Towards the Construction of New Democratic Theories

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in progress, with S. Giaccari

String Field Theory and String Perturbation Theory (SFT2019)

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Outline

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Introduction

A "Democratic" theory is one in which all pictures are present.

Democratic open SSFT (M.K 09)

- Relies on an alternative (equivalent) formulation of *the cohomology problem* (Berkovits 01).
- Cubic; large Hilbert space; single mid-point insertion.
- Includes and unifies the Ramond sector.
- BV (classical) master equation is formally straightforward.

Potential problems

- The space of string fields?
- Mid-point problems?
- Operators of arbitrarily negative conformal weight?

Outline

Formulating String Field Theory

- Identify the worldsheet cohomology problem.
- Extend vertex operators off-shell to string fields.
- Reinterpret the cohomology problem as defining e.o.m and gauge symmetry.
- Derive from an action.
- Add interaction terms: Non-linear e.o.m and gauge symmetry.
- Verify that a proper covering of moduli space is obtained.

Various Formulations of the Cohomology Problem

The cohomology problem for the open RNS string

- In the small space $\Psi \in H_S$ at a fixed picture number:
  $Q\Psi = 0, \qquad \delta\Psi = Q\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
- In the large space $\Psi \in H_L$ at a fixed picture number:
  $Q\eta\Psi = 0, \quad \delta\Psi = Q\Lambda_1 + \eta\Lambda_2, \quad pic(\Psi) = p, \quad gh(\Psi) = 0.$

These formulations used for defining most of the SSFT's.

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- In the large space $\Psi \in H_L$ at a fixed picture number:
  $(Q-\eta)\Psi = 0, \quad \delta\Psi = (Q-\eta)\Lambda, \quad pic(\Psi) = p, \quad gh(\Psi) = 1.$
- In the large space $\Psi \in H_L$ at an arbitrary picture range:
  $(Q-\eta)\Psi = 0, \quad \delta\Psi = (Q-\eta)\Lambda,$
  $p_1 < pic(\Psi) < p_2, \quad gh(\Psi) = 1.$
  In particular one can take $p_1 = -\infty$ and/or $p_2 = \infty$.

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  In particular one can take $p_1 = -\infty$ and/or $p_2 = \infty$.
  But there is a subtlety here.

The Cohomology Problem of $(Q-\eta)$ for $\Psi \in H_L$

If $pic(\Psi) = p$, the equation $(Q-\eta)\Psi = 0$ gives components at pictures p, $p-1$, which must vanish independently: $Q\Psi = \eta\Psi = 0$, i.e. $\Psi \in H_S$ and obeys the standard equation.

The gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ implies $\Lambda = \Lambda_1 + \Lambda_2$ with $pic(\Lambda_1) = p, \quad pic(\Lambda_2) = p+1, \quad \eta\Lambda_1 = Q\Lambda_2 = 0$. Then, $\Lambda_1 = \eta\tilde{\Lambda}_1, \quad \Lambda_2 = Q\tilde{\Lambda}_2$. All in all: $\delta\Psi = Q\eta\tilde{\Lambda}$.

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The equation $(Q-\eta)\Psi = 0$ and gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ define the standard cohomology without restricting the picture number.

Changing the picture is a gauge transformation: Let $pic(\Psi) = p$. Then, $(Q-\eta)\Psi = 0 \quad \Rightarrow \quad Q\Psi = \eta\Psi = 0 \quad \Rightarrow \quad \Psi = \eta\Phi = \eta(\xi\Psi)$. Define $\Lambda = \xi\Psi$. Then $\delta\Psi = (Q-\eta)(\xi\Psi) = X\Psi - \Psi$. So $p \to p+1$. $X = Q\xi$ is the picture changing operator (PCO).

The Cohomology Problem of $(Q-\eta)$ for Unbounded Picture

Starting from a bounded picture range we can send Ψ to any given picture. What if it is unbounded?

Let Ψ be a vertex operator at some picture. Define:

$$\tilde{\Psi} \equiv \Psi + \sum_{n=1}^{\infty} X^n \Psi + \sum_{n=1}^{\infty} Y^n \Psi$$

Y is the inverse PCO. Ignore OPE singularities for now.

Now, $X\tilde{\Psi} = \tilde{\Psi}$. It is an eigenmode of the PCO.

One has to restrict the space of string fields.

Criteria for Unbounded Picture

Let Ψ involve terms whose picture is unbounded.

One can try to move different picture components one by one.

$$\delta_1\Psi = (Q-\eta)(\xi\Psi_{p_0-1}) = X\Psi_{p_0-1} - \Psi_{p_0-1}$$

$$\delta_2\Psi = (Q-\eta)(\xi(1+X)\Psi_{p_0-2}) = X^2\Psi_{p_0-2} - \Psi_{p_0-2}$$

$$\vdots$$

Formally, all components at $p < p_0$ vanish. At p_0 we get:

$$\tilde{\Psi}_{p_0} = \Psi_{p_0} + X\Psi_{p_0-1} + X^2\Psi_{p_0-2} + \ldots$$

For $\Psi = X\Psi$ this would be $\infty\Psi$.

Similarly, at $p_0 - 1$ we get a not-absolutely convergence series.

Outline

Multi-Picture Changing Operators and Their Potentials

In H_L, Q and η have trivial cohomologies.

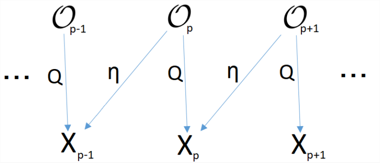
Contracting homotopy operators:

$\mathcal{O}_0 \equiv -c\xi\partial\xi e^{-2\phi}$, $\mathcal{O}_1 \equiv \xi$: $\quad Q\mathcal{O}_0 = \eta\mathcal{O}_1 = 1.$

These operators define the PCOs:

$Q\mathcal{O}_1 = X \equiv X_1, \quad \eta\mathcal{O}_0 = Y \equiv X_{-1}.$

This structure can be extended to arbitrary pinture:



In this infinite chain $X_0 \equiv 1$ and all picture changing operators X_p and their potentials $\mathcal{O}_p$ are weight zero primaries.

A Linearized Democratic Theory

The string field Ψ lives in the large Hilbert space within any desirable range of picture numbers.
Find an action from which the linearized e.o.m could be derived: $(Q-\eta)\Psi = 0$.

$$S = \frac{1}{2}\int \mathcal{O}\Psi(Q-\eta)\Psi$$

$\mathcal{O}$ is needed for *proper ghost number and parity*.
It should *include ξ and commute with $Q-\eta$*.

Choose: $\mathcal{O} \equiv \sum_{p=-\infty}^{\infty} \mathcal{O}_p$.
The range of summation is fixed by the requirement: $[Q-\eta, \mathcal{O}] = 0$.

Can be extended to a non-linear theory:

Action: $S = \int \mathcal{O}(\frac{1}{2}\Psi(Q-\eta)\Psi + \frac{1}{3}\Psi^3)$.

$\mathcal{O}$ is a mid-point insertion.

E.O.M: $(Q-\eta)\Psi + \Psi^2 = 0$.

Gauge symmetry: $\delta\Psi = (Q-\eta)\Lambda + [\Psi,\Lambda]$.

While $\mathcal{O}$ decouples from the E.O.M and gauge symmetry, it leads to problems with defining a propagator.

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Replace the kinetic term by: $S_0 = \frac{1}{2}\int \tilde{\mathcal{O}}\Psi(Q-\eta)\Psi$.

Here, e.g., $\tilde{\mathcal{O}} = \frac{1}{2\pi i}\oint \frac{dz\mathcal{O}(z)}{z}$.

The insertion is BPZ even.

Now, the free E.O.M is: $\tilde{\mathcal{O}}(Q-\eta)\Psi = 0$.

But $\tilde{\mathcal{O}}$ does not decouple, since it is not a mid-point insertion.

$\tilde{\mathcal{O}} \sim \xi \sum_{n=-\infty}^{\infty} X^n$.

ξ could be though of as being part of the measure, but we cannot invert even the sum.

Half Infinite Pictures

Define instead: $S_0 = \frac{1}{2}\int \tilde{\mathcal{O}}\Psi(Q-\eta)\Psi$ with $\tilde{\mathcal{O}} \sim \xi \sum_{n=0}^{\infty} X^n$.

Now, $[Q-\eta, \tilde{\mathcal{O}}] = -1$. Is it a problem?

Not if we take: $\Psi = \sum_{p=-\infty}^{-1} \Psi_p$,

since then this term does not contribute to the action.

The E.O.M is now: $\tilde{\mathcal{O}}(Q-\eta)\Psi + \xi\eta\Psi_{-1} = 0$.

$\eta - Q$ removes the insertion from the first term.

Also, we can write: $\sum_{n=0}^{\infty} X^n = \frac{1}{1-X}$.

Acting with $1-X$ also removes this term.

Gauge Transformations for Half Infinite Pictures

One reason for using all pictures was that a finite gauge transformation would introduce all of them (for simplicity consider a standard cubic vertex):
$\delta\Psi = (Q - \eta)\Lambda + [\Psi, \Lambda]$ contributes at $pic(\Psi) + pic(\Lambda)$.
Iterating we get $pic(\Psi) + 2pic(\Lambda)$ and so on.

But now we only allow $pic(\Lambda) \leq 0$ with the restriction $Q\Lambda_0 = 0$, so all contributions are at $pic < 0$ and the set of pictures is closed under gauge transformations.

Adding Interactions

Since now the insertion is not at the mid-point we cannot just insert it as it is on the three vertex.
Presumably A_∞ structure would emerge.

The structure would have insertions of the form:
$\xi(1 + X + X^2 + ...)^N$.
Now, we could make sense of such insertions.
For a two-directional infinite picture theory this is not defined.

Projection

It seems that the space of string fields would need to be further restricted to: $XY\Psi = \Psi$.
While XY is not necessarily a projector, this is a projection equation.
It is defined in each finite dimensional block of given g, p, h and it is linear, so all is well defined.

Furthermore, this would eliminate negative weight states.

If possible:

- Inclusion of the Ramond sector would remain straightforward.
- The BV master equation would automatically hold, and not only formally.
- Presumable, such a theory would give a framework from which other known theories could be derived.

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- Examine whether it is possible to obtain such a democratic theory.
- Consider new gauge fixings of the theory that would lead to new formulations.
- New expressions for scattering amplitudes?
- Extend to closed and to heterotic theories.

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THANK YOU!