Closed string symmetries in open string field theory: tachyon vacuum as sine-square deformation

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with Isao Kishimoto, Tomomi Kitade

and work in progress with I. Kishimoto, T. Kitade, S. Seki and A. Tsuda

Intoroduction

Open bosonic string field theories on a D-brane have a stable vacuum where a tachyon field is condensed and the D-brane disappears.

At the tachyon vacuum, it is proved that we have no open strings as physical degrees of freedom and there is a belief that closed strings living in a bulk space-time are found.

In string theories, it has been long considered that closed string dynamics is described by open string degrees of freedom.

As naive intuition, a closed string is given by a bound state of an open string and so an open string is regarded as a fundamental degree of freedom.

About a decade ago, Gendiar, Krcmar and Nishino made an important discovery concerning discretized open systems in condensed matter physics.

They considered an N-site system with open boundary condition described by the Hamiltonian

$$\hat{H} = -t \sum_{l=1}^{N-1} \sin^2 \frac{l\pi}{N} \left(\hat{c}_l^\dagger \hat{c}_{l+1} + \hat{c}_{l+1}^\dagger \hat{c}_l \right).$$

This is a deformed Hamiltonian of a uniform open system by the sine square factor.

They found that **this system is equivalent to a system with periodic boundary condition by calculating the ground state energy and correlation functions numerically.**

This result suggests that a discretized closed system can be described by degrees of freedom of a discretized open system, and, considering the continuum limit, the above idea is realized in a two dimensional field theory.

After their seminal papers, this deformation is called the sine-square deformation (SSD) and it has been actively studied.

The above correspondence between the open and closed systems is understood as a result of the equivalence of the ground states of the SSD and uniform periodic systems:

Hikihara-Nishino ('11), Katsura ('11)

$$\hat{H}_{\rm closed} = -t\sum_{l=1}^{N}\left(\hat{c}_l^\dagger\hat{c}_{l+1}+\hat{c}_{l+1}^\dagger\hat{c}_l\right), \qquad (\,\hat{c}_{N+1}=\hat{c}_1\,), \qquad |0\rangle_{\rm closed}$$

$$\hat{H}_{\rm SSD} = -t\sum_{l=1}^{N-1}\sin^2\frac{l\pi}{N}\left(\hat{c}_l^\dagger\hat{c}_{l+1}+\hat{c}_{l+1}^\dagger\hat{c}_l\right), \qquad |0\rangle_{\rm SSD}$$

Then,

$${}_{\rm SSD}\langle\,0\,|\,0\,\rangle_{\rm closed}=1$$

(Each state is calculated as $|\;\rangle = \sum A_{i_1\cdots i_n}\hat{c}_{i_1}^\dagger\cdots\hat{c}_{i_n}^\dagger\,|0\rangle_{\rm Fock}$.)

The SSD has been extended to deformation with other suppression of boundary effects and the generalized SSD has been applied not only to the free fermion system but also to various models.

Gendiar-Krcmar-Nishino ('09), Hikihara-Nishino ('11)
Katsura ('11), Shibata-Hotta ('11), Maruyama-Katsura-Hikihara ('11)

Moreover, SSD has been also studied in the context of field theoretical description.

Katsura ('12), Tada ('14,'15), Ishibashi-Tada ('15, '16)
Okunishi ('16), Wen-Ryu-Ludwig ('16)

These studies strongly suggest the equivalence of an SSD system and a periodic system, and, in the string theoretical viewpoint, a possibility of closed string theories in terms of open strings.

Interestingly, it has been pointed out by Ishibashi-Tada that there is an intimate relation between the SSD system and open string field theories.

In the string field theoretical side, it is known that one of the identity-based tachyon vacuum solutions provides a kinetic operator corresponding to a string Hamiltonian at the tachyon vacuum:

(Tanimoto-Takahashi-Kishimoto '02)

$$L' = \oint \frac{dz}{2\pi i} \frac{-1}{4z} \left(z^2 - 1\right)^2 \left(T'(z) - 6\right).$$

$T'(z)$ is the twisted ghost energy-momentum tensor.

If we change the variable as $z = \exp(i\theta)$, the weighting function in L' becomes $\sin^2\theta$ due to a Jacobian and a conformal factor from $T'(z)$.

Hence, the kinetic operator turns out to be related to an SSD Hamiltonian in CFT.

These facts about SSD and SFT indicate that we are able to formulate a pure closed string theory in terms of open string fields and the identity-based tachyon vacuum solution.

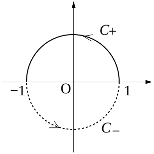
Motivated by this possibility, we will try to find closed string symmetries in the open string field theory at the identity-based tachyon vacuum.

To do so, we will strongly use the technique developed in dipolar quantization of two dimensional conformal field theories by Ishibashi-Tada.

Open string Hamiltonian

The Hamiltonian of an open string is given by

$$H_{\mathrm{O}} = \int_{C_+} \frac{dz}{2\pi i}\, zT(z) + \int_{C_-} \frac{dz}{2\pi i}\, zT(z),$$



$T(z)$: the energy-momentum tensor.

Each term corresponds to Hamiltonians of left and right moving modes, respectively, but they do not commute with each other due to open boundary conditions on $T(z)$.

The Hamiltonian is given by the zeroth component of the Virasoro operators: L_0. So, we do not encounter antiholomorphic Virasoro operators in the open string system.

Sine-squre-like deformation

Here, we consider the deformed Hamiltonian:

$$H_g = H_g^+ + H_g^-, \quad H_g^\pm = \int_{C_\pm} \frac{dz}{2\pi i}\, g(z) T(z),$$

where $g(z)$ is a holomorphic function satisfying $g(\pm 1) = \partial g(\pm 1) = 0$.

H_g^+ and H_g^- are left and right moving modes of H_g.

The simplest example of $g(z)$ is given by

$$g(z) = -\frac{1}{4z}(z^2-1)^2.$$

If we change the variable as $z = \exp(i\theta)$, the weighting function in H_g is changed to $z^{-1} g(z) = \sin^2\theta$. Hence, the deformed Hamiltonian provides a sort of generalization of the SSD Hamiltonian. In this sense, we call it the sine-square-like deformation, or SSLD for short.

$T(z)$ is expanded by holomorphic Virasoro operators only:

$$T(z) = \sum_{n=-\infty}^{\infty} L_n z^{-n-2}.$$

By using this expansion and the Virasoro algebra, we can obtain a commutation relation of $T(z)$:

$$[T(z), T(z')] = -(\,T(z) + T(z')\,)\,\partial\delta(z,z') - \frac{c}{12}\partial^3\delta(z,z'),$$

where c is the central charge of $T(z)$.

By this equation, we can calculate the commutation relation between H_g^+ and H_g^-.

The important point is that surface terms appear in the calculation as a result of derivatives of the delta function and these terms include a singular factor $\delta(\pm 1, \pm 1)$.

However, the singular surface terms turn out to vanish due to the factors $g(\pm 1)$ and $\partial g(\pm 1)$, which are set to zero in the definition of H_g.

As a result, we find

$$\left[\, H_g^+,\, H_g^- \,\right] = 0$$

and then the deformed system is decomposed into the left and right moving parts as in periodic systems.

Accordingly, it is concluded that the deformed system described by H_g corresponds not to an open string system, but to a closed string system, although the Hamiltonian is constructed by a single holomorphic energy-momentum tensor.

It should be noted that **the zeros of $g(z)$ and $\partial g(z)$ at open string boundaries cause the decoupling of the left and right moving sectors!**

Now, we will illustrate equal-time contours generated by the Hamiltonian for the simplest function

$$g(z) = -\frac{1}{4z}(z^2-1)^2.$$

with a focus on emergence of left and right moving sectors.

According to Ishibashi-Tada, we introduce the parameters, t and s, into the worldsheet generated by H_g:

$$t + is = \int^z \frac{dz}{g(z)} = \frac{2}{z^2-1},$$

where t denotes time and s parameterizes a string at a certain time.

If we introduce polar coordinates as $z = re^{i\theta}$, they can be expressed by the parameters t and s: on the upper half plane ($\mathrm{Im}\, z \geq 0$),

$$\begin{aligned}
\theta &= \begin{cases} \dfrac{\pi}{4} + \dfrac{1}{2}\arctan\dfrac{s^2+2t+t^2}{2s} & (s<0) \\[2ex] \dfrac{3\pi}{4} + \dfrac{1}{2}\arctan\dfrac{s^2+2t+t^2}{2s} & (s>0), \end{cases} \\
r &= \left(\frac{s^2+(t+2)^2}{s^2+t^2}\right)^{\frac{1}{4}},
\end{aligned}$$

and on the lower half plane ($\mathrm{Im}\, z \leq 0$),

$$\begin{aligned}
\theta &= \begin{cases} -\dfrac{\pi}{4} + \dfrac{1}{2}\arctan\dfrac{s^2+2t+t^2}{2s} & (s>0) \\[2ex] -\dfrac{3\pi}{4} + \dfrac{1}{2}\arctan\dfrac{s^2+2t+t^2}{2s} & (s<0), \end{cases} \\
r &= \left(\frac{s^2+(t+2)^2}{s^2+t^2}\right)^{\frac{1}{4}}.
\end{aligned}$$

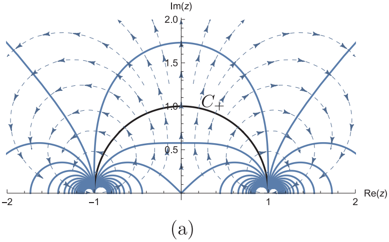
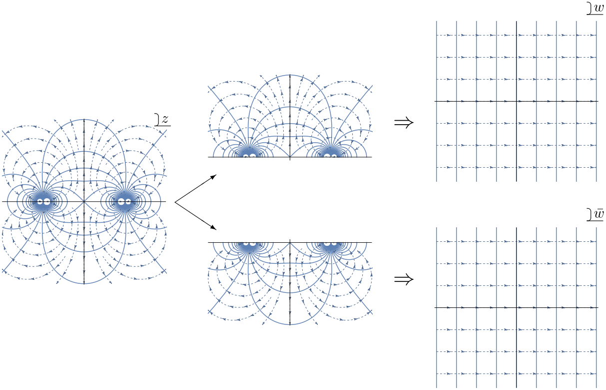


Figure: Equal-time contours on the z plane (solid lines). Dashed lines with arrows denote evolution of time t.

These contours have a remarkable feature that the string boundaries are fixed at $z = \pm 1$ during propagation of the string. One complex number $t + is$ corresponds to two points in the z plane.

Accordingly, we introduce a complex coordinate $w = t + is$ for the upper half z plane and $\bar{w} = t + is$ for the lower half plane.

By this mapping, the upper half plane corresponds to the whole w plane, and the lower half plane to the other $\bar{w}$ plane:



Hence, the equal-time contours by H_g lead us to the worldsheet which consists of two complex planes.

The two planes, w and $\bar{w}$, corresponding to the upper and lower half z planes are generated by the left and right moving Hamiltonian, H_g^+ and H_g^-, respectively.

Therefore, they can be regarded as holomorphic and antiholomorphic worldsheets of a closed string.

Now that we have obtained two decoupled Hamiltonians for the left and right moving sectors, we can construct two independent Virasoro operators according to Ishibashi-Tada:

$$\mathcal{L}_\kappa = \int_{C_+^t} \frac{dz}{2\pi i} g(z) f_\kappa(z) T(z), \qquad \tilde{\mathcal{L}}_\kappa = \int_{C_-^t} \frac{dz}{2\pi i} g(z) f_\kappa(z) T(z),$$

where $g(z)$ is the same function as that in the Hamiltonian H_g. $f_\kappa(z)$ is defined by the differential equation

$$g(z) \frac{\partial}{\partial z} f_\kappa(z) = \kappa f_\kappa(z).$$

For a constant time t, C_+^t and C_-^t denote integral contours along the equal-time line on the upper and lower half z plane, respectively.

We should note again that $T(z)$ **including in $\mathcal{L}_\kappa$ and $\tilde{\mathcal{L}}_\kappa$ is the same energy-momentum tensor of the open string system.**

$\mathcal{L}_0$ and $\tilde{\mathcal{L}}_0$ provide the left and right moving parts of the Hamiltonian, that is, $\mathcal{L}_0 = H_g^+$ and $\tilde{\mathcal{L}}_0 = H_g^-$.

$\mathcal{L}_\kappa$ satisfies continuous Virasoro algebra:

$$[\mathcal{L}_\kappa, \mathcal{L}_{\kappa'}] = (\kappa - \kappa')\mathcal{L}_{\kappa+\kappa'} \\ + \frac{c}{12}\int_{C_+^t} \frac{dz}{2\pi i}\left\{(\kappa - \kappa')\left(\frac{\partial^2 g}{\partial z^2} - \frac{1}{2g}\left(\frac{\partial g}{\partial z}\right)^2\right) + \frac{\kappa^3 - {\kappa'}^3}{2g}\right\} f_{\kappa+\kappa'}(z).$$

Ishibashi-Tada '16

The right moving sector of the Virasoro operator $\tilde{\mathcal{L}}_\kappa$ can be also defined by integration along the integration path on the lower half plane. Similarly, $\tilde{\mathcal{L}}_\kappa$ satisfies the continuous Virasoro algebra.

Moreover, since C^t_+ and C^t_- have no intersections, $\mathcal{L}_\kappa$ and $\tilde{\mathcal{L}}_\kappa$ commute with each other:

$$[\mathcal{L}_\kappa,\, \tilde{\mathcal{L}}_{\kappa'}] = 0.$$

Thus, we have found the **two independent Virasoro algebras in a deformed open string system, which can be regarded as the Virasoro algebras for closed strings, that is, the holomorphic and antiholomorphic parts.**

Kishimoto, Kitade and T.T ('18)

The continuous Virasoro algebra is explicitly calculated for the simplest weighting function:

$$\begin{aligned}\left[\mathcal{L}_\kappa,\,\mathcal{L}_{\kappa'}\right] &= (\kappa-\kappa')\mathcal{L}_{\kappa+\kappa'}\\ &\quad+\frac{c}{12}\kappa^3\delta(\kappa+\kappa')+\frac{3c}{96}(\kappa-\kappa')(1+|\kappa+\kappa'|)e^{-(\kappa+\kappa'+|\kappa+\kappa'|)},\\ \left[\tilde{\mathcal{L}}_\kappa,\,\tilde{\mathcal{L}}_{\kappa'}\right] &= (\kappa-\kappa')\tilde{\mathcal{L}}_{\kappa+\kappa'}\\ &\quad+\frac{c}{12}\kappa^3\delta(\kappa+\kappa')+\frac{3c}{96}(\kappa-\kappa')(1+|\kappa+\kappa'|)e^{-(\kappa+\kappa'+|\kappa+\kappa'|)}.\end{aligned}$$

The third term in each is characteristic in this weighting function.

But, it can be absorbed in a shift of $\mathcal{L}_\kappa$, since it is given as $\kappa-\kappa'$ times a function of $\kappa+\kappa'$.

In fact, in the w plane, we define a Virasoro operator associated with $\mathcal{L}_\kappa$:

$$\hat{\mathcal{L}}_\kappa = \int_{\hat{C}^t_+} \frac{dw}{2\pi i} e^{\kappa w} T(w),$$

where $\hat{C}^t_+$ is an image of C^t_+ by the conformal transformation.

Consequently, we can find a simpler form of continuous Virasoro algebra:

$$\left[\hat{\mathcal{L}}_\kappa, \hat{\mathcal{L}}_{\kappa'}\right] = (\kappa - \kappa')\hat{\mathcal{L}}_{\kappa+\kappa'} + \frac{c}{12}\kappa^3 \delta(\kappa + \kappa').$$

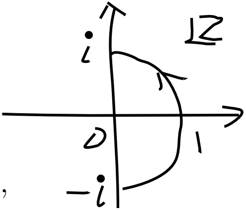
Similarly, we can find antiholomorphic Virasoro algebra by introducing a complex coordinate $\bar{w}$ in the lower half plane.

Here, it should be noted that the definition of $\hat{\mathcal{L}}_\kappa$ are independent of the choice of the weighting function $g(z)$.

Thus, this expression of the continuous Virasoro universal for sine-square-like deformed system irrelevant to $g(z)$.

Identity-based solutions

We consider cubic open bosonic string field theory;
$Q_{\rm B}\Psi + \Psi^2 = 0$.



A tachyon vacuum solution is given by

$$\Psi_0 \;=\; Q_L(e^h - 1)I - C_L((\partial h)^2 e^h)I,$$

$$Q_L(f) = \int_{C_{\rm left}} \frac{dz}{2\pi i} f(z) j_{\rm B}(z), \quad C_L(g) = \int_{C_{\rm left}} \frac{dz}{2\pi i} g(z) c(z).$$

Tanimoto-Takahashi-Kishimoto ('02)

The function $h(z)$ satisfies $h(-1/z) = h(z)$, $h(\pm i) = 0$, $(h(z))^* = h(1/z^*)$, and moreover $e^{h(z)}$ must have second or higher order zeros on $C_{\rm left}$

Expanding the string field around the solution Ψ_0, we obtain an action for fluctuation Ψ:

$$S[\Psi;Q'] = -\int \left(\frac{1}{2}\Psi * Q'\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right),$$

where the modified BRST operator Q' is given by

$$Q' \;=\; Q(e^h) - C((\partial h)^2 e^h).$$

The operators $Q(f)$ and $C(f)$ are defined as integrations along a whole unit circle.

(This vanishing cohomology provides an evidence that the solution correctly represents the tachyon vacuum solution.

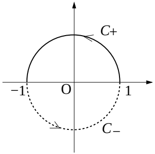
Kishimoto-Takahashi '02

Vacuum structure was evaluated numerically and then exactly.

Kishimoto, Takahashi '03, '09

Ishibashi '14, Kishimoto-Masuda-Takahashi '14)

First, we decompose the modified BRST operator Q' into two parts corresponding to the integrations along upper and lower semicircles:



$$Q' = Q'_+ + Q'_-,$$

where Q'_+ and Q'_- are defined by

$$Q'_\pm \equiv Q_\pm(e^h) - C_\pm((\partial h)^2 e^h),$$

$$Q_\pm(f) \equiv \int_{C_\pm} \frac{dz}{2\pi i} f(z) j_{\mathrm{B}}(z), \qquad C_\pm(f) \equiv \int_{C_\pm} \frac{dz}{2\pi i} f(z) c(z).$$

Assuming that $f(z)$ and $g(z)$ have zeros at $z = \pm 1$, we can find the following anti-commutation relations for $Q_\pm(f)$ and $C_\pm(f)$:

$$\begin{aligned}
\{Q_\pm(f), Q_\pm(g)\} &= 2\{Q_{\mathrm{B}}, C_\pm(\partial f \partial g)\}, \\
\{Q_\pm(f), C_\pm(g)\} &= \{Q_{\mathrm{B}}, C_\pm(fg)\}, \\
\{Q_\pm(f), Q_\mp(g)\} &= \{C_\pm(f), C_\mp(g)\} = \{Q_\pm(f), C_\mp(g)\} = 0.
\end{aligned}$$

Now, we consider the identity-based tachyon vacuum solution generated by $e^{h(z)}$ with second order zeros only at $z = \pm 1$.

In this case, owing to the above anti-commutation relations, the operator Q' splits into two anti-commutative nilpotent operators

$$(Q'_+)^2 = (Q'_-)^2 = \{Q'_+,\, Q'_-\} = 0.$$

It should be noted that this decomposition of the modified BRST operator occurs only for the tachyon vacuum solution.

We notice that this decomposition is similar to that of the SSLD Hamiltonian.

By analogy, Q'_+ and Q'_- can be regarded as holomorphic and antiholomorphic BRST operator of a closed string.

This interpretation is applied to closed string states in the open string field theory:

$$|\mathcal{V}\rangle_{\mathrm{OSFT}} = c(i)c(-i)\mathcal{V}(i,-i)\,|I\rangle\,,$$

where $\mathcal{V}(z,\bar{z})$ is a matter vertex operator for on-shell closed string states and $|I\rangle$ is the ket representation of the identity string field.

The state is invariant separately for Q'_+ and Q'_-:

$$Q'_+\,|\mathcal{V}\rangle_{\mathrm{OSFT}} = Q'_-\,|\mathcal{V}\rangle_{\mathrm{OSFT}} = 0,$$

On the other hand, in closed string theory, an on-shell closed string state is given by

$$|\mathcal{V}\rangle = c(0)\tilde{c}(0)\mathcal{V}(0,0)\,|0\rangle\,,$$

Since $\mathcal{V}$ is a $(1,1)$ primary operator, $|\mathcal{V}\rangle$ is also invariant under the action of Q_{B} and $\tilde{Q}_{\mathrm{B}}$:

$$Q_{\mathrm{B}}\,|\mathcal{V}\rangle = \tilde{Q}_{\mathrm{B}}\,|\mathcal{V}\rangle = 0.$$

In closed string field theories, a gauge transformation for a closed string field Φ is given by

$$\delta\Phi = (Q_{\rm B} + \tilde{Q}_{\rm B})\Lambda + \cdots ,$$

in which the summation of $Q_{\rm B}$ and $\tilde{Q}_{\rm B}$ is included.

Although an on-shell state is invariant separately, the quadratic term of the action for closed string field theory is not invariant under the transformation $\delta\Phi = Q_{\rm B}\Lambda + \tilde{Q}_{\rm B}\tilde{\Lambda}$.

Similarly, the open string field theory action at the tachyon vacuum is invariant under the gauge transformation

$$\delta\Psi = (Q'_+ + Q'_-)\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

but the quadratic term is not invariant for $\delta\Psi = Q'_+\Lambda_+ + Q'_-\Lambda_-$.

Thus, **the open string field theory at the tachyon vacuum has an analogous structure to closed string field theory under the correspondence of the BRST operators.**

Energy-momentum tensor

We define an operator at the tachyon vacuum:

$$\begin{aligned} \mathcal{T}(z) &\equiv e^{-h(z)}\{Q', b(z)\} \\ &= T(z) + \partial h(z)\, j_{\rm gh}(z) - (\partial h(z))^2 + \frac{3}{2} e^{-h(z)} \partial^2 e^{h(z)}. \end{aligned}$$

We find that $\mathcal{T}(z)$ satisfies the same OPE as $T(z)$ with zero central charge:

$$\mathcal{T}(y)\mathcal{T}(z) \sim \frac{2}{(y-z)^2}\mathcal{T}(z) + \frac{1}{y-z}\partial\mathcal{T}(z).$$

Here, it should be noted that $\mathcal{T}(z)$ includes not only operators but also a function in its form.

Since $h(z)$ is related to a coordinate frame of worldsheets, $\mathcal{T}(z)$ has an explicit dependence on the frame.

Virasoro algebra

By using $\mathcal{T}(z)$, we can define the continuous Virasoro operator at the tachyon vacuum:

$$\mathcal{L}_\kappa \equiv \int_{C_+} \frac{dz}{2\pi i} g(z) f_\kappa(z) \mathcal{T}(z), \qquad \tilde{\mathcal{L}}_\kappa \equiv \int_{C_-} \frac{dz}{2\pi i} g(z) f_\kappa(z) \mathcal{T}(z),$$

where the weighting function is related to $h(z)$ as $g(z) = z e^{h(z)}$.

Since $e^{h(z)}$ has second order zeros at $z = \pm 1$, $g(z)$ also has second order zeros at $z = \pm 1$.

These operators satisfy the holomorphic and antiholomorophic continuous Virasoro algebra for $c = 0$. ($\mathcal{L}_0 = H_+$ and $\tilde{\mathcal{L}}_0 = H_-$.)

By definition of $\mathcal{T}(z)$, these operators commute with $Q'_\pm$:

$$[Q'_\pm, \mathcal{L}_\kappa] = [Q'_\pm, \tilde{\mathcal{L}}_\kappa] = 0.$$

Thus, we have found the continuous Virasoro algebra at the tachyon vacuum.

Ghost numbers

It is better to define ghost and antighost fields at the tachyon vacuum as follows,

$$c'(z) \equiv e^{h(z)}c(z), \quad b'(z) \equiv e^{-h(z)}b(z).$$

These are also frame dependent operators as $\mathcal{T}(z)$, but they clearly satisfy $c'(y)b'(z) \sim 1/(y-z)$.

Moreover, it can be easily checked by the OPEs of $\mathcal{T}(z)$ that $c'(z)$ and $b'(z)$ are primary operators with the conformal weight -1 and 2, respectively.

We find that

$$\mathcal{T}(z) = \{Q', b'(z)\}.$$

We give a ghost number current at the tachyon vacuum by using $c'(z)$ and $b'(z)$:

$$\mathcal{J}_{\rm gh}(z) = c'\, b'(z).$$

Since it should be defined by the normal-ordering prescription

$$\mathcal{J}_{\rm gh}(z) = \lim_{y\to z}\left[c'(y)b'(z) - \frac{1}{y-z}\right],$$

the current $\mathcal{J}_{\rm gh}(z)$ is related to the conventional ghost number current as

$$\mathcal{J}_{\rm gh}(z) = j_{\rm gh}(z) + \partial h(z).$$

This is also a frame dependent operator due to $h(z)$ and we can easily find that it satisfies the same OPEs as those of the perturbative vacuum:

$$\begin{aligned}
\mathcal{T}(y)\mathcal{J}_{\rm gh}(z) &\sim \frac{-3}{(y-z)^3} + \frac{1}{(y-z)^2}\mathcal{J}_{\rm gh}(z) + \frac{1}{y-z}\partial\mathcal{J}_{\rm gh}(z), \\
\mathcal{J}_{\rm gh}(y)\mathcal{J}_{\rm gh}(z) &\sim \frac{1}{(y-z)^2}.
\end{aligned}$$

We can define an operator counting the ghost number:

$$Q'_{c\pm} \equiv \int_{C_\pm} \frac{dz}{2\pi i} \mathcal{J}_{\rm gh}(z).$$

These satisfy the following commutation relations with the BRST operators

$$[Q'_{c\pm}, Q'_{\pm}] = Q'_{\pm}, \qquad [Q'_{c\pm}, Q'_{\mp}] = 0.$$

These imply that Q'_{c+} and Q'_{c-} count the holomorphic and antiholomorphic ghost number, respectively.

Here, it is interesting to consider the relation of $Q'_{c\pm}$ to the conventional ghost number Q_c:

$$Q'_{c+} + Q'_{c-} = Q_c + \oint_{C_+ + C_-} \frac{dz}{2\pi i} \partial h(z).$$

However, the integration on the right-hand side is ill-defined.
This result reflects the fact that the ghost number for open strings is not definable at the tachyon vacuum since we have no open strings there.

BRST current

Similarly, we can define the BRST current on the tachyon vacuum:

$$\mathcal{J}_{\rm B}(z) = e^{h(z)} j_{\rm B}(z) + \{2\partial^2 h(z) + (\partial h)^2(z)\} e^{h(z)} c(z) + 2\partial h(z) e^{h(z)} \partial c(z).$$

As well as other operators, $\mathcal{J}_{\rm B}(z)$ satisfies the same OPEs as the conventional ones.

In addition, the modified BRST operator $Q' = Q'_+ + Q'_-$ can be rewritten by using $\mathcal{J}_{\rm B}(z)$:

$$Q'_\pm = \int_{C_\pm} \frac{dz}{2\pi i} \mathcal{J}_{\rm B}(z).$$

Concluding remarks

We have studied SSLD in open string system and clarified that the left and right moving modes in the SSLD system are decoupled and uncorrelated by zeros of the weighting function of the Hamiltonian.

We have shown that, as a result of the decoupling, the SSLD system is equivalent to a closed string system, in the sense that we find the holomorphic and antiholomorphic Virasoro algebra in the SSLD system.

Secondly, we have considered open SFT expanded around the identity-based tachyon vacuum solution. We have found that the modified BRST operator is decomposed to holomorphic and antiholomorphic parts, which are anti-commutative and nilpotent.

These operators are analogous to closed string BRST operators and so gauge symmetry of the theory is regarded as that of closed string field theories.

We have constructed the local operators in the tachyon vacuum, including the energy-momentum tensor, the ghost and anti-ghost fields, the ghost number current and the BRST current.

It is remarkable feature that these operators depend on the frame of worldsheet, which is chosen by the identity-based tachyon vacuum solution.

From these operators, we have found that the continuous Virasoro algebra and the ghost number operator. The important point is that these have holomorphic and antiholomorphic parts, which are realized by the SSD mechanism.

The theory at the identity-based tachyon vacuum solution possesses a gauge symmetry generated by the holomorphic and antiholomorphic BRST operators, which is identified with a gauge symmetry of closed string field theories.

Since gauge symmetry is an essential ingredient in SFT, **we conjecture that the theory at the tachyon vacuum provides a kind of closed string field theories.**

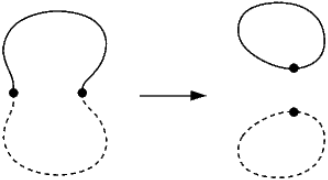


Figure: String pictures before and after SSLD. The solid and dashed lines correspond to holomorphic and antiholomorphic parts of a string. As a result of SSLD, open string boundaries (black dots) become joined and an open string divides to holomorphic and antiholomorphic strings.