Hadronic Form Factors

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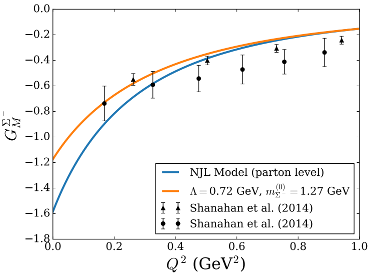
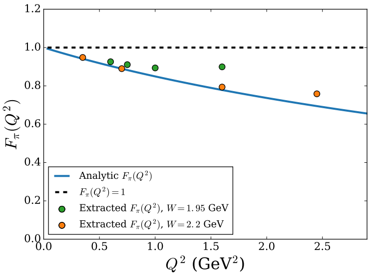
Talk Outline

- Motivation: Bridging QCD and low energy hadronic properties.
- Introduction to Form Factors
- Chiral Corrections to Baryon Electromagnetic Form Factors, arXiv:1703.01032
- Model Dependence of the Pion Form Factor Extracted from Pion Electro-production, arXiv:1811.09356
- Conclusion.





The QCD Lagrangian

- Lagrangian useful for understanding symmetries etc

$$\mathcal{L}_{\mathrm{QCD}} = \overline{q}(i\not{D} - m)q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Non-abelian, $SU(3)$ gauge field theory.
- Many questions remain.
- Masses of light quarks $\mathcal{O}(5\ \mathrm{MeV})$
- Lightest baryon ~ 1 GeV (approx. 2 orders of magnitude larger!)
- Where does this nucleon mass come from?
  - Emergent property of QCD.
  - Dynamical generation of mass contributes more than 95 percent of hadronic mass.

Strongly Coupled Physics

- In low energy region, QCD coupling runs, and theory becomes non-perturbative.
- Strongly coupled theory leads to emergent behavior.
  - Dynamical chiral symmetry breaking and mass generation.
  - Confinement.

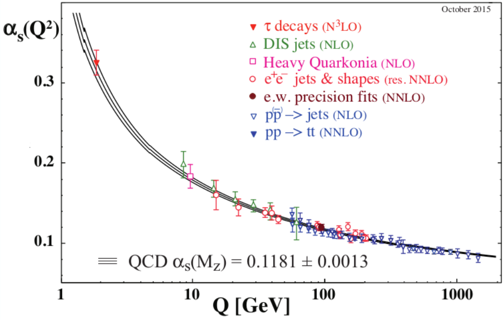


Figure 1: PDG, 2015

Open Questions Remain

The Central Goal of Hadronic Physics

- Central Goal of theoretical hadronic physics: Bridging the gap between $\mathcal{L}_{\text{QCD}}$ and observed hadronic properties.

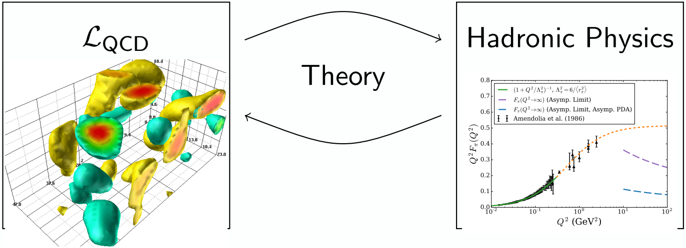


Figure 2: Image of gauge field configuration taken from J. Charvetto.

- Quark models, Chiral EFT, Lattice QCD, Schwinger-Dyson Equations.

Electromagnetic Form Factors

Historical Perspective

- Form factor introduced in 50's to explain proton scattering data.
- Introduce charge density $\rho(\vec{r})$.
- Form factor proportional to Fourier Transform of charge density (in NR limit): Extended structure.

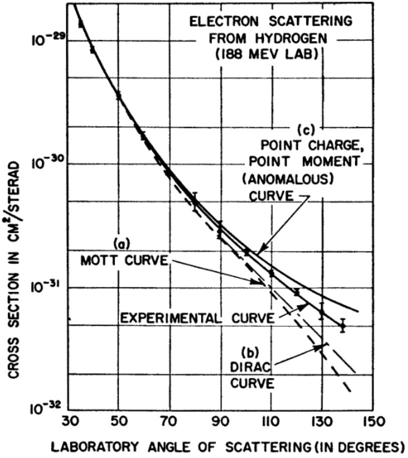
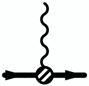


Figure 3: Figure taken from R. Hofstadter et al. (1958))

Baryon Electromagnetic Form Factors

- Contain information about the structure of the baryon.



$$= \overline{u}(p')\Gamma^{\mu}(p',p)u(p) = \overline{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

- $Q^2 = -q^2$
- Common to use the Sachs Parametrisation.

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- 3D charge Radius for $i = E, M$

$$\left\langle r^2 \right\rangle = -\frac{6}{G_i(0)}\frac{d}{dQ^2}G_i(Q^2)\Big|_{Q^2=0}$$

- Magnetic moment for $i = p, n$ (units of μ_N)

$$\mu_i = G_M^i(0)$$

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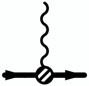

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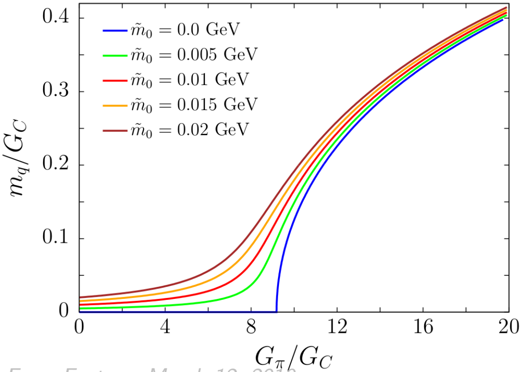
Calculating Electromagnetic Form Factors in the Nambu–Jona-Lasinio (NJL) Model

The Nambu–Jona-Lasinio (NJL) Model

- Low energy approximation of QCD: 4 fermion contact interaction

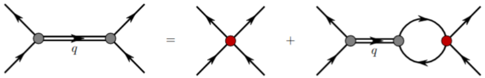
$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - \hat{m})\psi + \frac{1}{2}G_\pi\left[(\overline{\psi}\psi)^2 - (\overline{\psi}\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{2}G_\omega(\overline{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}G_\rho\left[(\overline{\psi}\gamma^\mu\lambda_i\psi)^2 + (\overline{\psi}\gamma^\mu\gamma_5\lambda_i\psi)^2\right]$$

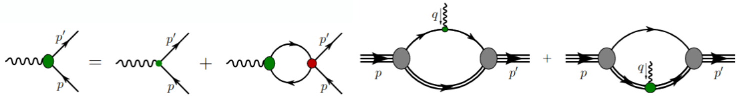


- Confinement failure of basic model, but imposed via Proper Time Regularisation & infra-red cutoff.

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau X}$$

- Prevents singularities in the spectrum from on-shell quarks $\Longrightarrow$ confinement
- Calculate BSE equation





Chiral Corrections to Baryon Electromagnetic Form Factors

arXiv:1703.01032

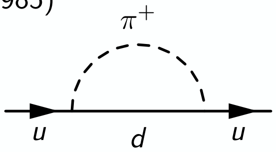
Pionic Corrections

- Long known that pion required dof in quark model calculations.
- Modern understanding of the pion as a pseudo-Goldstone Boson.
  - Result of dynamical chiral symmetry breaking.
- Formalized framework: χPT.
- Long distance (IR) properties are same as UV theory.
  - Must be respected in any model of QCD.
- A variety of ways to incorporate their effects.
  - Previously calculated in the NJL Model as a dressing on quark propagator.

Incorporating Pion Effects

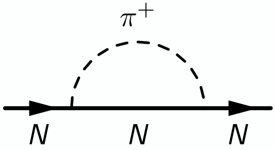
Quark Level

- Calculate pion effects from quark-pion coupling
- Idea goes back to Manohar and Georgi: *Chiral Quarks and the Non-Relativistic Quark Model* (1985)



Hadron Level

- Calculate pion loop corrections in (chiral) nucleon-pion EFT.



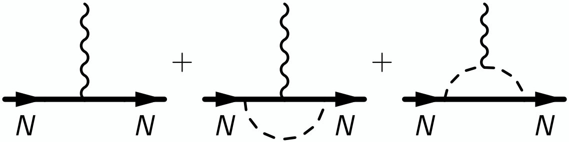
- Take guidance from χPT
- Correct LNA behvaior of nucleon mass only obtained in hadron level approach. (Model independent)
- We can examine the differences between the two approaches.

Pion-Nucleon Effective Field Theory

- Use chiral EFT.
- Work with a pseudoscalar pion-nucleon interaction:

$$\mathcal{L}_{N\pi} = -ig_{\pi N}\overline{\psi}_N\gamma_5\vec{\tau}\cdot\vec{\pi}\psi_N$$

- After minimal substitution, one has three diagrams at first loop order.



$$F_i^H(Q^2) = Z\left[F_{i,a}^H(Q^2) + F_{i,b}^H(Q^2) + F_{i,c}^H(Q^2)\right]$$

Baryon Self Energy

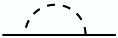
Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops



$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

- Must fit NJL model parameters to Bare Mass
- Related to physical mass via

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p} = m_N}$$

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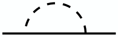

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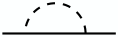 $= \frac{iZ}{\not{p} - m_N + i\epsilon}$

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Nucleon Results

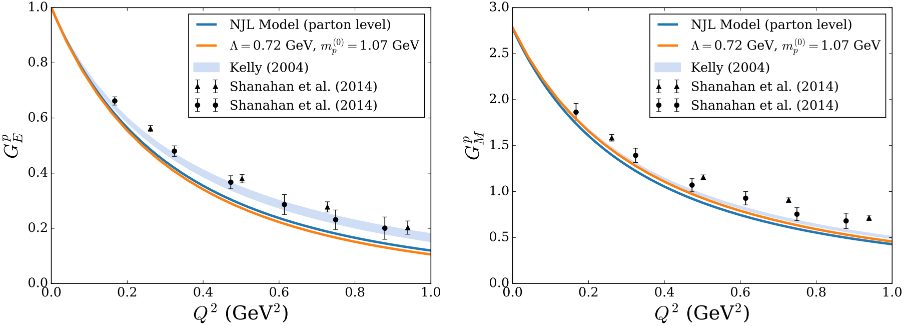


Figure 5: G_E^p and G_M^p. Data from lattice studies

- Similar!

Nucleon Results

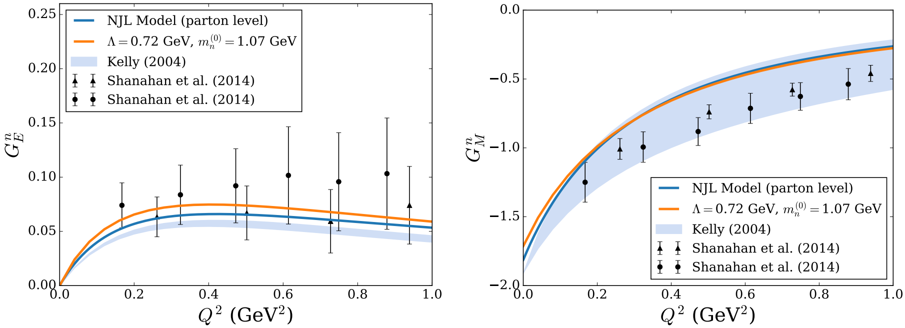


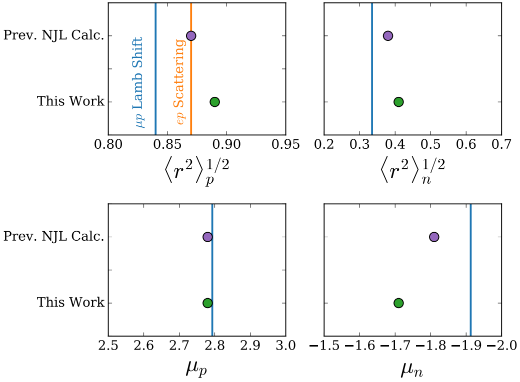
Figure 6: G_E^n and G_M^n. Data from lattice studies

- Similar!

Comparison

$\langle r^2 \rangle^{\frac{1}{2}}$	p	n
Prev. NJL Calc.	0.87	0.38
This Work	0.89	0.41
Exp.	0.84 [3]	0.335

μ	p	n
Prev. NJL Calc.	2.78	-1.81
This Work	2.78	-1.71
Exp.	2.793	-1.913



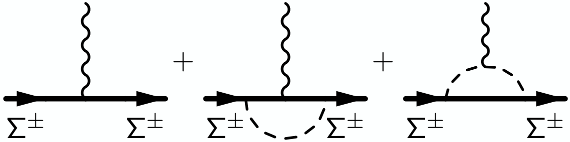
Generalizing Results to Hyperons

Generalization to Hyperons

- Due to approximate $SU(3)_F$ symmetry, one has relations between nucleon-pion and hyperon-pion couplings.

$$g_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}}(1-\alpha)g_{NN\pi}; \qquad g_{\Sigma\Sigma\pi} = 2\alpha g_{NN\pi}$$

- Although the particles themselves are different, topology of contributing diagrams are the same.



- Simple replacements in equations allows generalization of the equations to consider the hyperons.

Hyperon Results

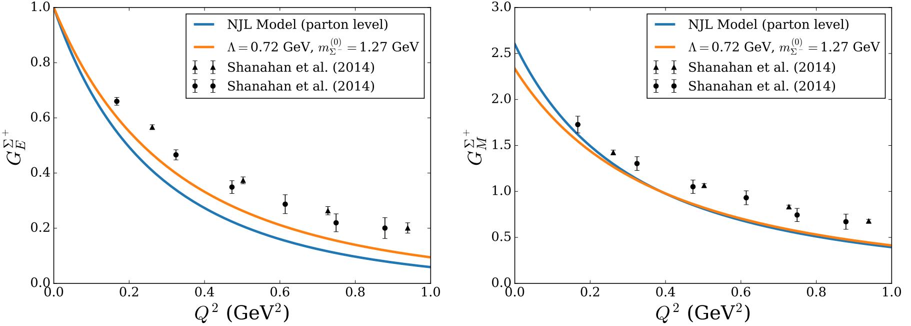


Figure 7: $G_E^{\Sigma^+}$ and $G_M^{\Sigma^+}$, data from lattice studies.

Hyperon Results

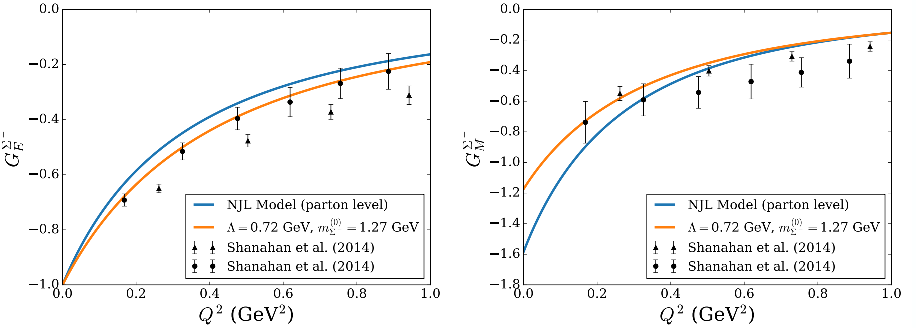
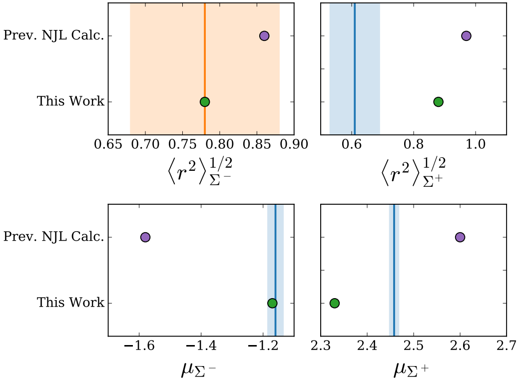


Figure 8: $G_E^{\Sigma^-}$ and $G_M^{\Sigma^-}$, data from lattice studies.

	$\langle r^2 \rangle^{\frac{1}{2}}$ Σ^-	Σ^+
Prev. Calc	0.86	0.97
This Work	0.78	0.88
Exp.	0.780	0.61(8) [4]

	μ Σ^-	Σ^+
Prev. Calc.	-1.58	2.60
This Work	-1.17	2.33
Exp.	-1.160(25)	2.458(10)



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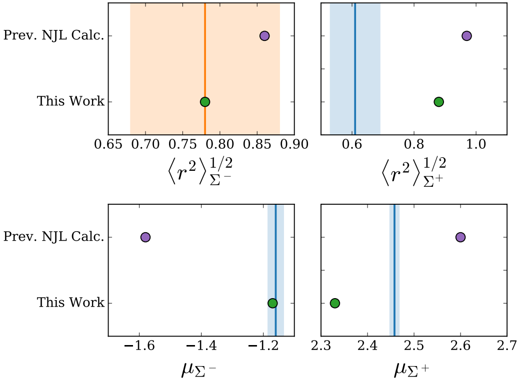

Quark Model

- Large contribution comes from π^- cloud on d quark ($e_{\pi^-} = -1$, $e_d = -1/3$).
- Sigma minus:

$$\left|\Sigma^-\right\rangle = \frac{1}{\sqrt{18}}[2\left|d_\uparrow d_\uparrow s_\downarrow\right\rangle + \text{perm.} - \left|d_\uparrow d_\downarrow s_\uparrow\right\rangle + \text{perm.}]$$

- Leads to coherent enhancement.

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- Leads to coherent enhancement.

Summary

To summarize...

- χPT gives model independent information on IR physics.
- Calculated chiral loop corrections to the NJL model at the Hadron Level.
- Nucleon system insensitive to approach, but
- Hyperon system sensitive to implementation of pion loops: improvement of Σ^- magnetic moment.

The Pion Electromagnetic Form Factor

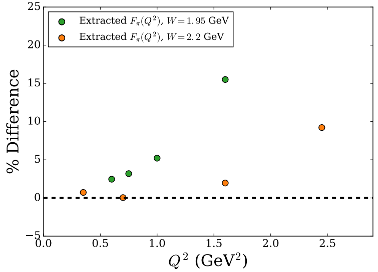


Figure 9: F_π extracted from simple model of pion electro-production.

The pion is special

- χPT $\implies$ important for low energy hadronic physics.
- Simplest QCD system: 'Hydrogen Atom of QCD': Excellent testing ground.
- Form factor spans large energy range: forces us to use a number of approaches.
- Must understand the model used to extract the form factor well.
  - Based on some theoretical arguments, we wanted to check the model dependence of the extracted pion form factor.

Theoretical Approaches

Low Energy

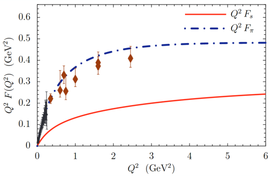


Figure 10: Cloët et al. (2014)

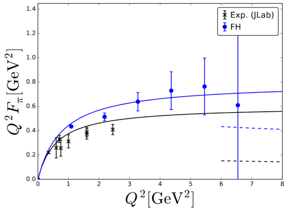


Figure 11: FH approach, $m_\pi = 470$ MeV, Chambers et al. (2017)

High Energy

Lepage and Brodsky:

$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)\omega_\phi^2, \text{ for } Q^2 > Q_0^2$$

Historically, $\omega_\phi = 1$.

$$\lim_{Q^2 \to \infty} \phi_\pi(x) = 6x(1-x)$$

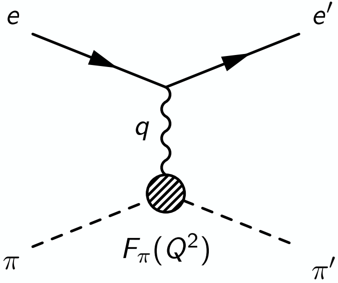
More recently (Chang et al., 2013):

$$\omega_\phi = \frac{1}{3}\int_0^1 dx \frac{1}{x}\phi_\pi(x)$$

Measuring the pion form factor

Experimental Measurements

- At low energy (~ 0.3 GeV^2), scatter pion beam from electrons in liquid hydrogen target.
- Measure recoiling pion and electron.

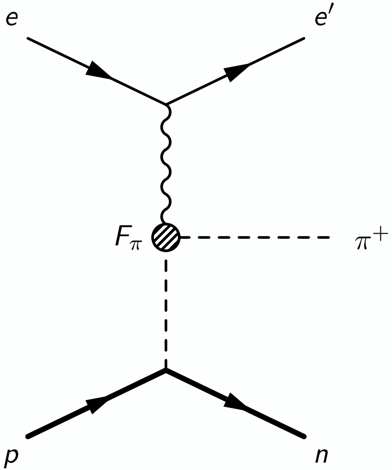


- Differential cross section is

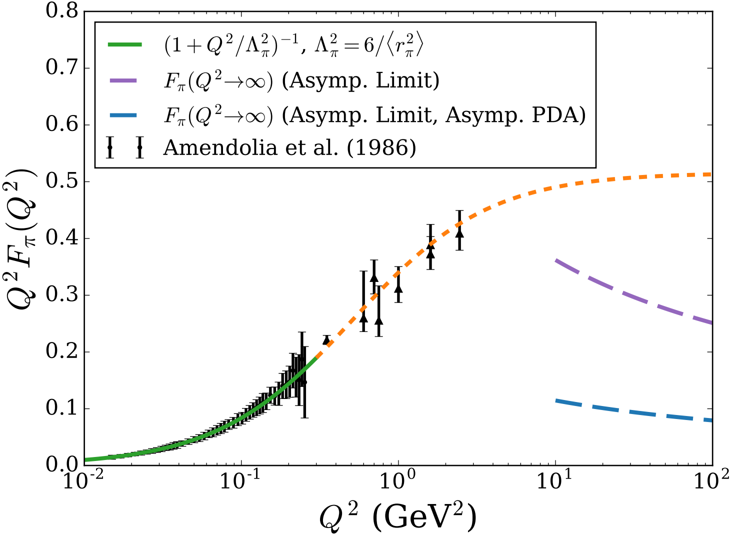
$$\frac{d\sigma}{dq^2} \propto |F_\pi|^2 \frac{1}{q^4}\left(1 - \frac{q^2}{q^2_{\text{max}}}\right)$$

Pion Electro-production

- Scatter electron off liquid hydrogen target.
- Knock pion out of nucleon's virtual meson cloud.
- Measure recoiling electron and produced pion.
- Two theoretical questions:
  1. How does F_π enter cross section?
  2. How does the 'off-shellness' effect the measurement of F_π?
- Must understand how extraction is currently performed.



The State of the Art



Understanding the F_π Measurement

PHYSICAL REVIEW C **78**, 045202 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. I. Measurements of the cross section for the ^{1}H$(e, e'\pi^+)n$ reaction

H. P. Blok,[1,2] T. Horn,[3,4] G. M. Huber,[5] E. J. Beise,[3] D. Gaskell,[4] D. J. Mack,[4] V. Tadevosyan,[6] J. Volmer,[7,8] D. Abbott,[4] K. Aniol,[9] H. Anklin,[4,10] C. Armstrong,[11] J. Arrington,[12] K. Assamagan,[13] S. Avery,[13] O. K. Baker,[4,13] B. Barrett,[14] C. Bochna,[15] W. Boeglin,[10] E. J. Brash,[5] H. Breuer,[3] C. C. Chang,[3] N. Chant,[3] M. E. Christy,[13] J. Dunne,[4] T. Eden,[4,16] R. Ent,[4] H. Fenker,[4] E. Gibson,[17] R. Gilman,[4,18] K. Gustafsson,[3] W. Hinton,[13] R. J. Holt,[12] H. Jackson,[12] S. Jin,[19] M. K. Jones,[11] C. E. Keppel,[4,13] P. H. Kim,[19] W. Kim,[19] P. M. King,[3] A. Klein,[20] D. Koltenuk,[21] V. Kovaltchouk,[5] M. Liang,[4] J. Liu,[3] G. J. Lolos,[5] A. Lung,[4] D. J. Margaziotis,[9] P. Markowitz,[10] A. Matsumura,[22] D. McKee,[23] D. Meekins,[4] J. Mitchell,[4] T. Miyoshi,[22] H. Mkrtchyan,[6] B. Mueller,[12] G. Niculescu,[24] I. Niculescu,[24] Y. Okayasu,[22] L. Pentchev,[11] C. Perdrisat,[11] D. Pitz,[25] D. Potterveld,[12] V. Punjabi,[16] L. M. Qin,[20] P. Reimer,[12] J. Reinhold,[10] J. Roche,[4] P. G. Roos,[3] A. Sarty,[14] I. K. Shin,[19] G. R. Smith,[4] S. Stepanyan,[6] L. G. Tang,[4,13] V. Tvaskis,[7] R. L. J. van der Meer,[5] K. Vansyoc,[20] D. Van Westrum,[26] S. Vidakovic,[5] W. Vulcan,[4] G. Warren,[4] S. A. Wood,[4] C. Xu,[5] C. Yan,[4] W.-X. Zhao,[27] X. Zheng,[12] and B. Zihlmann[4,28]
(Jefferson Lab F_π Collaboration)

PHYSICAL REVIEW C **78**, 045203 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. II. Determination of, and results for, the pion form factor

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(Jefferson Lab F_π Collaboration)

Decomposing the Cross Section

- Cross section described in terms of
  - Q^2: photon virtuality.
  - W: Invariant mass of virtual photon proton system.
  - $t = (p_\pi - q)^2$: Expresses virtuality of pion.
- Cross section may be decomposed into 4 structure functions.

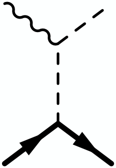
$$(2\pi)\frac{d^2\sigma}{dtd\phi} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi.$$

ϵ is a measure of the virtual photon polarization

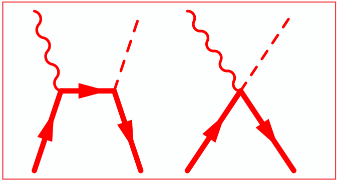
- Important, as is known that Longitudinal cross section dominated by t-channel pion exchange.
  - A good reconstruction of this structure function gives us a good change of extracting the pion form factor.
  - The modern extraction uses the Vanderhaeghen Guidal and Laget (VGL) Model.

Born Term Model of Electro-production

- VGL Model based on Born Term pion exchange diagram:



(a) t-channel pion exchange



(b) Required to restore gauge invariance

- Model is not gauge invariant, so one must include the s-channel diagram and KR term (when using a PV coupling) to restore gauge invariance.
- This is $i\mathcal{M}^{\mu}_{\mathrm{BTM}}$

Gauge Invariance in BTM Model

- WTI requires $q_\mu \mathcal{M}^\mu_{\mathrm{BTM}} = 0$

$$
\begin{aligned}
iq_\mu \mathcal{M}^\mu_{\mathrm{BTM}} \propto & \bar{u}_N \left[\gamma_5 \not{p}_\pi \frac{(\not{p}_s + m_N)}{s - m_N^2} \not{q} + \gamma_5 \not{p}_t \frac{q \cdot (p_t + p_\pi)}{t - m_\pi^2} - \gamma_5 \not{q} \right] u_N \\
\propto & \bar{u}_N \gamma_5 \left[\not{p}_\pi - (\not{p}_\pi - \not{q}) - \not{q} \right] u_N
\end{aligned}
$$

- Delicate cancellation required.
- Limits the ways we can modify this amplitude.

Improving Agreement with Data: Reggeizing Amplitude

- Agreement between the model and data may be improved by Reggeizing the amplitude.
  - Replace the Feynman Propagator for the t-channel pion exchange by its Reggeized version

$$S_{\mathrm{R}}^{\pi}(t) = i(\alpha'_{\pi} W^2)^{\alpha_{\pi}(t)} \frac{\pi \alpha'_{\pi} \phi(t)}{\sin(\pi \alpha_{\pi}(t) \Gamma(1 + \alpha_{\pi}(t)))}$$

- Unless the s-channel and contact terms are also modified, gauge invariance will be broken again.
- This is done in the VGL Model by multiplying these terms by a factor $S_F^{\pi-1}(t) S_{\mathrm{R}}^{\pi}(t)$
- One can also understand this Reggeization as multiplication of $i\mathcal{M}^{\mu}_{\mathrm{BTM}}$ by this overall factor:

$$i\mathcal{M}^{\mu}_{\mathrm{R}} = S_F^{\pi-1}(t) S_{\mathrm{R}}^{\pi}(t) \times [i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$$

The VGL Model

- The pion structure is incorporated my multiplying this amplitude by a factor of the pion form factor.
- To summarize:

$$\begin{aligned} i\mathcal{M}^{\mu}_{\text{VGL}} =& F_{\pi}(Q^2) \times [i\mathcal{M}^{\mu}_{\text{R}}] \\ =& F_{\pi}(Q^2) \times S_F^{\pi\,-1}(t) S^{\pi}_{\text{R}}(t) \times [i\mathcal{M}^{\mu}_{\text{BTM}}] \end{aligned}$$

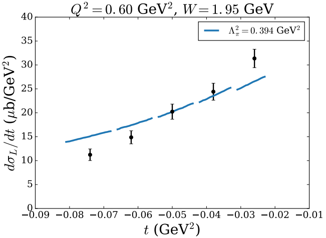
- In theory, one would expect s-channel diagram to be proportional to $F_1^p(Q^2)$, but this breaks gauge invariance.
- Only possible to have single form factor. Amounts to $F_1^p(Q^2) \approx F_{\pi}(Q^2)$.

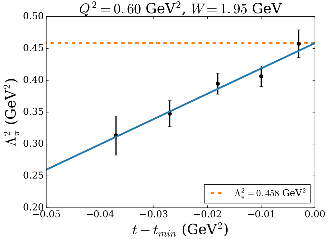
Extracting Pion Form Factor from Data

- Measure cross section at a range of t values for fixed Q^2 and W.
- Longitudinal cross section is

$$\frac{d\sigma_L}{dt} \propto |F_\pi|^2$$

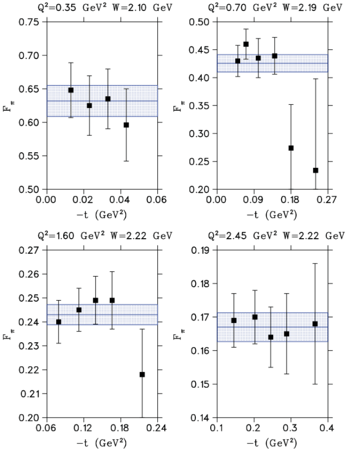
- $F_\pi(Q^2) = (1 + Q^2/\Lambda_\pi^2)^{-1}$
- Fit model to cross section.
- If required...
  - Fit each data point.
  - Extrapolate these points to $t = t_{\mathrm{min}}$, where there is least contamination from interfering backgrounds not included in the VGL model.





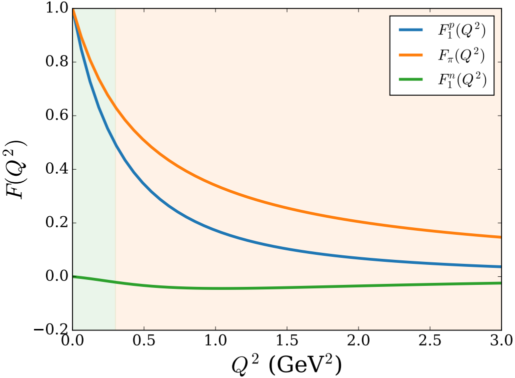
Sanity Checks of Extraction

- Clearly some simplifications in this model.
- How do we know we are extracting the pion form factor?



Key Questions

1. Currently, $F_1^p = F_\pi$: can we do better?
2. Can we incorporate the 'off-shellness' of particles?
3. What are the implications for the current measured data points?



Model Dependence of the Pion Form Factor Extracted from Pion Electro-production Data

arXiv:1811.09356

Our Approach

1. ~~Currently, form factors are all the same: can we do better?~~
2. ~~Can we incorporate the ‘off-shellness’ of particles?~~
3. What are the implications for the current measured data points?
   - Generate cross section in model (pseudodata), and then attempt to extract form factor using VGL-like Model.

	Current Extraction	This Analysis
Model	$i\mathcal{M}^{\mu}_{\mathrm{VGL}} = F_{\pi}(Q^2) S_F^{\pi -1} S_{\pi}^{\mathrm{R}}(t)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$	$i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^{1}\mathrm{H}(e, e'\pi^{+})n$	Pseudodata

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	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^{1}\text{H}(e, e'\pi^{+})n$	Pseudodata

A Bosonic Model of Pion Electro-production

- Inspired by a simple model due to Miller.

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Electromagnetic form factors and charge densities from hadrons to nuclei

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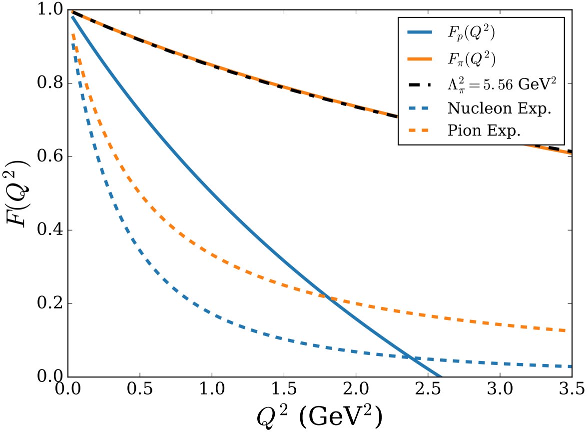
(Received 18 August 2009; published 22 October 2009)

A simple exact covariant model in which a scalar particle Ψ is modeled as a bound state of two different particles is used to elucidate relativistic aspects of electromagnetic form factors $F(Q^2)$. The model form factor is computed using an exact covariant calculation of the lowest order triangle diagram. The light-front

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_N)^2 - \frac{1}{2} m_N^2 \Psi_N^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 \\ - g_{\pi N} \Psi_N^\dagger \boldsymbol{\tau} \cdot \boldsymbol{\pi} \Psi_N$$

Form Factors in Simple Model



- F_π well described by monopole.

Our Approach

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3. What are the implications for the current measured data points?
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	Current Extraction	This Analysis
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	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^{1}\mathrm{H}(e, e'\pi^{+})n$	$i\mathcal{M}^{\mu}_{\text{1-Loop}}$

Pros and Cons

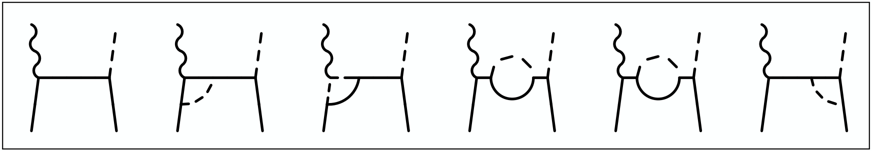
Pros

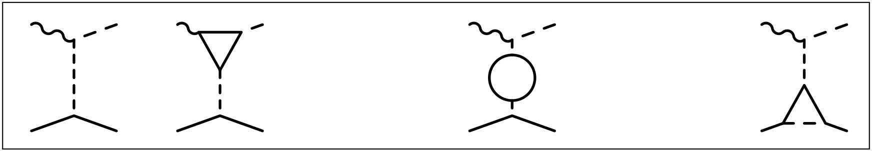
- Perturbative calculation: gauge invariant.
- Calculate to 1-loop order: obtain (different) form factors at vertices.
- Simple.

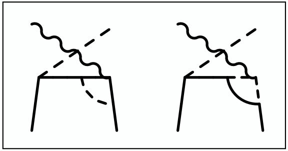
Cons

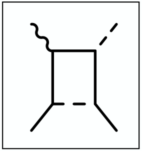
- Perturbative calculation doesn't generally give form factors enough q^2 dependence.
- Connection to QCD is tenuous
  - Prevents quantitative conclusions.

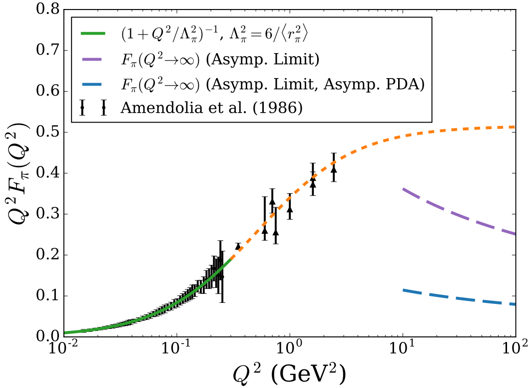
Generating Pseudodata









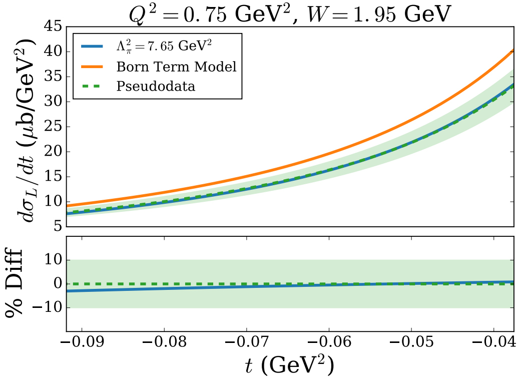


Q^2 (GeV^2)	W (GeV)
0.35	2.10
0.60	1.95
0.70	2.19
0.75	1.95
1.00	1.95
1.60	1.9
1.60	2.22
2.45	2.22

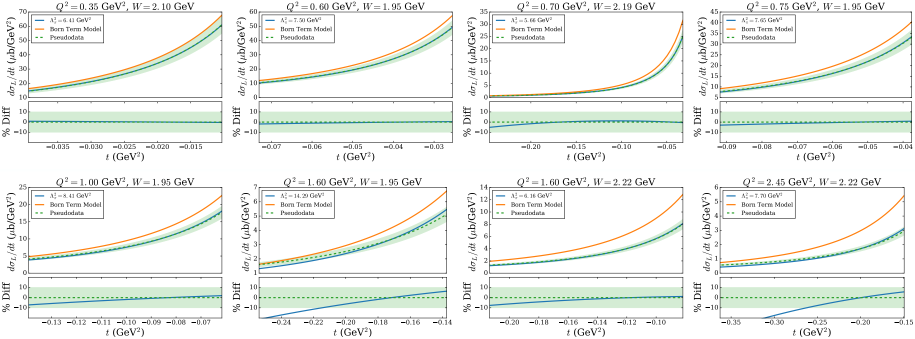
Pseudodata: A Specific Example

- Pseudodata: $i\mathcal{M}^{\mu}_{\text{1-Loop}}$
- Model:
  $i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\text{BTM}}]$

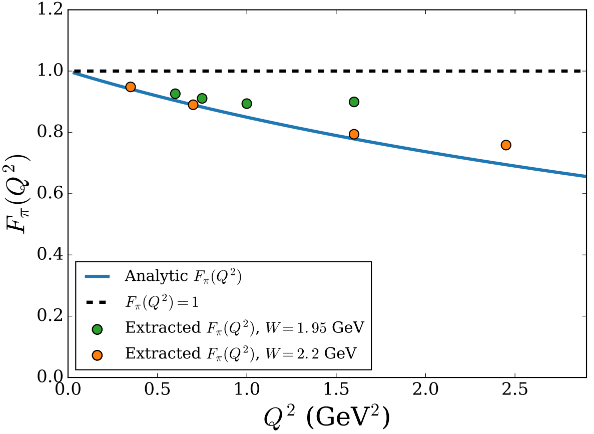
  $F_{\pi}(Q^2) = (1+Q^2/\Lambda_{\pi}^2)^{-1}$
- t range chosen to be same as experiment.



Cross Section

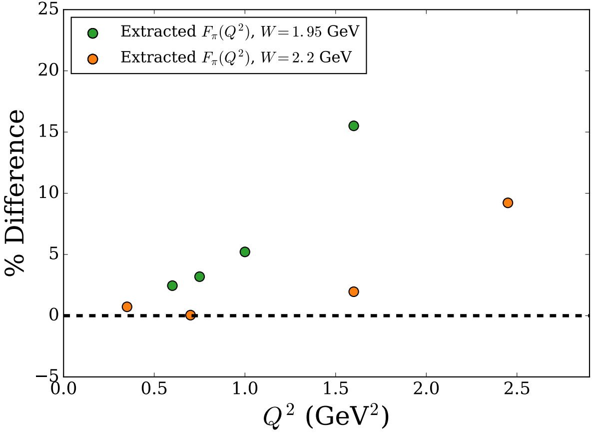


Results



- Apart from possibly point at $(Q^2, W) = (1.6, 1.95)$, results look ok.

Systematic Overestimate?



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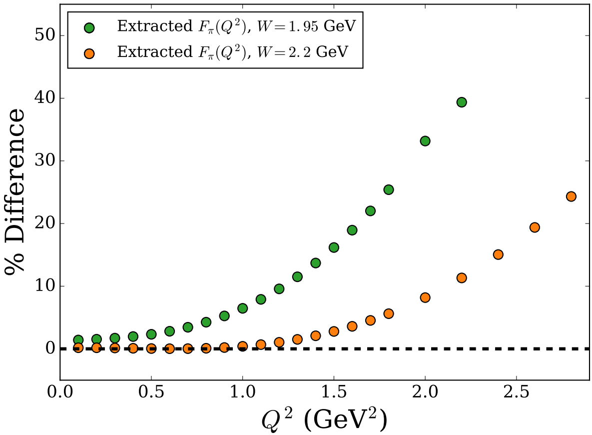
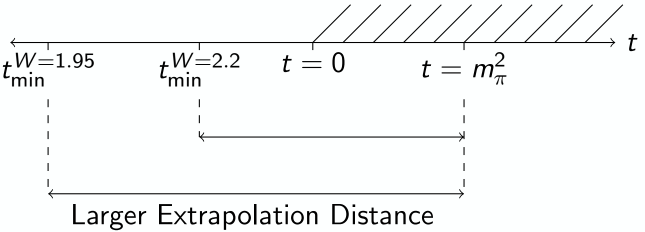


- Fit first 5% of allowed t.

How Do We Understand the W Dependence?

- A kinematic argument. Ideally, we would measure this process at $t = m_\pi^2$.
- $t < 0$ for electro-production.
- t_{min} more negative for increasing Q^2
- t_{min} more negative for decreasing W
- Larger W at the same Q^2 will allow a smaller (negative) $|t_{\text{min}}|$
- Closer to the pion pole. So interpretation of F_π as pion form factor better.



Conclusion

- Pion electro-production allows us to measure the pion form factor at higher Q^2.
- We tested extraction method in simple model.
- Results seem to imply a reasonably accurate extraction is possible, except at certain kinematics.
- Important to choose kinematics wisely to minimize extrapolation to pion pole.

Further Work

- Model extremely simple. A more complicated calculation including fermions is underway.
- Lattice QCD:

$$\left\langle N(p')\pi(p_\pi)\right| J^\mu(q)\left|N(p)\right\rangle$$

$$\left\langle \pi(k')\right| J^\mu(q)\left|\pi(k)\right\rangle = (k+k')F_\pi(Q^2)$$

Thanks