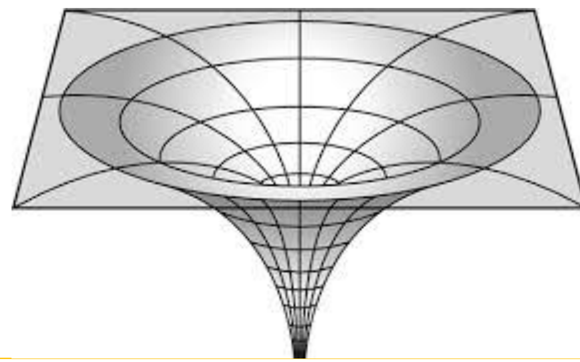
BOOTSTRAPPING NEWTONIAN GRAVITY



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R. Casadio, M. Lenzi and O. Micu, Phys. Rev.D 98, 104016 (2018)
R. Casadio, M. Lenzi and O. Micu, arXiv:1904.06752 [gr-qc]

Introduction

- Motivations
- Bootstrapped approach
- Main results

Motivations

- Gravitational collapse

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- Gravitational collapse → Singularity theorems

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- Gravitational collapse → Buchdal limit

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- Gravitational collapse → Singularity theorems
- Gravitational collapse → Buchdal limit

$$R > \frac{9}{4} G_{\rm N}\, M \equiv R_{\rm BL}$$

- Corpuscular model →

$$\lambda_{\rm G} \sim R_{\rm H} \qquad N_{\rm G} \sim \frac{M^2}{m_{\rm p}^2} \sim \frac{R_{\rm H}^2}{\ell_{\rm p}^2}$$

G. Dvali, C. Gomez, Fortschr. Phys. 63, 742(2013)

Bootstrapped approach

Effective field theory : bootstrap

$$L[V] \;=\; L_{\mathrm{N}}[V]$$

Newtonian part

$$\triangle V = 4\,\pi\,G_{\mathrm{N}}\,\rho$$

R. Casadio, A. Giugno, A. Giusti, and M. Lenzi, Phys. Rev.D 96, 044010 (2017).

Effective field theory : bootstrap

$$L[V] \;=\; L_{\rm N}[V] - 4\,\pi \int_0^\infty r^2\,{\rm d}r\,\left[q_\Phi\,J_V\,V + q_\rho\,J_\rho\,(\rho+p)\right]$$

Newtonian part

$$\triangle V = 4\,\pi\,G_{\rm N}\,\rho$$

Self-interaction terms

$$J_V \simeq \frac{{\rm d}U_{\rm N}}{{\rm d}\mathcal{V}} = -\frac{\left[V'(r)\right]^2}{2\,\pi\,G_{\rm N}}$$

$$J_\rho = -2\,V^2$$

R. Casadio, A. Giugno, A. Giusti, and M. Lenzi, Phys. Rev.D 96, 044010 (2017).

Effective field theory : bootstrap

$$L[V] = -4\,\pi \int_0^\infty r^2\,\mathrm{d}r \left[\frac{\left(V'\right)^2}{8\,\pi\,G_\mathrm{N}} \left(1 - 4\,V\right) + \left(\rho + p\right) V \left(1 - 2\,V\right) \right]$$

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$$\downarrow$$

$$\left\{ \begin{array}{rcl} \triangle V & = & 4\,\pi\,(\rho+p) + \dfrac{2\,\left(V'\right)^2}{1-4\,V} \\ \\ p' & = & -V'\,(\rho+p) \end{array} \right.$$

Effective field theory : bootstrap

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$$\downarrow$$

$$\begin{cases} \triangle V &=& 4\,\pi\,(\rho+p) + \dfrac{2\,(V')^2}{1-4\,V} \\[2ex] p' &=& -V'\,(\rho+p) \end{cases}$$

- Perturbative solution $\longrightarrow$ fails at $R \le R_\mathrm{H} \equiv 2\,G_\mathrm{N}\,M$

  $V \simeq V_0 + q_\Phi V_1$

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  $V \simeq V_0 + q_\Phi V_1$

- Non-perturbative solution ⟶ BOOTSTRAP

Main results

Homogeneous ball in vacuum

$$\rho = \frac{3\,M_0}{4\,\pi\,R^3}\,\Theta(R - r) \qquad \text{with} \qquad M_0 = 4\,\pi \int_0^R r^2\,\mathrm{d}r\,\rho(r)$$

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↓

$$\begin{cases} \triangle V = \dfrac{3\;M_0}{R^3}\,e^{V_R - V} + \dfrac{2\,(V')^2}{1-4\,V} \\[2ex] p = \rho_0\left[e^{V_R-V} - 1\right] \end{cases}$$

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Boundary conditions :

$$V'_{\rm in}(0) = 0$$

$$V_{\rm in}(R) = V_{\rm out}(R) \equiv V_R$$

$$V'_{\rm in}(R) = V'_{\rm out}(R) \equiv V'_R$$

$$\longrightarrow \qquad M_0 = M_0(M)$$

Outer vacuum solution

$$\triangle V = \frac{2\,(V')^2}{1 - 4\,V} \quad \longrightarrow \quad V_{\mathrm{c}} = \frac{1}{4}\left[1 - \left(1 + \frac{6\,G_{\mathrm{N}}\,M}{r}\right)^{2/3}\right]$$

Outer vacuum solution

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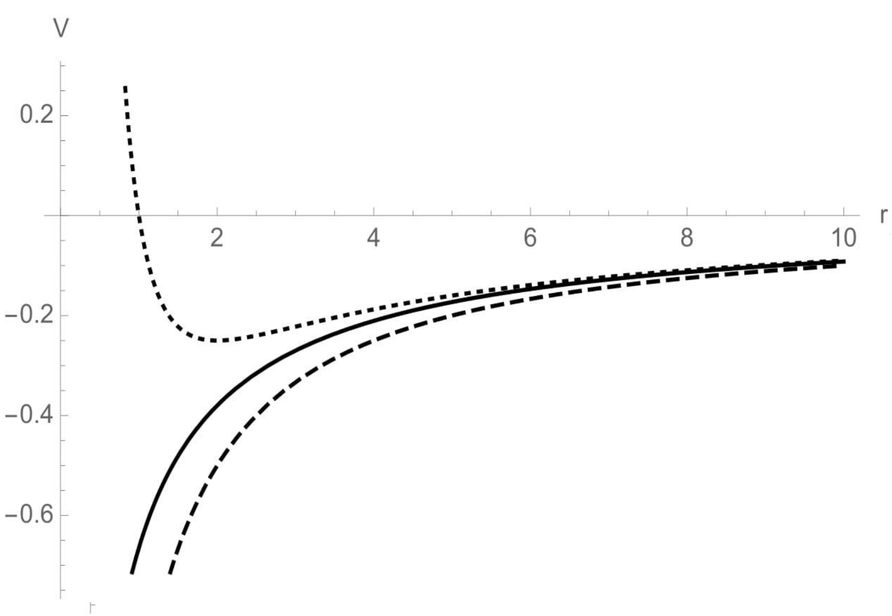
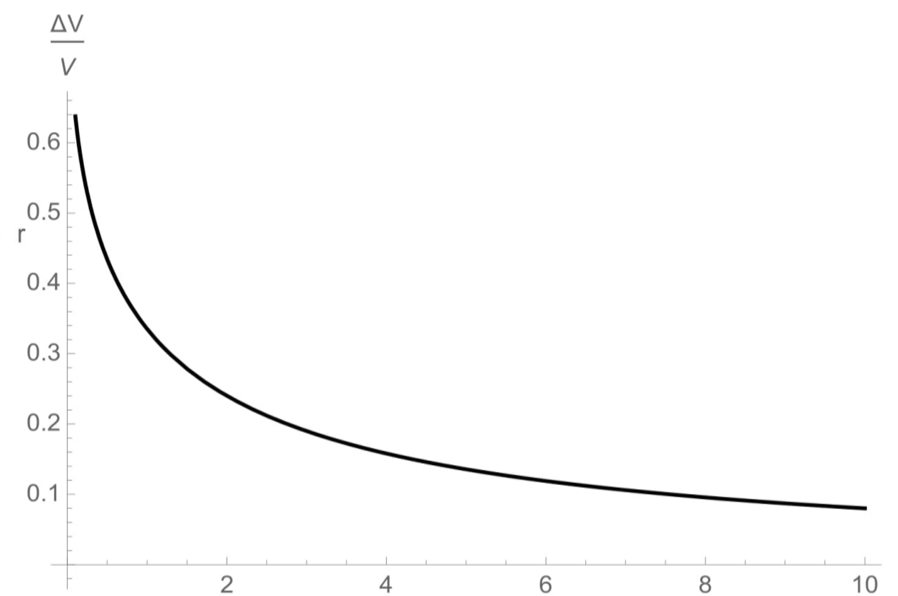
- Large r expansion $$V_c \underset{r\to\infty}{\simeq} -\frac{G_N\,M}{r} + \frac{G_N^2\,M^2}{r^2}$$

- Always attractive potential





Inner potential

$$\triangle V = \frac{3\ M_0}{R^3}\, e^{V_R - V} + \frac{2\,(V')^2}{1 - 4\,V}$$

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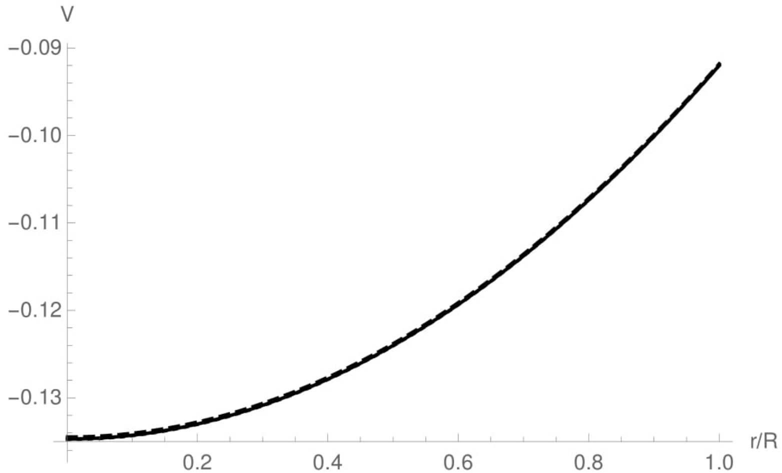
Small/intermediate compactness

$$\frac{G_\mathrm{N}\,M}{R}\ll 1 \qquad \frac{G_\mathrm{N}\,M}{R}\simeq 1$$



$$V_\mathrm{s}=V_0+\frac{G_\mathrm{N}\,M_0}{2\,R^3}\,e^{V_R-V_0}\,r^2$$

Inner potential

$$\triangle V = \frac{3\ M_0}{R^3}\, e^{V_R - V} + \frac{2\,(V')^2}{1 - 4\,V}$$

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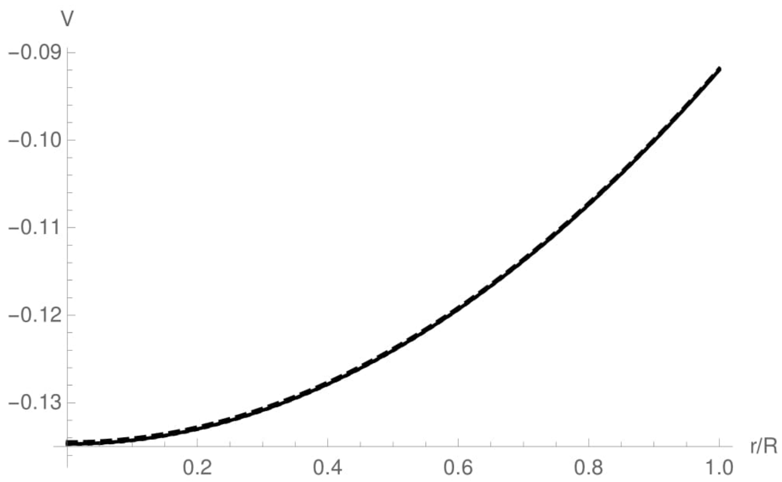
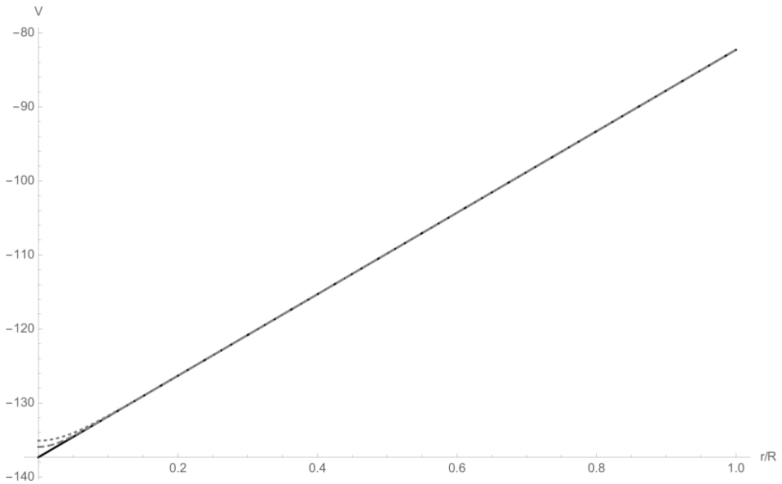


$$V_s = V_0 + \frac{G_N\,M_0}{2\,R^3}\, e^{V_R - V_0}\, r^2$$

High compactness

$$\frac{G_N\,M}{R} \gg 1$$



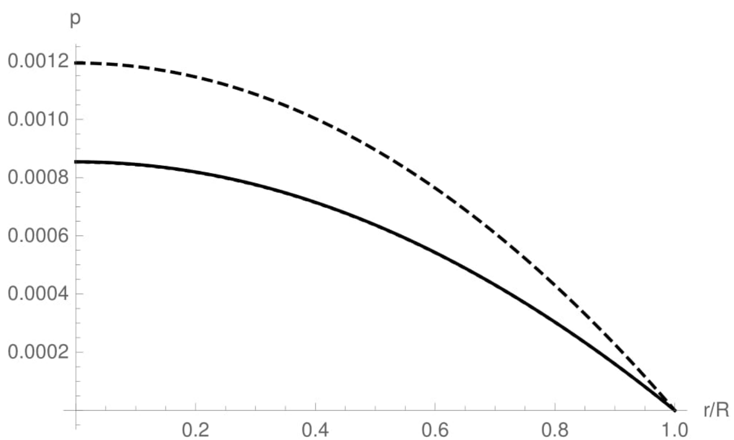
$$V_{\rm lin} \simeq V_R + V_R'\,(r - R)$$

$$\frac{G_N\,M_0}{R} \sim \left(\frac{G_N\,M}{R}\right)^{2/3}$$

Pressure

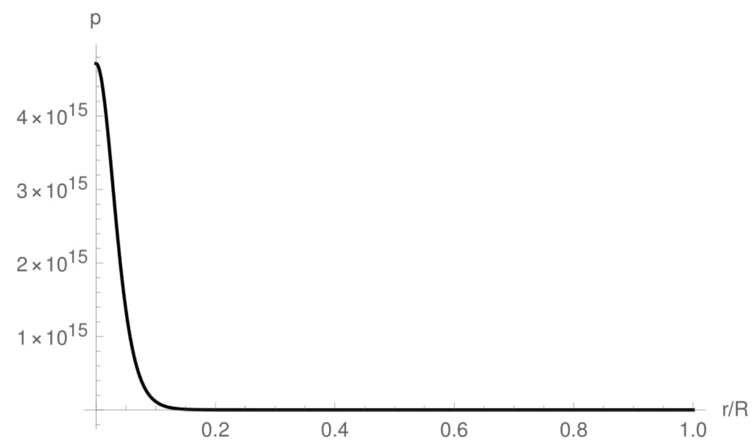
$$p = \rho_0 \left[e^{V_R - V} - 1 \right]$$

Small/intermediate compactness



$$p \simeq \frac{3\, G_{\rm N}\, M^2\, (R^2 - r^2)}{8\, \pi\, R^6}$$

High compactness



$$p \simeq \frac{M^2\, e^{\frac{1}{2}\left(\frac{G_{\rm N}\, M}{\sqrt{6}\, R}\right)^{2/3}\left(3 - \frac{5}{\tilde{C}}\right)}}{2\, \pi\, \tilde{C}^2\, (6\, G_{\rm N}\, M/R)^{2/3}} \left[e^{\left(\frac{G_{\rm N}\, M}{\sqrt{6}\, R}\right)^{2/3}\left(1 - \frac{r}{R}\right)} - 1 \right]$$

Horizon

- GR geometrical approach $\longrightarrow$ Schwarzschild radius $R_{\mathrm{H}} \equiv 2\, G_{\mathrm{N}}\, M$

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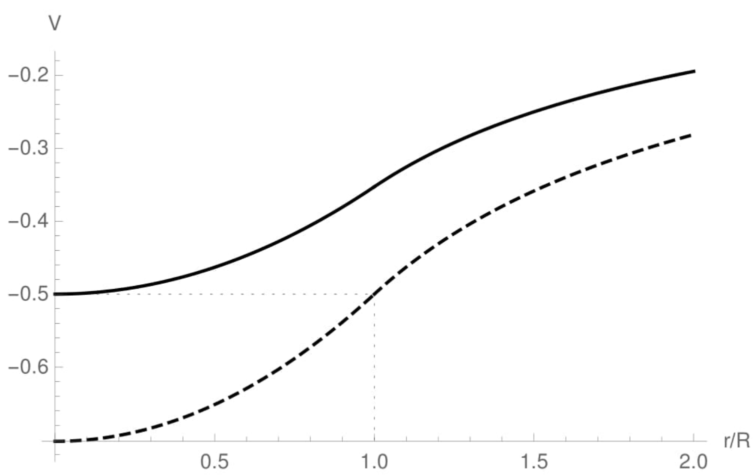

$$\longrightarrow \begin{cases} \text{no horizon} & \text{for} \quad G_{\rm N}\, M/R \lesssim 0.46 \\ 0 < r_{\rm H} \leq R \simeq 1.4\, G_{\rm N}\, M & \text{for} \quad 0.46 \lesssim G_{\rm N}\, M/R \leq 0.69 \\ r_{\rm H} \simeq 1.4\, G_{\rm N}\, M & \text{for} \quad G_{\rm N}\, M/R \gtrsim 0.69\ . \end{cases}$$

Conclusions

- Investigation of the quantum features of this potential
- Connection with the corpuscular model