SFT 2019 [May.9th]

Light-cone Reduction *of Witten's open String Field Theory*

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based on JHEP04(2019)143 [arXiv:1901.08555]

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Motivation

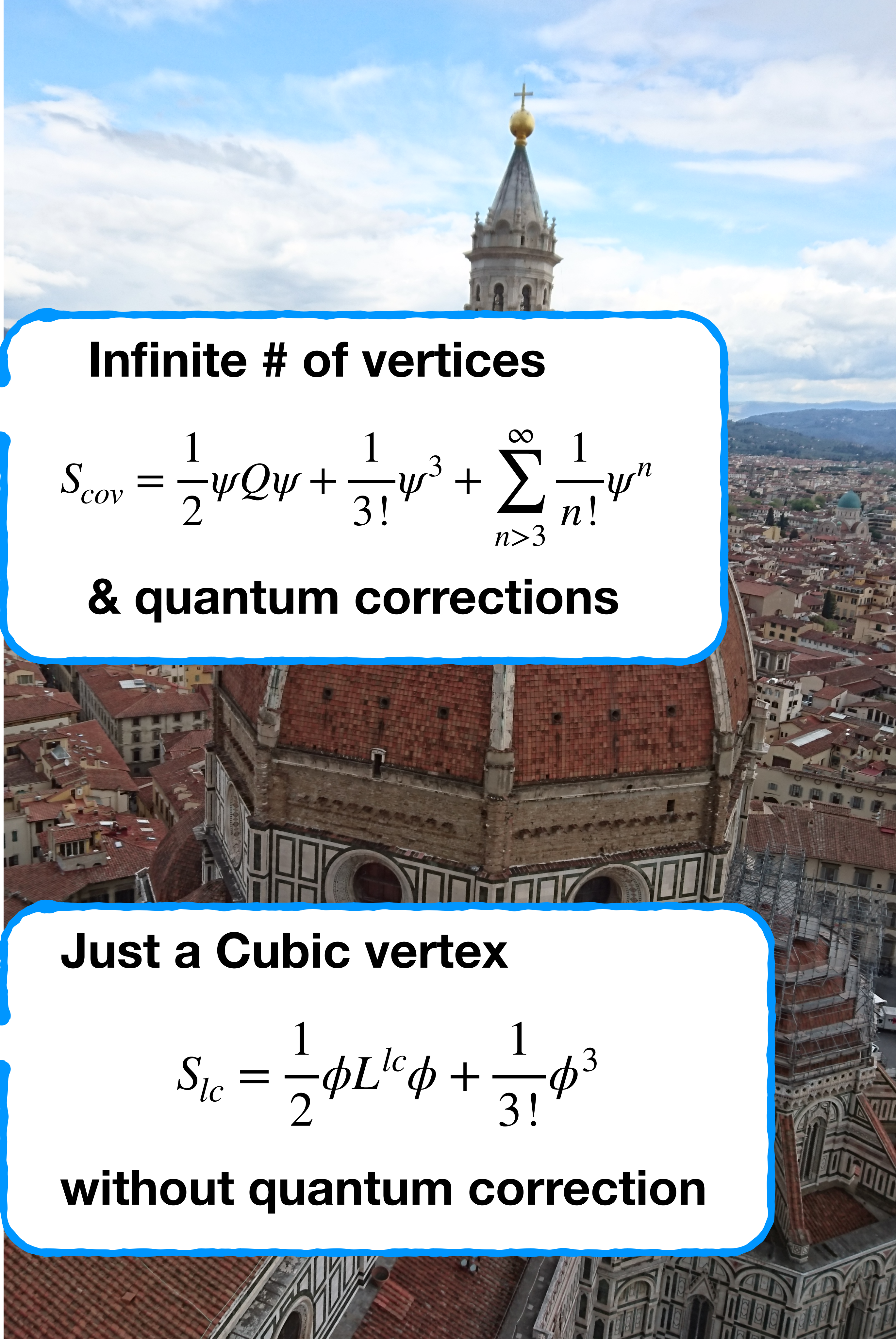
- We formulated cov. SFTs !!

  - Beautiful but complicated.

- We want more usable one
  as “a tool” ...

- As a tool, “light-cone SFT” may be better.

  - Easy to handle
  - Simple action
    even for closed strings
  - Quartic for open strings



Infinite # of vertices

$$S_{cov} = \frac{1}{2}\psi Q \psi + \frac{1}{3!}\psi^3 + \sum_{n>3}^{\infty} \frac{1}{n!}\psi^n$$

& quantum corrections

Just a Cubic vertex

$$S_{lc} = \frac{1}{2}\phi L^{lc}\phi + \frac{1}{3!}\phi^3$$

without quantum correction

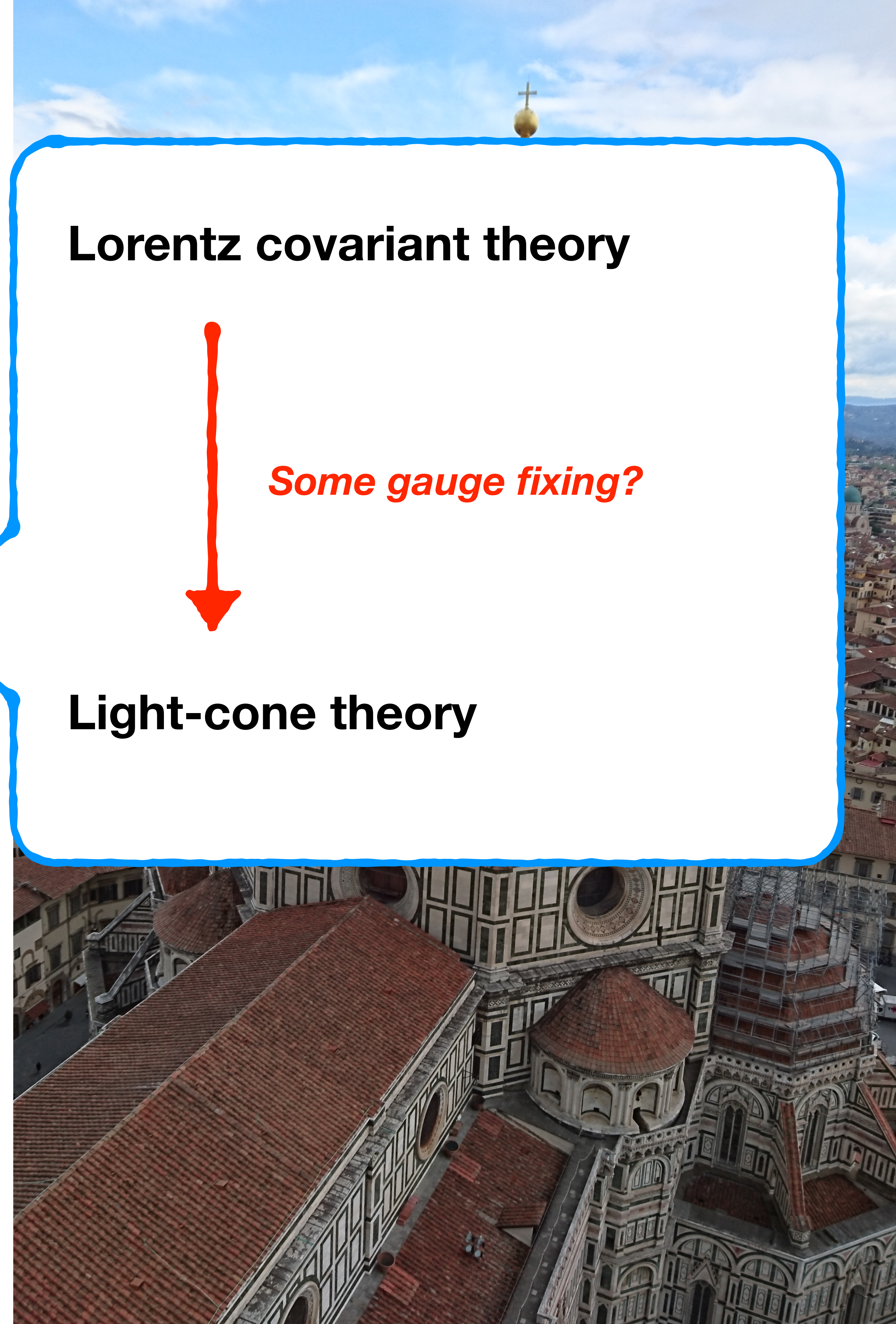
Our ultimate goal

- **Relating “light-cone SFT” with “covariant SFT”.**

  - People may expect that “light-cone theory” is a gauge fixed version of “covariant theory”.

  - No explicit construction. . .

- As a first step, we consider **how the light-cone string field appears from covariant SFT**



Two differences

- **“Field contents” (world-sheet variables) are different.**

  - Light-cone string field ⋯ Physical modes

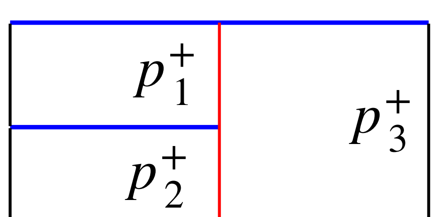
$$\Psi_{lc} = \Psi_{lc}[\, X^I(\sigma), p^+, x^- \,]$$

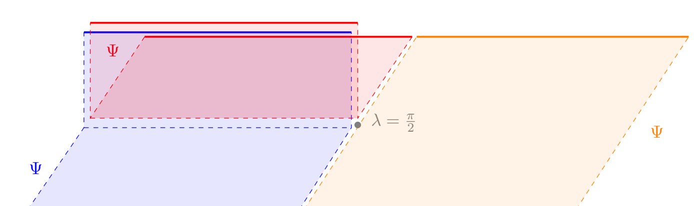
  - Covariant string field ⋯ (Phys + [gauge + unphys]) modes

$$\Psi_{cov} = \Psi_{cov}[\, X^\mu(\sigma), b(\sigma), c(\sigma) \,]$$

$$= \Psi_{cov}[\, X^I(\sigma), p^+, x^-;\ \underbrace{\partial X^\pm(\sigma), b(\sigma), c(\sigma)}_{\textit{BRST quartets}} \,]$$

- **“Neumann coefficients” (string-overlaps) are not equivalent.**

LC type : $\int dp^+$ 

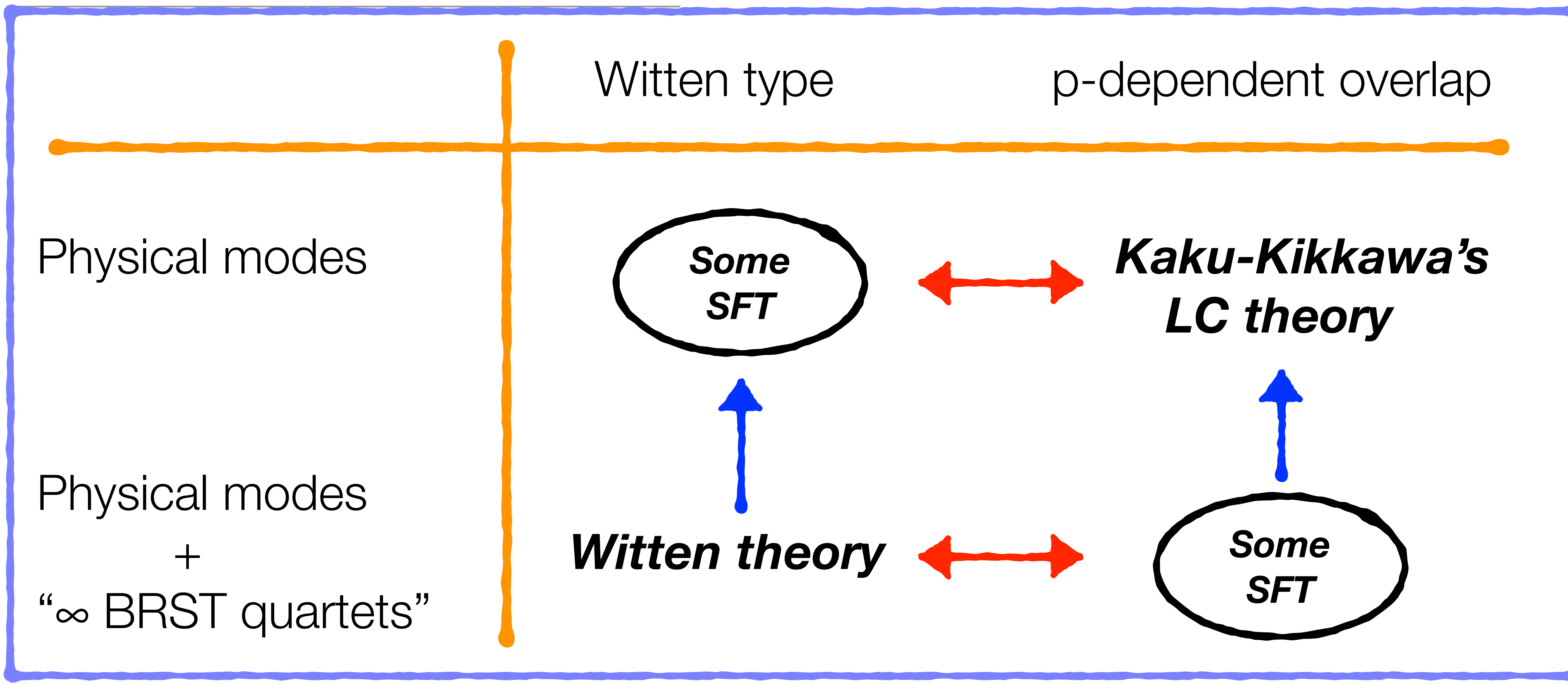
Witten type : 

Sketch of light-cone & covariant theories

	Witten type	p-dependent overlap
Physical modes		***Kaku-Kikkawa's LC theory***
Physical modes + "∞ BRST quartets"	***Witten theory***	

So we can expect . . .

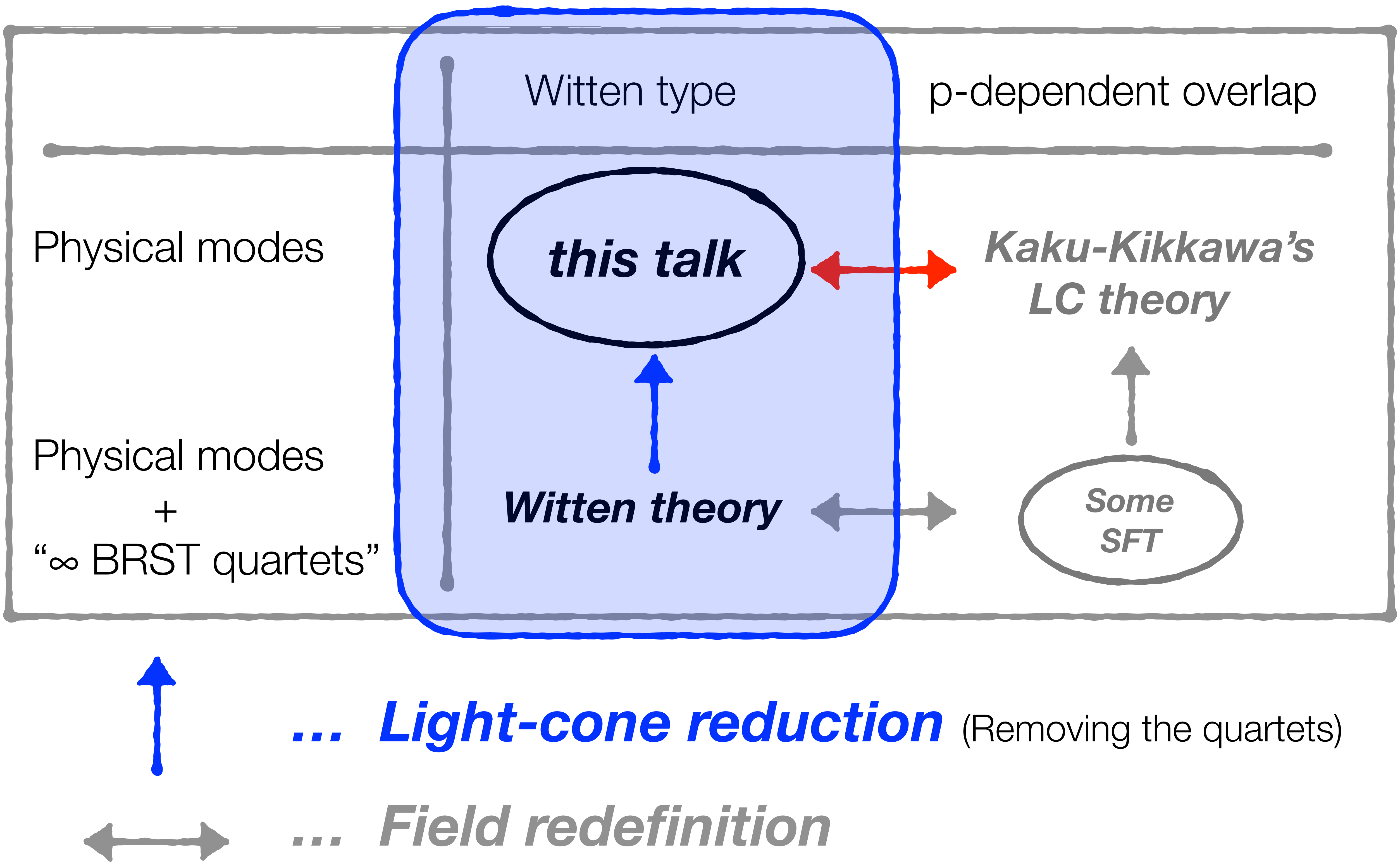
Sketch of light-cone & covariant theories



… **Light-cone reduction** (Removing the quartets)

… **Field redefinition**

Sketch of light-cone & covariant theories



Our strategy

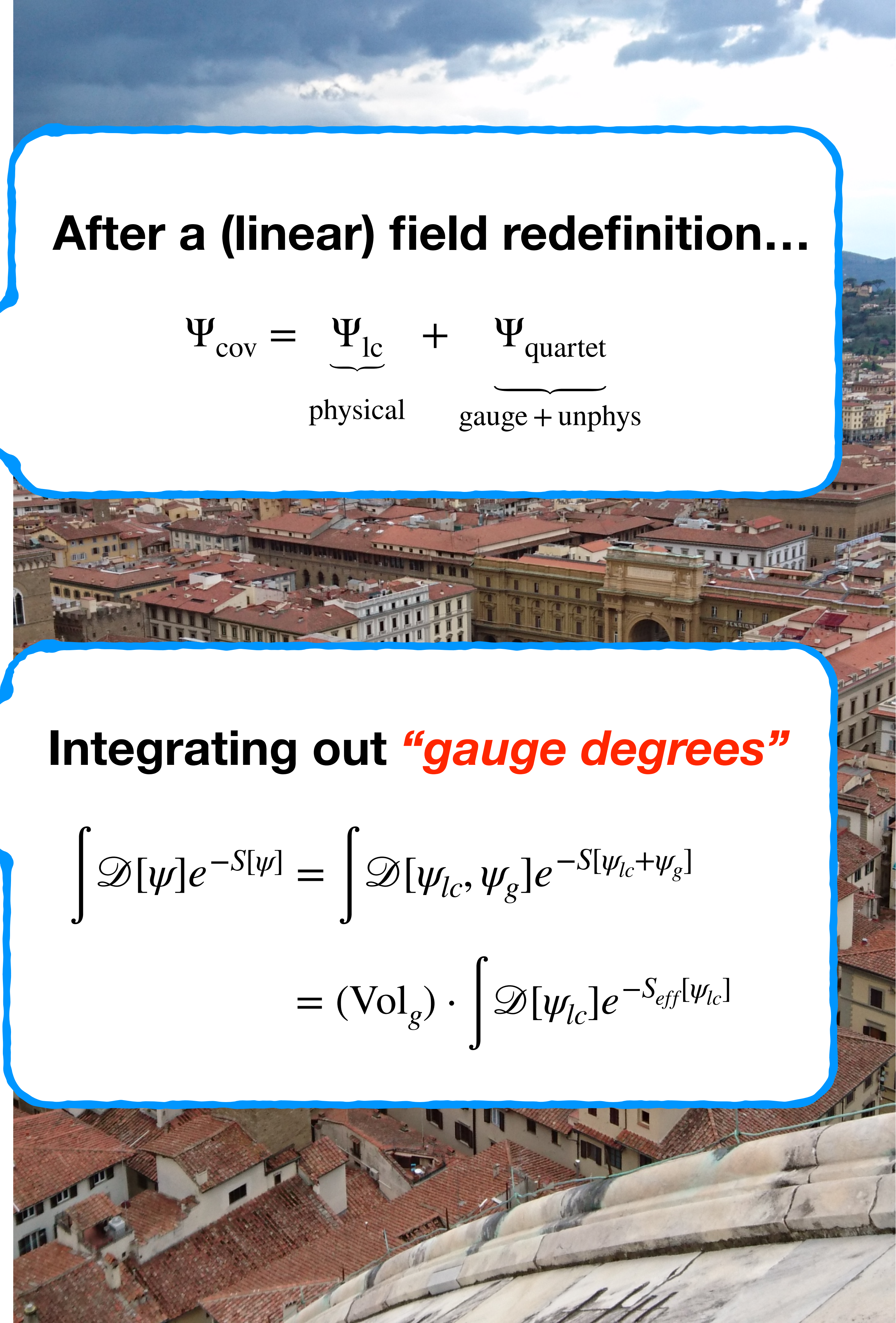
- Covariant String Field
  = Light-cone String Field
  + BRST quartets

- Gauge fixing as making
  an effective field theory

  - Classical part = our LC action

$$S_{eff}[\psi_{lc}] = \underline{S[\psi_{lc}]} + (\,\text{loop}\,)$$

- *In terms of math…*
  “Homological perturbation”
  for A_∞ / L_∞



After a (linear) field redefinition…

$$\Psi_{\rm cov} = \underbrace{\Psi_{\rm lc}}_{\text{physical}} + \underbrace{\Psi_{\rm quartet}}_{\text{gauge + unphys}}$$

Integrating out “*gauge degrees*”

$$\int \mathscr{D}[\psi] e^{-S[\psi]} = \int \mathscr{D}[\psi_{lc}, \psi_g] e^{-S[\psi_{lc}+\psi_g]}$$

$$= (\mathrm{Vol}_g) \cdot \int \mathscr{D}[\psi_{lc}] e^{-S_{eff}[\psi_{lc}]}$$

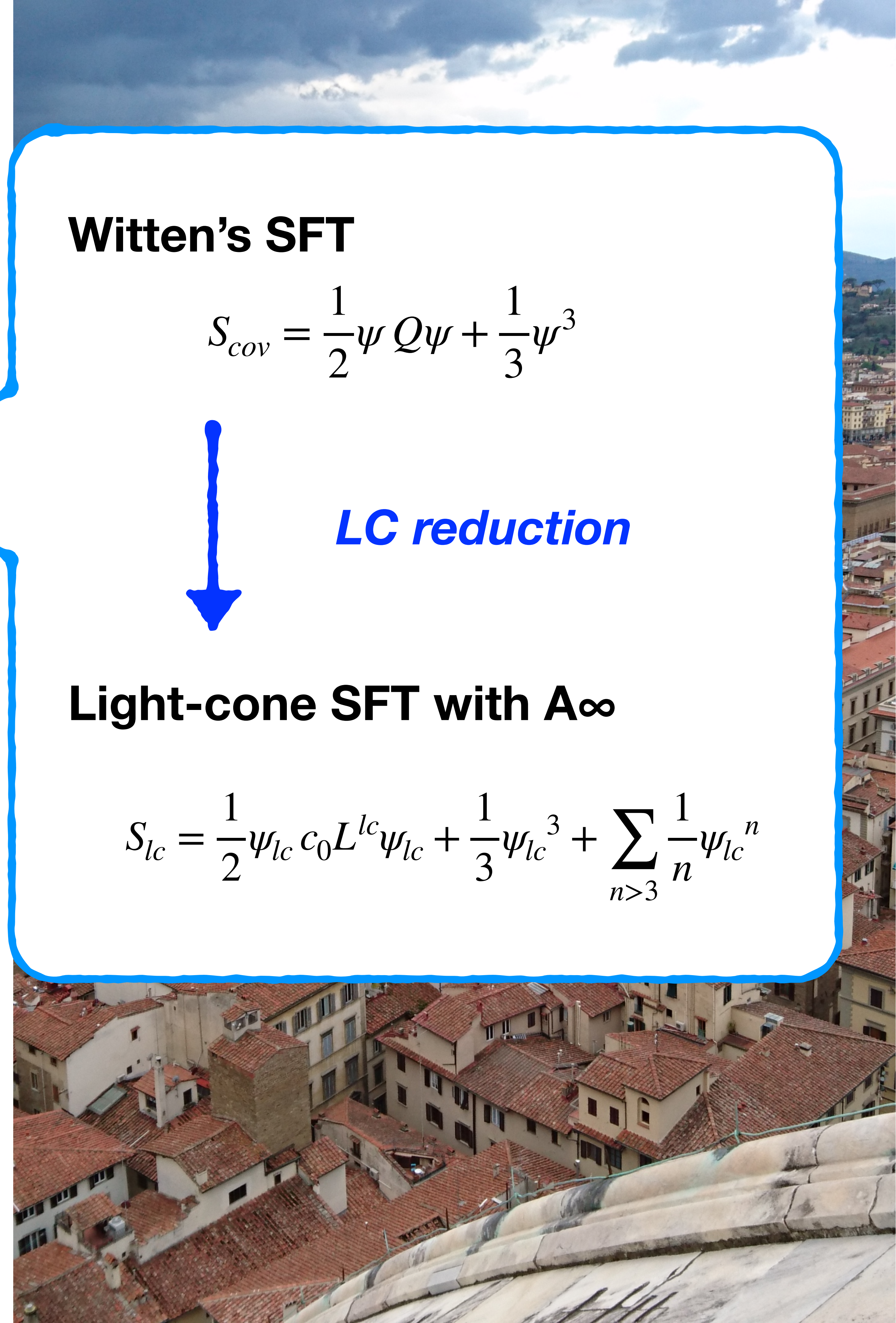
Our results

- Light-cone SFT from Witten's covariant SFT

  - A new type of theory

  - Different from the old Kaku-Kikkawa's one.

- Amplitudes are exactly the same as Witten's SFT.
  (by construction)



Witten's SFT

$$S_{cov} = \frac{1}{2}\psi Q \psi + \frac{1}{3}\psi^3$$

$$\Big\downarrow \ \textit{LC reduction}$$

Light-cone SFT with A∞

$$S_{lc} = \frac{1}{2}\psi_{lc}\, c_0 L^{lc} \psi_{lc} + \frac{1}{3}{\psi_{lc}}^3 + \sum_{n>3}\frac{1}{n}{\psi_{lc}}^n$$

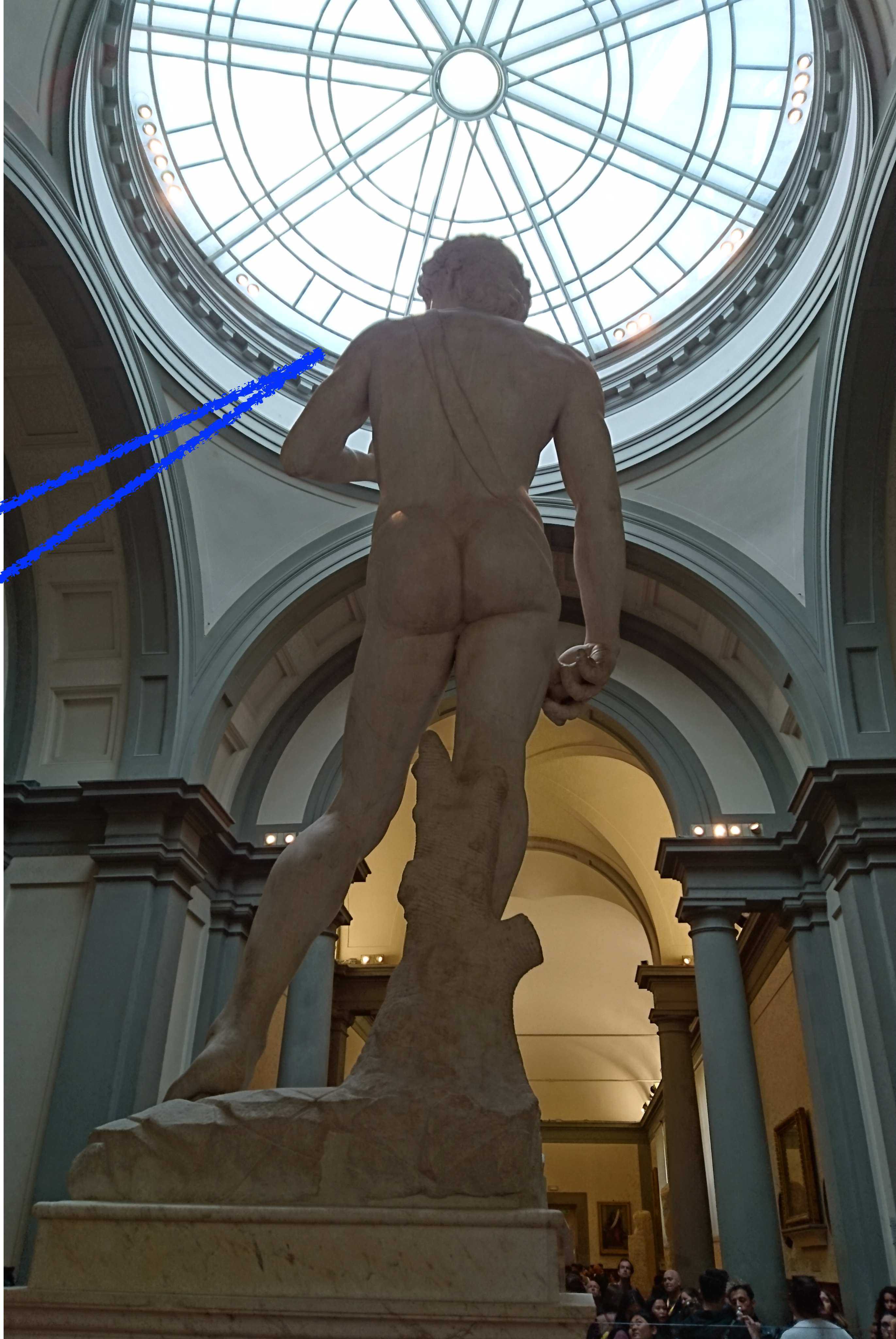
Plan

- Introduction & Summary ✓
- LC reduction of Witten's SFT

(a) Sketch of LC reduction
- Field redefinition
- Effective field theory

(b) More on the reduction
- Homological perturbation
  & path-integrating-out

- Summary + α



(a) Sketch of LC reduction

- Review of BRST quartets

- Find a field redefinition $\Psi_{\text{cov}} = \mathscr{U}\,\Psi$ s.t. $\Psi_{\text{cov}} = \underbrace{\Psi_{\text{lc}}}_{\text{physical}} + \underbrace{\Psi_{\text{quartet}}}_{\text{unphys + gauge}}$

- Integrating-out Ψ_{quartet} (= Homological Perturbation)

Review: BRST quartets

$$N = \left[\!\left[\, d \, , \frac{1}{p^+} \sum_{n\neq 0} a^+_{-n} b_n \right]\!\right]$$

- Virasoro zero = Light-cone + "quartets"

$$L_0 \equiv \underbrace{\frac{1}{2}p^2 + \sum_{n\geq 1} a^I_{-n} a^I_n - 1}_{=L_0^{lc}} + \underbrace{\sum_{n\neq 0} \Big[\, n : c_{-n} b_n : - : a^+_{-n} a^-_n : \Big]}_{=N}$$

- **N** counts the level of BRST quartets : $\{\, a^+_{-n}, c_{-n}, ; b_n, a^-_n \,\}$

- Nilpotent operator $d \equiv -p^+ \sum_{n\neq 0} c_{-n} a^-_n$ generates BRST transformations

$$\delta_{\mathrm{B}}(a^+_n) \equiv [\![\, d \, , a^+_n \,]\!] = -n(p^+ c_n) \qquad\qquad \delta_{\mathrm{B}}\Big(\frac{1}{p^+} b_{-n}\Big) \equiv \Big[\!\Big[\, d \, , \frac{1}{p^+} b_{-n} \Big]\!\Big] = a^-_{-n}$$

and ***N*** is a "***d***-exact operator".

→ Any "***d***-closed" state ***A*** satisfying ***N A = n A (n≠0)*** is "***d***-exact"

$$\boxed{\textit{No } \boldsymbol{d}\textit{-cohomology except for } \textbf{\textit{Ker}}\textit{[} \boldsymbol{N} \textit{] !}}$$

Light-cone & BRST

- ***bc*** zero-modes & LC modes

$$Q_1 \equiv \sum_{n\neq 0} c_{-n}\Big[\frac{1}{2}\sum_k a^i_{n-k}a^i_k - \sum_{k\neq 0} a^+_{n-k}a^-_k\Big] - \frac{1}{2}\sum_{\substack{n,m\neq 0\\ n+m\neq 0}} (m-n) : c_{-m}c_{-n}b_{n+m} :$$

$$Q \equiv c_0\, L_0 + [\, Q_{\text{non zero}}\,] - \underbrace{b_0 \sum_{n\neq 0} n \; : c_{-n}c_n :}_{=Q_2}$$

$$= c_0\big(L_0^{lc} + N\big) + \Big[\underbrace{-p^+\sum_{n\neq 0} c_{-n}a^-_n}_{=d} + Q_1 \underbrace{- p^-\sum_{n\neq 0} c_{-n}a^+_n}_{=Q_3}\Big] + Q_2$$

$$= d + c_0\,\big(L_0^{lc} + N\big) + \sum_{k=1} Q_k$$

- We find **the similarity transformation :** $Q = \mathcal{U}^{-1}\left(\, c_o L^{lc} + d\,\right)\mathcal{U}$

$$\mathcal{U} \equiv \exp\Big(-c_0\frac{1}{p^+}\sum_{n\neq 0} a^+_{-n}b_n\Big)\exp\Big(\frac{1}{p^+}\sum_{n\neq 0}\frac{1}{n}\Big[\!\Big[\, a^+_{-n}b_n,\; Q_1 + \frac{1}{2}Q_2\,\Big]\!\Big]\Big)$$

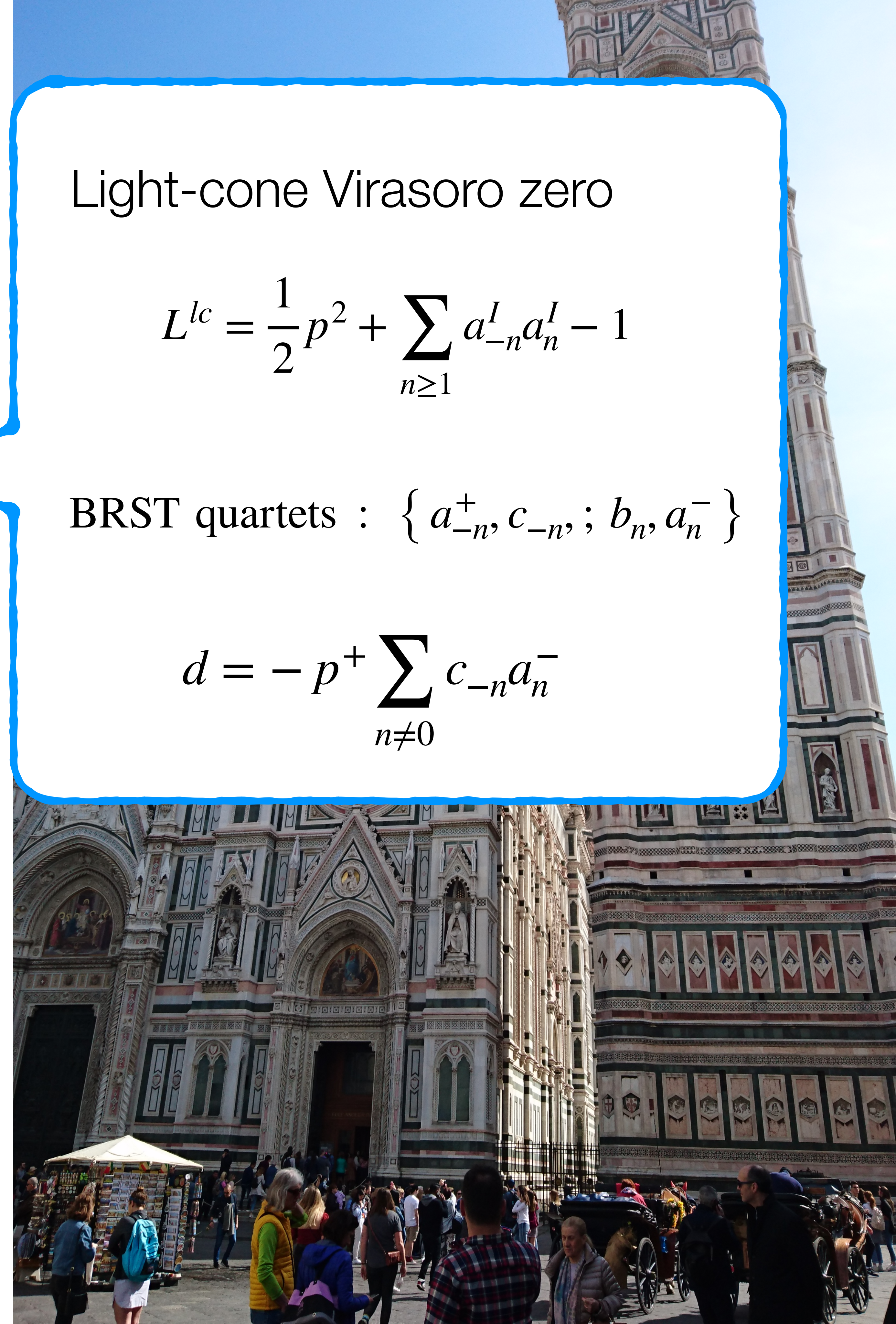
Field redefinition

- LC dec. of BRST op.

$$Q = \mathscr{U}^{-1} \left(c_o L^{lc} + d \right) \mathscr{U}$$

- Field redefinition

$$\Psi \;\rightarrow\; \mathscr{U}\,\Psi = \underbrace{\Psi_{\rm lc}}_{\text{physical}} + \underbrace{\Psi_{\text{BRS quartet}}}_{\text{gauge}}$$



Light-cone Virasoro zero

$$L^{lc} = \frac{1}{2}p^2 + \sum_{n\geq 1} a^I_{-n} a^I_n - 1$$

BRST quartets : $\left\{ a^+_{-n}, c_{-n}, ;\, b_n, a^-_n \right\}$

$$d = -\,p^+ \sum_{n\neq 0} c_{-n} a^-_n$$

Hodge type decomposition

- BRST quartets have no cohomology.

- We thus find a homotopy equivalence relation

$$d\,h + h\,d = 1 - \Pi$$

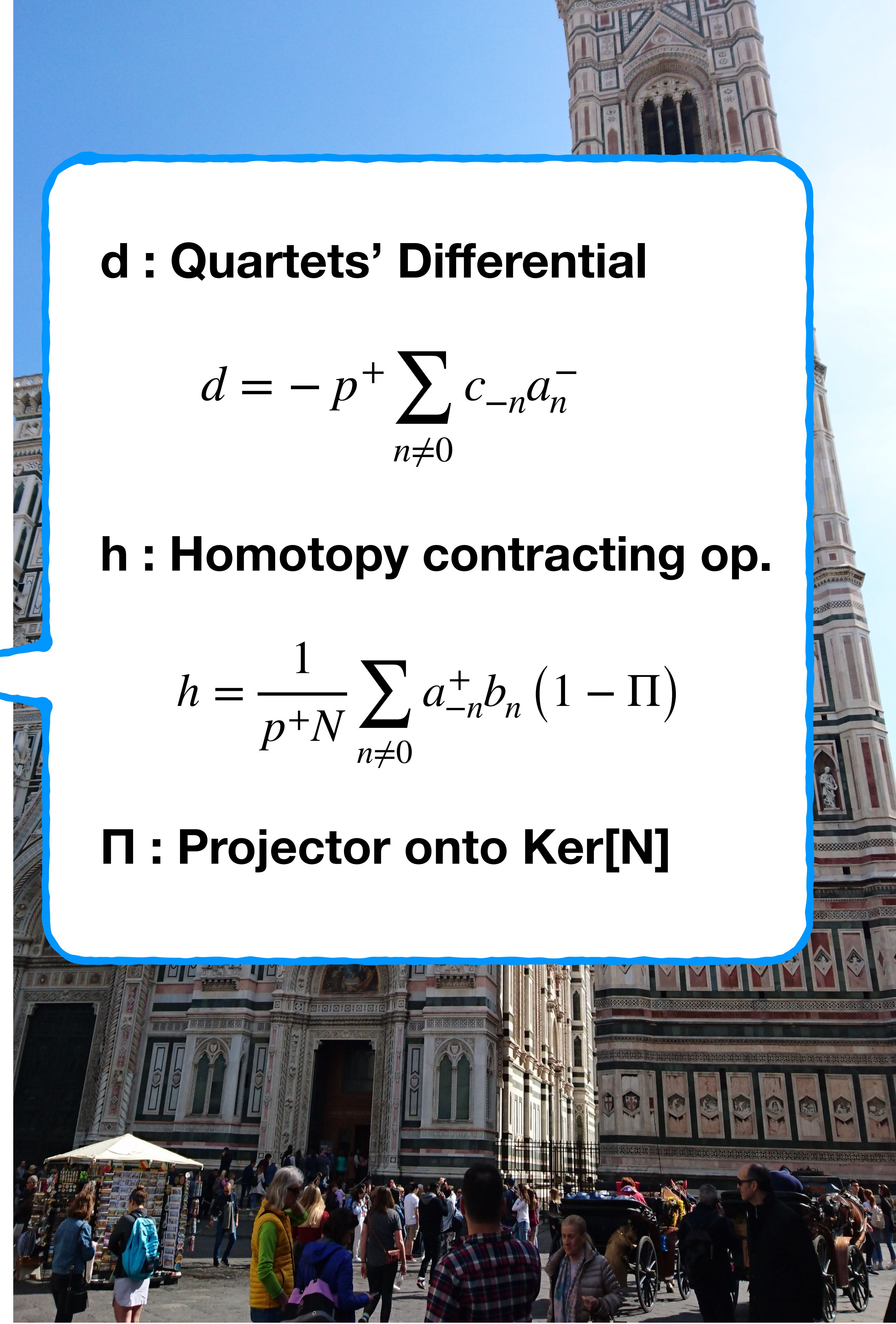
- Actually…

$$\Psi \;\to\; \mathscr{U}\,\Psi = \Pi\,\Psi + (d\,h + h\,d)\,\Psi$$

$$= \underbrace{\Psi_{\mathrm{lc}}}_{\text{physical}} + \underbrace{\Psi_{\text{BRS quartet}}}_{\text{gauge}}$$

- “h” is a kind of propagator.

$$Q\,(bL^{-1}) + (bL^{-1})\,Q = 1 - e^{-\infty L}$$



d : Quartets’ Differential

$$d = -\,p^{+} \sum_{n \neq 0} c_{-n} a_n^{-}$$

h : Homotopy contracting op.

$$h = \frac{1}{p^{+}N} \sum_{n \neq 0} a_{-n}^{+} b_n \,(1 - \Pi)$$

Π : Projector onto Ker[N]

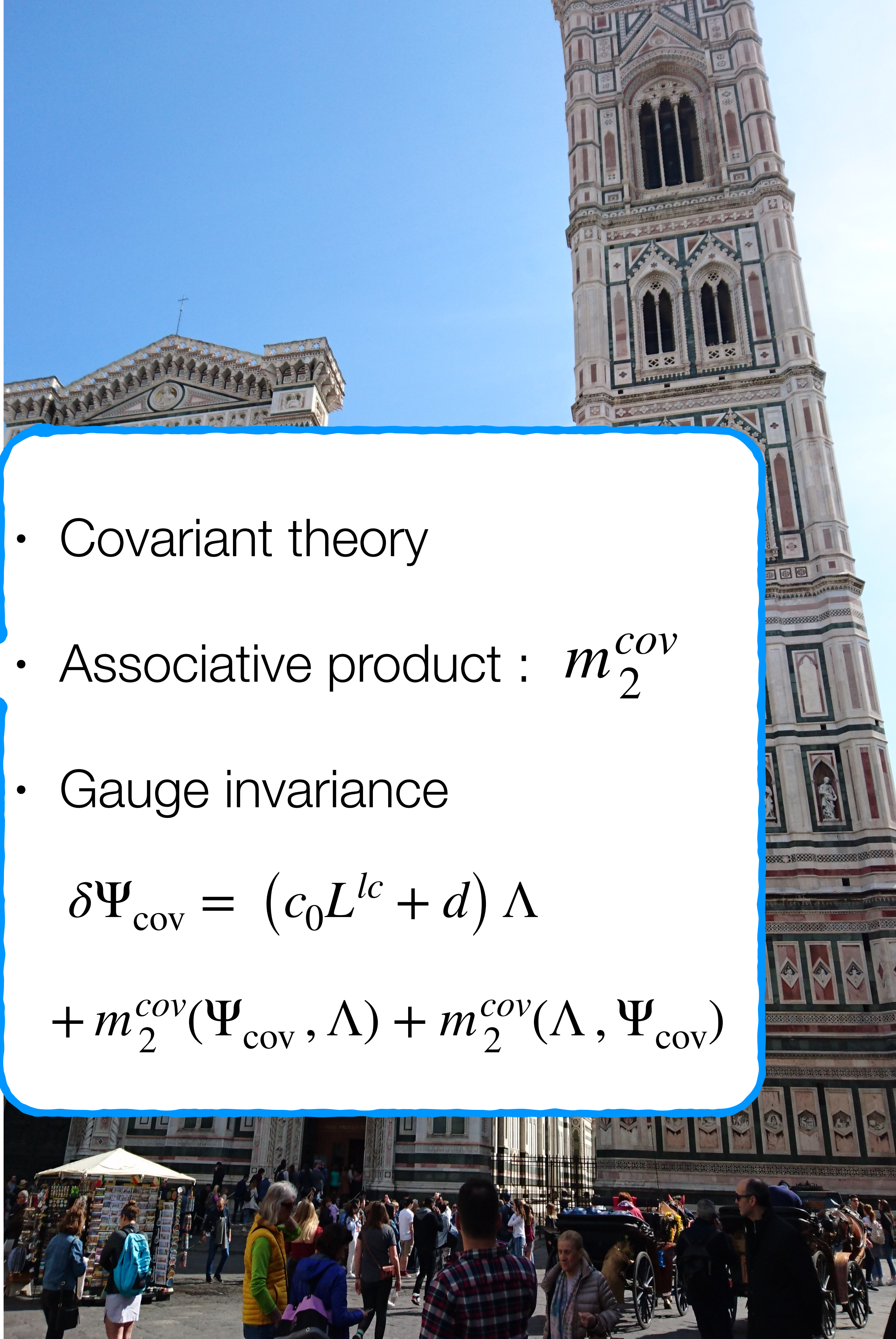
Witten theory

- String field redefinition

$$\Psi_{\rm cov} = \mathscr{U}\,\Psi$$

  - Witten theory becomes

$$S_{\rm Witten} = \frac{1}{2}\Psi_{\rm cov}\left(c_0 L^{lc} + d\right)\Psi_{\rm cov} + \frac{1}{3}\left(\Psi_{\rm cov}\right)^3$$



- Covariant theory
- Associative product : m_2^{cov}
- Gauge invariance

$$\delta\Psi_{\rm cov} = \left(c_0 L^{lc} + d\right)\Lambda$$

$$+\, m_2^{cov}(\Psi_{\rm cov}\,, \Lambda) + m_2^{cov}(\Lambda\,, \Psi_{\rm cov})$$

- Integrating-out all of the gauge & unphys degrees

  = By HP lemma for A∞

  (We will see later...)

An “effective” theory

- Roughly, in terms of field theory…
  - Expanding the action $\Psi_{\text{cov}} = \psi + c_0\,\chi$

$$S[\psi + c_0\chi] = \frac{1}{2}\big\langle \psi,\, c_0 K^{lc}\,\psi \big\rangle + \frac{1}{3}\big\langle \psi,\, m_2^{\text{cov}}\big(\psi,\psi\big)\big\rangle + \big\langle c_0\chi,\, m_2^{\text{cov}}\big(\psi,\psi\big)\big\rangle$$
$$+ \big\langle c_0\chi,\, d\,\psi \big\rangle + \big\langle \psi,\, m_2^{\text{cov}}\big(c_0\chi, c_0\chi\big)\big\rangle + \frac{1}{3}\big\langle c_0\chi,\, m_2^{\text{cov}}\big(c_0\chi, c_0\chi\big)\big\rangle$$

  - Solving the e.o.m. of gauge modes $\chi = (d\,h + h\,d)\,\chi$

$$c_0\Big[d\,\psi + m_2^{\text{cov}}(\psi, c_0\chi) + m_2^{\text{cov}}(c_0\chi, \psi) + m_2^{\text{cov}}(\psi,\psi) + m_2^{\text{cov}}(c_0\chi, c_0\chi)\Big] = 0$$

- LC SFT from covariant SFT
  - A new type of theory

$$S_{lc} = \frac{1}{2}\psi_{lc} c_0 L^{lc} \psi_{lc} + \frac{1}{3}{\psi_{lc}}^3 + \sum_{n>3} \frac{1}{n}{\psi_{lc}}^n$$

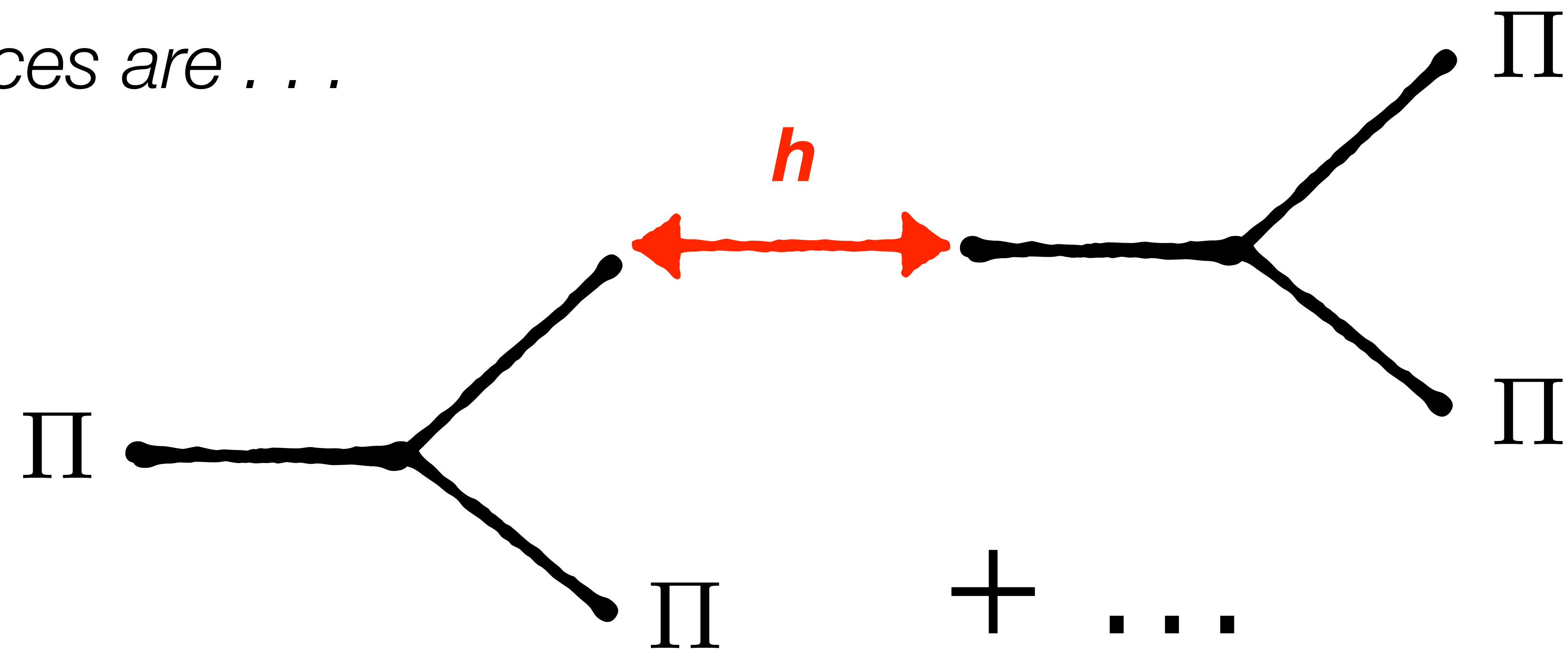
An “effective” theory

- Ro
  - E
  - S

S_{lc} = Classical part of the “effective” action

Higher vertices are . . .

$$\psi_{lc}{}^4 \sim$$



- LC SFT from covariant SFT
  - A new type of theory

$$S_{lc} = \frac{1}{2}\psi_{lc} c_0 L^{lc} \psi_{lc} + \frac{1}{3}\psi_{lc}{}^3 + \sum_{n>3} \frac{1}{n}\psi_{lc}{}^n$$

(b) More on the reduction

- Homological Perturbation lemma & Integrating out $\Psi_{\mathrm{unphys+gauge}}$

- Application to Other types of reduction

Review: HP lemma

- For given “Standard situation”

$$h \circlearrowright (V, \partial_V) \underset{g}{\overset{f}{\rightleftarrows}} (W, \partial_W)$$

$$1_V - g\, f = \partial_V\, h + h\, \partial_V$$

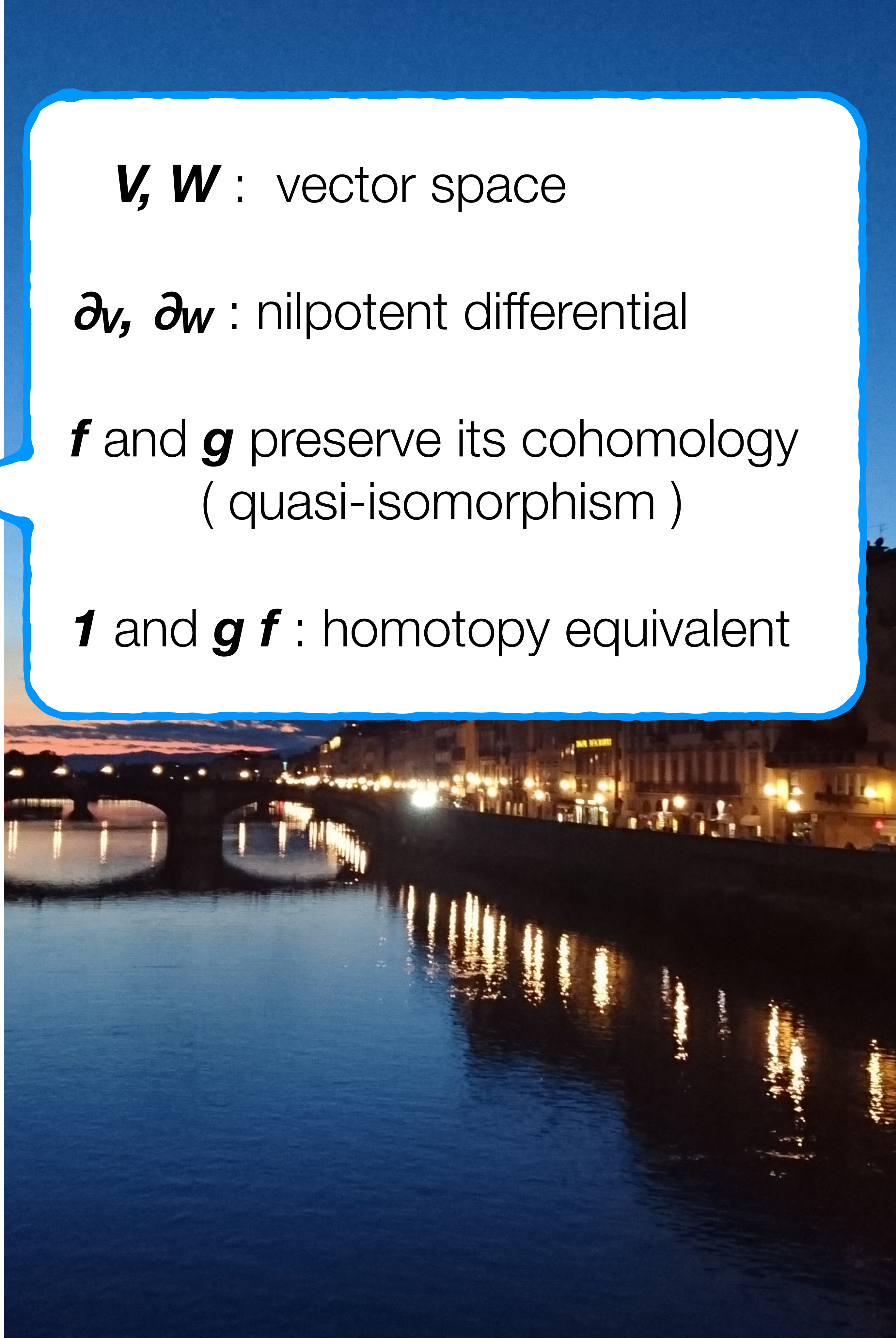
& given “perturbation”

$$\Delta \quad \text{s.t.} \quad (\partial_V + \Delta)^2 = 0$$

- **New** Standard Situation

$$\widetilde{h} \circlearrowright (V, \partial_V + \Delta) \underset{\widetilde{g}}{\overset{\widetilde{f}}{\rightleftarrows}} (W, \widetilde{\partial}_W)$$

$$1_V - \widetilde{g}\, \widetilde{f} = (\partial_V + \Delta)\, \widetilde{h} + \widetilde{h}\, (\partial_V + \Delta)$$



V*, *W : vector space

∂_V, ∂_W : nilpotent differential

f and ***g*** preserve its cohomology
(quasi-isomorphism)

1 and ***g f*** : homotopy equivalent

Review: HP lemma

- For given “Standard situation”

$$h \circlearrowright (V, \partial_V) \underset{g}{\overset{f}{\rightleftarrows}} (W, \partial_W)$$

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- **New** Standard Situation

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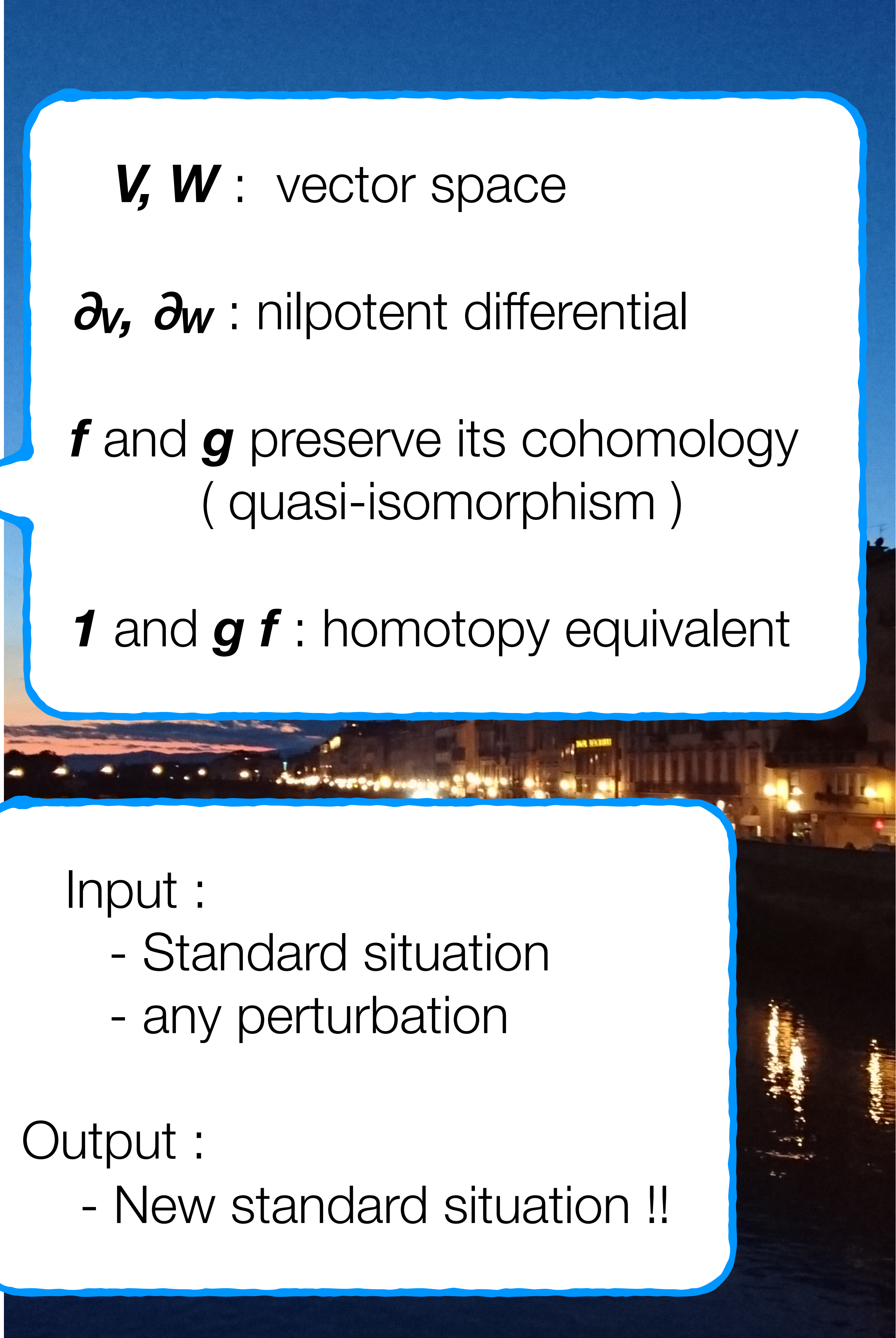

V*, *W : vector space

***∂*$_V$, *∂*$_W$** : nilpotent differential

f and ***g*** preserve its cohomology
(quasi-isomorphism)

1 and ***g f*** : homotopy equivalent

Input :
- Standard situation
- any perturbation

Output :
- New standard situation !!

Review: HP lemma

- For given “Standard situation”

$$h \circlearrowleft (V, \partial_V) \underset{g}{\overset{f}{\rightleftarrows}} (W, \partial_W)$$

$$1_V - g\,f = \partial_V\,h + h\,\partial_V$$

& given “perturbation”

$$\Delta \quad \text{s.t.} \quad (\partial_V + \Delta)^2 = 0$$

- **New** Standard Situation

$$\widetilde{h} \circlearrowleft (V, \partial_V + \Delta) \underset{\widetilde{g}}{\overset{\widetilde{f}}{\rightleftarrows}} (W, \widetilde{\partial}_W)$$

$$1_V - \widetilde{g}\,\widetilde{f} = (\partial_V + \Delta)\,\widetilde{h} + \widetilde{h}\,(\partial_V + \Delta)$$

Roughly, in terms of QFT …

- Homotopy equivalence -

We have a theory $S_V = \frac{1}{2}\psi_v\,\partial_V\psi_v$

and its field-contents split

$$\psi_v = \underbrace{(gf)\,\psi_v}_{\phi_W} + \underbrace{(\partial_V h + h\partial_V)\,\psi_v}_{\text{no cohomology}}$$

- Standard situation -

By integrating-out $(\partial_V h + h\partial_V)\,\psi_v$,

$$\int \mathscr{D}[\psi_v]e^{-S_V[\psi_v]} \longrightarrow \int \mathscr{D}[\phi_w]e^{-S_W[\phi_w]}$$

S_V reduces to an equivalent one

$$S_W = \frac{1}{2}\phi_w\,\partial_W\phi_w$$

Explicit form of A_∞ vertices

- Start with "quartets" $\{\, a^{+}_{-n}, c_{-n}, ;\, b_n, a^{-}_{n} \,\}_{n\neq 0}$

$$h \circlearrowright (\mathcal{H}_{\text{cov}},\, d) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (\, \underbrace{\text{Ker}[N]}_{=\mathcal{H}_{\text{lc}}},\, 0\,) \qquad \text{with} \qquad 1_{\mathcal{H}_{\text{cov}}} - \underbrace{\iota\,\pi}_{=\Pi} = d\,h + h\,d$$

- Add "LC physics" $c_0\, L_0^{lc}$ as a perturbation

$$h \circlearrowright (\mathcal{H}_{\text{cov}},\, \underbrace{d + c_0\, L_0^{lc}}_{\text{covariant}}\,) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (\,\mathcal{H}_{\text{lc}}\,,\, \underbrace{c_0\, L_0^{lc}}_{\text{light-cone}}\,)$$

In terms of coalgebra...

$$\boldsymbol{m^{lc} = \pi \left[c_0\, L_0^{lc} + m_2^{cov} \frac{1}{1 + h\, m_2^{cov}} \right] \iota}$$

- Add "interactions" m_2^{cov} as a perturbation

$$\widetilde{h} \circlearrowright (\mathcal{H}_{\text{cov}},\, \underbrace{(d + c_0\, L_0^{lc}) + m_2^{cov}}_{\text{Witten theory}}\,) \underset{\widetilde{\iota}}{\overset{\widetilde{\pi}}{\rightleftarrows}} (\,\mathcal{H}_{\text{lc}},\, \underbrace{c_0\, L_0^{lc} + \sum_{n=2}^{\infty} m_n^{lc}}_{A_\infty \text{ light-cone theory}}\,)$$

Example

- Removing **BRST quartets** : $\{ a^+_{-n}, c_{-n}, ; b_n, a^-_n \}$ except for "n=1"

  → Consider $d_{[1]} \equiv -p^+ \sum_{|n|>1} c_{-n} a^-_n$

- Then, we get string field theory which consists of tachyon, **Covariant massless modes** & transverse others

$$S[\phi] = \frac{1}{2} \langle \phi \, , \big[\, c_0 \, L_0^{lc} + (d - d_{[1]}) \, \big] \, \phi \, \rangle + \sum_{n=2}^{\infty} \frac{1}{n+1} \langle \, \phi \, , \, \widetilde{m}_n (\underbrace{\phi, \ldots, \phi}_{n}) \, \rangle$$

- Gauge transformation of the string field

$$\delta\phi = \big[\, c_0 \, L_0^{lc} + (d - d_{[1]}) \, \big] \, \lambda + \sum_{\text{cyclic}} \sum_{n=1}^{\infty} \widetilde{m}_{n+1} (\underbrace{\phi, \ldots, \phi}_{n}, \lambda \,)$$

Summary

- Our **light-cone SFT** has an **A-infinity** action.
- **Different** from **Kaku-Kikkawa's** old LC theory.
- Consistent as a LC theory & No gauge degree.
- Our LC SFT has **the same S-matrix** as Witten's SFT.

Summary + α

	Witten type	p-dependent overlap
Physical modes	***LC theory w/ A_∞***	***Kaku-Kikkawa's LC theory***
Physical modes + "∞ BRS quartets"	***Witten theory***	***Some SFT***

Summary + α

	Witten type	p-dependent overlap
Physical modes	***LC theory w/ A_∞***	***Kaku-Kikkawa's LC theory***
Physical modes + "∞ BRS quartets"	***Witten theory***	***$\alpha = p^+$ HIKKO*** [Kugo-Zwiebach 92']

*Unitary (but almost cov.) ver. of **HIKKO's SFT***

Summary + α

	Witten type	p-dependent overlap
Physical modes	***LC theory w/ A_∞***	***Kaku-Kikkawa's LC theory***
Physical modes + "∞ BRS quartets"	***Witten theory***	***$\alpha = p^+$ HIKKO*** [Kugo-Zwiebach 92']

… work in progress with Ted Erler

Summary + α

	Witten type	p-dependent overlap
Physical modes	***LC theory w/ A_∞***	***Kaku-Kikkawa's LC theory***
Physical modes + "∞ BRS quartets"	***Witten theory***	***$\alpha = p^+$ HIKKO*** [Kugo-Zwiebach 92']

Physical modes + "(∞**+1**) BRS quartets"	***Osp(26,2|2)-extended theory*** [Siegel-Zwiebach, Kugo]

Summary + α

Physical modes
+
"∞ BRS quartets"

Some covariant closed SFT w/ L_∞

$$\Psi_{cov} = \Psi_{cov}[\, X^\mu(\sigma), b(\sigma), c(\sigma) \,]$$

Removing extra quartet
… work in progress

$$\Psi = \Psi[\, X^\mu(\sigma), b(\sigma), c(\sigma) \,;\, \text{one quartet} \,]$$

Physical modes
+
"(∞**+1**) BRS quartets"

Osp(26,2|2)-extended theory

[Siegel-Zwiebach, Kugo]

"Thank you !!"

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