Quantum Aspects of Black Holes

Jan de Boer, Amsterdam

Padova, December 16, 2009

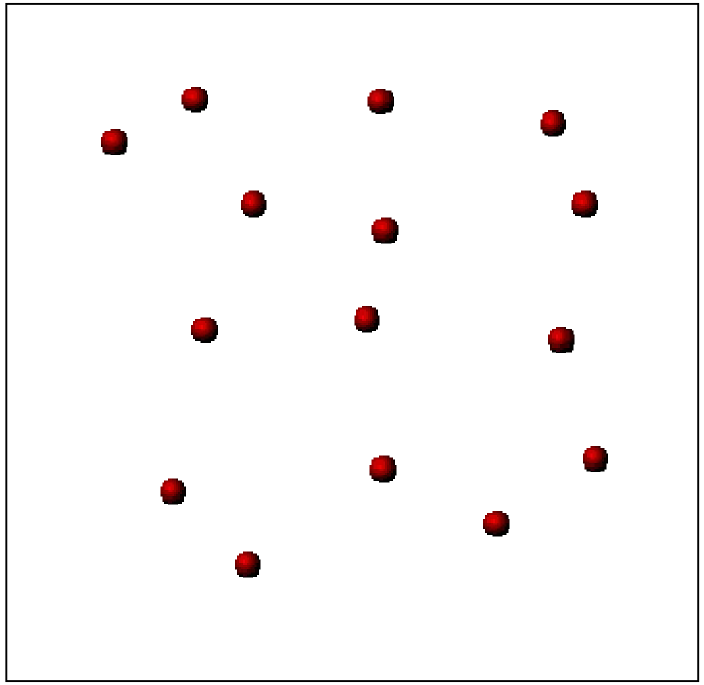
Roughly based on:

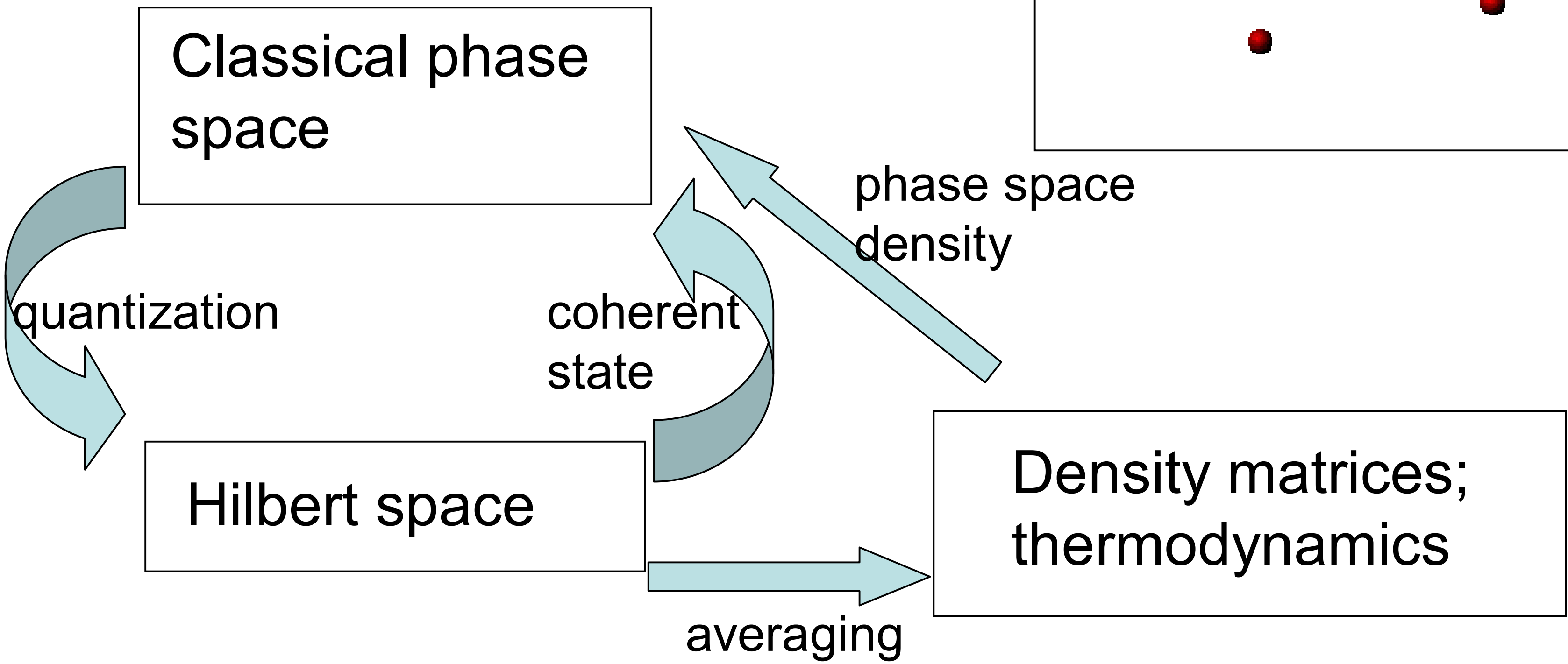
arXiv:0802.2257 - JdB, Frederik Denef, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
arXiv:0807.4556 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
arXiv:0811.0263 - Vijay Balasubramanian, JdB, Sheer El-Showk, Ilies Messamah
arXiv:0906.0011 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken.
arXiv:0906.3272 - Vijay Balasubramanian, JdB, S. Sheikh-Jabbari, J.Simon

Outline

- Introduction
- Microstates for large supersymmetric black holes
- Quantum effects in deep throats
- The number of smooth supergravity solutions
- Quantum effects in extreme Kerr (& various comments on Kerr/CFT)
- Conclusions

Idea of microstate program: develop a precise microscopic quantum statistical picture of black holes, just like we have for a gas of atoms in a box.





In certain cases it has been shown that for black holes the classical phase space of the atoms can equivalently be described by spaces of smooth horizonless solutions to the (super)gravity equations of motion. (pioneered by Mathur: fuzzballs).

Smoothness here is crucial: singularities arise after averaging (or coarse graining) over degrees of freedom. The smoothness requirement is also what makes this idea compatible with holography.

Pure states $\Longleftrightarrow$ smooth geometries $\Longleftrightarrow$ $S = 0$

Mixed states $\Longleftrightarrow$ singular geometries $\Longleftrightarrow$ $S \neq 0$

Many caveats:

-need to include smooth solutions with Planck-size curvature.

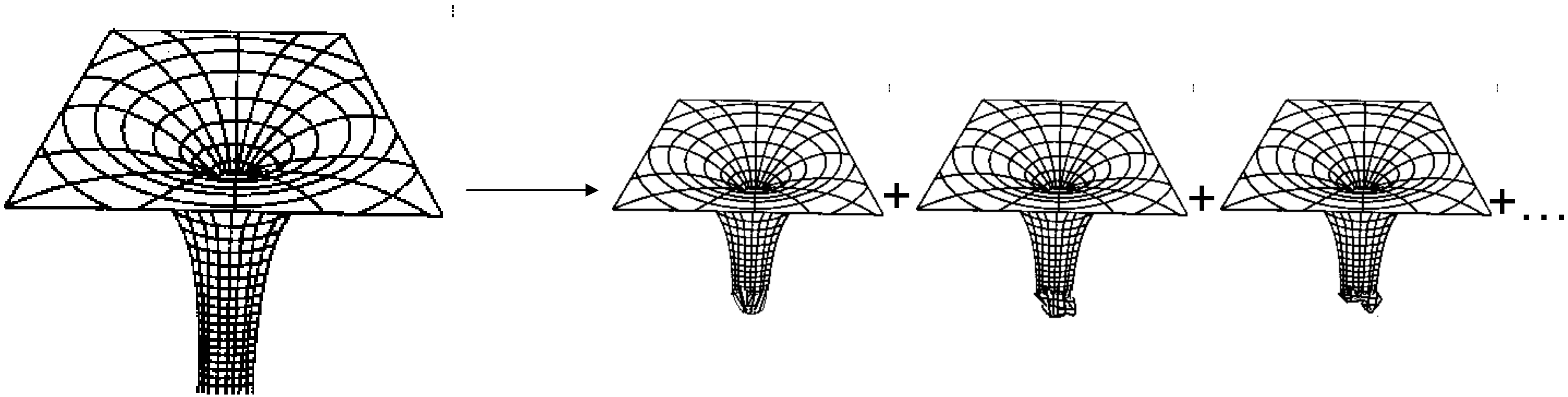
-not exactly clear what happens when higher derivative corrections are included.

-only works for certain small supersymmetric black holes.

-so far no complete description for any macroscopic black hole: strong evidence that stringy degrees of freedom are always necessary (see later)

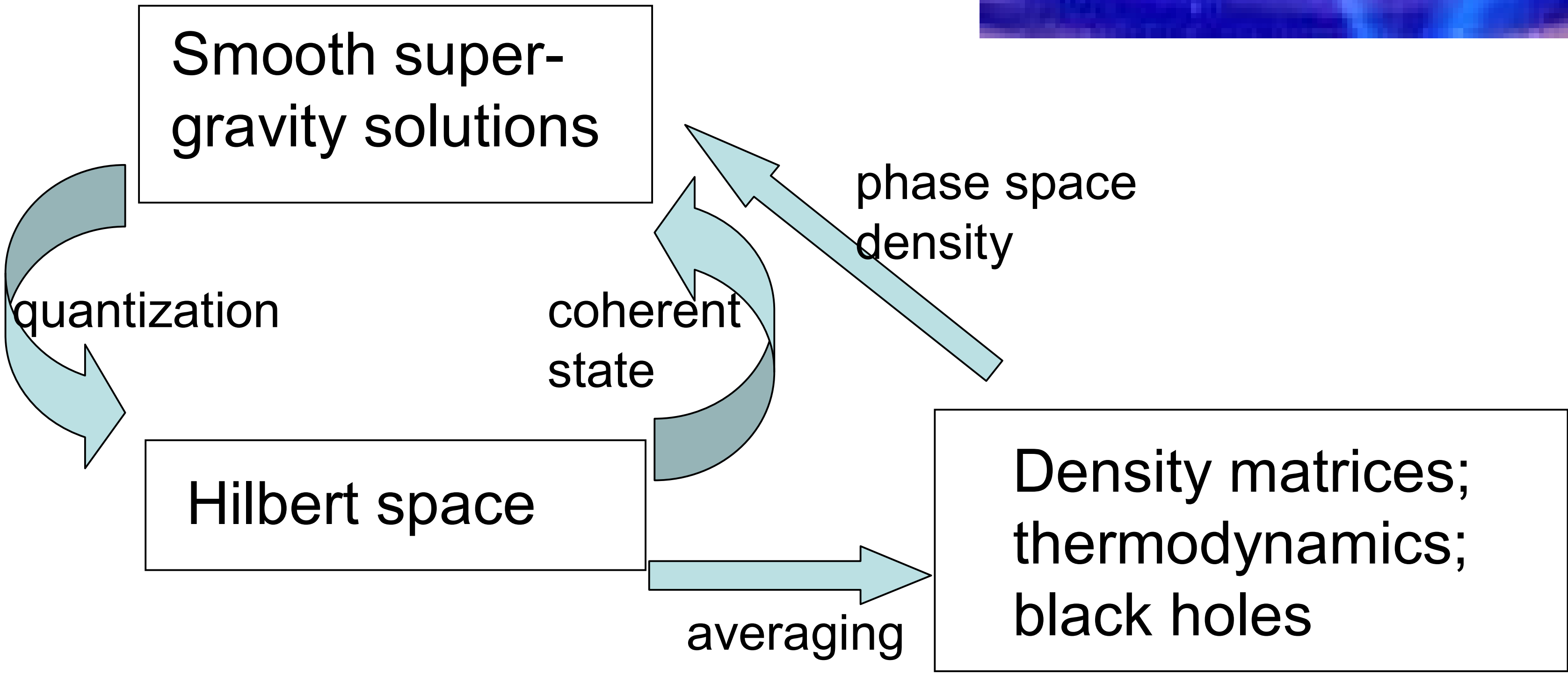
-recent work on 3d gravity?

This picture has been developed in great detail for "small" black holes. One can make sense of the notion of "adding" geometries, and show that



Of course, we would like to generalize this picture to large, macroscopic black holes.

Small black holes



Adding geometries in AdS/CFT:

An asymptotic AdS geometry is dual to a state. From asymptotics can read off the one-point functions of operators in the field theory.

Geometry 1

$$\langle \psi_1 | \mathcal{O}_k | \psi_1 \rangle = a_k$$

Geometry 2

$$\langle \psi_2 | \mathcal{O}_k | \psi_2 \rangle = b_k$$

Shortcut for BPS

$$\rho = \frac{1}{2}\left(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|\right)$$

Σ(Geometries)

$$\mathrm{Tr}(\rho \mathcal{O}_k) = (a_k + b_k)/2$$

Solve field equations with new boundary conditions

The small black holes for which this works are:

•½-BPS states in N=4 SYM, the relevant geometries are the LLM geometries, the relevant small black hole has not yet explicitly been constructed.

Lin, Lunin, Maldacena

•½-BPS states in the D1-D5 CFT, the relevant geometries are the LM geometries, the relevant small black hole is the M=0 BTZ black hole.

Lunin, Mathur

Actually, these black holes remain singular even if α' corrections are included…

Sen

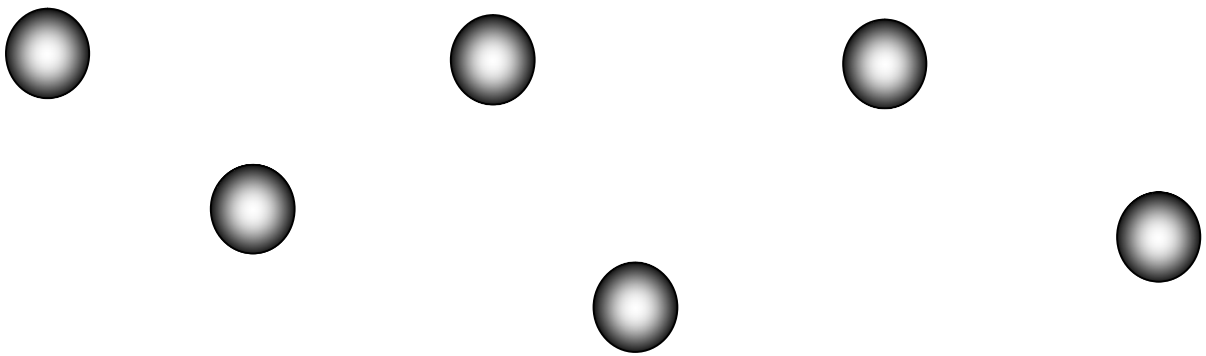
⟹ Turn to large supersymmetric black holes instead.

Large supersymmetric black holes carrying electric charge Q and magnetic charge P exist in four dimensions. (P and Q can be vectors with many components).

There exists however a much larger set of solutions of the gravitational field equations, which includes bound states of black holes, and also many smooth solutions.

Lopes Cardoso, de Wit, Kappeli, Mohaupt; Denef; Bates, Denef; Balasubramanian, Gimon, Levi

Put black holes with charges $\Gamma_i = (P_i, Q_i)$ at locations $\vec{x}_i \in \mathbb{R}^3$



There are corresponding solutions of the field equations only if (necessary, not sufficient)

$$\langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_j - \vec{x}_i|} = 0$$

Here, $\langle \Gamma_1, \Gamma_2 \rangle = P_1 \cdot Q_2 - P_2 \cdot Q_1$ is the electric-magnetic duality invariant pairing between charge vectors. The constant vector h determines the asymptotics of the solution.

Solutions are stationary with angular momentum

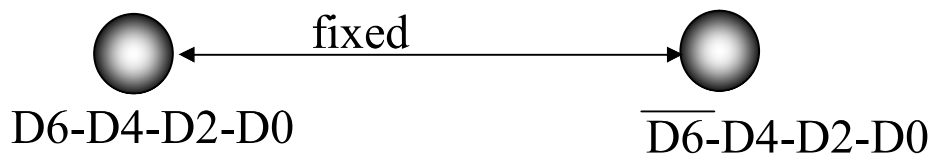
$$\vec{J} = \frac{1}{4} \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

Typical setup: type IIA on CY

Magnetic charges: D6,D4

Electric charges: D0,D2

Example:



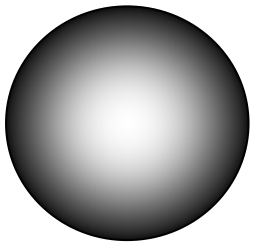
- Whenever the total D6-brane charge of a solution vanishes, one can take a decoupling limit so that the geometry (after uplifting to d=5) becomes asymptotic to $AdS_3 x S^2 x CY$. (dual=MSW (0,4) CFT)

Maldacena, Strominger, Witten

- When the centers correspond to pure branes with only a world-volume gauge field, the 5d uplift is a smooth geometry. The space of all such solutions will be our candidate phase space.

- Uplift of a D4-D2-D0 black hole yields the BTZ black hole, and can apply Cardy.

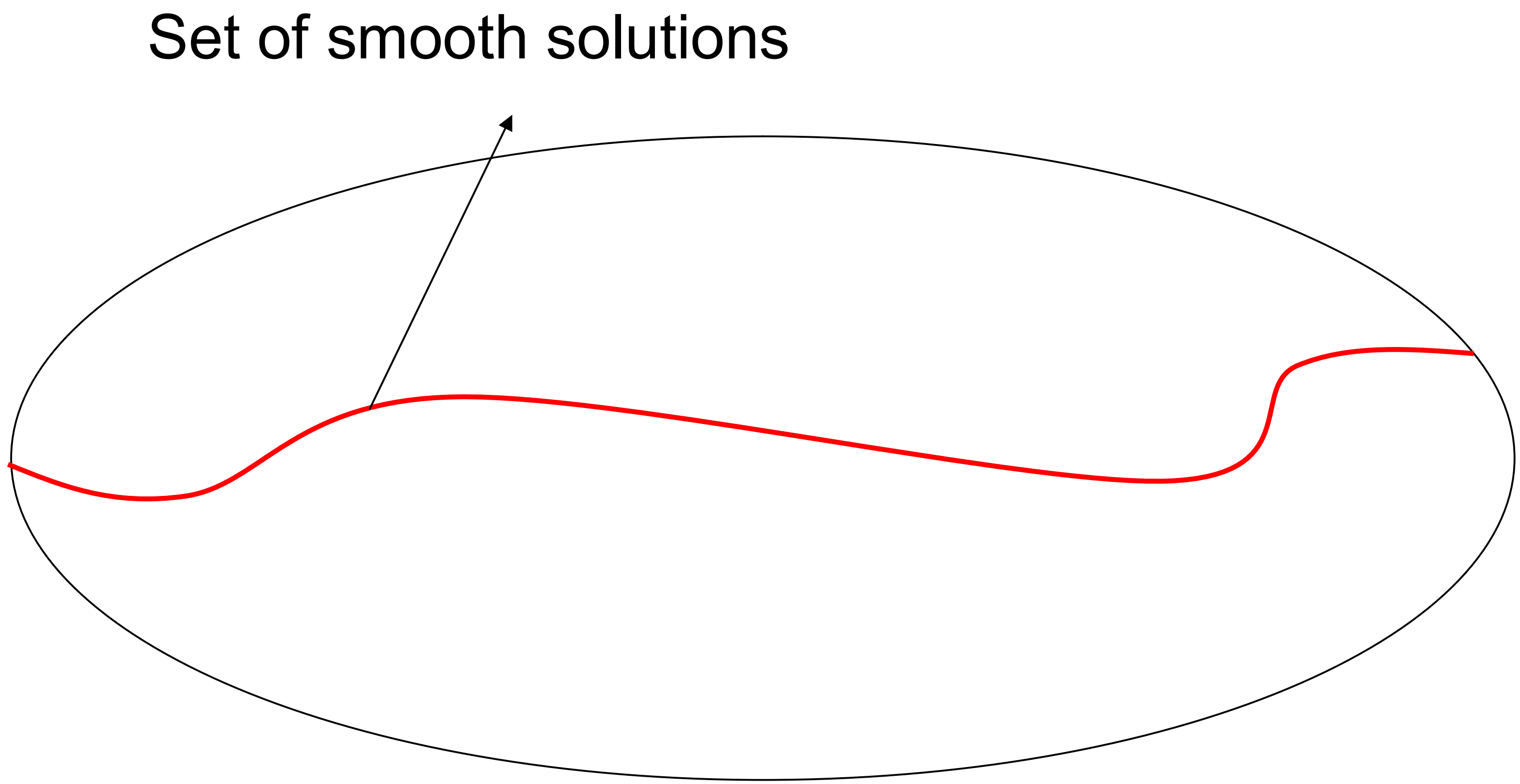
The BMPV black hole (D6-brane charge ≠ 0) does not admit a decoupling limit to AdS_3. Cannot use CFT methods to compute its entropy. But



BMPV

$\overline{\text{D6}}$ + flux

can be put in AdS_3. Dual to a sector of the CFT which we do not know how to characterize. In Cardy regime single centered black hole dominates entropy, but numerical evidence suggests that for $L_0 < c/24$ the above configuration dominates (entropy enigma). May in principle be able to microscopically determine BMPV entropy in this way.



Full phase space=set of all solutions of the equations of motion.

$$\omega \sim \int d\Sigma^{\mu} \left(\delta \frac{\delta \mathcal{L}}{\delta(\partial^{\mu}\phi)} \wedge \delta\phi \right)$$

Result:

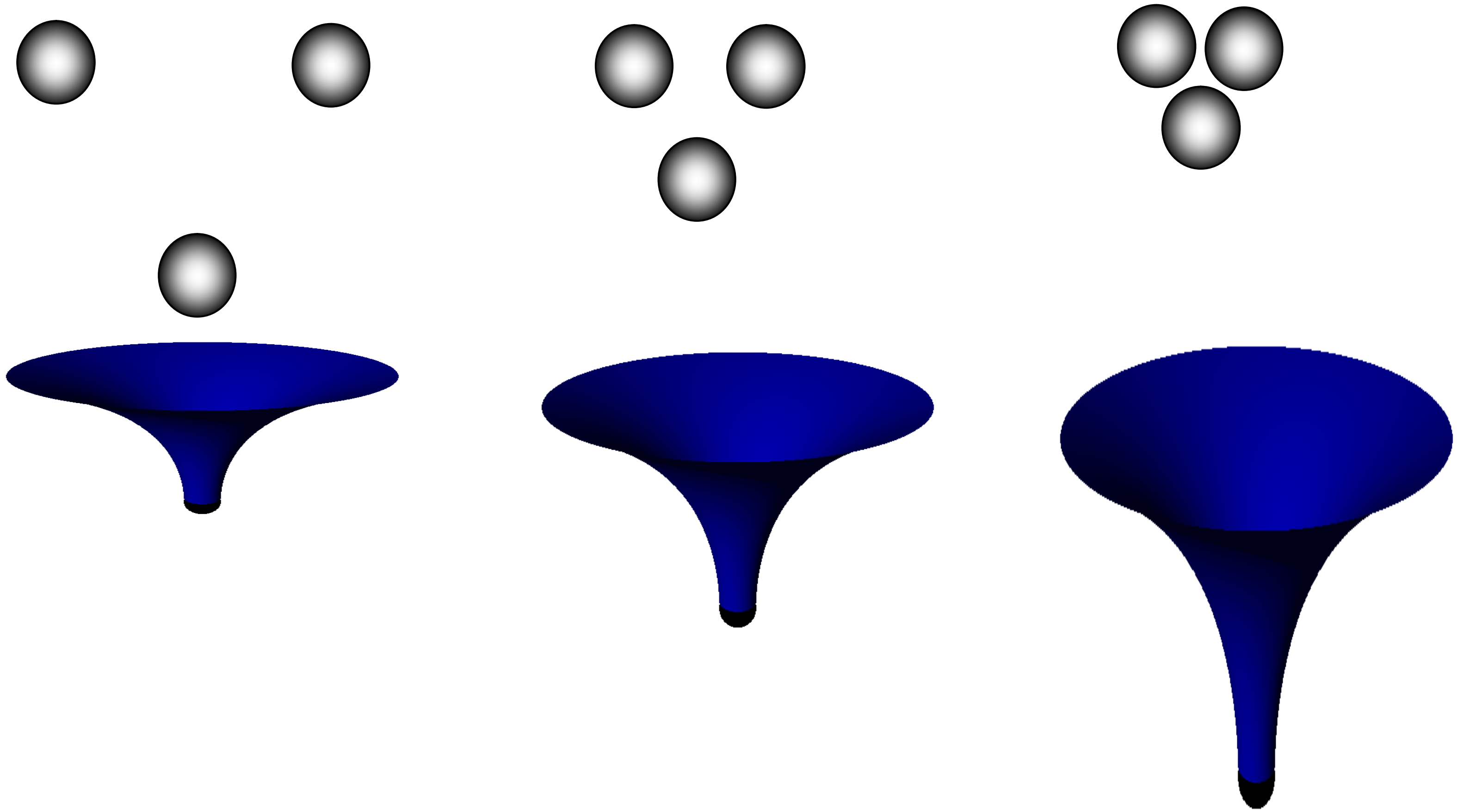
$$\omega = \frac{1}{4}\sum_{p\neq q}\langle\Gamma_i,\Gamma_j\rangle \frac{\epsilon_{ijk}(\delta(x_p-x_q)^i\wedge\delta(x_p-x_q)^j)\,(x_p-x_q)^k}{|\mathbf{x}_p-\mathbf{x}_q|^3}$$

Can now use various methods to quantize the phase space, e.g. geometric quantization. Can explicitly find wavefunctions for various cases.

In particular, one can use this to reproduce and extend the wall-crossing formula.

Bena, Wang, Warner; Denef, Moore

Of particular interest: scaling solutions: solutions where the constituents can approach each other arbitrarily closely.



In space-time, a very deep throat develops, which approximates the geometry outside a black hole ever more closely.

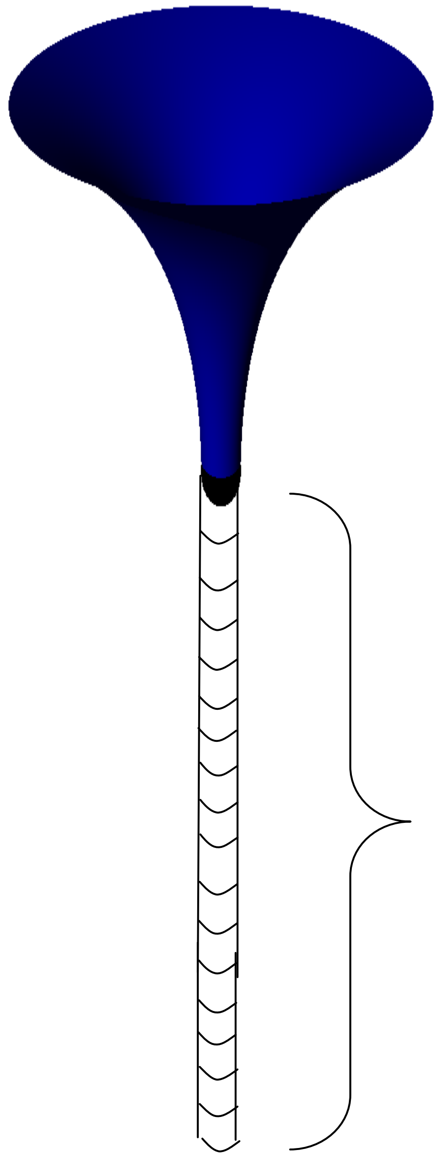
None of these geometries has large curvature: they should all be reliably described by general relativity.

However, this conclusion is incorrect!

The symplectic volume of this set of solutions is finite. Throats that are deeper than a certain critical depth are all part of the same $\hbar$-size cell in phase space: wave-functions cannot be localized on such geometries.

Quantum effects become highly macroscopic and make the physics of very deep throats nonlocal.

This is an entirely new breakdown of effective field theory.



Wave functions have support
on all these geometries

As a further consistency check of this picture, it also resolves an apparent inconsistency that emerges when embedding these geometries in AdS/CFT.

This is related to the fact that very deep throats seem to support a continuum of states as seen by an observer at infinity, while the field theories dual to AdS usually have a gap in the spectrum.

Bena, Wang, Warner

The gap one obtains agrees with the expected gap 1/c in the dual field theory (the dual 2d field theory appears after lifting the solutions to five dimensions and taking a decoupling limit).

This non-local breakdown of effective field theory near the horizon is perhaps exactly the sort of thing one needs in order to reconcile the information paradox with effective field theory?

(It has been argued that the information paradox cannot be resolved in perturbation theory)

Notice that the scale that appears is 1/c, which is a scale that appears in many different contexts in AdS_3. Evidence for a universal underlying long string picture in all 2d CFT's dual to AdS_3?

What about a recent suggestion by Sen that smooth geometries can never be thought of as microscopic degrees of freedom of a black hole (motivated by D1D5 case)?

$$Z = Z_{\rm hair} + Z_{\rm hair} Z_{\rm BH} + Z_{\rm hair} Z_{\rm BH} Z_{\rm BH} + \dots$$

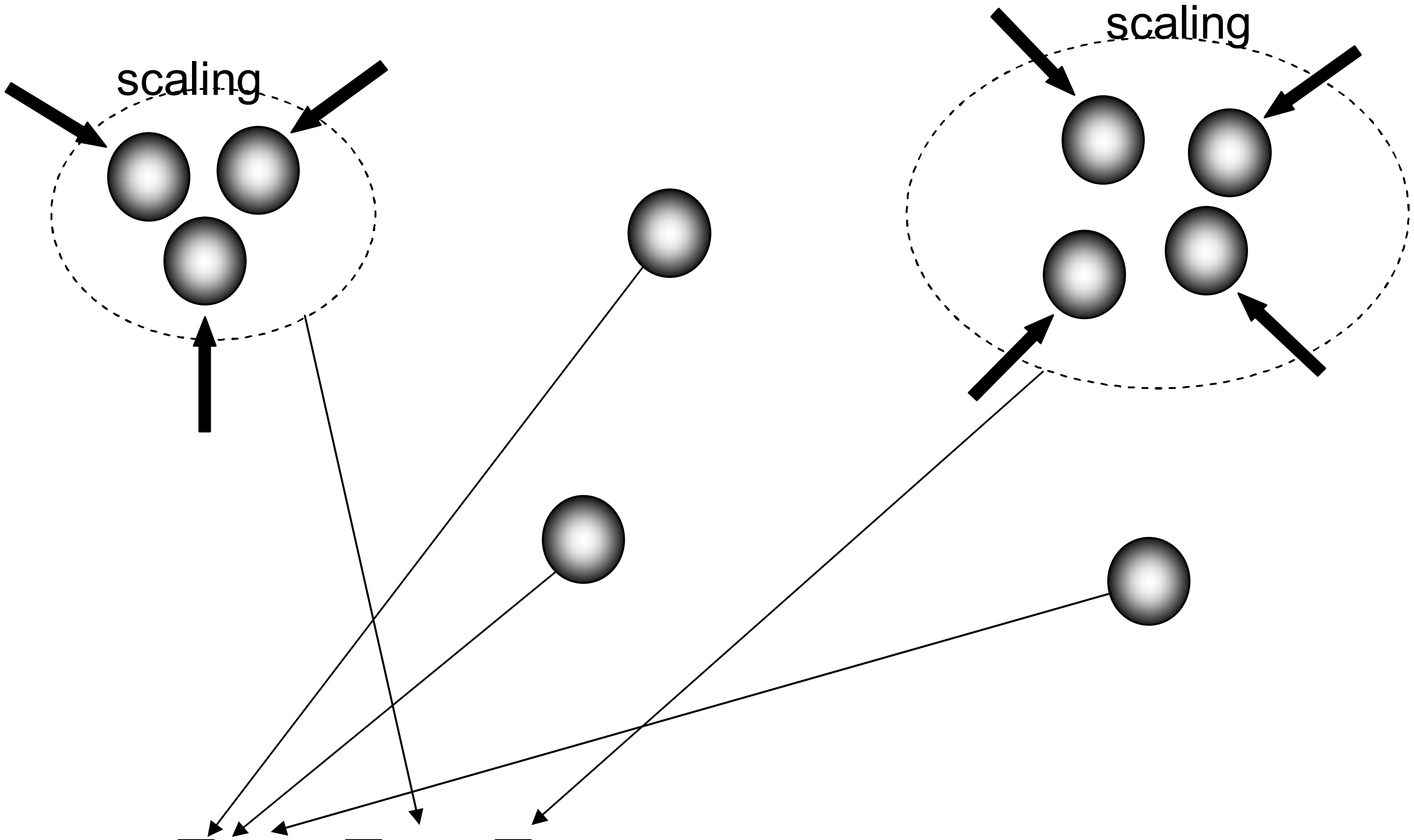
I do not quite understand the precise justification of this expression:

•This is not a sum over Euclidean saddle points, smooth solutions can typically not be Wick rotated.

•Smooth solutions are also not obviously related to one-loop determinants.

•Reminiscent of Farey tail expansion, except there $Z_{\rm hair}$ only contains polar states which never coexist with black holes.

For the ½-BPS black holes we considered, there is natural split into "hair" and "black holes".



$$Z_{\text{hair}} Z_{\text{BH}} Z_{\text{BH}} + \dots$$

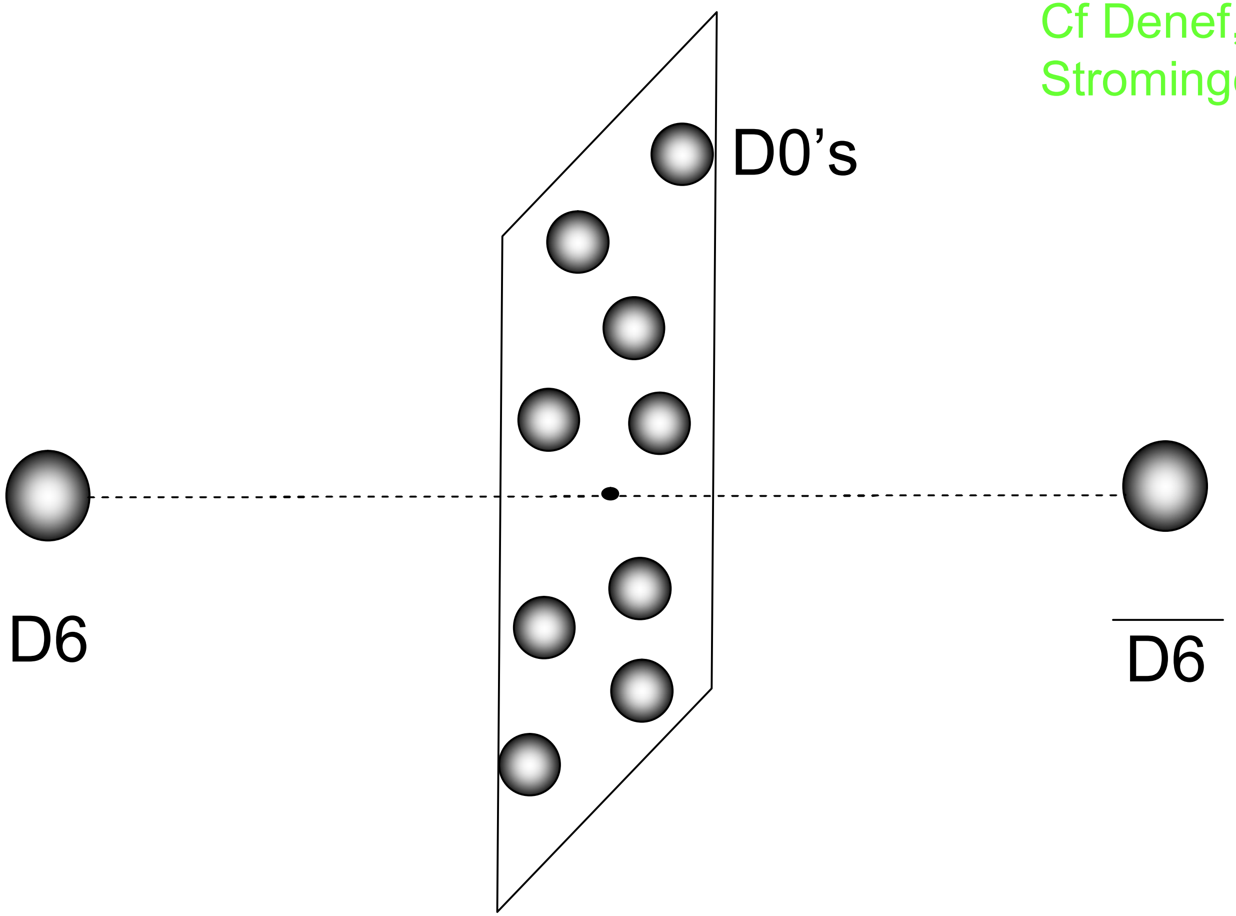
This is also suggested by considering split attractor flows.

Are there sufficiently many smooth supergravity solutions to account for the black hole entropy?

This is not a prediction of AdS/CFT.

Largest set we have been able to find:

Cf Denef, Gaiotto, Strominger, vdBleeken, Yin



In terms of standard 2d CFT quantum numbers we find the following number of states:

$$\left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$\left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \qquad L_0 \geq c/6$$

This is less than the black hole entropy, which scales as

$$S \sim 2\pi \left(\tfrac{c}{6}L_0\right)^{1/2}$$

Perhaps we are simply missing many solutions?

Try to find upper bound: count the number of states in a gas of BPS supergravitons. Idea is that all smooth BPS solutions are obtained by taking a superposition of free BPS supergravitons and letting the system backreact. Because of the BPS bound, the energy of the system cannot become be lowered.

Thus we compute the partition function of a gas of BPS supergravitons; spectrum can be read off from the the KK modes of M-theory on CYxS^2. Result:

$$\mathcal{Z} = \mathrm{Tr}_{\mathrm{NS,BPS}}(-1)^F q^{L_0} y^{\tilde{L}_0 - 1/2}$$

equals

$$\mathcal{Z} = Z_{\{\frac{1}{2},\frac{1}{2}\}}^{2h^{1,2}+2}\, Z_{\{0,1\}}^{h^{1,1}-1}\, Z_{\{1,0\}}^{h^{1,1}-1}\, Z_{\{-1,2\}}\, Z_{\{0,2\}}\, Z_{\{1,1\}}\, Z_{\{2,1\}}$$

where

$$Z_{\{s,\tilde{h}_{\min}\}} = \prod_{n\geq 0} \prod_{m\geq 0} (1 - y^{m+\tilde{h}_{\min}-1/2} q^{n+m+\tilde{h}_{\min}+s})^{(-1)^{2s+1}}$$

We put y=1 and compute the asymptotics of this partition function. Result:

$$S \sim \left(\frac{3}{16}\zeta(3)L_0^2\right)^{1/3}$$

Clearly backreaction will be important. Difficult to deal with, but can impose one dynamical feature: stringy exclusion principle.

Maldacena, Strominger

The stringy exclusion principle is related to the fact that the spins of primaries in a level k SU(2) WZW cannot exceed k/2. Thus we reinstate y and keep only the terms where the power of y is at most c/6.

Now we find precisely the same result as before:

$$S \sim \left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$S \sim \left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \quad L_0 \geq c/6$$

Strongly suggests supergravity is not sufficient to account for the entropy.

Stringy exclusion principle is visible in classical supergravity (and not so stringy).

In particular, this suggests that all attempts to quantize gravity on its own are futile and will never lead to a consistent unitary theory with black holes.

This is of course perfectly fine: string theory was invented to yield a consistent quantum theory of gravity, so it would have been somewhat disappointing if we could get away with gravity alone.

This statement is also supported by the N=4 case, where one can show that multicentered configurations can never contribute to the index.

Dabholkar, Guica, Murthy, Nampuri

Notice however that not all solutions of supergravity are small perturbations of AdS, e.g. wormhole solutions with a lot of structure behind the horizon or not of this type.

Question: should we really only be looking for smooth solutions? This is not entirely clear.

First of all, states are not just characterized by one-point functions (e.g. collapsing shell of matter looks like a black hole outside the shell). So adding geometries is actually not so straightforward.

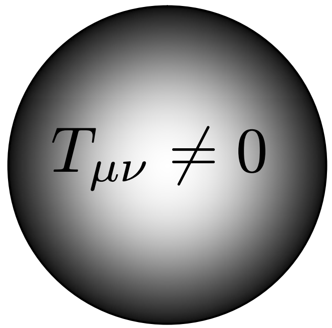
Second, averaging over geometries leads to geometries with sources. What sources are allowed?

Even without averaging over geometries, we sometimes have to include sources. For example, we showed that

JdB, Papadodimas, Verlinde

$T_{\mu\nu} = 0$

Very heavy fermionic operator



holographic neutron star

Towards more realistic black holes? Try to repeat the arguments e.g. for extremal Kerr. Dual to a CFT?

Guica, Hartman, Song, Strominger

The near-horizon limit of extremal Kerr is Bardeen, Horowitz

$$ds^2 = 2G_4 J\, \Omega(\theta)^2 \left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2 \, (d\varphi + r dt)^2 \right] ,$$

with $\Omega(\theta)^2 = \frac{1+\cos^2\theta}{2}, \qquad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}.$

Diffeomorphisms of the form: $\zeta_\lambda = \lambda(\varphi)\partial_\varphi - r\lambda(\varphi)'\partial_r$

Generate a Virasoro algebra ⟹ "chiral CFT"

Cardy reproduces entropy of extremal Kerr.

What does this “NHEK” geometry represent?

1. It is a geometric representation of a set of ground states but it is has no dynamics, just like AdS_2. (AdS_2 fragments when perturbed by any finite energy excitation)

   Maldacena, Strominger

2. It is dual to some quantum mechanical system which carries representations of a single Virasoro algebra: would not explain why one can use Cardy.

3. It is dual to a chiral CFT, i.e. a CFT with one sector frozen and the other dynamical (so Cardy is OK).

4. It is dual to a full CFT.

Answer will depend on the precise choice of boundary conditions.

Evidence in favor of 1 and 2: The NHEK geometry is unique up to diffeomorphisms with suitable boundary conditions. Single Virasoro is still there as these are diffeo's.

Amsel, Horowitz, Marolf, Roberts;
Diaz, Reall, Santor

Evidence against 4: the AdS_2 cannot be excited (it would fragement again) so there can be at most one Virasoro algebra.

Evidence for 3: Near-horizon limit of extremal BTZ is the chiral half of a CFT (more precisely, it is the DLCQ of a CFT).

Balasubramanian, JdB, Sheikh-Jabbari, Simon

Evidence for 3,4: Agreement for greybody factors/superradiance scattering (not clear whether agreement is kinematical or dynamical though).

Bredberg, Hartman, Song, Strominger
Cvetic, Larsen

If option 2 is correct, we should be able to add a thermal gas of L_{-k} gravitons, and in option 3, we should be able to change the temperature of the chiral CFT. Puzzle: in NHEK, there is no room for an extra parameter. Both the temperature and the central charge are fixed. What geometry describes heating up NHEK?

$$ds^2 = 2G_4 J\,\Omega(\theta)^2 \left[-r^2 dt^2 + \tfrac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2\ \left((2\pi T)d\varphi + r dt\right)^2\right]\ ,$$

This metric has conical singularities at the north and south poles. (Kerr threaded by a cosmic string?) If either 2 or 3 is correct, we should allow for such types of metrics.

For now simply assume that option 3 is correct, and that the mass gap of this chiral CFT as 1/c (as it was in extremal BTZ).

This puts the radius for quantum fluctuations in

$$ds^2 = 2G_4 J\, \Omega(\theta)^2 \left[-r^2 dt^2 + \tfrac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2 \, (d\varphi + r dt)^2 \right] ,$$

at r of order $1/J$. For GRS 1915+105, $J \sim 2 \times 10^{79}$. This is a very small distance and seems related to quantization of angular momenta….

Open question: If NHEK is dual to a chiral CFT, does the full CFT then also admit a gravitational dual? If so, what is it?

OUTLOOK:

Several naïve black hole expectations have been made precise in extremal supersymmetric situations. (coarse graining microstates, typicality,….)

Extend to other (cosmological) singularities? New interpretation of the Hartle-Hawking no-boundary proposal? Entropy of cosmological horizon is sum over smooth cosmologies?

Understand role and meaning of geometries with sources.

Extend breakdown of effective field theory and discussion of quantum effects to generic Schwarzschild black holes: AdS/CFT may allow us to make some progress in this direction.

Can we understand anything about the stringy degrees of freedom that we need to account for the entropy of a large black hole?

What happens when you fall into a black hole? Fluctuations in the metric are larger than you would naively expect and just enough for information to come out. Eventually classical geometry will cease to exist and you will thermalize……

Explore the open string picture in more detail (this involves some quantum mechanical gauge theory and interesting connections between the Coulomb and Higgs branch).

Understand relation between non-local breakdown of effective field theory and information paradox.

Finally, try to address more complicated dynamical black hole questions…