# Rolling Near the Tachyon Vacuum

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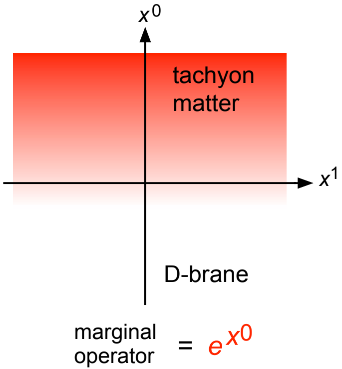
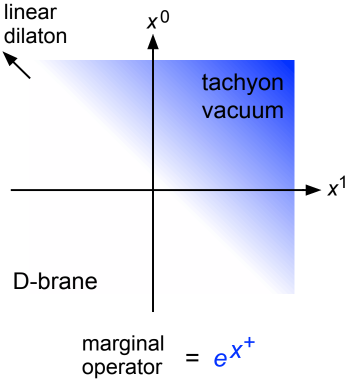
D-branes in bosonic string theory are unstable. What is the nature of the decay process?




(Sen)




(Hellerman & Schnabl)

For the light-like deformation, the endpoint of the decay process is the tachyon vacuum.

Paradox:

How can the decay continuously evolve towards a final state which does not admit physical deformation?

In the conventional description of string theory, it is difficult to think clearly about physics near the tachyon vacuum, since there is no worldsheet theory there.

In open string field theory, the tachyon vacuum is a finite field configuration. Physics near the tachyon vacuum (if there is any) should be described by perturbations of this field configuration.

Exact Solution ($\mathcal{B}_0$ gauge)

$$\Psi = \sqrt{\Omega} c\, e^{X^+} \frac{B}{1 + \frac{1-\Omega}{K} e^{X^+}} c\sqrt{\Omega},$$

$e^{x^+}$ = marginal operator; $\Omega = e^{-K}$ = CFT vacuum; etc.

Early times: $\Psi = \sqrt{\Omega} c e^{X^+} \sqrt{\Omega} - \sqrt{\Omega}\, cB e^{X^+} \frac{1-\Omega}{K} e^{X^+} c\sqrt{\Omega} + \ldots$

Late times: $\Psi = \underbrace{\sqrt{\Omega} c \frac{KB}{1-\Omega} c\sqrt{\Omega}}_{\text{Schnabl's Solution}} - \sqrt{\Omega} cB \frac{K}{1-\Omega} e^{-X^+} \frac{K}{1-\Omega} c\sqrt{\Omega} + \ldots$

The leading contribution to the late time expansion is Schnabl's solution for the tachyon vacuum. Subleading contributions represent "fluctuations" of Schnabl's solution

However, the tachyon vacuum has no fluctuations. The subleading corrections represent time-dependent gauge transformations of the tachyon vacuum.

What to make of this?

Some possibilities:

- The decay process ends at the tachyon vacuum in finite time, exactly when the late time expansion becomes convergent.
- Early/late time expansions represent two consistent but incompatible interpretations of the same algebraic expression.
- $\vdots$

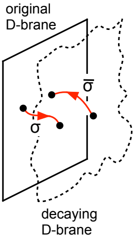
What actually happens:

1. The decay process smoothly connects the unstable D-brane to the tachyon vacuum.
2. The late time expansion around the tachyon vacuum has vanishing radius of convergence. Therefore, while the fluctuations of the tachyon vacuum are order-by-order pure gauge, it does not follow that the solution is equivalent to the tachyon vacuum at any finite time.
3. The late time asymptotic expansion is hiding a nonperturbative effect, representing a physical fluctuation of the tachyon vacuum which persists through the decay process and finally vanishes in the infinite future.

# 1. The final state

Exact Solution (different gauge)

$$\Psi = \underbrace{c(1+K)Bc\frac{1}{1+K}}_{\text{tachyon vacuum}} - \underbrace{c(1+K)\sigma\frac{B}{1+K}\overline{\sigma}(1+K)c\frac{1}{1+K}}_{\substack{\text{decaying configuration}\\ \text{on top of tachyon vacuum.}}}$$




$\sigma, \overline{\sigma} =$ boundary condition changing operators connecting unstable D-brane to copy of itself in process of decay.

$x^+ \to -\infty$: $\sigma = \overline{\sigma} = 1$, implies $\Psi = 0$

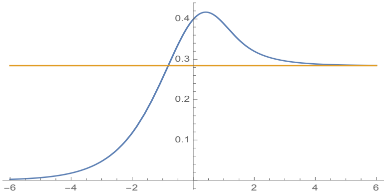
$x^+ \to +\infty$: $\sigma = \overline{\sigma} = 0$, implies $\Psi =$ tachyon vacuum

We can expand solution in a basis of eigenstates of $L_0$:

$$\Psi = \int \frac{d^2k}{(2\pi)^2}\, T(k)\, c e^{ik\cdot X}(0)|0\rangle + \ldots,$$

The coefficient of the state with lowest $L_0$ eigenvalue for a given momentum is the tachyon field $T(x)$.

This is how the tachyon evolves in lightcone time:




The higher $m^2$ fields show similar behavior.

The solution really does approach the tachyon vacuum in the infinite future.

## 2. The late time expansion

Ghost number zero model (makes life easier):

$$\Gamma = \frac{1}{1+K} - \sigma\frac{1}{1+K}\overline{\sigma}$$

The rational function of $K$ can be written as a continuous superposition of star-algebra powers of the CFT vacuum (wedge states):

$$\frac{1}{1+K} = \int_0^\infty dt\, e^{-t}\Omega^t$$

The integration variable $t > 0$ is a "Schwinger parameter."

$t \to 0$: $\Omega^t$ approaches the identity string field

$t \to \infty$: $\Omega^t$ approaches the sliver state

Complex $t \in \mathbb{C}$ will be called the Schwinger plane. From the point of view of the perturbative expansion at late times and nonperturbative effects, the Schwinger parameter plays a role analogous to a field variable in the path integral.

Expand in a basis of $L_0$ eigenstates:

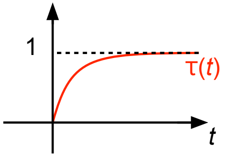
$$\Gamma = \int \frac{d^2k}{(2\pi)^2} \gamma(k)\, e^{ik\cdot X}(0)|0\rangle + ...,$$

The coefficient of the state with lowest $L_0$ eigenvalue for a given momentum will be called the ghost number zero tachyon $\gamma(x)$.

Exact formula:

$$\gamma(x) = 1 - \int_0^\infty dt\, e^{-t} \exp\left[-\alpha\tau(t)\right]$$

- $\alpha \propto e^{x^+}$ is an expansion parameter.
  $\alpha = 0$ is the infinite past.
  $|\alpha| \to \infty$ is the infinite future.
- $\tau(t)$ is a function of $t > 0$
  which monotonically increases from 0 to 1.



$$\gamma(x) = 1 - \int_0^\infty dt\, e^{-t} \exp\left[-\alpha\tau(t)\right]$$

Comments:

- In the infinite past ($\alpha = 0$) the two terms cancel, giving $\gamma = 0$ representing the original unstable D-brane.
- In the infinite future ($\alpha = \infty$) the second term vanishes, giving $\gamma = 1$. This is the ghost number zero analogue of the expectation value at the tachyon vacuum.
- For large $\alpha$, the integrand is highly suppressed except near $t = 0$, where $\tau(t)$ vanishes. Therefore the late time behavior near the tachyon vacuum is characterized by the identity string field.

Expansion around $\alpha = 0$:

$$\gamma(x) = -\sum_{n=1}^{\infty} \frac{1}{n!}(-\alpha)^n \underbrace{\int_0^{\infty} dt\, e^{-t} \tau(t)^n}_{\text{positive and } <1}$$

Therefore the early time expansion has infinite radius of convergence.

Furthermore, the late time expansion has vanishing radius of convergence.

Transform integration variable from $t$ to $\tau$:

$$\gamma(x) = \alpha \int_0^1 d\tau\, e^{-t(\tau)} e^{-\alpha\tau}$$

$\tau \in \mathbb{C}$ is the Borel plane.

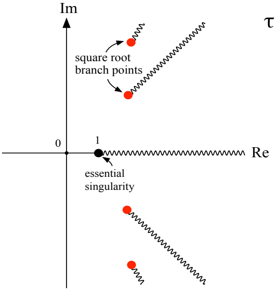
$$\gamma(x) = \alpha \int_0^1 d\tau \, e^{-t(\tau)} e^{-\alpha\tau}$$

The factor $e^{-t(\tau)}$ in the integrand is the Borel transform of the late time expansion of the ghost number zero tachyon in inverse powers of $\alpha$.

Since the late time expansion has vanishing radius of convergence, expansion of $t(\tau)$ around $\tau = 0$ must have finite radius of convergence.

$t(\tau)$ will have a singularity for any finite $\tau = \tau(t)$ where $t$ is infinite. In particular, it must be singular at $\tau = 1$.

A full analysis of the singularities of $e^{-t(\tau)}$ in the Borel plane reveals:

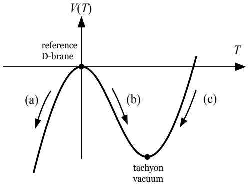



The singularity at $\tau = 1$ implies that the late time expansion around the tachyon vacuum is not Borel resummable.

The late time behavior of the solution must receive important contribution from nonperturbative effects.

# 3. Nonperturbative effects I: Rolling to the wrong side of the potential.

The effective potential for the tachyon field can roughly be visualized as a cubic curve.




The solution we have been discussing corresponds to pushing the tachyon in the positive direction, towards the local minimum of the effective potential.

What happens if we push the tachyon to negative values, where the tachyon effective potential is unbounded from below?

Pushing towards negative values corresponds to switching the sign in front of the marginal operator $e^{X^+}$, or equivalently considering negative $\alpha$.

The formal argument given at the beginning suggests that the string field will still condense to the tachyon vacuum.

Field theory model:

$$S = -\int d^2x\, e^{x^-} \left( \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}\phi^2 + \frac{1}{3}\phi^3 \right)$$

Note the linear dilaton coupling and cubic potential.

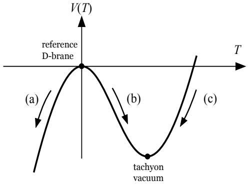
Solution:

$$\phi(x) = 1 - \frac{1}{1 \pm e^{x^+}}$$

with $\pm$ rolling in positive or negative direction from unstable maximum.

Same as string field theory solution after setting $K = 0$.

Rolling in negative direction, the field hits a singularity at finite time. After we can continue to another branch where the field approaches the local minimum in the infinite future.



The finite time singularity in the field theory model is possible since the expansion in powers of $e^{x^+}$ has finite radius of convergence.

In string field theory, the expansion in powers of $e^{x^+}$ has infinite radius of convergence, so a finite time singularity is not possible.

## What happens in string theory is quite different

For $\alpha << 0$ the late time behavior ghost number zero tachyon

$$\gamma(x) = 1 - \int_0^{\infty} dt\, e^{-t} \exp\left[-\alpha\tau(t)\right]$$

is dominated by a sliver-like critical point at

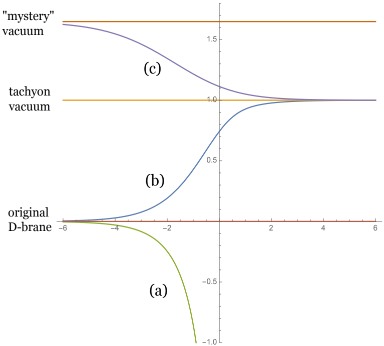
$$t \propto (-\alpha)^{1/3}$$

which leads to the asymptotic behavior

$$\gamma(x) \sim e^{e^{x^+}}$$

The solution diverges super-exponentially at late times as it rolls down the unbounded side of the tachyon effective potential.

Strangely, the late time behavior near the tachyon vacuum still makes sense as an asymptotic expansion even if $\alpha < 0$.



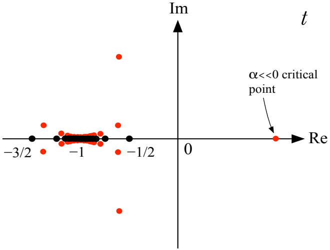

The asymptotic expansion is actually Borel resumable for $\alpha < 0$, so we can we can derive a curious solution which rolls to the tachyon vacuum from the "other side" of the local minimum of the tachyon effective potential.

# Nonperturbative effects II: Steepest descent

Nonperturbative effects are related to saddle points of the "action functional" defining the ghost number zero tachyon:

$$S(t) = t + \alpha\tau(t)$$

The Schwinger parameter is analogous to a field in the path integral, and $\alpha^{-1}$ is analogous to a coupling constant.




For $\alpha < 0$ the singularities and saddle points of the action functional are shown to the left. The red dot on the real axis is the sliver-like critical point which determines the superexponential growth when the tachyon rolls to the wrong side of the potential.

It is natural to guess that the sliver-like critical point for $\alpha < 0$ is related to a saddle point contribution to the late time behavior when $\alpha > 0$.

To see this we consider complex $\alpha$

$$\alpha = -e^{i\theta}|\alpha|$$

and track the saddle point contribution to the asymptotics as $\theta$ increases from 0 to $\pi$.

The saddle point contribution to the late time behavior is defined by the method of steepest descent.

Idea: Decompose the contour $t \in [0, \infty]$ into a homotopically equivalent contour consisting of segments which follow paths of steepest descent. Each segment will produce a distinguished contribution to the late time behavior.

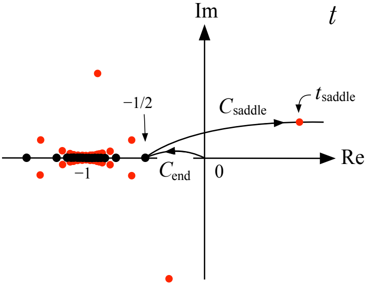
One can show that $t \in [0, \infty]$ is homotopically equivalent to a steepest descent contour consisting of two segments:

$$C_{\text{end}} : \ \mathrm{Im}(S(t)) = 0, \qquad C_{\text{saddle}} : \ \mathrm{Im}(S(t)) = \mathrm{Im}(S(t_{\text{saddle}}))$$

$C_{\text{end}}$ is the steepest descent contour emanating from the origin $t = 0$, and $C_{\text{saddle}}$ is the steepest descent contour passing through the sliver-like saddle point at

$$t_{\text{saddle}} \propto e^{i\theta/3} |\alpha|^{1/3}$$

Rolling (in a complex direction) towards the side of the tachyon effective potential which is unbounded from below corresponds to $\theta \in [0, \pi/2]$. In this case the steepest descent contour looks like this:



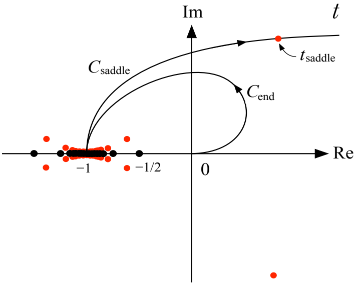

The saddle point contour produces the dominant super-exponential growth at late times. The contribution from the endpoint contour is vanshingly insignificant.

The endpoint contour, however, is exactly the same as the strange solution which approaches the tachyon vacuum from the opposite side of the local minimum.

Crossing $\theta = \pi/2$ the saddle point contribution switches from super-exponentially dominant to super-exponentially suppressed. This is an anti-Stokes line.

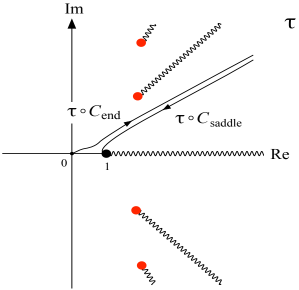
For $\pi/2 < \theta \lesssim .65\pi$ we have some complicated transitional Stokes phenomena from other saddle points. This is unphysical and we do not care about it.

Rolling (in a complex direction) towards the tachyon vacuum corresponds to $.65\pi \lesssim \theta < \pi$. In this case the steepest descent contour looks like this:




The real decay process towards the tachyon vacuum occurs at $\theta = \pi$. This is a Stoke's line, where the relevant saddle point suddenly shifts from above to below the real axis. This is a reflection of the fact that the late time expansion for $\alpha > 0$ is not Borel resummable.

Transform from the Schwinger to the Borel plane:




The image of the endpoint contour represents an upper lateral Borel sum of the late time asymptotic series. The image of the saddle point contour partially cancels this to produce the correct integration $0 < \tau < 1$.

In the context of resurgence theory, nonperturbative corrections corresponds to whatever needs to be added to a lateral Borel transform to obtain the correct nonperturbative result.

In this way we have identified the nonperturbative fluctuation hiding underneath the pure gauge asymptotic expansion around the tachyon vacuum.

# More...

- Through its contribution to the boundary state, it is possible to show that the nonperturbative effect is not gauge trivial, but represents a physical fluctuation of the tachyon vacuum.
- The strange solution which approaches the tachyon vacuum from the "other side" has no nonperturbative corrections, since it is defined by Borel summation. Therefore this solution is gauge trivial. $\rightarrow$ The tachyon effective potential terminates at the tachyon vacuum.
- Using resurgence, it is possible to show that the asymptotic expansion around the tachyon vacuum—though formally gauge trivial—uniquely determines the form of the physical nonperturbative fluctuation around the tachyon vacuum.

Thank you!