Unconventional D-branes on T^4

arXiv:1903.00487 & ongoing work

J. Vošmera[1]

in collaboration with

M. Kudrna, M. Schnabl, B. Stefański

[1]CEICO, Institute of Physics, AS CR, Prague

GGI, Firenze

May 6, 2019

Upshot

$\exists$ non-trivial stable D-branes for trivial type II superstring backgrounds

Upshot

$\exists$ non-trivial stable D-branes for trivial type II superstring backgrounds

- weak closed string coupling

Upshot

$\exists$ non-trivial stable D-branes for trivial type II superstring backgrounds

- weak closed string coupling
- toroidal compactifications $\implies$ free worldsheet SCFT

Upshot

$\exists$ non-trivial stable D-branes for trivial type II superstring backgrounds

- weak closed string coupling
- toroidal compactifications $\Longrightarrow$ free worldsheet SCFT
- $g_{\mathrm{s}} \to 0 \Longrightarrow$ superconformal boundary states

Upshot

$\exists$ non-trivial stable D-branes for trivial type II superstring backgrounds

- weak closed string coupling
- toroidal compactifications $\Longrightarrow$ free worldsheet SCFT
- $g_{\mathrm{s}} \to 0 \Longrightarrow$ superconformal boundary states
- $\neq$ Dp-branes (with or without flux)

Upshot

∃ non-trivial stable D-branes for trivial type II superstring backgrounds

- weak closed string coupling
- toroidal compactifications $\Longrightarrow$ free worldsheet SCFT
- $g_\mathrm{s} \to 0 \Longrightarrow$ superconformal boundary states
- $\neq$ Dp-branes (with or without flux)
- no tachyons in the open string (boundary) spectrum

Conventions

$$\alpha' = 1$$

Motivation

Exotic classical solutions of bosonic OSFT on Dp-branes wrapping a T^2

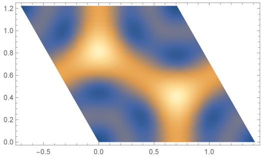
Kudrna, Schnabl, Vošmera; to appear soon

Motivation

Exotic classical solutions of bosonic OSFT on Dp-branes wrapping a T^2

Kudrna, Schnabl, Vošmera; to appear soon

Example: D2 brane, $R_1 = R_2 = R$, $2/\sqrt{3} < R < \sqrt{3}$, $\theta = 2\pi/3$, $B = 0$



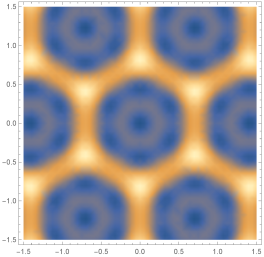


Figure: Energy density from Ellwood invariants in level truncation at $\infty(14)$

Motivation

Exotic classical solutions of bosonic OSFT on Dp-branes wrapping a T^2

Kudrna, Schnabl, Vošmera; to appear soon

Example: D2 brane, $R_1 = R_2 = R$, $2/\sqrt{3} < R < \sqrt{3}$, $\theta = 2\pi/3$, $B = 0$

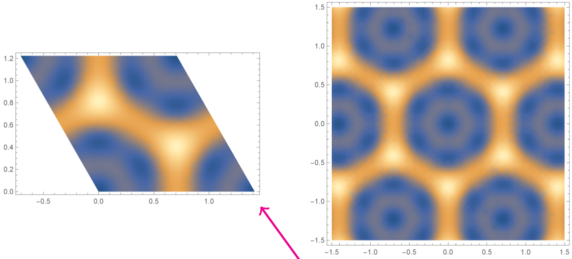


Figure: Energy density from Ellwood invariants in level truncation at $\infty(14)$

$$\rho(x) = T^{00}(x) = \sum_{k \in \Lambda^*} e^{ik\cdot x} \langle k | \alpha_1^0 \overline{\alpha}_1^0 || B \rangle\!\rangle_{\text{matter}}$$

Motivation

Exotic classical solutions of bosonic OSFT on Dp-branes wrapping a T^2

Kudrna, Schnabl, Vošmera; to appear soon

Example: D2 brane, $R_1 = R_2 = R$, $2/\sqrt{3} < R < \sqrt{3}$, $\theta = 2\pi/3$, $B = 0$

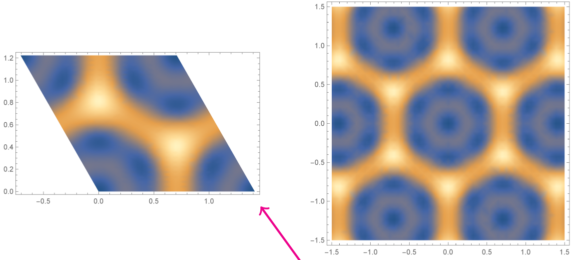


Figure: Energy density from Ellwood invariants in level truncation at $\infty(14)$

$$\rho(x) = T^{00}(x) = \sum_{k \in \Lambda^*} e^{ik\cdot x} \langle k | \alpha_1^0 \overline{\alpha}_1^0 \| B \rangle\!\rangle_{\text{matter}}$$

BCFT: exact boundary state at $R = \sqrt{2}$ from "bosonic Gepner construction"

Motivation

Extended (rational) chiral algebra at $R = \sqrt{2}$: '00 Affleck, Oshikawa, Saleur

$$\mathcal{A}_{\rm ext} = \mathcal{W}_3^{c_1=\frac{4}{5}} \oplus \mathcal{W}_3^{c_2=\frac{6}{5}} \supset \mathcal{V}ir^{c=2} \qquad \text{s.t.} \qquad \mathcal{A}_{\rm b} = \{\partial X^1, \partial X^2\} \notin \mathcal{A}_{\rm ext}$$

Motivation

Extended (rational) chiral algebra at $R = \sqrt{2}$: '00 Affleck, Oshikawa, Saleur

$$\mathcal{A}_{\text{ext}} = \mathcal{W}_3^{c_1=\frac{4}{5}} \oplus \mathcal{W}_3^{c_2=\frac{6}{5}} \supset \mathcal{V}ir^{c=2} \qquad \text{s.t.} \qquad \mathcal{A}_{\text{b}} = \{\partial X^1, \partial X^2\} \notin \mathcal{A}_{\text{ext}}$$

Bulk spectrum:

$$\mathcal{H}_{\text{b}} = \bigoplus_{i,\bar{\imath}} M_{i\bar{\imath}} \, (\mathcal{H}_{i_1}^{c_1=\frac{4}{5}} \otimes \mathcal{H}_{i_2}^{c_2=\frac{6}{5}}) \otimes (\overline{\mathcal{H}}_{\bar{\imath}_1}^{c_1=\frac{4}{5}} \otimes \overline{\mathcal{H}}_{\bar{\imath}_2}^{c_2=\frac{6}{5}})$$

$i_1, i_2, \bar{\imath}_1, \bar{\imath}_2$ run over $\mathcal{W}_3$ MM irreps with $c_1 = 4/5$ and $c_2 = 6/5$

bulk field multiplicities $M_{i\bar{\imath}}$ non-diagonal

Motivation

Extended (rational) chiral algebra at $R = \sqrt{2}$: '00 Affleck, Oshikawa, Saleur

$$\mathcal{A}_{\rm ext} = \mathcal{W}_3^{c_1=\frac{4}{5}} \oplus \mathcal{W}_3^{c_2=\frac{6}{5}} \supset \mathcal{V}ir^{c=2} \qquad \text{s.t.} \qquad \mathcal{A}_{\rm b} = \{\partial X^1, \partial X^2\} \notin \mathcal{A}_{\rm ext}$$

Bulk spectrum:

$$\mathcal{H}_{\rm b} = \bigoplus_{i,\bar{\imath}} M_{i\bar{\imath}} \, (\mathcal{H}_{i_1}^{c_1=\frac{4}{5}} \otimes \mathcal{H}_{i_2}^{c_2=\frac{6}{5}}) \otimes (\overline{\mathcal{H}}_{\bar{\imath}_1}^{c_1=\frac{4}{5}} \otimes \overline{\mathcal{H}}_{\bar{\imath}_2}^{c_2=\frac{6}{5}})$$

$i_1, i_2, \bar{\imath}_1, \bar{\imath}_2$ run over $\mathcal{W}_3$ MM irreps with $c_1 = 4/5$ and $c_2 = 6/5$

bulk field multiplicities $M_{i\bar{\imath}}$ non-diagonal

$\mathcal{A}_{\rm ext}$-symmetric boundary states:

$$[(W_r)_n + \Omega_r (\overline{W}_r)_{-n}] \| B \rangle\rangle = 0 \quad \left\{ \begin{array}{ll} \Longrightarrow & (L_n - \overline{L}_{-n}) \| B \rangle\rangle = 0 \\ \not\Longrightarrow & (\alpha_n^\mu + \Omega^\mu{}_\nu \overline{\alpha}_{-n}^\nu) \| B \rangle\rangle = 0 \end{array} \right.$$

$$\Omega_r = \pm 1 \,, \quad r = 1,2 \qquad\qquad \Omega^\mu{}_\rho \Omega^\nu{}_\sigma g_{\mu\nu} = g_{\rho\sigma} \,, \quad \mu = 1,2$$

Superstring setup

Type II superstring on $\mathbb{R}^{1,5} \times T^4$:

$$\mathbb{R}^{1,5}: \qquad \overbrace{X^0, X^1}^{\text{light-cone}}, \overbrace{X^2, X^3, X^4, X^5}^{\text{transverse external}}$$

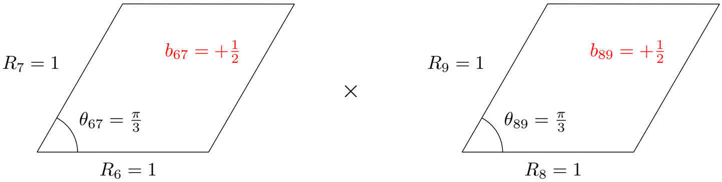
$$T^4 = T^2 \times T^2: \qquad \underbrace{X^6, X^7}_{T^2}, \underbrace{X^8, X^9}_{T^2}$$

Superstring setup

Type II superstring on $\mathbb{R}^{1,5} \times T^4$:

$$\mathbb{R}^{1,5} : \quad \overbrace{X^0, X^1}^{\text{light-cone}}, \overbrace{X^2, X^3, X^4, X^5}^{\text{transverse external}}$$

$$T^4 = T^2 \times T^2 : \quad \underbrace{X^6, X^7}_{T^2}, \underbrace{X^8, X^9}_{T^2}$$



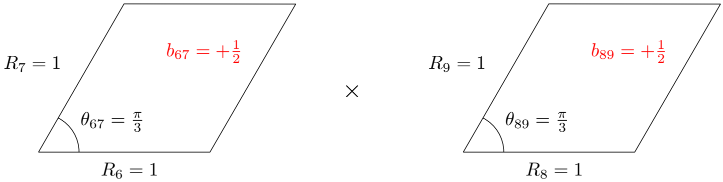
$$T^2_{SU(3)} \quad \times \quad T^2_{SU(3)}$$

Superstring setup

Type II superstring on $\mathbb{R}^{1,5} \times T^4$:

$$\mathbb{R}^{1,5} : \quad \overbrace{X^0, X^1}^{\text{light-cone}}, \overbrace{X^2, X^3, X^4, X^5}^{\text{transverse external}}$$

$$T^4 = T^2 \times T^2 : \quad \underbrace{X^6, X^7}_{T^2}, \underbrace{X^8, X^9}_{T^2}$$



$$T^2_{SU(3)} \times T^2_{SU(3)}$$

Could alternatively consider rectangular T^4 with $R_i = 1$ and $B = 0$

$\mathcal{N} = 2$ superconformal algebra in two dimensions

Generators:

$$L_m\,,\quad J_m\,,\quad G^{\pm}_r$$

$\mathcal{N} = 2$ superconformal algebra in two dimensions

Generators:

$$L_m \,, \quad J_m \,, \quad G_r^{\pm}$$

Commutation relations:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$

$$[L_n, J_m] = -mJ_{n+m}$$

$$[L_n, G^{\pm}_{r\pm a}] = (\frac{n}{2} - (r \pm a))G^{\pm}_{n+r\pm a}$$

$$[J_n, J_m] = \frac{c}{3}n\delta_{n+m,0}$$

$$[J_n, G^{\pm}_{r\pm a}] = \pm G^{\pm}_{n+r\pm a}$$

$$\{G^{+}_{r+a}, G^{-}_{s-a}\} = 2L_{r+s} + (r-s+2a)J_{r+s} + \frac{c}{3}\left((r+a)^2 - \frac{1}{4}\right)\delta_{r+s,0}$$

NS sector: $a = 1/2$

R sector: $a = 0$

$\mathcal{N} = 2$ minimal models

Coset construction:

$$\mathrm{MM}_k \equiv \frac{\widehat{SU}(2)_k \times U(1)_4}{U(1)_{2k+4}}$$

$\mathcal{N} = 2$ minimal models

Coset construction:

$$\mathrm{MM}_k \equiv \frac{\widehat{SU}(2)_k \times U(1)_4}{U(1)_{2k+4}}$$

Central charge:

$$c = \frac{3k}{k+2}\,, \qquad k = 1, 2, \ldots$$

$\mathcal{N} = 2$ minimal models

Coset construction:

$$\mathrm{MM}_k \equiv \frac{\widehat{SU}(2)_k \times U(1)_4}{U(1)_{2k+4}}$$

Central charge:

$$c = \frac{3k}{k+2}\,, \qquad k = 1, 2, \ldots$$

Primaries:

$$\Phi^l_{m,s}: \qquad 0 \leqslant l \leqslant k\,, \quad m \in \mathbb{Z}_{2(k+2)}\,, \quad s \in \mathbb{Z}_4\,, \quad l+m+s \in 2\mathbb{Z}\,,$$

$$\text{coset field id.}: \qquad (l,m,s) \sim (k-l, m+k+2, s+2)$$

NS sector: s even

R sector: s odd

$\mathcal{N} = 2$ minimal models

Coset construction:

$$\mathrm{MM}_k \equiv \frac{\widehat{SU}(2)_k \times U(1)_4}{U(1)_{2k+4}}$$

Central charge:

$$c = \frac{3k}{k+2}\,, \qquad k = 1, 2, \ldots$$

Primaries:

$$\Phi^l_{m,s}: \qquad 0 \leqslant l \leqslant k\,, \quad m \in \mathbb{Z}_{2(k+2)}\,, \quad s \in \mathbb{Z}_4\,, \quad l+m+s \in 2\mathbb{Z}\,,$$

$$\text{coset field id.}: \qquad (l, m, s) \sim (k-l, m+k+2, s+2)$$

NS sector: s even

R sector: s odd

Weights:

$$h^l_{m,s} = \frac{l(l+2) - m^2}{4(k+2)} + \frac{s^2}{8} \mod 1\,, \qquad q^l_{m,s} = \frac{m}{k+2} - \frac{s}{2} \mod 2$$

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Extended chiral algebra: '88 Gepner; '97 Recknagel, Schomerus; '98 Gutperle, Satoh

$$\mathcal{A}_{\text{ext}} \equiv \underbrace{\left(\bigoplus_{r=1}^{2} \widehat{\mathfrak{so}}(2)_1\right)}_{\text{external fermions}} \oplus \underbrace{\left(\bigoplus_{a=1}^{6} \mathrm{MM}_{k=1}\right)}_{\text{internal } T^2_{SU(3)} \times T^2_{SU(3)} \text{ SCFT}} \supset \mathcal{SV}ir^{c=12}$$

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Extended chiral algebra: '88 Gepner; '97 Recknagel, Schomerus; '98 Gutperle, Satoh

$$\mathcal{A}_{\rm ext} \equiv \underbrace{\left(\bigoplus_{r=1}^{2} \widehat{\mathfrak{so}}(2)_1 \right)}_{\text{external fermions}} \oplus \underbrace{\left(\bigoplus_{a=1}^{6} \mathrm{MM}_{k=1} \right)}_{\text{internal } T^2_{SU(3)} \times T^2_{SU(3)} \text{ SCFT}} \supset \mathcal{SV}ir^{c=12}$$

$\mathcal{A}_{\rm ext}$ primaries:

$$\boldsymbol{\lambda} = (\overbrace{l_1, l_2, l_3}^{T^2_{SU(3)}}, \overbrace{l_4, l_5, l_6}^{T^2_{SU(3)}}) \qquad l_a, m_a, s_a \quad \ldots \quad \mathrm{MM}_{k=1} \text{ primaries}$$

$$\boldsymbol{\mu} = (\underbrace{\sigma_1, \sigma_2}_{2}; \underbrace{m_1, \ldots, m_6}_{6}; \underbrace{s_1, \ldots, s_6}_{6}) \qquad \sigma_1, \sigma_2 \quad \ldots \quad \widehat{\mathfrak{so}}(2)_1 \text{ primaries}$$

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Extended chiral algebra: '88 Gepner; '97 Recknagel, Schomerus; '98 Gutperle, Satoh

$$\mathcal{A}_{\text{ext}} \equiv \underbrace{\left(\bigoplus_{r=1}^{2} \widehat{\mathfrak{so}}(2)_1 \right)}_{\text{external fermions}} \oplus \underbrace{\left(\bigoplus_{a=1}^{6} \mathrm{MM}_{k=1} \right)}_{\text{internal } T^2_{SU(3)} \times T^2_{SU(3)} \text{ SCFT}} \supset \mathcal{SV}ir^{c=12}$$

$\mathcal{A}_{\text{ext}}$ primaries:

$$\boldsymbol{\lambda} = (\overbrace{l_1, l_2, l_3}^{T^2_{SU(3)}}, \overbrace{l_4, l_5, l_6}^{T^2_{SU(3)}})$$

$$\boldsymbol{\mu} = (\underbrace{\sigma_1, \sigma_2}_{2}; \underbrace{m_1, \ldots, m_6}_{6}; \underbrace{s_1, \ldots, s_6}_{6})$$

l_a, m_a, s_a ... $\mathrm{MM}_{k=1}$ primaries

σ_1, σ_2 ... $\widehat{\mathfrak{so}}(2)_1$ primaries

$\mathcal{A}_{\text{ext}}$ characters:

$$\chi^{\boldsymbol{\lambda}}_{\boldsymbol{\mu}}(q) = \chi_{\sigma_1}(q) \chi_{\sigma_2}(q) \prod_{a=1}^{6} \chi^{l_a}_{m_a, s_a}(q)$$

tensor product characters

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Partition function:

$$Z^{\text{IIA, Gep}}_{T^2_{SU(3)} \times T^2_{SU(3)}} = \frac{1}{8} \sum_{\boldsymbol{\lambda},\boldsymbol{\mu}}^{\beta} \sum_{\zeta_{\pm} \in \mathbb{Z}_{12}} \sum_{\nu_p \in \mathbb{Z}_2} (-1)^{\zeta_+ + \zeta_-} \chi^{\boldsymbol{\lambda}}_{\boldsymbol{\mu}} \, \overline{\chi}^{\boldsymbol{\lambda}}_{-\boldsymbol{\mu} + \zeta_+ \boldsymbol{\beta}_0^+ + \zeta_- \boldsymbol{\beta}_0^- + \sum_p \nu_p \boldsymbol{\beta}_p}$$

Gepner-like β-constraints:

$\boldsymbol{\beta}_0^{\pm} \bullet \boldsymbol{\mu} \in \mathbb{Z} + 1/2$	locality w.r.t. supercharges
$\boldsymbol{\beta}_p \bullet \boldsymbol{\mu} \in \mathbb{Z}$	spin structure alignment

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Partition function:

$$Z^{\text{IIA, Gep}}_{T^2_{SU(3)}\times T^2_{SU(3)}} = \frac{1}{8}\sum_{\boldsymbol{\lambda},\boldsymbol{\mu}}{}^{\beta} \sum_{\zeta_\pm\in\mathbb{Z}_{12}} \sum_{\nu_p\in\mathbb{Z}_2} (-1)^{\zeta_+ + \zeta_-} \chi^{\boldsymbol{\lambda}}_{\boldsymbol{\mu}}\, \overline{\chi}^{\boldsymbol{\lambda}}_{-\boldsymbol{\mu}+\zeta_+\boldsymbol{\beta}_0^+ + \zeta_-\boldsymbol{\beta}_0^- + \sum_p \nu_p \boldsymbol{\beta}_p}$$

Gepner-like β-constraints:

$\boldsymbol{\beta}_0^\pm \bullet \boldsymbol{\mu} \in \mathbb{Z} + 1/2$ locality w.r.t. supercharges

$\boldsymbol{\beta}_p \bullet \boldsymbol{\mu} \in \mathbb{Z}$ spin structure alignment

Notation:

$$\boldsymbol{\beta}_0^\pm = (\underbrace{1,\pm 1}_{2};\underbrace{1,1,1,\pm1,\pm1,\pm1}_{6};\underbrace{1,1,1,\pm1,\pm1,\pm1}_{6})$$

$$\boldsymbol{\beta}_p = \begin{cases} (\underbrace{2,2}_{2};\underbrace{0,0,0,0,0,0}_{6};\underbrace{0,0,0,0,0,0}_{6}) & p=2 \\ (\underbrace{2,0}_{2};\underbrace{0,0,0,0,0,0}_{6};\underbrace{0,\ldots,\overset{(6+p)^{\text{th}}\text{ position}}{\overset{\downarrow}{2}},\ldots,0}_{6}) & p>2 \end{cases}$$

$$\boldsymbol{\mu}\bullet\boldsymbol{\mu}' \equiv \sum_{r=1}^{2}\frac{\sigma_r\sigma_r'}{4} + \frac{1}{2}\sum_{a=1}^{6}\left(-\frac{m_a m_a'}{3} + \frac{s_a s_a'}{2}\right)$$

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

<u>Can show that:</u>

$$Z^{\text{IIA, Gep}}_{T^2_{SU(3)} \times T^2_{SU(3)}}(q,\bar{q}) = \frac{1}{4|\eta|^8} \left|\theta_3(q)^4 - \theta_4(q)^4 - \theta_2(q)^4\right|^2 \left[Z^{\text{bos}}_{T^2_{SU(3)}}(q,\bar{q})\right]^2$$

where

$$Z^{\text{bos}}_{T^2_{SU(3)}}(q,\bar{q}) = \frac{1}{|\eta(q)|^4} \sum_{m,n,r,s \in \mathbb{Z}} q^{h^{\text{L}}_{m,n,r,s}} \bar{q}^{h^{\text{R}}_{m,n,r,s}}$$

$T^2_{SU(3)} \times T^2_{SU(3)}$: $(k=1)^6$ Gepner-like model

Can show that:

$$Z^{\text{IIA, Gep}}_{T^2_{SU(3)} \times T^2_{SU(3)}}(q,\bar{q}) = \frac{1}{4|\eta|^8} \left|\theta_3(q)^4 - \theta_4(q)^4 - \theta_2(q)^4\right|^2 \left[Z^{\text{bos}}_{T^2_{SU(3)}}(q,\bar{q})\right]^2$$

where

$$Z^{\text{bos}}_{T^2_{SU(3)}}(q,\bar{q}) = \frac{1}{|\eta(q)|^4} \sum_{m,n,r,s\in\mathbb{Z}} q^{h^{\mathsf{L}}_{m,n,r,s}} \bar{q}^{h^{\mathsf{R}}_{m,n,r,s}}$$

where we define

$$h^{\mathsf{L}}_{m,n,r,s} = \frac{1}{4} k^{\mathsf{L}}_{m,n,r,s} \cdot k^{\mathsf{L}}_{m,n,r,s}\,, \qquad (k^{\mathsf{L}}_{m,n,r,s})_\mu = (k_{m,n})_\mu + (g+B)_{\mu\nu}(w_{r,s})^\nu$$

$$h^{\mathsf{R}}_{m,n,r,s} = \frac{1}{4} k^{\mathsf{R}}_{m,n,r,s} \cdot k^{\mathsf{R}}_{m,n,r,s}\,, \qquad (k^{\mathsf{R}}_{m,n,r,s})_\mu = (k_{m,n})_\mu - (g-B)_{\mu\nu}(w_{r,s})^\nu$$

$$(k_{m,n})_\mu = (m,n)_\mu\,, \quad m,n\in\mathbb{Z} \qquad g_{\mu\nu} = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}, \qquad B_{\mu\nu} = \begin{pmatrix} 0 & +\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$$

$$(w_{r,s})^\mu = (r,s)^\mu\,, \quad r,s\in\mathbb{Z}$$

Gluing conditions on $\mathcal{N} = 2$ SCAs

A-type:

$$(J_m - \overline{J}_{-m})\|b,\eta\rangle\rangle = 0\,, \qquad (G^{\pm}_r + i\eta\overline{G}^{\mp}_{-r})\|b,\eta\rangle\rangle = 0$$

Gluing conditions on $\mathcal{N} = 2$ SCAs

A-type:

$$(J_m - \overline{J}_{-m})\|b, \eta\rangle\!\rangle = 0\,, \qquad (G_r^{\pm} + i\eta \overline{G}_{-r}^{\mp})\|b, \eta\rangle\!\rangle = 0$$

B-type:

$$(J_m + \overline{J}_{-m})\|b, \eta\rangle\!\rangle = 0\,, \qquad (G_r^{\pm} + i\eta \overline{G}_{-r}^{\pm})\|b, \eta\rangle\!\rangle = 0$$

Gluing conditions on $\mathcal{N} = 2$ SCAs

A-type:

$$(J_m - \overline{J}_{-m})\|b,\eta\rangle\rangle = 0\,, \qquad (G_r^{\pm} + i\eta\overline{G}_{-r}^{\mp})\|b,\eta\rangle\rangle = 0$$

B-type:

$$(J_m + \overline{J}_{-m})\|b,\eta\rangle\rangle = 0\,, \qquad (G_r^{\pm} + i\eta\overline{G}_{-r}^{\pm})\|b,\eta\rangle\rangle = 0$$

Permutations: '02 Recknagel

$$\underbrace{\mathrm{MM}_k \oplus \ldots \oplus \mathrm{MM}_k}_{n \text{ times}}: \qquad (J_m^{(\alpha)} + \Omega_\alpha \overline{J}_{-m}^{\pi(\alpha)})\|b,\eta\rangle\rangle = 0 \quad \Omega_\alpha = \pm 1\,, \pi \in S_n$$

in total $n! \times 2^n$ possibilities

Gluing conditions on $\mathcal{N} = 2$ SCAs

A-type:

$$(J_m - \overline{J}_{-m})\|b,\eta\rangle\rangle = 0\,, \qquad (G_r^{\pm} + i\eta\overline{G}_{-r}^{\mp})\|b,\eta\rangle\rangle = 0$$

B-type:

$$(J_m + \overline{J}_{-m})\|b,\eta\rangle\rangle = 0\,, \qquad (G_r^{\pm} + i\eta\overline{G}_{-r}^{\pm})\|b,\eta\rangle\rangle = 0$$

Permutations: '02 Recknagel

$$\underbrace{\mathrm{MM}_k \oplus \ldots \oplus \mathrm{MM}_k}_{n \text{ times}}: \qquad (J_m^{(\alpha)} + \Omega_\alpha \overline{J}_{-m}^{\pi(\alpha)})\|b,\eta\rangle\rangle = 0 \quad \Omega_\alpha = \pm 1\,, \pi \in S_n$$

in total $n! \times 2^n$ possibilities

Notation $(n = 6,\ k = 1)$:

e.g. $\omega = (1_\mathrm{A} 3_\mathrm{B} 4_\mathrm{A})(2_\mathrm{A} 5_\mathrm{B})(6_\mathrm{B}) \iff$

$$\begin{aligned}
(J_m^{(1)} - \overline{J}_{-m}^{(3)})\|b,\eta\rangle\rangle &= 0\\
(J_m^{(3)} + \overline{J}_{-m}^{(4)})\|b,\eta\rangle\rangle &= 0\\
(J_m^{(4)} - \overline{J}_{-m}^{(1)})\|b,\eta\rangle\rangle &= 0\\
(J_m^{(2)} - \overline{J}_{-m}^{(5)})\|b,\eta\rangle\rangle &= 0\\
(J_m^{(5)} + \overline{J}_{-m}^{(2)})\|b,\eta\rangle\rangle &= 0\\
(J_m^{(6)} + \overline{J}_{-m}^{(6)})\|b,\eta\rangle\rangle &= 0
\end{aligned}$$

$\omega_0 = (1_{\rm B})(2_{\rm B})(3_{\rm B})(4_{\rm B})(5_{\rm B})(6_{\rm B})$

Boundary states: '97 Recknagel, Schomerus

$$\|\alpha\rangle\rangle_{\omega_0} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_0} = \sum_{\boldsymbol{\lambda},\boldsymbol{\mu}} {}^{\beta}B^{\alpha,\omega_0}_{\boldsymbol{\lambda},\boldsymbol{\mu}} |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_0}$$

where

$$B^{\alpha,\omega_0}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{8}{9}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi\boldsymbol{\mu}\cdot\boldsymbol{M}} \prod_{a=1}^{6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \,.$$

$\omega_0 = (1_{\rm B})(2_{\rm B})(3_{\rm B})(4_{\rm B})(5_{\rm B})(6_{\rm B})$

Boundary states: '97 Recknagel, Schomerus

$$\|\alpha\rangle\rangle_{\omega_0} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_0} = \sum_{\boldsymbol{\lambda},\boldsymbol{\mu}}{}^{\beta} B^{\alpha,\omega_0}_{\boldsymbol{\lambda},\boldsymbol{\mu}} |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_0}$$

where

$$B^{\alpha,\omega_0}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{8}{9}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi \boldsymbol{\mu}\cdot\boldsymbol{M}} \prod_{a=1}^{6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \,.$$

Open string spectrum:

$$Z^{\omega_0}_{\alpha\alpha}(q)_{\rm NS} = \frac{\theta_3(q)^4 - \theta_4(q)^4}{2\eta(q)^4} Z^{\rm bos}_{\rm D0}(q) = 8 + 224q + 2976q^2 + \ldots$$

$$Z^{\omega_0}_{\alpha\alpha}(q)_{\rm R} = \frac{\theta_2(q)^4}{2\eta(q)^4} Z^{\rm bos}_{\rm D0}(q) = 8 + 224q + 2976q^2 + \ldots$$

where

$$Z^{\rm bos}_{\rm D0}(q) = \frac{1}{\eta(q)^4}\left(\frac{1}{\eta(q)^2}\sum_{m,n\in\mathbb{Z}} q^{m^2+n^2+mn}\right)^2$$

$\omega_0 = (1_{\rm B})(2_{\rm B})(3_{\rm B})(4_{\rm B})(5_{\rm B})(6_{\rm B})$

Can match massless NSNS and RR couplings, conserved spacetime supercharges, tensions and open string spectra to identify

$$
\begin{aligned}
\|\mathbf{0},\mathbf{0}\rangle\rangle_{\omega_0} &= \|{\rm D0}\rangle\rangle \\
\|L_1 = +M_1 = 1\rangle\rangle_{\omega_0} &= \|{\rm D0}/\overline{{\rm D2}}_{67}\rangle\rangle \\
\|L_1 = -M_1 = 1\rangle\rangle_{\omega_0} &= \|{\rm D2}_{67}\rangle\rangle \\
\|L_4 = +M_4 = 1\rangle\rangle_{\omega_0} &= \|{\rm D0}/\overline{{\rm D2}}_{89}\rangle\rangle \\
\|L_4 = -M_4 = 1\rangle\rangle_{\omega_0} &= \|{\rm D2}_{89}\rangle\rangle \\
\|L_{1,4} = +M_{1,4} = 1\rangle\rangle_{\omega_0} &= \|{\rm D0}/\overline{{\rm D2}}_{67}/\overline{{\rm D2}}_{89}/{\rm D4}\rangle\rangle \\
\|L_{1,4} = -M_{1,4} = 1\rangle\rangle_{\omega_0} &= \|{\rm D4}\rangle\rangle \\
\|L_{1,4} = +M_1 = -M_4 = 1\rangle\rangle_{\omega_0} &= \|{\rm D2}_{89}/\overline{{\rm D4}}\rangle\rangle \\
\|L_{1,4} = -M_1 = +M_4 = 1\rangle\rangle_{\omega_0} &= \|{\rm D2}_{67}/\overline{{\rm D4}}\rangle\rangle
\end{aligned}
$$

$\omega_0 = (1_{\rm B})(2_{\rm B})(3_{\rm B})(4_{\rm B})(5_{\rm B})(6_{\rm B})$

Can match massless NSNS and RR couplings, conserved spacetime supercharges, tensions and open string spectra to identify

$$\begin{aligned}
\|\mathbf{0},\mathbf{0}\rangle\!\rangle_{\omega_0} &= \|{\rm D0}\rangle\!\rangle \\
\|L_1 = +M_1 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D0}/\overline{\rm D2}_{67}\rangle\!\rangle \\
\|L_1 = -M_1 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D2}_{67}\rangle\!\rangle \\
\|L_4 = +M_4 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D0}/\overline{\rm D2}_{89}\rangle\!\rangle \\
\|L_4 = -M_4 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D2}_{89}\rangle\!\rangle \\
\|L_{1,4} = +M_{1,4} = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D0}/\overline{\rm D2}_{67}/\overline{\rm D2}_{89}/{\rm D4}\rangle\!\rangle \\
\|L_{1,4} = -M_{1,4} = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D4}\rangle\!\rangle \\
\|L_{1,4} = +M_1 = -M_4 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D2}_{89}/\overline{\rm D4}\rangle\!\rangle \\
\|L_{1,4} = -M_1 = +M_4 = 1\rangle\!\rangle_{\omega_0} &= \|{\rm D2}_{67}/\overline{\rm D4}\rangle\!\rangle
\end{aligned}$$

$\Longrightarrow$ 1/2-BPS (bound states of) Dp-branes

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Boundary states: '02 Recknagel

$$\|\alpha\rangle\rangle_{\omega_1} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_1} \quad = \sum_{\substack{\boldsymbol{\lambda},\boldsymbol{\mu} \\ l_3=l_4,\, m_3=m_4}}^{\beta} B^{\alpha,\omega_1}_{\boldsymbol{\lambda},\boldsymbol{\mu}} \, |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_1}$$

where

$$B^{\alpha,\omega_1}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{4}{3}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi\boldsymbol{\mu}\cdot\boldsymbol{M}} \prod_{a=1,2,5,6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \times$$

$$\times \frac{\sin[\frac{\pi}{3}(l_3+1)(L_3+1)]}{\sin[\frac{\pi}{3}(l_3+1)]} e^{-\frac{i\pi}{3} m_4 M_4}$$

$\omega_1 = (1_\mathrm{B})(2_\mathrm{B})(3_\mathrm{B}4_\mathrm{B})(5_\mathrm{B})(6_\mathrm{B})$

Boundary states: '02 Recknagel

$$\|\alpha\rangle\rangle_{\omega_1} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_1} = \sum_{\substack{\boldsymbol{\lambda},\boldsymbol{\mu} \\ l_3=l_4,\, m_3=m_4}}^{\beta} B^{\alpha,\omega_1}_{\boldsymbol{\lambda},\boldsymbol{\mu}} \, |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_1}$$

where

$$B^{\alpha,\omega_1}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{4}{3}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi \boldsymbol{\mu}\cdot\boldsymbol{M}} \prod_{a=1,2,5,6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \times$$

$$\times \frac{\sin[\frac{\pi}{3}(l_3+1)(L_3+1)]}{\sin[\frac{\pi}{3}(l_3+1)]} e^{-\frac{i\pi}{3} m_4 M_4}$$

Open string spectrum:

$$Z^{\omega_1}_{\alpha\alpha\,\mathsf{NS,R}} = 12 + 16q^{\frac{1}{3}} + 48q^{\frac{2}{3}} + 368q + 368q^{\frac{4}{3}} + 864q^{\frac{5}{3}} + \mathcal{O}(q^2)$$

- tachyon-free, bose-fermi degenerate
- 12 massless states in both NS and R sector
- $12 \neq 8 \implies$ cannot be Dp-branes

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Claim: ω_1 gives 1/4-BPS (truly) bound states of Dp-branes

$$\begin{aligned} \|\mathbf{0},\mathbf{0}\rangle\rangle_{\omega_1} &= \|{\rm D2}_{67}/{\rm D2}_{89}/\overline{\rm D4}\rangle\rangle \\ \|L_1 = +M_1 = 1\rangle\rangle_{\omega_1} &= \|{\rm D0}/\overline{\rm D4}\rangle\rangle \\ \|L_1 = -M_1 = 1\rangle\rangle_{\omega_1} &= \|\overline{\rm D0}/{\rm D2}_{67}/{\rm D2}_{89}\rangle\rangle \end{aligned}$$

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Claim: ω_1 gives 1/4-BPS (truly) bound states of Dp-branes

$$\begin{aligned} \|\mathbf{0},\mathbf{0}\rangle\!\rangle_{\omega_1} &= \|{\rm D2}_{67}/{\rm D2}_{89}/\overline{\rm D4}\rangle\!\rangle \\ \|L_1\!=\!+M_1\!=\!1\rangle\!\rangle_{\omega_1} &= \|{\rm D0}/\overline{\rm D4}\rangle\!\rangle \\ \|L_1\!=\!-M_1\!=\!1\rangle\!\rangle_{\omega_1} &= \|\overline{\rm D0}/{\rm D2}_{67}/{\rm D2}_{89}\rangle\!\rangle \end{aligned}$$

For non-zero, non-selfdual B-field, have

$$\mathcal{M}_{{\rm D0}/\overline{\rm D4}} < \mathcal{M}_{\rm D0} + \mathcal{M}_{\overline{\rm D4}}$$

i.e. $\|{\rm D0}/\overline{\rm D4}\rangle\!\rangle \neq \|{\rm D0}\rangle\!\rangle + \|\overline{\rm D4}\rangle\!\rangle$

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Check #1: RR charges

e.g. $\;{}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| L_1 = M_1 = 1 \rangle\!\rangle_{\omega_1} = {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| {\rm D0} \rangle\!\rangle + {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| \overline{\rm D4} \rangle\!\rangle$

$$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$$

Check #1: RR charges

e.g. $${}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| L_1 = M_1 = 1 \rangle\!\rangle_{\omega_1} = {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| \mathrm{D0} \rangle\!\rangle + {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| \overline{\mathrm{D4}} \rangle\!\rangle$$

Check #2: tensions

$$\mathcal{M}_{\textsf{boundary state}} = \sqrt{3}\mathcal{M}_{\rm D0} = \mathcal{M}_{\textsf{BPS-mass}}$$

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Check #1: RR charges

e.g. ${}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| L_1 = M_1 = 1\rangle\rangle_{\omega_1} = {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| {\rm D0}\rangle\rangle + {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| \overline{\rm D4}\rangle\rangle$

Check #2: tensions

$$\mathcal{M}_{\text{boundary state}} = \sqrt{3}\mathcal{M}_{\rm D0} = \mathcal{M}_{\text{BPS-mass}}$$

Check #3: spacetime supersymmetries

$$(Q^{\sf L}_{\tau_1,\tau_2,\tau_3,\tau_3} + e^{-\frac{2i\pi}{3}\tau_3(-\frac{1}{2}+\sum_{a\neq 4} M_a)} Q^{\sf R}_{-\tau_1,\tau_2,\tau_3,\tau_3}) \| \boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_1} = 0$$

$Q^{\sf L,R}$ act on the boundary states by spectral flow '98 Gutperle, Satoh

$\omega_1 = (1_{\rm B})(2_{\rm B})(3_{\rm B}4_{\rm B})(5_{\rm B})(6_{\rm B})$

Check #1: RR charges

$$\text{e.g.} \quad {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| L_1 = M_1 = 1 \rangle\!\rangle_{\omega_1} = {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| {\rm D0} \rangle\!\rangle + {}_{\omega_0}\langle \tau_1\tau_2\tau_3\tau_4 \| \overline{\rm D4} \rangle\!\rangle$$

Check #2: tensions

$$\mathcal{M}_{\text{boundary state}} = \sqrt{3}\mathcal{M}_{\rm D0} = \mathcal{M}_{\text{BPS-mass}}$$

Check #3: spacetime supersymmetries

$$(Q^{\sf L}_{\tau_1,\tau_2,\tau_3,\tau_3} + e^{-\frac{2i\pi}{3}\tau_3(-\frac{1}{2}+\sum_{a\neq 4} M_a)} Q^{\sf R}_{-\tau_1,\tau_2,\tau_3,\tau_3}) \| \boldsymbol{\Lambda}, \boldsymbol{M} \rangle\!\rangle_{\omega_1} = 0$$

$Q^{\sf L,R}$ act on the boundary states by spectral flow '98 Gutperle, Satoh

Check #4: moduli space dimension

$$\#\ \text{massless open string states} \quad = \quad \underbrace{4kN}_{\text{ADHM}} + \underbrace{8}_{\text{CoM}} = 4 \times 1 \times 1 + 8 = 12$$

where

$$kN = |ZZ^{6789} - Z^{67}Z^{89}|$$

$\omega_1 = (1_\mathrm{B})(2_\mathrm{B})(3_\mathrm{B}4_\mathrm{B})(5_\mathrm{B})(6_\mathrm{B})$

Check #5: IIB SUGRA solution for D1/D5 on $T^2_{SU(3)} \times T^2_{SU(3)}$:

'99 Dhar, Mandal, Wadia, Yogendran

$$
\begin{aligned}
ds^2 &= f(r)^{-1}[-(dx^0)^2 + (dx^5)^2] + f(r)(dr^2 + r^2 d\Omega_3^2) + \\
&\qquad + (dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \,, \\
e^{2\phi} &= 1 \,, \\
B &= \frac{1}{\sqrt{3}}(dx^6 \wedge dx^7 + dx^8 \wedge dx^9) \,,
\end{aligned}
$$

where

$$
f(r) = 1 + \frac{8\sqrt{3}\pi^4 g N}{r^2}
$$

$\omega_1 = (1_\mathrm{B})(2_\mathrm{B})(3_\mathrm{B}4_\mathrm{B})(5_\mathrm{B})(6_\mathrm{B})$

Check #5: IIB SUGRA solution for D1/D5 on $T^2_{SU(3)} \times T^2_{SU(3)}$:

'99 Dhar, Mandal, Wadia, Yogendran

$$
\begin{aligned}
ds^2 &= f(r)^{-1}[-(dx^0)^2 + (dx^5)^2] + f(r)(dr^2 + r^2 d\Omega_3^2) + \\
&\qquad + (dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \,, \\
e^{2\phi} &= 1 \,, \\
B &= \frac{1}{\sqrt{3}}(dx^6 \wedge dx^7 + dx^8 \wedge dx^9) \,,
\end{aligned}
$$

where

$$f(r) = 1 + \frac{8\sqrt{3}\pi^4 gN}{r^2}$$

This gives precisely the asymptotic metric and dilaton deviation profiles predicted from the boundary state in $\mathbb{R}^{1,5}$ '97 Di Vecchia et al.

$$
\begin{aligned}
h^\alpha_{ij}(r) &= \frac{8\sqrt{3}\pi^4 gN}{r^2}\,\mathrm{diag}\,[+1,+1,+1,+1,+1,-1]_{ij} \,, \quad i,j = 0,\ldots,5 \\
\phi^\alpha(r) &= 0
\end{aligned}
$$

$\omega_1 = (1_\mathrm{B})(2_\mathrm{B})(3_\mathrm{B}4_\mathrm{B})(5_\mathrm{B})(6_\mathrm{B})$

Check #5: IIB SUGRA solution for D1/D5 on $T^2_{SU(3)} \times T^2_{SU(3)}$:

'99 Dhar, Mandal, Wadia, Yogendran

$$
\begin{aligned}
ds^2 &= f(r)^{-1}[-(dx^0)^2 + (dx^5)^2] + f(r)(dr^2 + r^2 d\Omega_3^2) + \\
&\qquad + (dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \,, \\
e^{2\phi} &= 1 \,, \\
B &= \frac{1}{\sqrt{3}}(dx^6 \wedge dx^7 + dx^8 \wedge dx^9) \,,
\end{aligned}
$$

where

$$f(r) = 1 + \frac{8\sqrt{3}\pi^4 gN}{r^2}$$

This gives precisely the asymptotic metric and dilaton deviation profiles predicted from the boundary state in $\mathbb{R}^{1,5}$ '97 Di Vecchia et al.

$$
\begin{aligned}
h^\alpha_{ij}(r) &= \frac{8\sqrt{3}\pi^4 gN}{r^2}\, \mathrm{diag}\,[+1,+1,+1,+1,+1,-1]_{ij} \,, \quad i,j = 0,\ldots,5 \\
\phi^\alpha(r) &= 0
\end{aligned}
$$

Similarly for RR forms

$$\omega_2 = (1_{\rm B}4_{\rm B})(2_{\rm A}5_{\rm A})(3_{\rm B})(6_{\rm B})$$

Boundary states: '02 Recknagel

$$\|\alpha\rangle\rangle_{\omega_2} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_2} \quad = \sum_{\substack{\boldsymbol{\lambda},\boldsymbol{\mu} \\ l_1=l_4,\, m_1=+m_4 \\ l_2=l_5,\, m_2=-m_5}}^{\beta} B^{\alpha,\omega_2}_{\boldsymbol{\lambda},\boldsymbol{\mu}} \; |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_2}$$

where

$$B^{\alpha,\omega_2}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{1}{2}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi \boldsymbol{\mu}\cdot\boldsymbol{M}} e^{-\frac{i\pi}{3}m_4 M_4} e^{-\frac{i\pi}{3}m_5 M_5} \prod_{a=3,6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \times$$

$$\times \prod_{a=1,2} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin[\frac{\pi}{3}(l_a+1)]}$$

$\omega_2 = (1_{\mathrm{B}}4_{\mathrm{B}})(2_{\mathrm{A}}5_{\mathrm{A}})(3_{\mathrm{B}})(6_{\mathrm{B}})$

Boundary states: '02 Recknagel

$$\|\alpha\rangle\rangle_{\omega_2} \equiv \|\boldsymbol{\Lambda}, \boldsymbol{M}\rangle\rangle_{\omega_2} = \sum_{\substack{\boldsymbol{\lambda},\boldsymbol{\mu} \\ l_1=l_4,\, m_1=+m_4 \\ l_2=l_5,\, m_2=-m_5}}^{\beta} B^{\alpha,\omega_2}_{\boldsymbol{\lambda},\boldsymbol{\mu}} \; |\boldsymbol{\lambda}, \boldsymbol{\mu}\rangle\rangle_{\omega_2}$$

where

$$B^{\alpha,\omega_2}_{\boldsymbol{\lambda},\boldsymbol{\mu}} = \frac{1}{2}(-1)^{\frac{\sigma_1^2}{2}} e^{-2i\pi\boldsymbol{\mu}\cdot\boldsymbol{M}} e^{-\frac{i\pi}{3}m_4 M_4} e^{-\frac{i\pi}{3}m_5 M_5} \prod_{a=3,6} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin^{\frac{1}{2}}[\frac{\pi}{3}(l_a+1)]} \times$$
$$\times \prod_{a=1,2} \frac{\sin[\frac{\pi}{3}(l_a+1)(L_a+1)]}{\sin[\frac{\pi}{3}(l_a+1)]}$$

Open string spectrum:

$$Z^{\omega_2}_{\alpha\alpha}(q)_{\mathrm{NS}} = 28 + 44q^{\frac{1}{3}} + 192q^{\frac{2}{3}} + 884q + 1288q^{\frac{4}{3}} + 3456q^{\frac{5}{3}} + \mathcal{O}(q^2)$$
$$Z^{\omega_2}_{\alpha\alpha}(q)_{\mathrm{R}} = 16 + 80q^{\frac{1}{3}} + 192q^{\frac{2}{3}} + 704q + 1648q^{\frac{4}{3}} + 3456q^{\frac{5}{3}} + \mathcal{O}(q^2)$$

- tachyon-free $\Longrightarrow$ stable
- massless level: 28 bosons, 16 fermions $\Longrightarrow$ no supersymmetry

$\omega_2 = (1_{\mathrm{B}} 4_{\mathrm{B}})(2_{\mathrm{A}} 5_{\mathrm{A}})(3_{\mathrm{B}})(6_{\mathrm{B}})$

Massless couplings:

NSNS sector:	mass	$\mathcal{M}_{\omega_2} = 3\mathcal{M}_{\mathrm{D0}}$
RR sector:	forbidden	

$\Longrightarrow$ ω_2 branes uncharged

$\omega_2 = (1_B 4_B)(2_A 5_A)(3_B)(6_B)$

Massless couplings:

NSNS sector:	mass	$\mathcal{M}_{\omega_2} = 3\mathcal{M}_{D0}$
RR sector:	forbidden	

$\Longrightarrow$ ω_2 branes uncharged

Interbrane potential:

($Z^{\omega_2}_{\alpha\alpha}(q)_{NS} - Z^{\omega_2}_{\alpha\alpha}(q)_R \neq 0 \implies$ no-force condition violated)

$$V_p(r) = -\int_0^\infty \frac{dt}{t} e^{-\frac{r^2 t}{2\pi}} (8\pi^2 t)^{-\frac{p+1}{2}} \eta(q)^{-4} Z^{\omega_2}_{\alpha\alpha}(q)$$

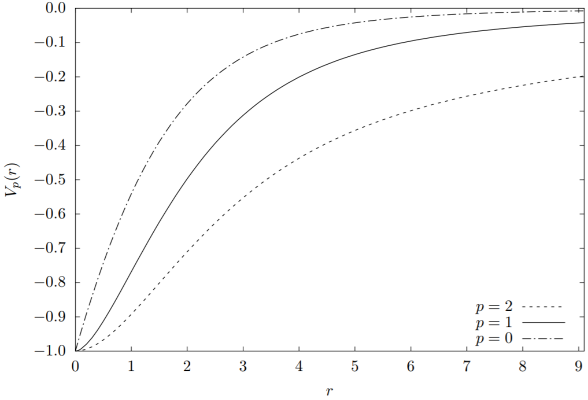
where $q = e^{-2\pi t}$

p	...	number of N directions in $\mathbb{R}^{1,5}$
r	...	separation in $\mathbb{R}^{1,5}$

with

$$Z^{\omega_2}_{\alpha\alpha}(q) = Z^{\omega_2}_{\alpha\alpha}(q)_{\text{NS}} - Z^{\omega_2}_{\alpha\alpha}(q)_{\text{R}} = 12 - 36q^{\frac{1}{3}} + 180q - 360q^{\frac{4}{3}} + \mathcal{O}(q^2)$$

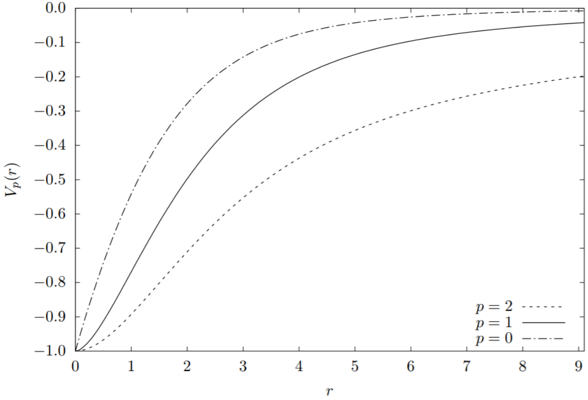
$\omega_2 = (1_B 4_B)(2_A 5_A)(3_B)(6_B)$



$$V_p(r) \propto r^{p-3} \quad \text{as} \quad r \to \infty$$

$$V_p(r) \propto \begin{cases} r^2 & p = 2 \\ r^2 & p = 1 \\ r & p = 0 \end{cases} \quad \text{as} \quad r \to 0$$

$\omega_2 = (1_B 4_B)(2_A 5_A)(3_B)(6_B)$



$$V_p(r) \propto r^{p-3} \quad \text{as} \quad r \to \infty$$

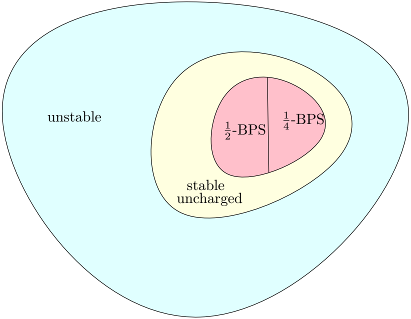
$$V_p(r) \propto \begin{cases} r^2 & p = 2 \\ r^2 & p = 1 \\ r & p = 0 \end{cases} \quad \text{as} \quad r \to 0$$

Dilaton profile from SUGRA:

$$e^{\phi} = \left[\frac{1 - K r^{p-3}}{1 + K r^{p-3}} \right]^{\frac{1}{\sqrt{2}}(p-1)\sqrt{\frac{4-p}{3-p}}} \xrightarrow{r \to r_*} \begin{cases} 0 & p = 2 \\ \text{const.} & p = 1 \\ \infty & p = 0 \end{cases}$$

General $(k=1)^6$ Gepner-like gluing conditions

Computer algorithm was developed to classify boundary states for all $6! \times 2^6 = 46\,080$ Gepner-like gluing conditions:



- no unstable RR charged boundary states
- all RR charged boundary states are supersymmetric