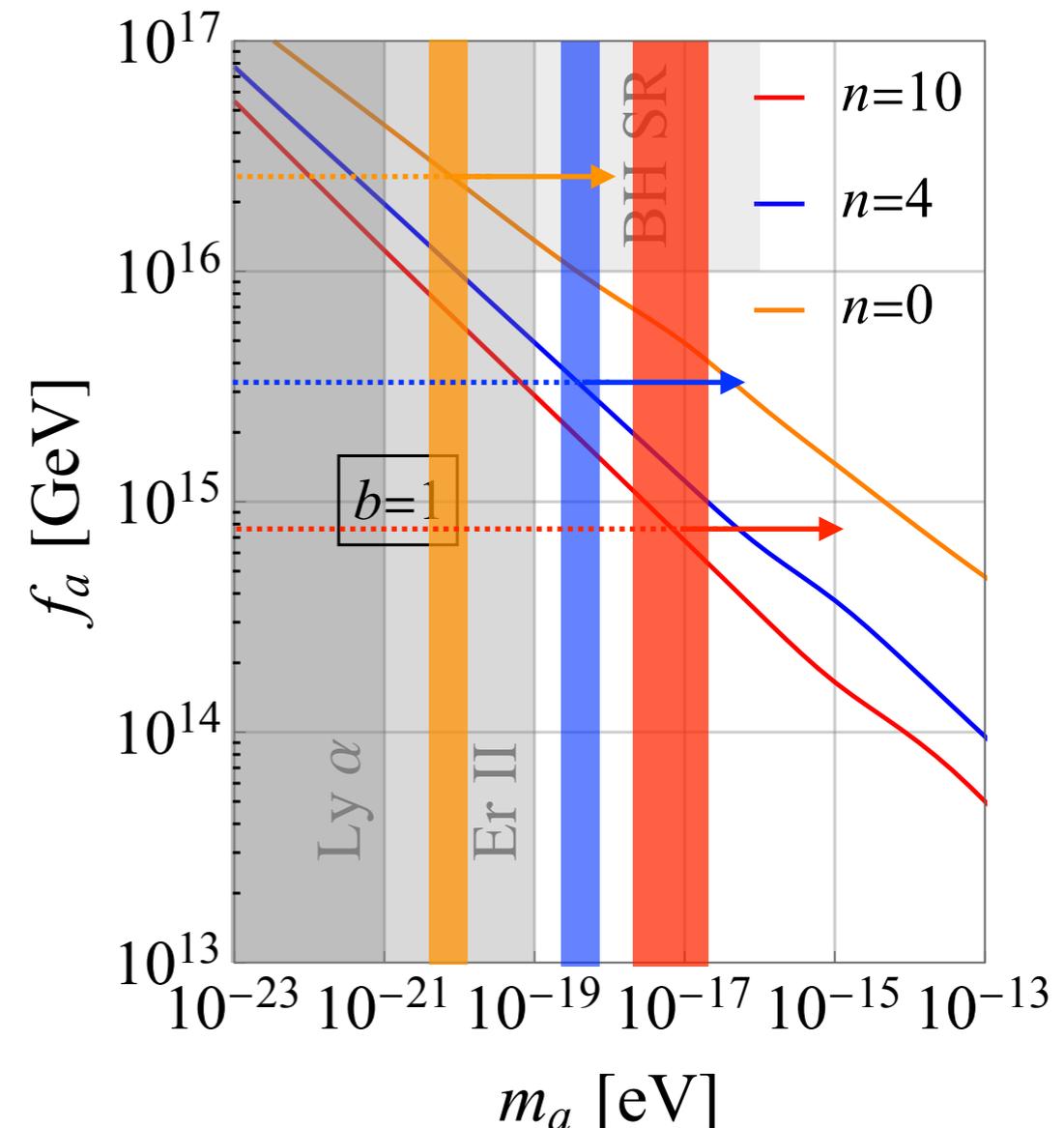
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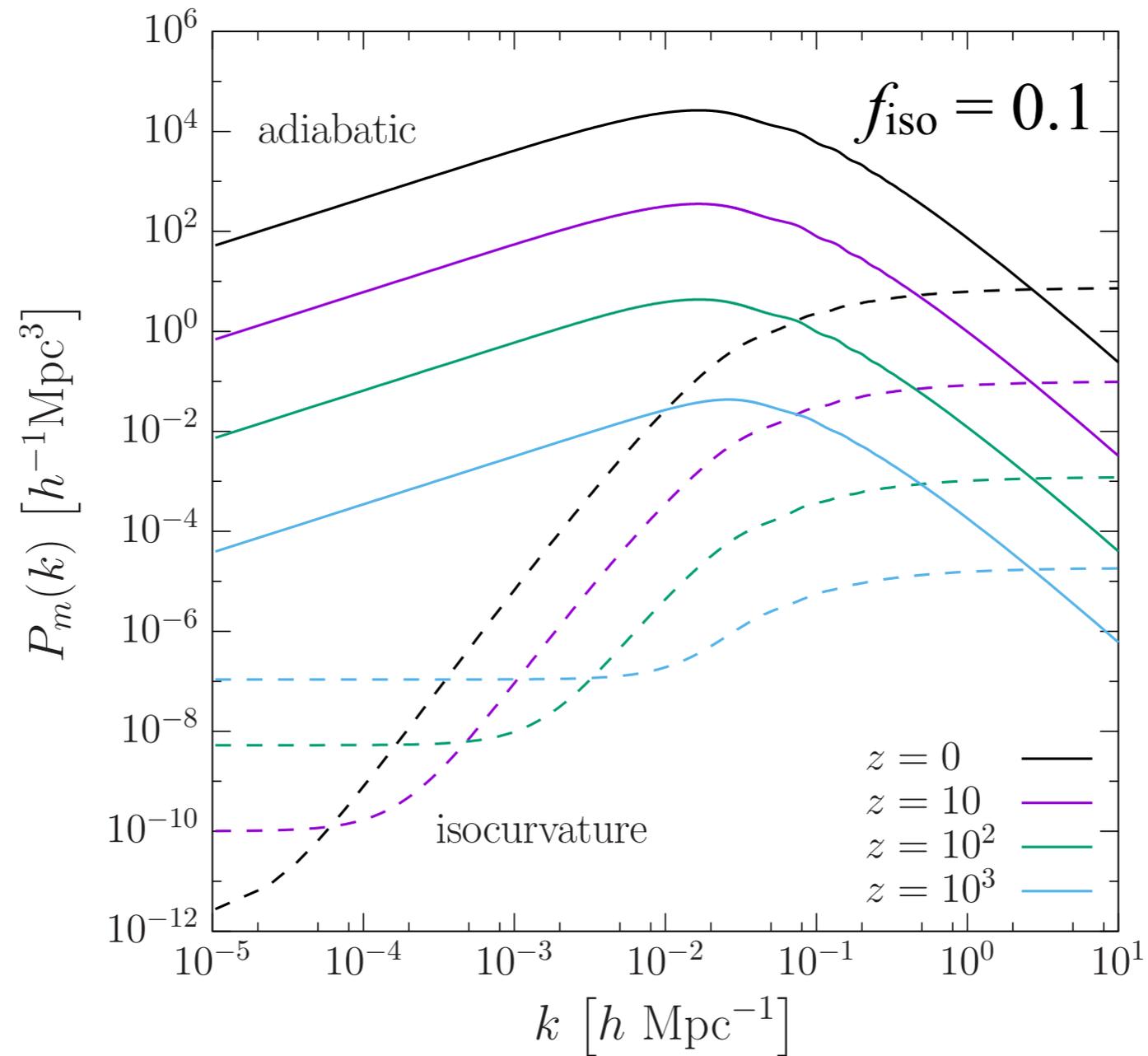
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# Outlook

explore signatures in the matter power spectrum



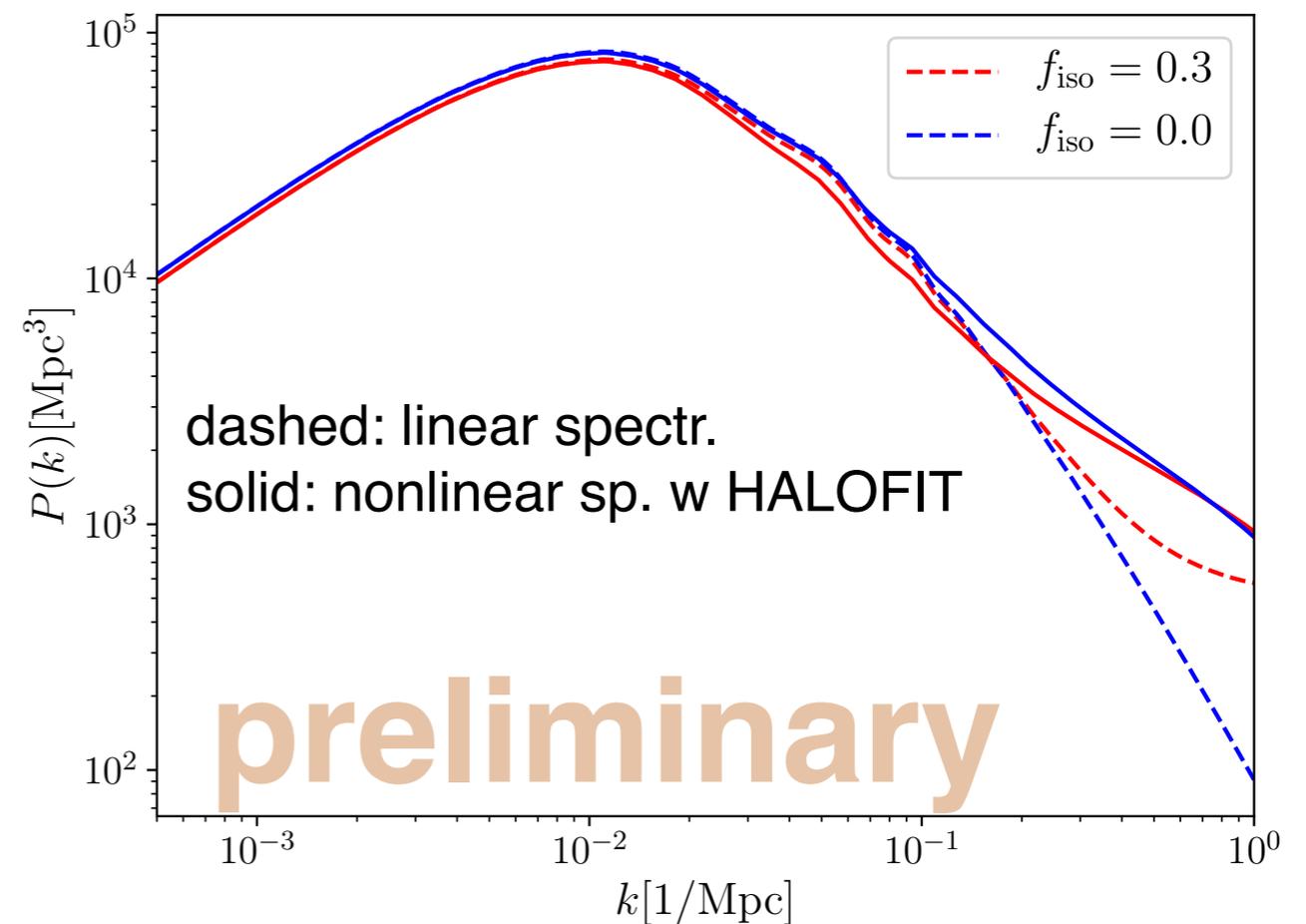
Feix et al. 1903.06194

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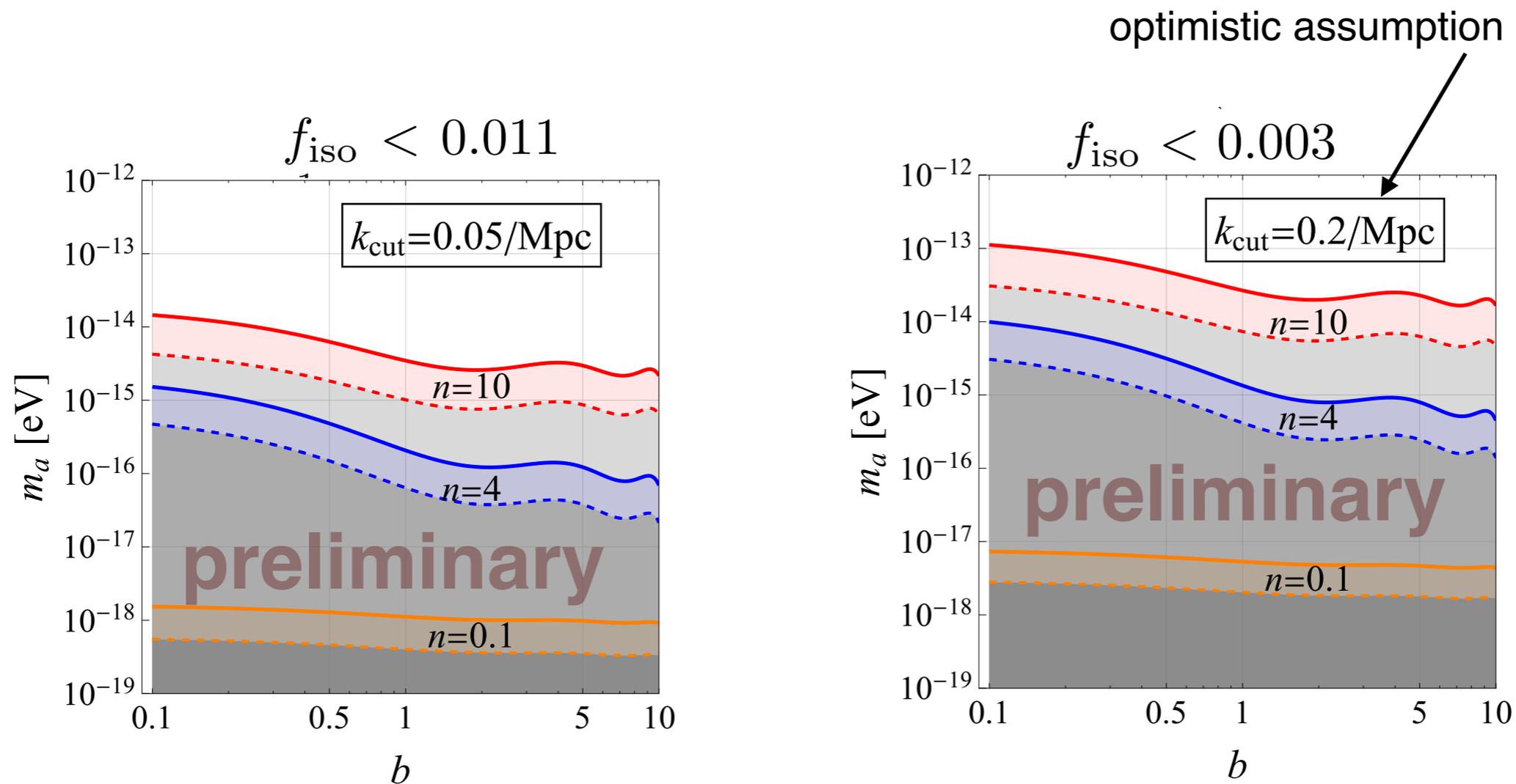
Feix, Hagstotz, Pargner, Reischke, Schäfer, TS, in prep.

- cosmic shear
- CMB lensing
- galaxy clustering
- galaxy counts
  
- crucial: how far can we trust linear predictions?



# CMB lensing, galaxy clustering, cosmic shear

Feix, Hagstotz, Pargner, Reischke, Schäfer, TS, in prep.



combined sens. at  $1\sigma$  from CMB lensing (S4 experiment) +  
galaxy clustering & cosmic shear (EUCLID)  
sensitivity dominated by **cosmic shear**

# Summary

- isocurvature fluctuations are generated in post-inflationary ALP DM scenario
- white noise power spectrum with cut-off due to uncorrelated field values per Hubble patch
- lower limit on ALP mass in range  $10^{-19}$ — $10^{-16}$  eV from Planck CMB data (weaker limits for temp.-indep. ALP mass)
- purely gravitational effect
- significant improvement expected from S4 CMB, 21cm maps, EUCLID LSS (cosmic shear)

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**Thank you for your attention!**