

String Field Theory and Self-dual Forms

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This talk will not be on string field theory, ...

... but on how insights from (super-)string field theory may help formulate the theory of self-dual forms in $4n+2$ space-time dimensions.

References:

A.S., arXiv:1511.08220, arXiv:1903.12196

Plan:

1. A brief introduction to the issues
2. A brief review of closed superstring field theory
3. Action for self-dual forms
4. Hamiltonian formulation
5. Some speculations

In $4n+2$ space-time dimensions we can consider theories of $2n$ -form fields C with self-dual field strength F :

$$F = dC, \quad *F = F$$

*****: Hodge dual

There is no conventional manifestly Lorentz invariant action describing this theory.

Normally we impose the self-duality constraint by hand.

There are several proposals to get around this problem.

1. Give up manifest Lorentz invariance

Henneaux, Teitelboim; Schwarz; Belov, Moore; . . .

2. Introduce infinite number of auxiliary fields

McClain, Yu, Wu; Wotzasek; Martin, Restuccia; Devecchi, Henneaux

Faddeev, Shatashvili; Bengtsson, Kleppe; Berkovits

3. Introduce a non-polynomial action containing $1/(\partial_\mu \phi \partial^\mu \phi)$ multiplying a term in the action

Pasti, Sorokin, Tonin, Bandos, Lechner, Nurmagambetov, Dall'Agata, . . .

4. Other approaches

Siegel; Floreanini, Jackiw; Castellani, Pesando; Witten

In this talk we shall follow a new approach motivated by superstring field theory.

A brief review of the structure of classical type II superstring field theory:

$\mathcal{H}_{m,n}$: A subspace of the full Hilbert space of matter and ghost world-sheet CFT, defined by the constraints:

$$\begin{aligned} \mathbf{b}_0^- |\psi\rangle &= 0, & \mathbf{L}_0^- |\psi\rangle &= 0, & \mathbf{n}_g |\psi\rangle &= 2|\psi\rangle \\ \mathbf{b}_0^\pm &= \mathbf{b}_0 \pm \bar{\mathbf{b}}_0, & \mathbf{L}_0^\pm &= \mathbf{L}_0 \pm \bar{\mathbf{L}}_0, & \mathbf{c}_0^\pm &= \frac{1}{2}(\mathbf{c}_0 \pm \bar{\mathbf{c}}_0) \\ & & \mathbf{n}_g &= \text{ghost number} \end{aligned}$$

(left, right) picture number: (m,n)

Matter CFT: Any SCFT with $c=15$.

Classical string field has two components, $|\psi\rangle$ and $|\phi\rangle$

$$|\psi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1/2,-1/2}$$

$$|\phi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-3/2,-3/2}$$

$$|\psi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1/2,-1/2}$$

$$|\phi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-3/2,-3/2}$$

$$\mathbf{S} = \langle \phi | \mathbf{c}_0^- \mathbf{Q}_B | \psi \rangle - \frac{1}{2} \langle \phi | \mathbf{c}_0^- \mathbf{Q}_B \mathbf{G} | \phi \rangle + \sum_{n \geq 3} \frac{1}{n!} \{ \psi^n \}$$

G: identity in NSNS sector, \mathcal{X}_0 in NSR sector,

$\bar{\mathcal{X}}_0$ in RNS sector, $\mathcal{X}_0 \bar{\mathcal{X}}_0$ in RR sector

$$\mathcal{X}_0 \equiv \oint d\mathbf{z} \mathbf{z}^{-1} \mathcal{X}(\mathbf{z}), \quad \bar{\mathcal{X}}_0 \equiv \oint d\bar{\mathbf{z}} \bar{\mathbf{z}}^{-1} \bar{\mathcal{X}}(\bar{\mathbf{z}})$$

$\mathcal{X}(\mathbf{z}), \bar{\mathcal{X}}(\bar{\mathbf{z}})$: Picture changing operator

Friedan, Martinec, Shenker; Knizhnik

$\{\mathbf{A}_1 \cdots \mathbf{A}_N\}$: Some number constructed from integrals of correlation functions of $\mathbf{A}_1, \cdots, \mathbf{A}_N$.

$$S = \langle \phi | \mathbf{c}_0^- \mathbf{Q}_B | \psi \rangle - \frac{1}{2} \langle \phi | \mathbf{c}_0^- \mathbf{Q}_B \mathbf{G} | \phi \rangle + \sum_{n \geq 3} \frac{1}{n!} \{ \psi^n \}$$

Derive equations of motion from S:

$$\mathbf{Q}_B(\mathbf{G}|\phi\rangle - |\psi\rangle) = 0$$

$$\mathbf{Q}_B|\phi\rangle + \sum_{N=2}^{\infty} \frac{1}{N!} [\psi^N] = 0$$

$[\mathbf{A}_1 \cdots \mathbf{A}_N] \in \mathcal{H}$ is defined via:

$$\langle \mathbf{A} | \mathbf{c}_0^- [\mathbf{A}_1 \cdots \mathbf{A}_N] \rangle = \{ \mathbf{A} \mathbf{A}_1 \cdots \mathbf{A}_N \}$$

Combine two equations to get the interacting field equation:

$$\mathbf{Q}_B|\psi\rangle + \sum_{N=2}^{\infty} \frac{1}{N!} \mathbf{G}[\psi^N] = 0$$

$$Q_B |\psi\rangle + \sum_{N=2}^{\infty} \frac{1}{N!} G[\psi^N] = 0$$

$$Q_B |\phi\rangle + \sum_{N=2}^{\infty} \frac{1}{N!} [\psi^N] = 0$$

Once we solve the first equation for $|\psi\rangle$, the solution for $|\phi\rangle$ to the second equation is fixed up to addition of solutions to free field equations:

$$Q_B |\phi\rangle = 0$$

Furthermore which solution we choose does not affect the equations of motion for $|\psi\rangle$.

Therefore even though we have doubled the number of degrees of freedom, ...

... one set of degrees of freedom describe free fields that completely decouple from the interacting sector, and are unobservable

This construction holds also for type IIB string theory.

It contains a self-dual 5-form field strength in $D=10$.

Furthermore compactification of type IIB on $K3$ has self-dual 3-form field strengths in $D=6$.

IIB compactification on $K3 \times K3$ has chiral scalar fields in $D=2$.

Therefore the low energy effective action of superstring field theory must contain Lorentz invariant action for these self-dual fields.

This suggests that we should be able to construct an action for self-dual fields in $4n+2$ dimensions by doubling the degrees of freedom.

Proposed field content:

– a $2n$ -form potential P (like $|\phi\rangle$)

– a $(2n+1)$ -form self-dual field strength Q (like $|\psi\rangle$)

Proposed action:

$$S = \frac{1}{2} \int dP \wedge *dP - \int dP \wedge Q + \int \mathcal{L}_I(Q, \chi),$$

χ : all other fields including the metric

Q satisfies $*Q = Q$, $*$: Hodge dual with metric $\eta_{\mu\nu}$

\mathcal{L}_I : A general interacting Lagrangian density, assumed to not contain derivatives of Q

$$\mathbf{S} = \frac{1}{2} \int \mathbf{dP} \wedge * \mathbf{dP} - \int \mathbf{dP} \wedge \mathbf{Q} + \int \mathcal{L}_1(\mathbf{Q}, \chi),$$

Equations of motion:

$$\mathbf{d}(*\mathbf{dP} - \mathbf{Q}) = \mathbf{0}, \quad \mathbf{dP} - *\mathbf{dP} + \mathbf{R}(\mathbf{Q}, \chi) = \mathbf{0}, \quad \frac{\delta}{\delta \chi} \int \mathcal{L}_1 = \mathbf{0}$$

R is an anti-selfdual (2n+2)-form defined through:

$$\delta \int \mathcal{L}_1 = -\frac{1}{2} \int \mathbf{R} \wedge \delta \mathbf{Q}$$

Adding the exterior derivative of the second equation to the first equation we get

$$\mathbf{d}(\mathbf{R} - \mathbf{Q}) = \mathbf{0}.$$

$$d(*dP - Q) = 0, \quad dP - *dP + R(Q, \chi) = 0, \quad d(Q - R) = 0$$

Once we find Q by solving the third equation, the anti-self-dual part of dP is fixed by the middle equation.

Writing the first equation as

$$d(dP + *dP - Q) = 0$$

we see that the self-dual part of dP is also fixed except for the freedom of adding a self-dual form A satisfying

$$dA = 0$$

This is a free field degree of freedom.

Furthermore the choice of A does not affect the interacting field equations $d(Q - R) = 0$.

Net content: An unobservable free field and an observable interacting field Q .

$$S = \frac{1}{2} \int d\mathbf{P} \wedge *d\mathbf{P} - \int d\mathbf{P} \wedge \mathbf{Q} + \int \mathcal{L}_1(\mathbf{Q}, \chi),$$

Note that the decoupling does not happen at the level of the action but only at the level of the equations of motion

– may seem a little unsatisfactory.

We shall now show that the same decoupling can be seen at the level of the Hamiltonian.

In the following we shall treat \mathbf{P} and \mathbf{Q} in the canonical formulation but the other fields χ in the Lagrangian formulation.

Result: Routhian (although we shall denote it by H)

Conjugate momenta:

$$\Pi_{\mathbf{P}}^{i_1 \dots i_{2n}} = -(\partial_0 \mathbf{P}_{i_1 \dots i_{2n}} + \partial_{[i_1} \mathbf{P}_{i_2 \dots i_{2n}]} 0) - \sum_{\substack{i_1, \dots, i_{2n+1} \\ i_1 < i_2 < \dots < i_{2n+1}}} \epsilon^{i_1 \dots i_{2n} j_1 \dots j_{2n+1}} \mathbf{Q}_{j_1 \dots j_{2n+1}},$$

$$\Pi_{\mathbf{P}}^{0 i_1 \dots i_{2n-1}} = 0, \quad \Pi_{\mathbf{Q}}^{i_1 \dots i_{2n+1}} = 0.$$

Note: The last two are constraint equations

$$\mathbf{H} = \sum_{\substack{i_1, \dots, i_{2n} \\ i_1 < i_2 < \dots < i_{2n}}} \int \mathbf{d}^{4n+1} \mathbf{x} \Pi_{\mathbf{P}}^{i_1 \dots i_{2n}} \partial_0 \mathbf{P}_{i_1 \dots i_{2n}} - \mathbf{L}$$

up to addition of constraints.

1. Compute the Poisson brackets of the constraints with \mathbf{H} to generate more constraints

2. Analyze Poisson brackets of the constraints with each other to sort out first and second class constraints.

Define new variables:

$$\Pi_{\pm}^{i_1 \dots i_{2n}} \equiv \frac{1}{2} \left(\Pi_{\mathbf{P}}^{i_1 \dots i_{2n}} \pm \epsilon^{i_1 \dots i_{2n} j_1 \dots j_{2n+1}} \partial_{[j_1} \mathbf{P}_{j_2 \dots j_{2n+1}]} \right)$$

Result: Second class constraints:

$$\Pi_{\mathbf{Q}}^{i_1 \dots i_{2n+1}} = 0$$

$$\mathbf{Q}_{j_1 \dots j_{2n+1}} + 2 \epsilon^{i_1 \dots i_{2n} j_1 \dots j_{2n+1}} \Pi_{-}^{i_1 \dots i_{2n}} + \frac{\partial \mathcal{L}_1}{\partial \mathbf{Q}_{j_1 \dots j_{2n+1}}} = 0$$

– can be used to eliminate $\mathbf{Q}_{j_1 \dots j_{2n+1}}$ and $\Pi_{\mathbf{Q}}^{i_1 \dots i_{2n+1}}$.

First class constraints generating gauge transformations:

$$\Pi_{\mathbf{P}}^{0i_1 \dots i_{2n-1}} = 0, \quad \partial_{i_1} \Pi_{\mathbf{P}}^{i_1 \dots i_{2n}} = 0$$

The second constraint is equivalent to:

$$\Rightarrow \partial_{i_1} \Pi_{\pm}^{i_1 \dots i_{2n}} = 0$$

Π_{\pm} are gauge invariant.

The Routhian is given by $H = H_+ + H_-$ where

$$H_+ = - \int \mathbf{d}^{4n+1} \mathbf{x} \prod_+^{i_1 \dots i_{2n}} \prod_+^{i_1 \dots i_{2n}},$$

$$H_- = \int \mathbf{d}^{4n+1} \mathbf{x} \prod_-^{i_1 \dots i_{2n}} \prod_-^{i_1 \dots i_{2n}} + \int \mathbf{d}^{4n+1} \mathbf{x} g(\Pi_-, \chi)$$

$$g(\Pi_-, \chi) = \left[-\frac{1}{2} \sum_{\substack{j_1 \dots j_{2n+1} \\ i_1 < i_2 < \dots < i_{2n+1}}} \left(\frac{\partial \mathcal{L}_1}{\partial \mathbf{Q}_{j_1 \dots j_{2n+1}}} \right)^2 - \mathcal{L}_1(\mathbf{Q}, \chi) \right]$$

evaluated at the solution to the constraint:

$$\mathbf{Q}_{j_1 \dots j_{2n+1}} + 2 \epsilon^{i_1 \dots i_{2n} j_1 \dots j_{2n+1}} \prod_-^{i_1 \dots i_{2n}} + \frac{\partial \mathcal{L}_1}{\partial \mathbf{Q}_{j_1 \dots j_{2n+1}}} = 0$$

Dirac brackets:

$$\{\Pi_+^{i_1 \dots i_{2n}}(\mathbf{t}, \vec{\mathbf{x}}), \Pi_+^{k_1 \dots k_{2n}}(\mathbf{t}, \vec{\mathbf{y}})\}_{\text{DB}} = \frac{1}{2} \epsilon^{i_1 \dots i_{2n} j_1 k_1 \dots k_{2n}} \frac{\partial}{\partial \mathbf{x}^{j_1}} \delta(\mathbf{x} - \mathbf{y}),$$

$$\{\Pi_-^{i_1 \dots i_{2n}}(\mathbf{t}, \vec{\mathbf{x}}), \Pi_-^{k_1 \dots k_{2n}}(\mathbf{t}, \vec{\mathbf{y}})\}_{\text{DB}} = -\frac{1}{2} \epsilon^{i_1 \dots i_{2n} j_1 k_1 \dots k_{2n}} \frac{\partial}{\partial \mathbf{x}^{j_1}} \delta(\mathbf{x} - \mathbf{y}),$$

$$\{\Pi_+^{i_1 \dots i_{2n+1}}(\mathbf{t}, \vec{\mathbf{x}}), \Pi_-^{j_1 \dots j_{2n+1}}(\mathbf{t}, \vec{\mathbf{y}})\}_{\text{DB}} = 0.$$

Recall that $\mathbf{H} = \mathbf{H}_+ + \mathbf{H}_-$ where

$$\mathbf{H}_+ = - \int \mathbf{d}^{4n+1} \mathbf{x} \Pi_+^{i_1 \dots i_{2n}} \Pi_+^{i_1 \dots i_{2n}},$$

$$\mathbf{H}_- = \int \mathbf{d}^{4n+1} \mathbf{x} \Pi_-^{i_1 \dots i_{2n}} \Pi_-^{i_1 \dots i_{2n}} + \int \mathbf{d}^{4n+1} \mathbf{x} \mathbf{g}(\Pi_-, \chi)$$

– sum of a free Hamiltonian \mathbf{H}_+ and an interacting Routhian \mathbf{H}_-

– confirms the result based on the analysis of equations of motion.

Special case: coupling to metric (included in χ)

In the usual formulation of a self-dual form coupled to the metric, we consider an action of the form:

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2(\chi)$$

$$\mathbf{S}_1 \equiv -\frac{1}{2} \int (\mathbf{dC} + \mathbf{Y}) \wedge *_g(\mathbf{dC} + \mathbf{Y}) + \int \mathbf{dC} \wedge \mathbf{Y},$$

Constraint

$$(\mathbf{dC} + \mathbf{Y}) = *_g(\mathbf{dC} + \mathbf{Y})$$

Y: a $(2n+1)$ -form constructed out of the other fields χ

\mathbf{S}_2 is a function of the other fields χ , including the metric

$*_g$: Hodge dual with respect to the metric $g_{\mu\nu}$.

Question: How do we replace the self-duality constraint with $g_{\mu\nu}$ for $\mathbf{dC} + \mathbf{Y}$ by self-duality constraint with respect to $\eta_{\mu\nu}$ for \mathbf{Q} ?

It turns out that it is possible, via a non-standard coupling of the metric to \mathbf{Q} .

$$\mathbf{S}_1 \equiv -\frac{1}{2} \int (\mathbf{dC} + \mathbf{Y}) \wedge *_g(\mathbf{dC} + \mathbf{Y}) + \int \mathbf{dC} \wedge \mathbf{Y},$$

can be replaced by

$$\begin{aligned} \mathbf{S}'_1 &= \frac{1}{2} \int \mathbf{dP} \wedge *_g \mathbf{dP} - \int \mathbf{dP} \wedge \mathbf{Q} \\ &+ \int \mathbf{d}^{4n+2}x \left[\frac{1}{16} \mathbf{Q}^T \mathcal{M} \mathbf{Q} + \frac{1}{2} \mathbf{Q}^T \left\{ \frac{1}{2} \mathcal{M} \mathbf{Y} - (\zeta - \varepsilon) \mathbf{Y} \right\} - \frac{1}{2} \mathbf{Y}^T \zeta \mathbf{Y} + \frac{1}{4} \mathbf{Y}^T \mathcal{M} \mathbf{Y} \right]. \end{aligned}$$

\mathcal{M} : a $\binom{4n+2}{2n+1} \times \binom{4n+2}{2n+1}$ matrix determined in terms of the metric.

ζ, ε : constant matrices

Identification of variables: $\mathbf{Q} = \mathbf{dC} + *_g \mathbf{dC}$

$*\mathbf{dY}$ acts as the source for the $(2n+1)$ -form field strength.

Using this formalism one can address some apparent puzzles involving compactification of theories of self-dual forms.

e.g. if we start with a self-dual 3-form in $D=6$, upon compactification on T^2 we should get $U(1)$ gauge theory in $D=4$

In standard compactification the dimensionally reduced action is multiplied by the area $R_1 R_2$ of T^2

– should act as the inverse coupling.

But the expected coupling from S-duality is R_1/R_2 !

Witten

In the new formulation, due to the non-standard coupling of the metric, the dimensionally reduced action no longer has $R_1 R_2$ as an overall multiplicative factor.

A detailed analysis shows that the dimensionally reduced action and the associated Routhian are indeed S-duality invariant.

This formalism allows us to construct the action of a single M5-brane

What about multiple M5-branes?

– expected to have tensionless strings carrying charge under the three form field strength

The possibility of including these degrees of freedom exist in this formulation since $*dY$ acts as the source for the self-dual field.

The rest of the talk will be some speculations on how we might construct appropriate Y representing tensionless string source.

Go back to type IIB on K3.

The self-dual 3-forms arise from components of self-dual 5-form along a harmonic two form on K3.

D3-branes wrapped on the dual 2-cycles are charged under the resulting 3-form field strength in 6D.

The area modulus X of the 2-cycle is part of the same supermultiplet as the 3-form field.

When X approaches zero, the D3-brane wrapped on the 2-cycle has zero tension

– a tensionless string

– related to the interacting part of the theory of multiple M5-branes.

Question: Is there a field theory that can describe these strings, but does not introduce other fields?

Strings can be generated as solitons, e.g. instanton solutions in 6D Yang-Mills theory.

... , Kim, Kim, Park

However this also introduces Yang-Mills fields in the theory.

One example where this does not happen is tachyon condensation in open string theory.

Tachyons can produce solitons without generating perturbative excitations around the vacuum.

In type IIB on K3, we can consider a pair of D7-branes and a pair of anti-D7 branes wrapped on the 2-cycle of size X.

Tachyon condensation on this system can produce a D3-brane wrapped on the 2-cycle – the string that we want to generate.

Open string field theory on the D7- $\overline{D7}$ brane system, coupled to the abelian tensor multiplet, will contain the desired CFT

The relevant part of the action:

$$X g_s^{-1} S_{\text{open}}$$

g_s : string coupling (eventually taken to 0)

S_{open} : Open string field theory action

* dY : related to Ellwood invariant measuring D3-brane charge wrapped on 2-cycle of size X

Question: Is there a truncation to field theory with finite number of fields?

Can we write down a tachyon action of the form

$$S_{\text{tachyon}} = X S(T, \dots)$$

with T a 2×2 complex matrix valued scalar, describing the physics of tachyon condensation?

We could then identify the source dY appearing earlier to the 4-form constructed from T that measures the D3-brane charge

$$dY \propto \text{Tr}(dT \wedge dT^\dagger \wedge dT \wedge dT^\dagger)$$

At present the only known perturbatively UV finite theory of this kind is p-adic open bosonic string field theory with action

$$\int \left[-\frac{1}{2} T^p \square^{p/2} T + \frac{1}{p+1} T^{p+1} \right], \quad \mathbf{p : a prime number,}$$

with a real field T.

Freund, Olson; Freund, Witten; Brekke, Freund, Olson, Witten

– captures the physics of tachyon condensation in open bosonic string theory quite well

Ghoshal, A.S.

A generalization of this to superstring theory could give us a quantum field theory on multiple M5-branes