Classical solutions in string field theory

a review

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Motivation

- There are many problems in string theory which can be addressed most naturally and possibly only within the context of ***string field theory***

- This talk will focus on constructions of classical backgrounds which are needed to understand fate of unstable systems, relationships between backgrounds, time-dependent processes etc.

Summary of classical solutions

Open bosonic SFT

Open supersymmetric SFT

Closed bosonic SFT

Closed supersymmetric SFT

Summary of classical solutions

Open bosonic SFT	Open supersymmetric SFT
Numerical: Sen and Zwiebach and many others (1999-2018); (non-)universal solutions of all sorts Analytic : wedge based, KBc algebra and generalizations	
Closed bosonic SFT	**Closed supersymmetric SFT**
	

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Numerical: tachyon vacuum, lumps, D0/D4 (mostly 2000-2001)

Analytic: Tachyon vacuum - Erler 2013, Marginal deformations (see talk by Ivo Sachs)

Closed bosonic SFT

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Outline of this talk

- 1-slide OSFT review
- Universal solutions: numerical approaches
- Non-universal solutions:
  BCFT–OSFT relation and numerical approaches
- Analytic solutions in OSFT
- Comment on defects

Bosonic OSFT: Witten's version

Open string field theory is defined using the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle . \rangle .$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle \Psi * Q_B \Psi \rangle + \frac{1}{3}\langle \Psi * \Psi * \Psi \rangle\right],$$

This action has a huge gauge symmetry

$$\delta\Psi = Q_B\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

provided certain natural axioms are obeyed (DGA with trace)

Universal solutions
- numerical approaches

Based on Kudrna, MS, arXiv:1812.03221

20 years of Sen-Zwiebach calculation

- In 1999 Sen and Zwiebach decided to test a conjecture by Sen, that the value of the OSFT action at the critical point corresponding to the condensed tachyon exactly cancels the tension of a D-brane.

- For a string field truncated to e.g. level 2 in the universal subspace of the underlying BCFT we take

$$|T\rangle = tc_1|0\rangle + uc_{-1}|0\rangle + v \cdot \frac{1}{\sqrt{13}} L_{-2}c_1|0\rangle$$

and one has to find stationary points of the OSFT action

20 years of Sen-Zwiebach calculation

- The OSFT action equals in this case

$$
\begin{aligned}
f^{(4)}(T) \;=\; & 2\pi^2\Big(-\frac{1}{2}\,t^2+\frac{3^3\sqrt{3}}{2^6}\,t^3 \\
& -\frac{1}{2}\,u^2\;+\;\frac{1}{2}\,v^2\;+\;\frac{11\cdot 3\sqrt{3}}{2^6}\,t^2\,u-\frac{5\cdot 3\sqrt{39}}{2^6}\,t^2\,v \\
& +\frac{19}{2^6\sqrt{3}}\,t\,u^2+\frac{7\cdot 83}{2^6\cdot 3\sqrt{3}}\,t\,v^2-\frac{11\cdot 5\sqrt{13}}{2^5\cdot 3\sqrt{3}}\,t\,u\,v\Big)\,.
\end{aligned}
$$

- They found a critical point

$$t_c \simeq 0.542,\; u_c \simeq 0.173,\; v_c \simeq 0.187$$

for which the D-brane tension is cancelled within 95%

20 years of Sen's conjectures

- First evidence, however, has been provided by a numerical approach – level truncation

0	-0.684616		
2	-0.959377		
4	-0.987822		Sen, Zwieabach 1999
6	-0.995177		
8	-0.997930		
10	-0.999182		Moeller, Taylor 2000
12	-0.999822		
14	-0.999826		
16	-1.000375		
18	-1.000494		Gaitotto, Rastelli 2002
20	-1.000563		Kishimoto, Takahashi 2009
22	-1.000602		
24	-1.000623		
26	-1.000631		Kishimoto 2011
28	-1.000632		
30	-1.000627		

$$-\frac{2^{12}}{3^{10}}\pi^2$$

Actually they used (L,2L) scheme, so their numbers are a bit different

Overshooting observed by Gaiotto and Rastelli

Lots of other references for superstring and/or lump solutions etc.

Turns back !

Tricks for level truncation

Critical ingredients for a useful computerized level truncation

1. Convenient basis of states
   universality, twist condition, gauge condition, SU(1,1) condition,…
2. Conservation laws for vertex computation
3. Finding a good starting point for Newton’s method
4. Algorithmic tricks (parallelism)
5. Having good observables
6. Fits to infinite level

Observables for universal solutions

- Energy computed from the action *E=V+1*

- The only independent Ellwood invariant

$$E_0 = -4\pi i \langle E[c\bar{c}\partial X^0 \bar{\partial} X^0] | \Psi \rangle + 1$$

For solutions equals *E*, see Baba, Ishibashi 2012

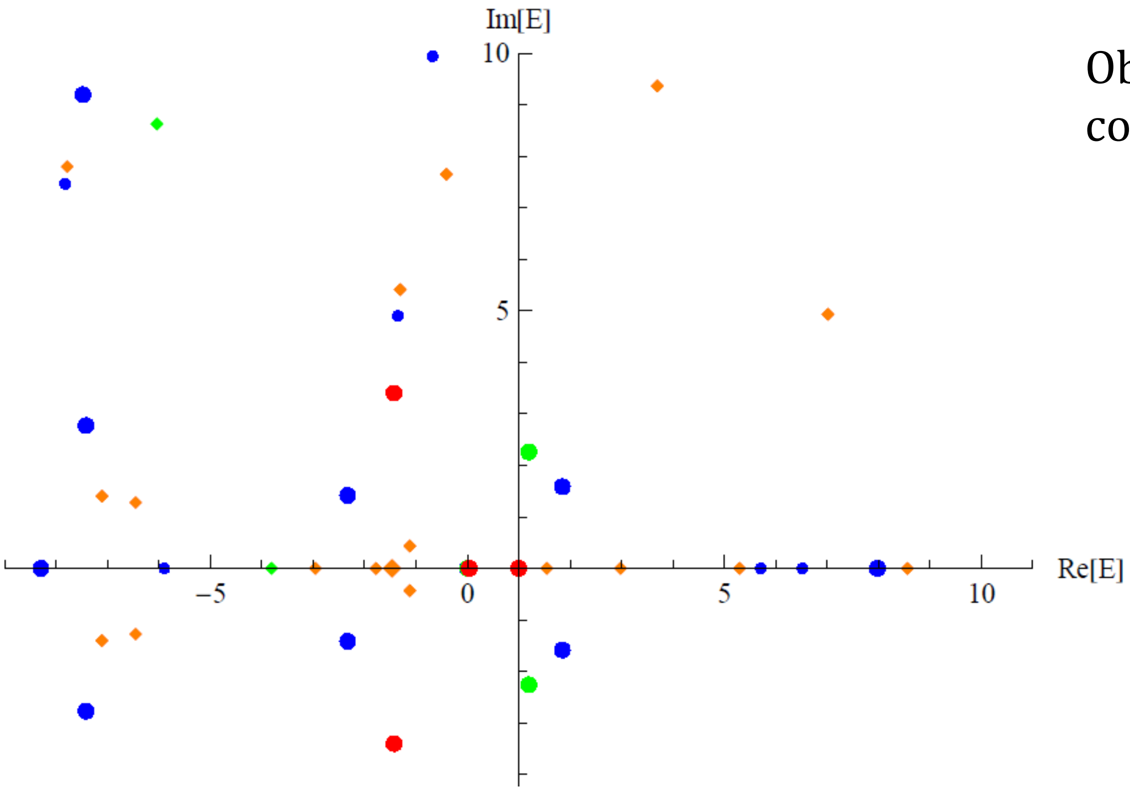
- Out-of-Siegel-gauge equations (we take just the first one)

$$\Delta_S = \left| \langle 0 | c_{-1} j_2^{gh} \, | Q\Psi + \Psi * \Psi \rangle \right| = |\langle 0 | c_{-1} c_0 b_2 \, | Q\Psi + \Psi * \Psi \rangle |$$

- Ratios

$$R_n = (-1)^n \frac{54}{65} \frac{\langle \Psi | c_0 L^m_{2n} | \Psi \rangle}{\langle \Psi | c_0 L_0 | \Psi \rangle}$$

Siegel gauge universal solutions



Obtained via homotopy continuation method

Solutions (starting points) at level 2, level 4, level 5, level 6.
Twist even •, non-even ⧫ , SU(1,1) singlets: bigger symbols

Siegel gauge universal solutions

Our results:

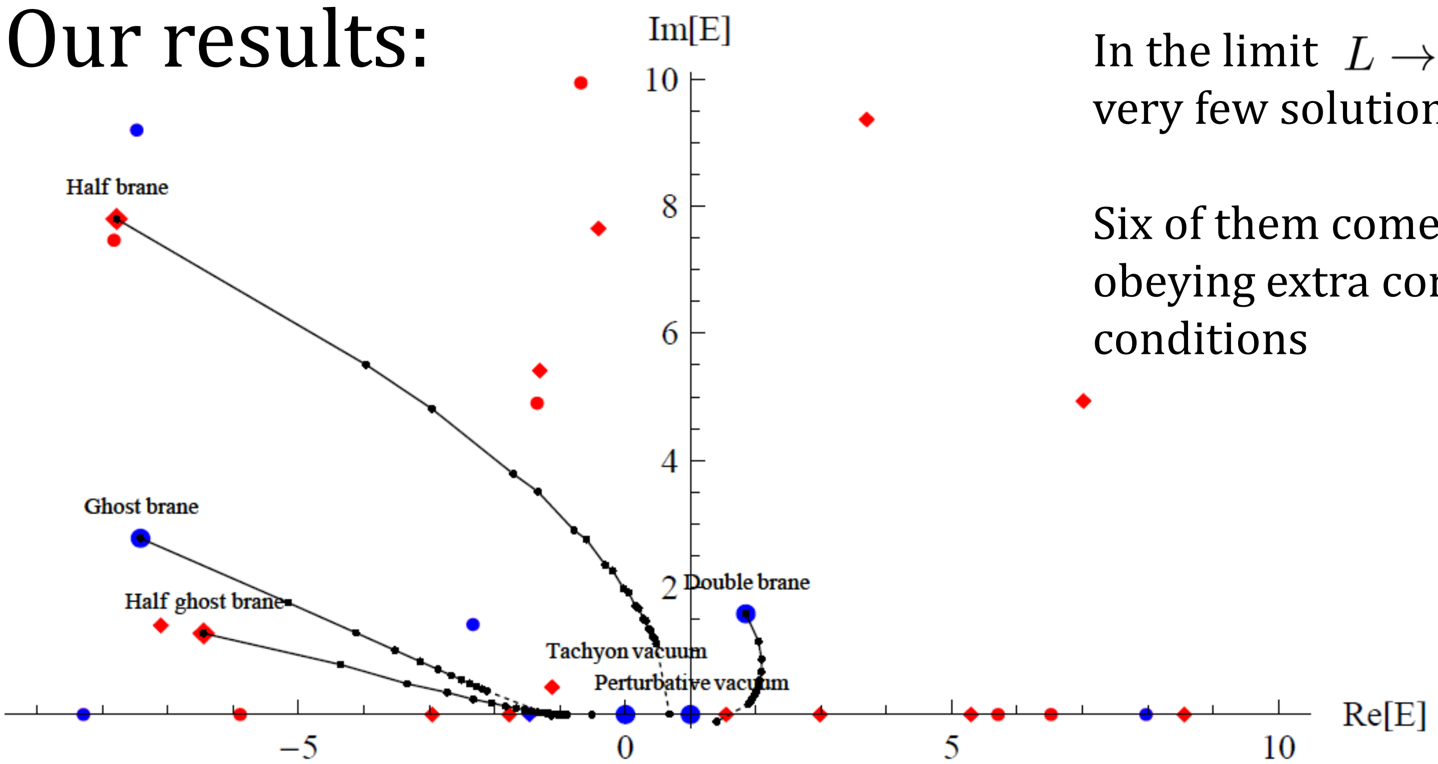


Figure 1: Twist even solutions at level 6 and twist non-even solutions at level 5 represented as dots or diamonds respectively in the energy complex plane. SU(1,1) singlet solutions are represented by blue color, non-singlet solutions by red. By black line and dots we show the improvement of the interesting solutions to higher levels, the dashed part of the line is infinite level extrapolation.

In the limit $L \to \infty$
very few solutions survive.

Six of them come close to obeying extra consistency conditions

Siegel gauge universal solutions

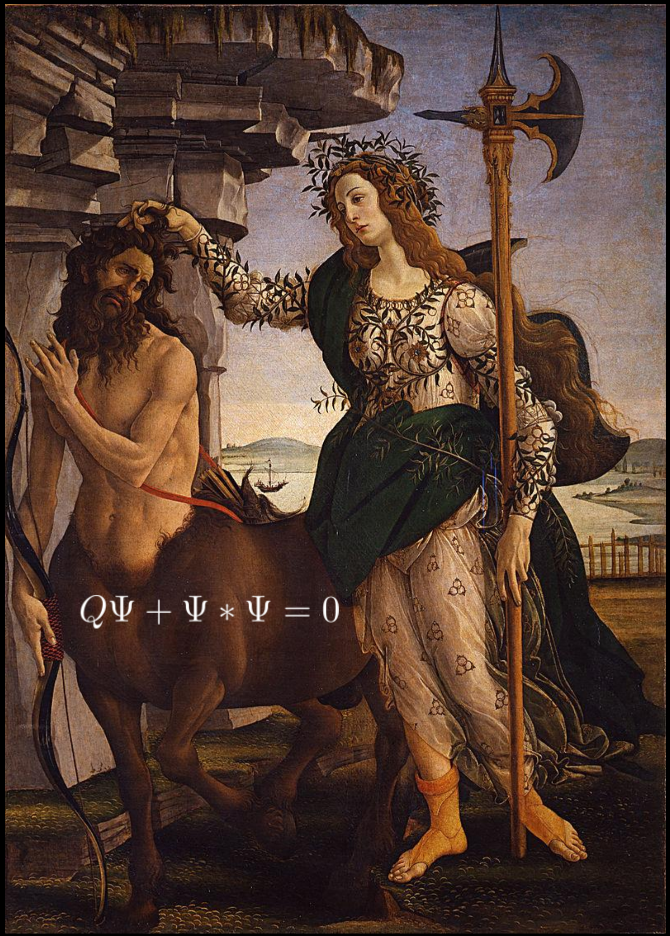
Our results:

Solution	$\text{Energy}^{L=\infty}$	$E_0^{L=\infty}$	$\Delta_S^{L=\infty}$	Reality	Twist even	SU(1,1) singlet
tachyon vacuum	-8×10^{-6}	0.0004	-7×10^{-6}	yes	yes	yes
single brane	1	1	0	yes	yes	yes
"ghost brane"	$-1.13+0.024i$	$-1.01+0.11i$	0.08	no	yes	yes
"double brane"	$1.40+0.11i$	$1.23+0.04i$	0.20	possibly*	yes	yes
"half ghost brane"	-0.51	-0.66	0.17	pseudoreal**	no	no
"half brane"	$0.68-0.01i$	$0.54+0.1i$	0.23	no	no	no

* as $L\to\infty$

** for $L\geq 22$

Unphysical objects fortunately do not obey reality condition, not even asymptotically. Could the ghost and meronic branes play a role in non-perturbative OSFT ?



Non-universal solutions - numerical approaches

Kudrna, Maccaferri, MS, JHEP 1307 (2013) 033; Kudrna, Rapčák, MS, : arXiv:1401.7980; Kudrna, MS, arXiv:1812.03221; Kudrna, Schnabl, Vošmera, to appear

BCFT's from OSFT: Brief review

- String field theory is a great tool to learn new things about BCFT from a novel perspective.

BCFT's from OSFT: Brief review

- Let us consider OSFT for strings 'propagating' in a background given by $\text{BCFT}_c \otimes \text{BCFT}_{26\text{-}c}$ and look for classical solutions which do not excite any primaries in $\text{BCFT}_{26\text{-c}}$. Such solutions will describe new BCFT_c^*.

BCFT's from OSFT: Brief review

- The full boundary state can be constructed rather explicitly as

$$|B_\Psi\rangle = \sum_\alpha B_\Psi^\alpha |V_\alpha\rangle\rangle$$

where $|V_\alpha\rangle\rangle$ are the Ishibashi states and

$$B_\Psi^\alpha = 2\pi i \langle I|\mathcal{V}^\alpha(i,-i)|\Psi - \Psi_{TV}\rangle^{\mathrm{BCFT}_c \otimes \mathrm{BCFT}_{26-c} \otimes \mathrm{BCFT}_{\mathrm{aux}}}$$

$$\mathcal{V}^\alpha = c\bar{c}V^\alpha \, e^{2i\sqrt{1-h_\alpha}\,Y} \, w$$

See: Kudrna,Maccaferri, M.S. (2012)
Alternative attempt:
Kiermaier, Okawa, Zwiebach (2008)

BCFT's from OSFT: Brief review

- Side comment: the OSFT formula for the boundary state makes manifest the BCFT relation

$$\frac{B_x^\beta B_y^\beta}{B_0^\beta} = \sum_z N_{xy}{}^z B_z^\beta$$

- To see that, let us assume that the solution $|\Psi\rangle$ describes the boundary state $||B_x\rangle\rangle$ as seen from $||B_0\rangle\rangle$. Assuming that the Verma modules "turned on" are present also on $||B_y\rangle\rangle$ and the structure of boundary operators is identical, the claim readily follows.

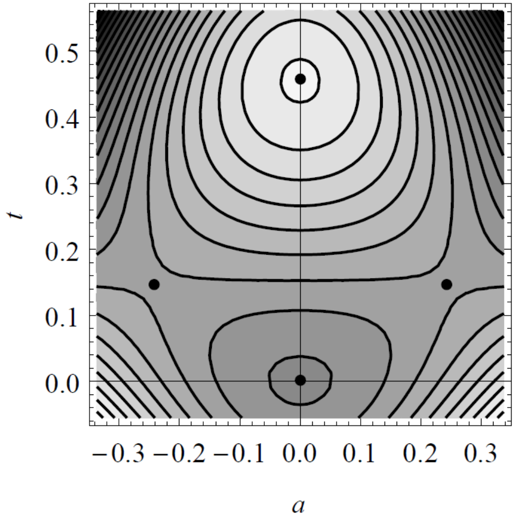
OSFT for the Ising model

- OSFT can be conveniently used for constructing e.g. the fixed boundary condition out of the free one. For the σ-brane at level ½ the string field

$$\Psi = tc_1|0\rangle + ac_1|\varepsilon\rangle$$

leads to a potential

$$\mathcal{V}(t,a) = -\frac{1}{2}t^2 - \frac{1}{4}a^2 + \frac{1}{3}K^3t^3 + K^2a^2t$$

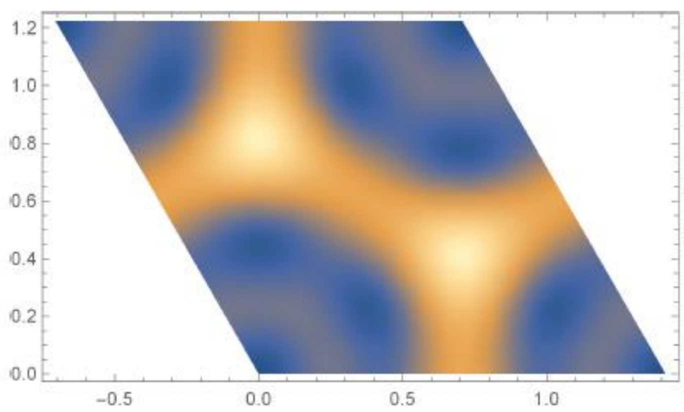


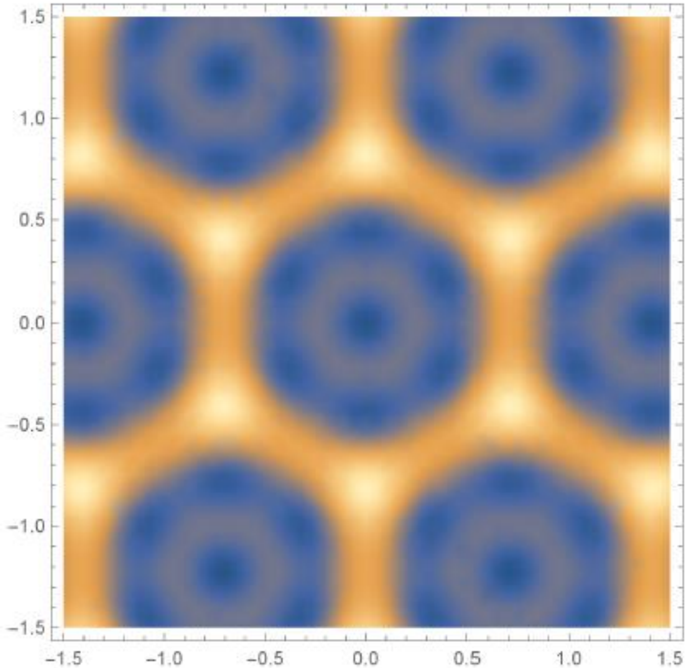
The various critical points have been interpreted as perturbative and tachyon vacua, and $\mathbb{1}$ or ε branes

Kudrna, Rapčák, M.S. 2014

BCFT's from OSFT

- Sometimes we discover really new boundary conditions. On a Z_3 torus with $\frac{2}{\sqrt{3}} < R < \sqrt{3}$ we find by condensing a Z_3 symmetric tachyon momentum mode on a wrapped D2 brane new unexpected state





See talk by Vošmera
and upcoming work Kudrna, MS, Vošmera

Analytic solutions in OSFT

Universal: MS (2005), Okawa (2006), …….., Mertes, MS, JHEP 1612 (2016) 151
Non-universal: MS, KORZ (2007); …, Erler, Maccaferri (2014)….

Simple subsector of the star algebra

- The star algebra is formed by vertex operators and the operator *K*. The simplest subalgebra relevant for tachyon condensation is therefore spanned by *K* and *c.* Let us be more generous and add an operator *B* such that *QB=K.*

- The building elements thus obey

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$
$$[K, B] = 0, \quad [K, c] = \partial c$$

- The derivative *Q* acts as $Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc.$

Universal classical solutions

This new understanding lets us construct solutions to OSFT equations of motion $Q_B\Psi + \Psi * \Psi = 0$ easily.

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \cancel{\alpha^2 c^2} - \cancel{\alpha c^2} K - \alpha cKc + (cK)(cK)$$

More general solutions are of the form

$$\Psi = Fc\frac{KB}{1-F^2}cF,$$

Here *F=F(K)* is arbitrary

M.S. 2005, Okawa, Erler 2006

Universal classical solutions

- What do these solutions correspond to?
- In 2011 with Murata we succeeded in computing their energy

$$E = \frac{1}{2\pi^2}\oint_C \frac{dz}{2\pi i}\frac{G'(z)}{G(z)}$$

in terms of the function $G(z) = 1 - F^2(z)$

- For simple choices of G, one can get perturbative vacuum, tachyon vacuum, or exotic multibrane solution. At the moment the multibrane solutions appear to be a bit singular. (see also follow-up work by Hata and Kojita, and talk by Kojita)

Universal classical solutions

- The KBc can be naturally extended by adding string fields

$$\mathbb{L}_n = \mathcal{L}_n^L \,|I\rangle \qquad \text{Note that:} \qquad K = \mathcal{L}_{-1}^L \,|I\rangle = \mathbb{L}_{-1}$$

$$[\mathbb{L}_m, \mathbb{L}_n] = (m-n)\mathbb{L}_{m+n}$$

- Considering n=0,-1 is a consistent truncation. Example of a solution is

$$\Psi = F(K,L)cB\frac{1}{1-F^2(K,L)}KcF(K,L)$$

- To show this, it is useful to use star product relations of kappa-deformed spacetime obeying identities such as

$$KF(K,L) = F(K,L-1)K$$

Mertes, M.S. 2016

Universal classical solutions

- Masuda, Noumi, Takahashi (2012) solution

$$\Psi = F(K)c\frac{KB}{1-F(K)^2}cF(K) + H(K)c\frac{KB}{1-H(K)^2}cH(K)$$

for $F(K)H(K) = 1$
It seems to describe ghost D-brane, but is too identity-like

- Most general tachyon vacuum solution due to Erler (see Jokel 2017)

$$\Psi = T\frac{KB}{1-F}T + Q_{\mathrm{B}}(BT) \qquad [B,T] = F$$

Non-universal solutions

- Going beyond wedge based or KBc ansatz for the string field we add new elementary string fields particular to the BCFT in question

- Popular choices:
  K, B, c, J for exactly marginal deformations
  $K, B, c, \sigma, \bar{\sigma}$ for generic background changes
  $K, B, c, \sigma, \bar{\sigma}, \gamma, Q\sigma, Q\bar{\sigma}$ for the superstring

Exactly marginal solutions

- Solutions based on K, B, c, J

  When the perturbing operator dimension 1 operator has regular OPE $J(z)J(0) \sim \text{finite}$ the resulting solution can be readily found (MS; Kiermaier, Okawa, Rastelli, Zwiebach 2007)

$$\Psi = FcJ\frac{B}{1+\frac{1-F^2}{K}J}cF$$

  For the typical case $J(z)J(0) \sim \text{singular}$ one has to resort to subtraction procedure (KORZ 2007) or to use integrated vertex operators Fuchs, Kroyter, Potting 2007 or Maccaferri 2014, or much more complex Kiermaier Okawa 2009

Erler-Maccaferri solution

In 2014 Erler and Maccaferri searched for a solution in the form

$$\Psi = \Psi_{\rm tv} - \Sigma\Psi_{\rm tv}\overline{\Sigma},$$

Precursors: Kiermaier, Okawa, Soler 2010
MS 2001 and VSFT papers

where

$$Q_{\Psi_{\rm tv}}\Sigma = Q_{\Psi_{\rm tv}}\overline{\Sigma} = 0, \quad \overline{\Sigma}\,\Sigma = 1$$

guarantee that the e.o.m. are satisfied. Interestingly

$$\Psi_{\rm tv} = \sqrt{F}\left(c\frac{B}{H}c + B\gamma^2\right)\sqrt{F},$$

$$\begin{aligned}\Sigma &= Q_{\Psi_{\rm tv}}(\sqrt{H}\sigma B\sqrt{H}),\\ \overline{\Sigma} &= Q_{\Psi_{\rm tv}}(\sqrt{H}B\overline{\sigma}\sqrt{H}).\end{aligned}$$

give a solution provided $\overline{\sigma}\sigma = 1$

Erler-Maccaferri solution

- In the follow-up work Erler, Maccaferri and Noris (2019) argue that for $F(K) \sim K^\nu, \quad K \to \infty,$ various ambiguities disappear for $\nu < -1$ (superstring) or $\nu < 0$ (bosonic)

  To satisfy $\bar{\sigma}\sigma = 1$ one cannot use simply the boundary condition changing operators, EM dress them by e^{kX^0} factors to cancel the OPE divergences.

- Interesting application by Ishibashi, Kishimoto, Masuda, Takahashi (2018) for solutions describing constant magnetic flux on D-brane

Classical solutions in super OSFT

Classical solutions in super OSFT

- In 2013 there was a significant progress by Erler, who found the tachyon vacuum for Berkovits super-OSFT

$$S=\frac{1}{2g^2}\Big\langle\Big\langle(e^{-\Phi}Q_Be^{\Phi})(e^{-\Phi}\eta_0e^{\Phi})-\int_0^1 dt(e^{-t\Phi}\partial_te^{t\Phi})\{(e^{-t\Phi}Q_Be^{t\Phi}),(e^{-t\Phi}\eta_0e^{t\Phi})\}\Big\rangle\Big\rangle$$

where Φ is picture-number and ghost-number zero string field. This action can be also written as

$$S=-\int_0^1 dt\,\mathrm{Tr}[(\eta\Psi_t)\Psi_Q] \qquad \Psi_Q\equiv g(t)^{-1}Qg(t), \qquad \Psi_\eta\equiv g(t)^{-1}\eta g(t), \qquad \Psi_t\equiv g(t)^{-1}\partial_t g(t)$$

The equation of motion takes the form $\eta_0(e^{-\Phi}Q_Be^{\Phi})=0$

Classical solutions in super OSFT

- On a non-BPS D-brane the solution can be conveniently looked for in the basis K, B, c, γ and γ^{-1} suggested by Berkovits and M.S.

- Erler looked for solutions such that

$$\Psi = c\frac{1}{1+K} - Q\left(c\frac{B}{1+K}\right)$$

Corresponding g can be constructed by $g = Q_{0\Psi}\beta$

With a clever choice of β he found:

$$g = (1+\zeta)\left(1+Q\zeta\frac{B}{1+K}\right) \qquad g^{-1} = \left(1-Q\zeta\frac{B}{1+K+V}\right)(1-\zeta)$$

$$\zeta \equiv \gamma^{-1}c \qquad V \equiv \frac{1}{2}\gamma^{-1}\partial c$$

for which he could have computed the energy etc.

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Open problems

Open bosonic SFT

Find a well behaved anomaly-free solutions for multiple branes

Find a well behaved anomaly-free solution for a generic RG flow (cf. BMT 2010)

Open supersymmetric SFT

Closed bosonic SFT

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Open bosonic SFT

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Find a well behaved anomaly-free solution for a generic RG flow (cf. BMT 2010)

Open supersymmetric SFT

Find an analog of EM solution

Resolve the controversy surrounding the marginal deformations for the D0/D4 system (see talk by Ivo Sachs)

Closed bosonic SFT

Closed supersymmetric SFT

Open problems

Open bosonic SFT	Open supersymmetric SFT
Find a well behaved anomaly-free solutions for multiple branes Find a well behaved anomaly-free solution for a generic RG flow (cf. BMT 2010)	Find an analog of EM solution in Berkovits theory Resolve the controversy surrounding the marginal deformations for the D0/D4 system (see talk by Ivo Sachs)
Closed bosonic SFT Find an analog of Sen's conjecture for the closed string and construct good observables. Understand action of defects. Are we confined to Witten vertex?	**Closed supersymmetric SFT**