

Hyperbolic vacuum decay

Hristu Culețu

Ovidius University, Romania

Introduction

- Does the vacuum energy (the energy of quantum fluctuations of empty space) gravitate?
- Its gravitational effects - the cosmological constant Λ - 120 orders of magnitude larger than what observed cosmologically.
- the trouble comes from the mixing of scales (S. Carlip, arXiv: 1905.05216; 1809.08277).
- Λ is generated at Planck scale but observed at cosmological scale.
- "Spacetime foam" picture (J. A. Wheeler, PRD 97, 511 (1955)) \rightarrow nontrivial topological structure of the spacetime at the Planck scale (WHS, domain walls, monopoles).
- "Minkowski WH geometry" - a model for topological fluctuations at Planck scale. (I. Redmount and W.M.-Suen, PRD 47, R 2163 (1993); arXiv: gr-qc/9210014) - their WH-unstable against growth to macroscopic size.

Equation of motion of the throat

- we take a slightly modified version of the dynamic RS wormhole, viewed as an expanding "bubble" with a time-dependent mass.
- Minkowski WFF - by excising a sphere of radius $r = R(t)$ from two copies of the Minkowski space and then identifying the two boundaries. Σ (the WFF throat).

The Lanczos equations (M. Visser,
PRD 39, 3182 (1989))

$$[K_{ij}^i] - \delta_{ij}^i [K_{\ell\ell}^\ell] = -8\pi S_{ij}^i$$

$$i,j = r, \theta, \varphi; [K_{ij}^i] \approx K_{ij}^r - K_{ij}^r = 2K_{ij}^r$$

The metric on the surface

$$ds_\Sigma^2 = -dt^2 + R^2 d\Omega^2,$$

$$\text{where } dt = \sqrt{1 - \dot{R}^2} dt, \dot{R} = dR/dt.$$

The velocity 4-vector

$$u^b = \left(\frac{1}{\sqrt{1 - \dot{R}^2}}, \frac{\dot{R}}{\sqrt{1 - \dot{R}^2}}, 0, 0 \right)$$

and the unit normal to Σ

$$n_b = \left(-\frac{\dot{R}}{\sqrt{1 - \dot{R}^2}}, \frac{1}{\sqrt{1 - \dot{R}^2}}, 0, 0 \right),$$

$$\text{with } n^b n_b = 1, n^b u_b = 0; u^b u_b = -1$$

The 2-nd fundamental form of Σ

$$K_{ij} = \frac{\partial R^a}{\partial x^i} \cdot \frac{\partial x^b}{\partial x^j} \nabla_a n_b,$$

where ' ∇ ' is applied in Minkowski Space in 4 dimension. One obtains

$$K_{rr} = -\frac{\ddot{R}}{(1-\dot{R}^2)^{3/2}}, \quad K_{\theta\theta} = \frac{R}{\sqrt{1-\dot{R}^2}}$$

$$K \equiv K_{ii} = \frac{\ddot{R}}{(1-\dot{R}^2)^{3/2}} + \frac{2}{R\sqrt{1-\dot{R}^2}}$$

whence

$$S_{rr} = -\frac{1}{2\pi R\sqrt{1-\dot{R}^2}}, \quad S_{\theta\theta} = \frac{R}{4\pi\sqrt{1-\dot{R}^2}} + \frac{R^2\ddot{R}}{k\pi(1-\dot{R}^2)^{3/2}}$$

Take $S_{ij} = (\rho_s + \sigma) u_i u_j + p_s h_{ij}$, with
 $h_{ij} = \text{diag}(-1, R^2, R^2 \sin^2 \theta)$. We have

$$\sigma = S_{rr}, \quad p_s = S_{\theta\theta}/R^2.$$

RS equation of state : $\sigma = -k\rho_s$. Our choice
 $p_s = -\sigma$, as for a domain wall. (S. J. Gers, P. Sikivie, PRD 30, 712 (1984)) - an appropriate choice for a Lorentz-invariant vacuum. Hence

$$R\ddot{R} + \dot{R}^2 - 1 = 0,$$

which gives the solution

$$R(t) = \sqrt{t^2 + b^2}, \quad R(0) = b$$

Free particle energy

From the previous equations

$$\sigma = -\beta_s = -\frac{1}{2\pi b} = \text{const.}$$

and

$$1 - \dot{R}^2 = \frac{b^2}{R^2}, \quad \ddot{R} = \frac{b^2}{R^3}$$

The normal acceleration of the throat

$$A_n = v_b A^b = v_b (v^a \nabla_a v^b) = -K_{rr} = -2\pi\sigma = \frac{1}{b},$$

as for IS planar domain wall.

- the throat turns out to play the role of a de Broglie pilot wave, dragging the null particles with it (H.C., Phys. Scripta 90, 12, (2015); arXiv: gr-qc/1407.3588)

The corresponding action

$$S = - \int \frac{b^2}{R} \sqrt{1 - \dot{R}^2} dt,$$

with the Lagrangian

$$L = -\frac{b^2}{R} \sqrt{1 - \dot{R}^2}$$

When $\dot{R} \ll 1$, L acquires the form

$$L \approx -\frac{b^2}{R} \left(1 - \frac{\dot{R}^2}{2}\right) = \frac{M v^2}{2} - M c^2$$

where $v(t) = \dot{R}$, $M(t) = \frac{b^2}{R(t)}$.

$M(t)c^2$ - as a "rest" potential energy.

The canonical momentum

$$p = \frac{\partial L}{\partial \dot{R}} = \frac{M v}{\sqrt{1 - v^2}},$$

which yields the Hamiltonian

$$H = \frac{1}{2} \dot{R} - L = \frac{M}{\sqrt{1-v^2}} = \sqrt{p^2 + M^2}$$

Inserting all fundamental constants

$M(t) = \frac{\hbar}{c R(t)}$, whence $R(t)$ — as the Compton wavelength associated to $M(t)$
For $t \gg b$, $R(t) \approx t$ and $M c^2 t = k$ — an uncertainty relation.

WH geometry — a spacetime foam structure unstable against growth to macroscopic size.

When $R(t) = \sqrt{t^2 + b^2}$, we get $H = \hbar = E_{\text{Planck}}$
We have $p(t)$, $M(t)$, but $H = \text{const.}$

The cosmological constant

WH embedded in a de Sitter space.

deS-Schwarzschild junction conditions:

BGG, PRD 35(6), 1447 (1987).

We take again $p_s = -\sigma > 0$ and, from the Gauss-Codazzi equations (for Koe):

$$2\sqrt{1-x^2 R^2 + R'^2} = -4\pi\sigma R,$$

where $R' = dR/dx$, $x^2 = \Lambda/3$, and $\Lambda > 0$.

One obtains $R(x) = b \cosh(x/b)$, with $R(0) = b$, provided

$$b^2(x^2 + g^2) = 1,$$

with the acceleration $g = 2\pi/\sigma$, $x, g \leq 1/b$

The WH throat metric will be

$$ds^2_{\Sigma} = -dx^2 + b^2 \cosh^2 \frac{x}{b} d\Omega^2,$$

which is the closed deS metric in three dimensions.

It acquires Planck values when $g \ll 1/b$.

Let us express now the throat radius in terms of the coordinate time t .

We have now $r = R(t)$ and from the static deS metric

$$\frac{dt}{dx} = \frac{\sqrt{1-x^2 R^2 + R'^2}}{1-x^2 R^2}$$

whence

$$\frac{x}{g} \sinh \frac{x}{b} = \tanh xt$$

that yields

$$R(t) = b \sqrt{1 + \frac{g^2}{x^2} \tanh^2 xt}$$

If we take the limit $x \rightarrow 0$ in the above equation, one obtains

$$R(t) = b \sqrt{1 + g^2 t^2}$$

But $x \rightarrow 0$ gives $g \rightarrow 1/b$ and so the Minkowski case is recovered.

Because of the horizon, we must have $R < 1/x$. Using also $x < 1/b$, we get $R_{\text{max}} = -(1+bx)/2x$.

Check now the energy constraint equation, from the 3+1 decomposition of Einstein's equations

$${}^3R - K^2 + K_{ij}K^{ij} - 2\Lambda = 0$$

(the matter contribution is overlooked w.r.t. Λ). Due to $R(\epsilon) = b \cosh(\epsilon/b)$,

$$K_\epsilon^2 = \frac{R'' - \epsilon^2 R}{\sqrt{1 - \epsilon^2 R^2 + R'^2}} = g; \quad K_0^0 = \frac{\sqrt{1 - \epsilon^2 R^2 + R'^2}}{R} = g,$$

whence $K = 3g$ and $K_{ij}K^{ij} = 3g^2$. We have also ${}^3R = 6/b^2$. We found that $\Lambda = 3(1/b^2 - g^2)$, so that the energy constraint equation is satisfied.

In addition

$$\sigma_{ij} = K_{ij} - \frac{1}{3} \delta_{ij} K = 0; \quad \Theta = K = 3g$$

Conclusions

- we investigated a dynamical WH, embedded in:
 - Minkowski space
 - de Sitter space.
- in the both situation: $p_s + \epsilon = 0$, as equation of state on the throat (as for the IS domain wall).

$$\left. \begin{aligned} R(t) &= \sqrt{t^2 + b^2} \quad (\text{Minkowski}) \\ R(t) &= b \sqrt{1 + \frac{g^2}{x^2} \tanh^2 xt} \quad (\text{dS}) \end{aligned} \right\} R(\epsilon) = b \cosh \frac{\epsilon}{b}$$
- $\Lambda = \frac{3}{b^2} - 3g^2; {}^3R = \frac{6}{b^2}; g = 2\pi |\epsilon|$
- from energy constraint, $\Lambda \approx 1/b^2$, when $K^2 \ll {}^3R$ (Λ is hidden at very small scales)