Nonperturbative definition of closed string theory via open string field theory

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Introduction

One of the most important problems in theoretical physics is to formulate quantum gravity in a consistent manner.

While the quantization of general relativity in the framework of quantum field theory turned out to be difficult, it was found that string theory consistently describes on-shell scattering amplitudes involving gravitons.

However, string theory only provides a perturbative definition of such on-shell scattering amplitudes with respect to the string coupling constant.

One possible approach to a nonperturbarive formulation of string theory would be to introduce a spacetime field for each oscillation mode of the string and construct an action of those spacetime fields. The resulting theory in this approach is called string field theory.

Since gravitons are described as states of the closed string, a natural approach to quantum gravity would be to construct closed string field theory.

For closed bosonic string field theory, the gauge invariance of the classical action turned out to be anomalous and we need quantum corrections to the action at each loop order to recover the gauge invariance.

The existence of such closed string field theory is useful when we handle phenomena such as vacuum shift and mass renormalization in the perturbative string theory.

arXiv:1703.06410, de Lacroix, Erbin, Kashyap, Sen and Verma

However, formulating closed string field theory at the quantum level non-perturbatively by the path integral does not seem to be promising because of these quantum corrections to the action.

As the origin of the quantum corrections is related to the decomposition of the moduli space of Riemann surfaces, we do not expect any improvement of the situation in the generalization to closed superstring field theory.

Then how can we formulate string theory nonperturbatively?

The typical origin of the string perturbation theory is the $1/N$ expansion of gauge theories with $N \times N$ matrix degrees of freedom.

Nucl. Phys. B72 (1974) 461, 't Hooft

The long history of research on string theory indicates that string theory can be defined nonperturbatively in terms of such gauge theory.

Question 1

What kind of closed string theory should we consider?

While the $1/N$ expansion in the gauge theories has a structure of the genus expansion in string theory, we do not see the smooth world-sheet picture in Feynman diagrams of matrix fields written in the double-line notation.

One attempt to generate the smooth world-sheet picture was to take the double scaling limit of matrix models. This successfully defines string theory nonperturbatively, but it worked out only for low dimensions where physical degrees of freedom of gravitons are absent.

The AdS/CFT correspondence can be regarded as providing a nonperturbative definition of closed string theory in terms of a quantum field theory without containing gravity.

Type IIB superstring theory on $AdS_5 \times S^5$, for example, is conjectured to be defined nonperturbatively by $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions, and the string coupling constant of type IIB superstring theory on $AdS_5 \times S^5$ is given by $1/N$.

Let us recall the explanation of the AdS/CFT correspondence.

Consider type IIB superstring theory on a flat spacetime in ten dimensions with N coincident D3-branes.

In the low-energy region where the energy of the system is much lower than the string scale $1/\sqrt{\alpha'}$, closed strings and open strings are decoupled.

Then closed string theory becomes a free theory in ten dimensions and open string theory becomes $\mathcal{N} = 4$ $U(N)$ super Yang-Mills theory in four dimensions.

Next consider type IIB superstring theory on the three-brane solution of supergravity.

Because of the redshift factor, an object brought closer and closer to the three-brane appears to have lower and lower energy for the observer at infinity.

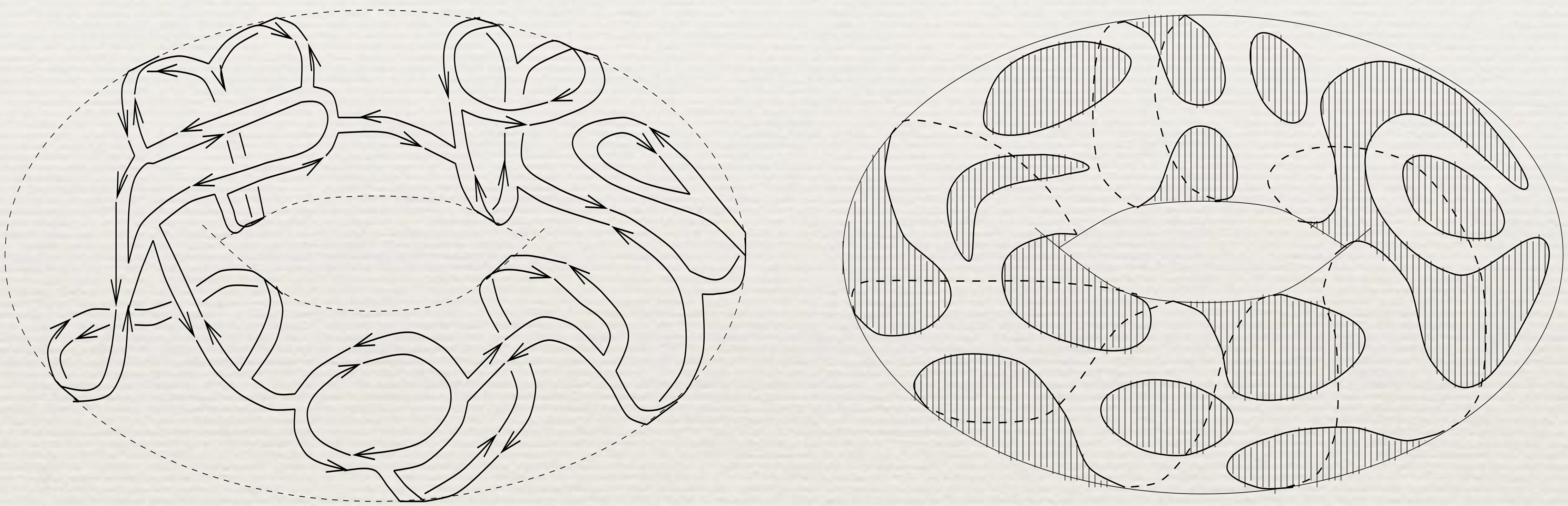
In the same low-energy limit, excitations propagating in ten dimensions and excitations in the near horizon region are decoupled, and we have a free theory in ten dimensions and type IIB superstring theory on $AdS_5 \times S^5$, which is the near horizon geometry of the three-brane solution.

We are then led to the conjecture that $\mathcal{N} = 4$ $U(N)$ super Yang-Mills theory in four dimensions is the same as type IIB superstring theory on $AdS_5 \times S^5$.

While there are closed strings and open strings in the presence of D-branes, we consider the closed string sector in this explanation of the AdS/CFT correspondence, but the world-sheet of closed strings contains holes.

The AdS/CFT correspondence tells us that this closed string theory with holes in the world-sheet is equivalent to a closed string theory without holes on a curved background in the low-energy limit.

This was discussed in the context of the large N duality of the topological string. hep-th/0205297, Ooguri and Vafa



Figures taken from hep-th/0205297 by Ooguri and Vafa

This was also discussed recently using the pure-spinor formalism.

arXiv:1903.08264, Berkovits

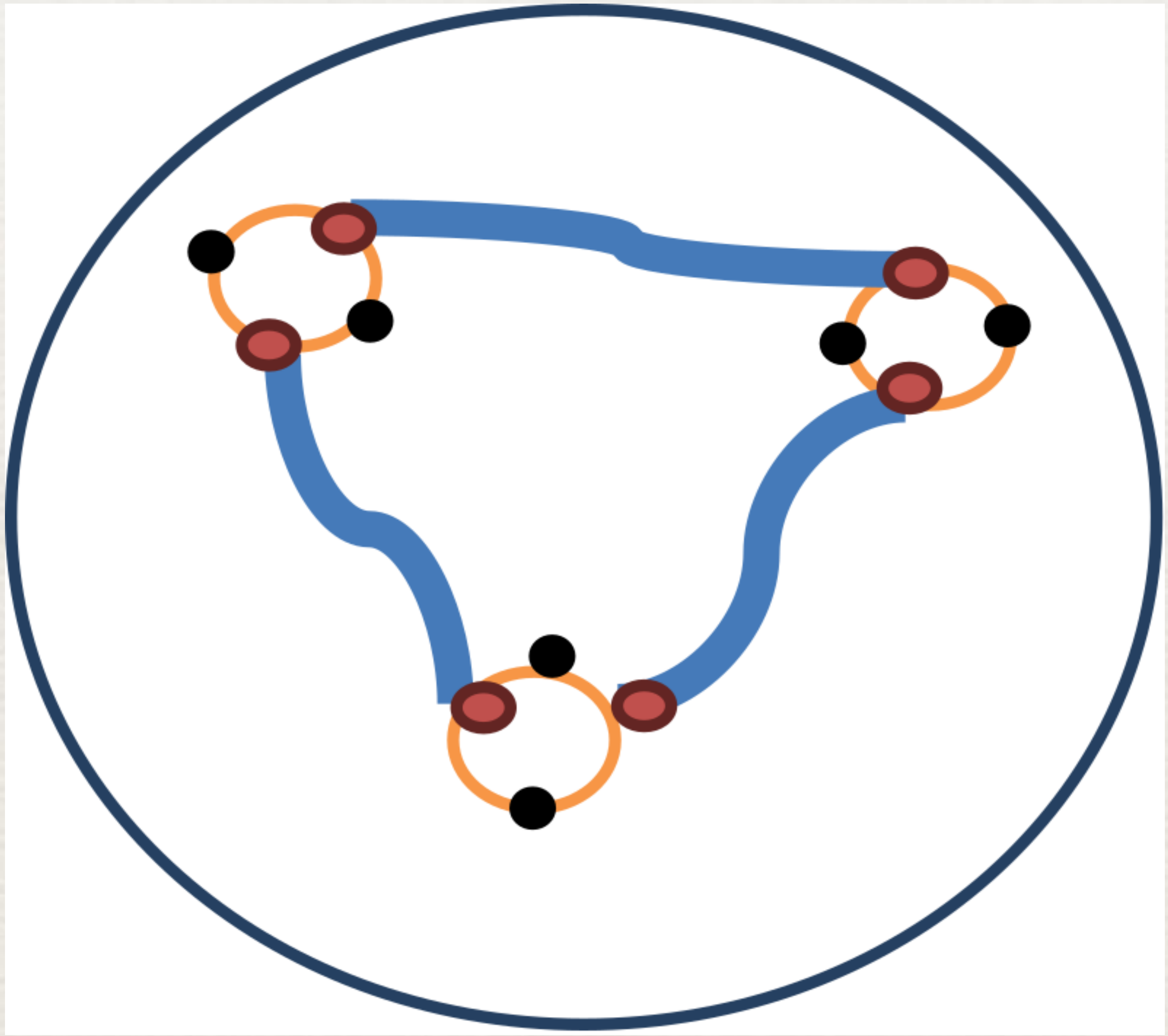


Figure taken from arXiv:1903.08264 by Berkovits

In this talk we do not discuss this problem and we instead concentrate on two other aspects.

The first aspect is to see that closed string theory with holes in the world-sheet is a consistent perturbation theory.

The second aspect is how such closed string theory with holes in the world-sheet can be reproduced by a theory from the open string sector.

Let us begin with the question of whether the closed string theory with holes in the world-sheet is a consistent perturbation theory.

Compared with the discussion in the topological string, we have to be more careful in the physical string theory.

First of all, the moduli space of Riemann surfaces with holes must be covered for consistency. In the moduli space there are regions where two boundaries are close. Such a region corresponds to a long propagation of an open string, so we may naively think that open strings are necessary for unitarity.

We claim that this is not necessarily the case because of the following arguments.

First, the closed string sector is analogous to the gauge-invariant sector of gauge theory, and when we consider correlation functions of gauge-invariant operators in gauge theory, we never see poles from fields which are not gauge invariant.

Second, correlation functions of one closed-string vertex operator and one open-string vertex operator on the disk are generically nonvanishing so that the closed-string propagation and the open-string propagation are mixed when the interaction is turned on.

On-shell states in the interacting theory need to be identified by diagonalizing such propagators and the long-propagation of an open string does not necessarily generates an on-shell pole.

The situation is analogous to the mass renormalization of closed string theory.

$$\frac{1}{k^2 - m^2 - \delta m^2} = \frac{1}{k^2 - m^2} + \frac{1}{k^2 - m^2}\,\delta m^2\,\frac{1}{k^2 - m^2} + \frac{1}{k^2 - m^2}\delta\, m^2\,\frac{1}{k^2 - m^2}\,\delta m^2\,\frac{1}{k^2 - m^2} + \ldots$$

Once we convince ourselves that degrees of freedom from open strings can be integrated out, one approach to see that the closed string theory with holes in the world-sheet is a consistent perturbation theory would be to start with open-closed string field theory.

It is a consistent perturbation theory, and the gauge invariance of the action is related to the decomposition of the moduli space of Riemann surfaces with punctures where the Riemann surface can have boundaries and we consider both bulk and boundary punctures.

After integrating out the open string field, we obtain closed string field theory with holes in the world-sheet.

When there are N coincident D-branes, we write the action in terms the 't Hooft coupling constant as usual. Then the coupling constant of closed string field theory with holes in the world-sheet is given by $1/N$.

This is the perturbation theory including gravity that we want to reproduce by a theory without gravity.

<u>Question 1</u>

What kind of closed string theory should we consider?

<u>Answer</u>

We should consider closed string theory with holes in the world-sheet.

We emphasize that the covering of the moduli space of Riemann surfaces with holes is important for the consistent world-sheet picture, and we have this world-sheet picture for any number of holes. This should be contrasted with the world-sheet picture which appears in the double scaling limit of the matrix models.

It would be difficult to see this world-sheet picture in $\mathcal{N} = 4$ super Yang-Mills theory because it is the theory after taking the low-energy limit. Before taking the low-energy limit, the dynamics on the D-branes is described by open string field theory. In open string field theory, it would be more promising to see the world-sheet picture.

Question 2

What quantities should we consider in open string field theory?

In the context of the AdS/CFT correspondence, we consider correlation functions of gauge-invariant operators in $\mathcal{N} = 4$ super Yang-Mills theory.

In string field theory, it is in general difficult to construct gauge-invariant operators. It is in fact an important feature of string field theory and it is part of the reason that the interacting string field theory is believed to be unique up to field redefinition given a free theory.

In open bosonic string field theory with the cubic interaction in terms of the star product there are a class of gauge-invariant operators and we can define a gauge-invariant operator for each on-shell closed string vertex operator.

hep-th/0111092, Hashimoto and Itzhaki
hep-th/0111129, Gaiotto, Rastelli, Sen and Zwiebach

These gauge-invariant operators have been mainly discussed in the context of the classical theory. They were evaluated for a classical solution to extract information on the boundary conformal field theory corresponding to the classical solution.

arXiv:0804.1131, Ellwood
arXiv:1207.4785, Kudrna, Maccaferri and Schnabl

In that context they were called gauge-invariant observables, gauge-invariant overlaps, Ellwood invariants, and so on. We call them gauge-invariant operators as we consider them in the quantum context.

The action is given by

$$S = -\frac{1}{2}\langle \Psi, Q\Psi \rangle - \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle ,$$

where Ψ is the open string field and Q is the BRST operator.

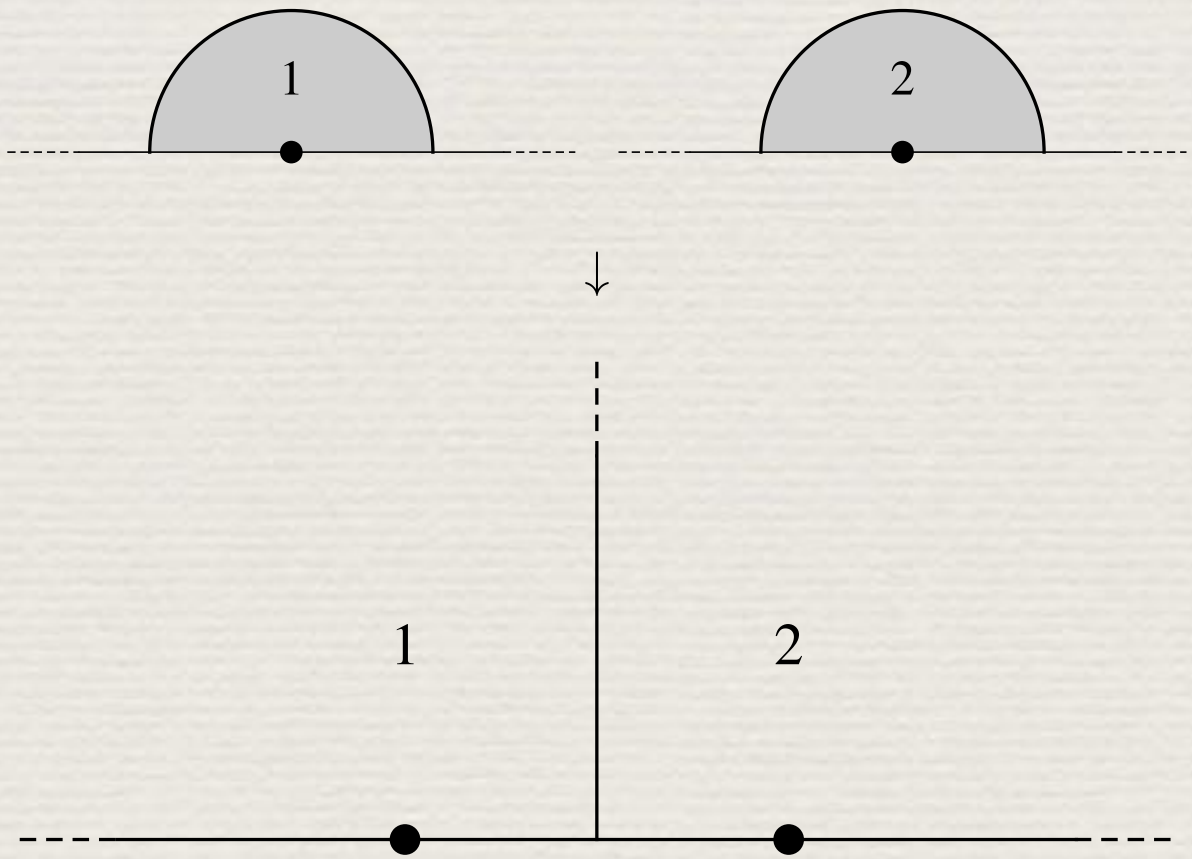
BPZ inner product

$$\langle\, A_1, A_2 \,\rangle = \langle\, h_1 \circ A_1(0)\, h_2 \circ A_2(0) \,\rangle_{\rm UHP}$$

with

$$h_1(z) = \tan\Big(\arctan z - \frac{\pi}{4} \Big) = \frac{z-1}{z+1}\,,$$

$$h_2(z) = \tan\Big(\arctan z + \frac{\pi}{4} \Big) = -\frac{z+1}{z-1}\,.$$

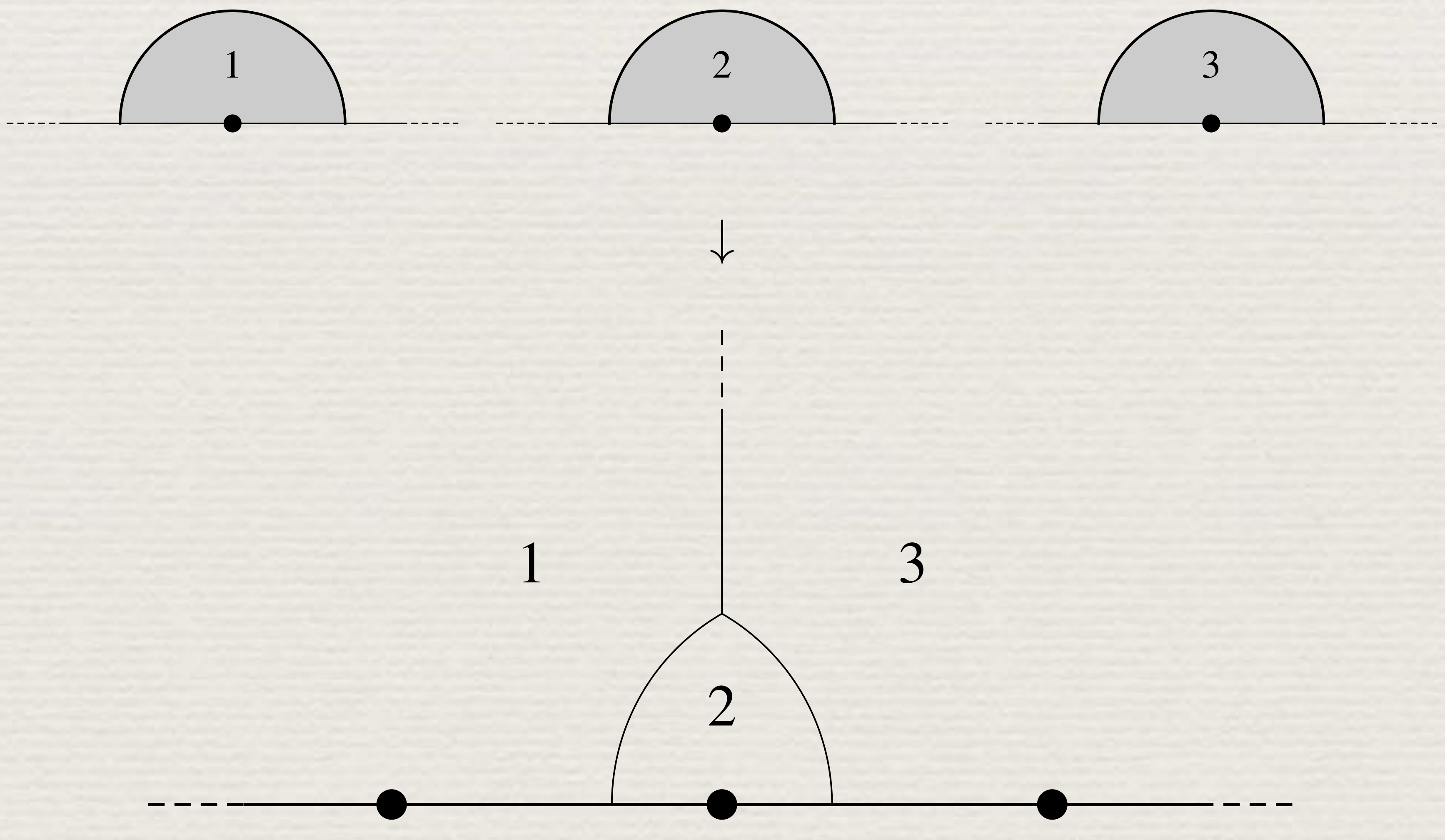


star product

$$\langle\, A_1, A_2 * A_3 \,\rangle = \langle\, f_1 \circ A_1(0)\, f_2 \circ A_2(0)\, f_3 \circ A_3(0) \,\rangle_{\rm UHP}$$

with

$$f_1(z) = \tan\left[\,\frac{2}{3}\left(\arctan z - \frac{\pi}{2}\right)\right], \qquad f_2(z) = \tan\left(\,\frac{2}{3}\,\arctan z\,\right),$$
$$f_3(z) = \tan\left[\,\frac{2}{3}\left(\arctan z + \frac{\pi}{2}\right)\right].$$

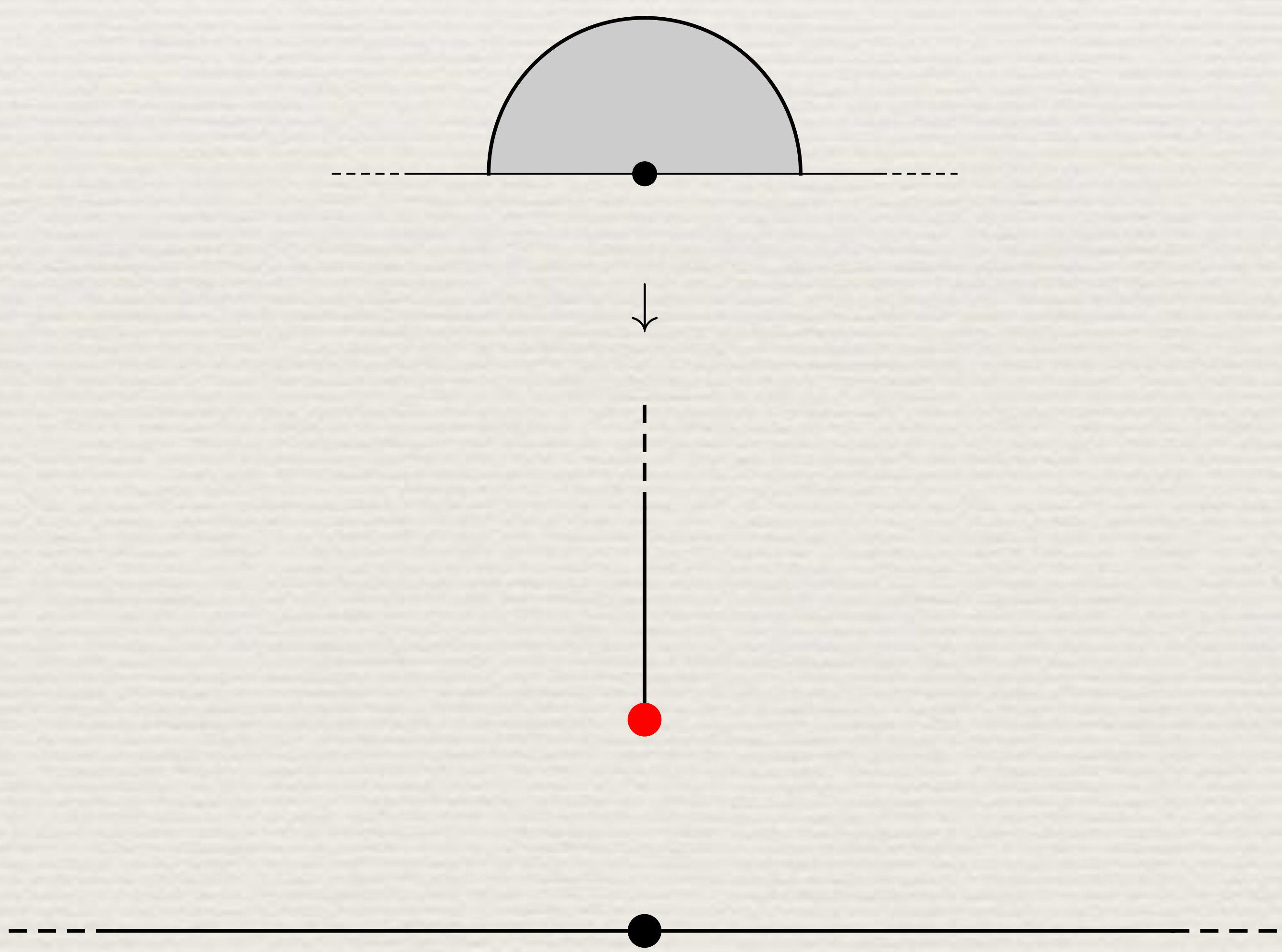


The gauge-invariant operator $\mathcal{A}_{\mathcal{V}}[\,\Psi\,]$ for an on-shell closed string vertex operator $\mathcal{V}$ is defined by

$$\mathcal{A}_{\mathcal{V}}[\,\Psi\,] = \langle\, \mathcal{V}(i)\, f_I \circ \Psi(0)\,\rangle_{\mathrm{UHP}}$$

with

$$f_I(z) = \tan\Big(\, 2\, \arctan z\,\Big) = \frac{2\,z}{1-z^2}\,.$$



These gauge-invariant operators have an interesting origin in open-closed string field theory.

A one-parameter family of formulations for open-closed bosonic string field theory were constructed, and it was observed that in a singular limit the action reduces to that of the cubic open bosonic string theory with an additional vertex which couples one off-shell open string field and one on-shell closed string field.

hep-th/9202015, Zwiebach

$$S = -\frac{1}{2}\langle\,\Psi, Q\Psi\,\rangle - \frac{1}{3}\langle\,\Psi, \Psi * \Psi\,\rangle + \langle\,V(\Phi), \Psi\,\rangle\,,$$

where Φ is the on-shell closed string field and $V(\Phi)$ is a map from a closed string field to an open string field.

The kinetic term of the closed string field is absent so that the resulting theory is no longer open-closed string field theory.

It is open string field theory and the on-shell closed string field can be regarded as a source for a set of gauge-invariant operators.

An important consequence from this relation of the gauge-invariant operators and open-closed string field theory is that Feynman diagrams for correlation functions of the gauge-invariant operators are given by Riemann surfaces containing holes with bulk punctures and the moduli space of such Riemann surfaces is covered.

Let us illustrate how the moduli space of disks with two bulk punctures is covered in the singular limit.

Note that open bosonic string field theory with the cubic vertex in terms of the star product plays a distinguished role in this context.

Let us now consider the theory on N coincident D-branes. If we evaluate correlation functions of the gauge-invariant operators in the $1/N$ expansion, by construction it reproduces the closed-string perturbation theory with holes in the world-sheet we mentioned before.

Question 2

What quantities should we consider in open string field theory?

Answer

We should consider correlation functions of the gauge-invariant operators. They can be evaluated in the $1/N$ expansion in terms of Feynman diagrams of closed string theory with holes in the world-sheet.

In open string field theory we can also consider dynamics of open strings in addition to the gauge-invariant operators. This may be related to the discussion on non-singlet sectors in the matrix models.

hep-th/0503112, Maldacena

However, Riemann surfaces associated with such Feynman diagrams contain at least one hole, and contributions from Riemann surfaces without any holes are missing.

In the context of on-shell scattering amplitudes, they are necessary for factorization.

Question 3

What do we lose in the missing Feynman diagrams?

Let us again recall the explanation of the AdS/CFT correspondence.

Both in the description with D-branes and in the description with the three-brane solution of supergravity, there are two decoupled sectors in the low-energy limit.

One of them is a free theory in ten dimensions from the closed string sector, and we identified the two descriptions of the other sector.

In the low-energy limit, the missing contributions correspond to those of the free theory. In the low-energy limit, the contributions from open string field theory are exactly what we want.

Question 3

What do we lose in the missing Feynman diagrams?

Answer

Nothing in the low-energy limit!

Previously there were some attempts to reproduce closed string theory *without* holes in the world-sheet from correlation functions of the gauge-invariant operators, for example, in the context of tachyon condensation.

We emphasize that our approach is different and we are not trying to reproduce closed string theory on a flat spacetime.

Note that after taking the low-energy limit the quantities we are considering are no longer on-shell scattering amplitudes.

There may be a physical interpretation about correlation functions of the gauge-invariant operators before taking the low-energy limit, but we have not figured it out.

In the low-energy limit, the on-shell condition for massive states is no longer satisfied. We emphasize that they are massive states in the flat spacetime in ten dimensions with D-branes, and massive closed string states in $AdS_5 \times S^5$ arise from the massless sector in the flat spacetime by taking the low-energy limit.

Under the assumption of the equivalence between closed string with holes in the world-sheet and closed string theory in a curved background, the $1/N$ expansion of correlation functions of the gauge-invariant operators should incorporate interactions in terms of massive closed string states in the curved background.

To summarize, we claim that the evaluation of correlation functions of the gauge-invariant operators in the $1/N$ expansion can be interpreted as a closed string perturbation theory in the low-energy limit.

Therefore, if open string field theory for finite N is a consistent quantum theory, it provides a nonperturbative definition of closed string theory.

Question 4

Is open string field theory
a consistent quantum theory?

We do not expect open bosonic string field theory to be a consistent quantum theory in general.

It is interesting to consider topological strings. For example, three-dimensional Chern-Simons gauge theory can be formulated as open string field theory. hep-th/9207094, Witten

It is also interesting to consider the Kontsevich model in this context. hep-th/0312196, Gaiotto and Rastelli

On the other hand, open superstring field theory can be a consistent quantum theory.

When we quantize open superstring field theory, we know that both the Neveu-Schwarz sector and the Ramond sector are necessary for consistency in such quantum treatment.

While the action of open superstring field theory involving the Ramond sector had not been constructed for many years, this problem was recently overcome and we now have several formulations of open superstring field theory which are complete at the classical level.

arXiv:1508.00366, Kunitomo and Okawa
arXiv:1508.05387, Sen
arXiv:1602.02582, Erler, Okawa and Takezaki
arXiv:1602.02583, Konopka and Sachs

We consider that the formulations of open superstring field theory need to be developed further and it is an important question to address whether or not open superstring field theory is consistent as a quantum theory.

At the same time, however, we consider that we are in a position to discuss how we use open superstring field theory to understand the mechanism which realizes the AdS/CFT correspondence. This is one of the main messages of the talk.

There might be a distinguished theory as in the case of open bosonic string field theory.

Question 4

Is open string field theory a consistent quantum theory?

Answer

Open string field theory for the topological string or the non-critical string can be a consistent quantum theory. Open superstring field theory can also be a consistent quantum theory, which motivates us to extend our discussion to the superstring.

Question 5

Can we make sense of the path integral of open string fields?

As we mentioned before, we do not consider the path integral of closed string field theory to be promising for a nonperturbative definition of closed string theory.

On the other hand, there will be a better chance of making sense of the path integral of open string field theory. However, it may still be difficult because the open string field contains infinite component fields.

Therefore, let us consider integrating out massive fields to obtain a theory in terms of massless fields. arXiv:1609.00459, Sen

While the resulting action for the massless fields will be complicated, we can show that it has an A_∞ structure if the original action has an A_∞ structure.

The A_∞ structure severely constrains the form of the interactions. The cubic interactions can be written explicitly, and the interactions which are higher than the cubic interaction might be determined by the A_∞ structure including α' corrections.

Since there are only a finite number of fields, the path integral of those fields can be defined in the same way as in ordinary quantum field theories.

We also need to include the source term for the gauge-invariant operators.

In the process of integrating out the massive fields, couplings of the closed string and multiple open string fields are generated, and the gauge-invariant operators in terms of the massless fields will resemble single-trace operators of $U(N)$ gauge theories in the low-energy limit.

We have to throw away Feynman diagrams involving only propagators for the massive fields to obtain an action for the massless fields, but we do not expect such Feynman diagrams to give nonvanishing contributions in the low-energy limit.

Question 5

Can we make sense of the path integral of open string fields?

Answer

We can integrate out massive fields. The path integral of a finite number of massless fields can be defined in the same way as in ordinary quantum field theories.

In the case of D3-branes, the resulting theory will be equivalent to $\mathcal{N} = 4$ super Yang-Mills theory in the low-energy limit.

In our approach we keep track of the relation to closed string theory with holes in the world-sheet, and this can be useful in proving the AdS/CFT correspondence.

As we mentioned before, the world-sheet picture can be seen in each order of the expansion in the 't Hooft coupling constant. In the hexagon approach based on the integrability, the world-sheet picture appears even for the weak-coupling region, and our approach might provide an explanation of this feature.

arXiv:1505.06745, Basso, Komatsu and Vieira

We believe that there are several advantages in our approach.

First, we define correlation functions of the gauge-invariant operators before taking the low-energy limit, and this can provide a well-defined setting to discuss the correspondence after taking the limit such as the AdS/CFT dictionary between correlation functions on the boundary and supergravity calculations in the bulk.

hep-th/9802109, Gubser, Klebanov and Polyakov
hep-th/9802150, Witten

Second, we have a relation between gauge-invariant operators and closed string states from the beginning, and this can be an advantage, although it might be difficult to keep track of the relation after taking the low-energy limit.

Third, our discussion can be applied to any background by taking an appropriate decoupling limit.

hep-th/9608024, Douglas, Kabat, Pouliot and Shenker

In particular, our discussion does not directly rely on conformal symmetry in the limit or supersymmetry.

Summary

Question 1

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Answer

We should consider closed string theory with holes in the world-sheet.

Question 2

What quantities should we consider in open string field theory?

Answer

We should consider correlation functions of the gauge-invariant operators. They can be evaluated in the $1/N$ expansion in terms of Feynman diagrams of closed string theory with holes in the world-sheet.

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What do we lose in the missing Feynman diagrams?

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Nothing in the low-energy limit!

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Roadmap

← Noi siamo qui.

Take *Via String Field Theory*

to get to the AdS/CFT correspondence!