A study on MUonE experimental systematics

A. Principe

Weekly meeting 30/01/2019

Method to propagate systematics on $\Delta\alpha_{had}(t)$ (1)

- From cross section ratio, at NLO VP:

$$R_{syst}(t) \equiv \frac{d\sigma_{syst}/dt}{d\sigma_{ref}/dt} \sim \frac{|1-\Delta\alpha_{lep}(t)-\Delta\alpha_{had}(t)|^2}{|1-\Delta\alpha_{lep}(t)-\Delta\alpha'_{had}(t)|^2} = \frac{|1-\Delta\alpha_{ref}(t)|^2}{|1-\Delta\alpha_{lep}(t)-\Delta\alpha'_{had}(t)|^2}$$

- Developing in had running, modified by systematic effects, we can evaluate directly systematics on final fit:

$$R_{syst}(t) \sim |1-\Delta\alpha_{ref}(t)|^2 \cdot [1+2(\Delta\alpha_{lep}(t)+\Delta\alpha'_{had}(t)) + 3(\Delta\alpha_{lep}(t)+\Delta\alpha'_{had}(t))^2 + 4(\Delta\alpha_{lep}(t)+\Delta\alpha'_{had}(t))^3]$$

- In concepts, at least there are two important ratios:

$$R_{\text{effect}}(O) \equiv \frac{d\sigma_{\text{effect}}(O)}{d\sigma_{\text{no effect}}(O)} \Rightarrow \text{knowledge (measurements, models)}$$

$$\Rightarrow R_{syst}(O) \equiv \frac{d\sigma_{\text{reco effect}}(O)}{d\sigma_{\text{effect}}(O)},$$

Method to propagate systematics on $\Delta\alpha_{had}(t)$ (2)

- Reference formulas:

$$t = t_{ee} = 2m_e\left(m_e - E'_e(\theta_e)\right) = 2m_e^2\left(1 - \frac{1 + r^2\cos^2\theta_e}{1 - r^2\cos^2\theta_e}\right) = \frac{4m_e^2 r^2\cos^2\theta_e}{r^2\cos^2\theta_e - 1},$$

$$\boxed{\frac{dt}{d\theta_e} = \frac{4m_e^2 r^2 \sin 2\theta_e}{\left(r^2\cos^2\theta_e - 1\right)^2}}$$

- Ratio Rsyst in electron scattering angle, in general:

$$R_{syst}(\theta_e) \equiv \frac{d\sigma_{syst}/d\theta_e}{d\sigma_{ref}/d\theta_e} = \frac{d\sigma_{syst}/dt}{d\sigma_{ref}/dt}\frac{(dt/d\theta_e)_{syst}}{(dt/d\theta_e)_{ref}} = R_{syst}[t(\theta_e)]\cdot\frac{(dt/d\theta_e)_{syst}}{(dt/d\theta_e)_{ref}}$$
$$\sim \frac{\left|1 - \Delta\alpha_{ref}[t(\theta_e)]\right|^2}{\left|1 - \Delta\alpha_{lep}[t(\theta_e)] - \Delta\alpha'_{had}[t(\theta_e)]\right|^2}\frac{r_{syst}^2}{r_{ref}^2}\frac{\left(r_{ref}^2\cos^2\theta_e - 1\right)^2}{\left(r_{syst}^2\cos^2\theta_e - 1\right)^2}.$$

- Ratio in case of no variation of beam energy (average value):

$$R_{syst}(\theta_e) = R_{syst}[t(\theta_e)] \sim \frac{\left|1 - \Delta\alpha_{ref}[t(\theta_e)]\right|^2}{\left|1 - \Delta\alpha_{lep}[t(\theta_e)] - \Delta\alpha'_{had}[t(\theta_e)]\right|^2}$$

Method to propagate systematics on $\Delta\alpha_{had}(t)$ (3)

- Systematics effects I evaluated in the present work:

$$R_{syst} = \frac{d\sigma(\text{reco energy})}{d\sigma(\text{nom energy})} = \frac{d\sigma(E_0 + \Delta E)}{d\sigma(E_0)} \qquad (\text{beam energy}),$$

$$R_{syst} = \frac{d\sigma(\text{reco width})}{d\sigma(\text{nom width})} = \frac{d\sigma(w_{beam} \oplus w_{BSM})}{d\sigma(w_{beam})} \qquad (\text{beam spread}),$$

$$R_{syst} = \frac{d\sigma(\text{reco MS})}{d\sigma(\text{nom MS})} = \frac{d\sigma(MS + \Delta\%)}{d\sigma(MS)} \qquad (\text{multiple scattering}),$$

- Method for final pseudo-fit:

I assumed an usual parametrization for $\Delta\alpha_{had}(t)$, namely this pol3:

$$\Delta\alpha_{had}(t) = [0] \cdot t + [1] \cdot t^2 + [2] \cdot t^3, \tag{13}$$

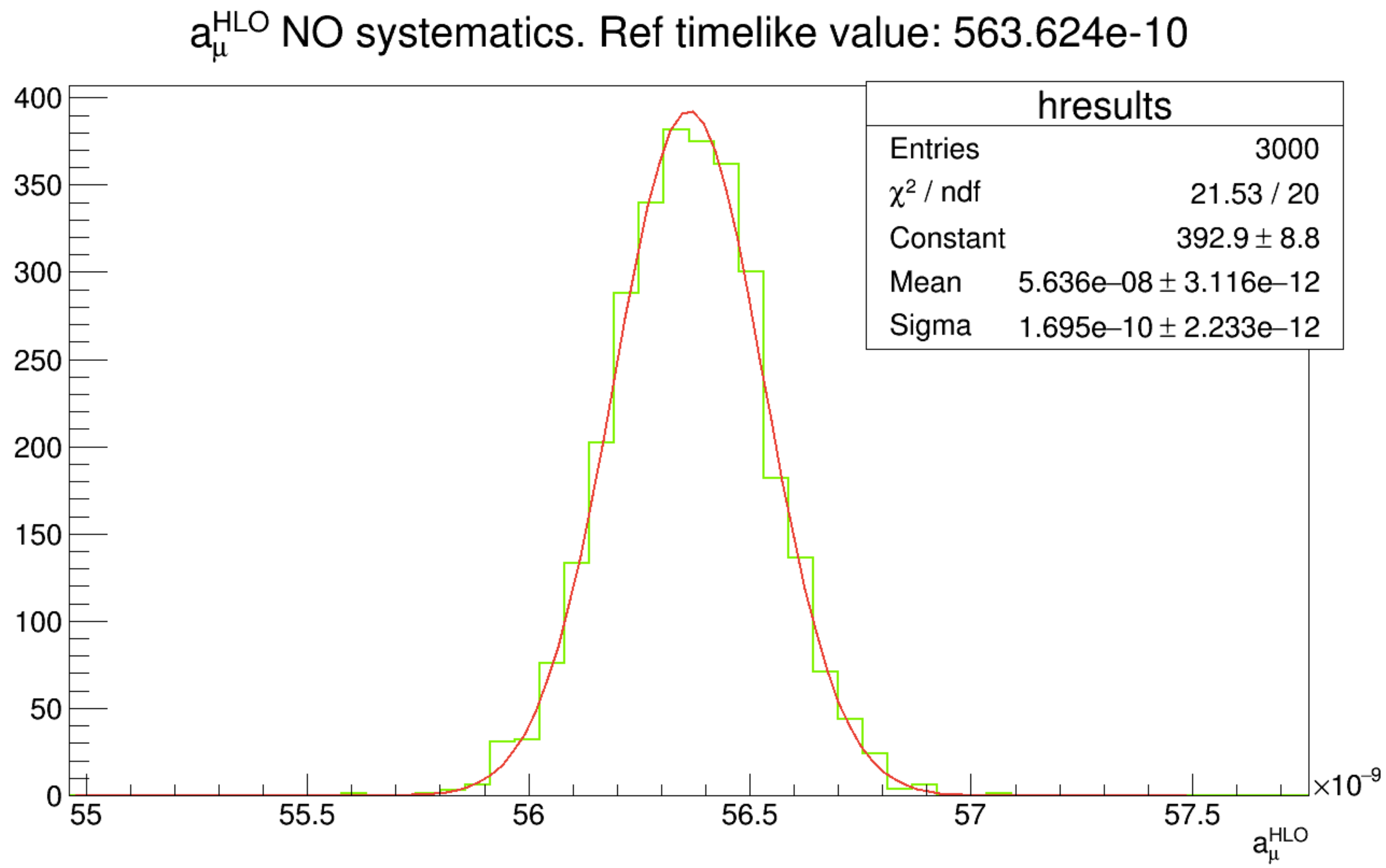
fitting distributions of pseudo-points in t, but also in x for cross check, using MUonE master formula:

$$I(x) = \frac{\alpha_0}{\pi}(1-x)\Delta\alpha_{had}[t(x)] = \frac{\alpha_0}{\pi}(1-x)\left([0]t(x) + [1]t^2(x) + [2]t^3(x)\right), \tag{14}$$

with

$$t(x) = \frac{x^2 m_\mu^2}{x-1}. \tag{15}$$

Zero systematics check: only stat, R_{syst}=1

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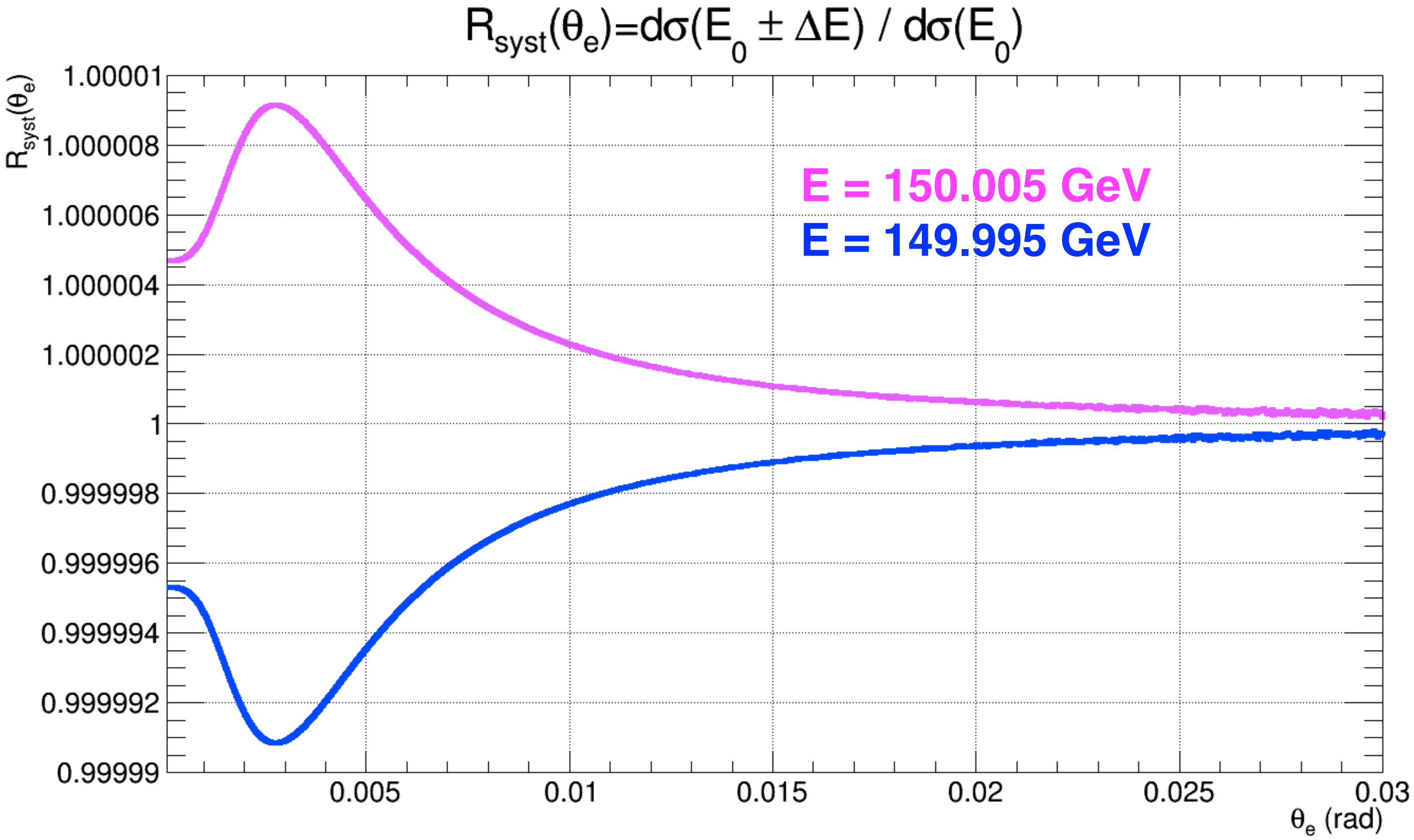
FCN=21.531 FROM MINOS     STATUS=SUCCESSFUL     21 CALLS        145 TOTAL
                     EDM=7.21333e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant   3.92895e+02   8.84723e+00  -6.05216e-02  -7.86928e-09
  2  Mean       5.63623e-08   3.11553e-12   4.28947e-15  -2.53976e+05
  3  Sigma      1.69454e-10   2.23295e-12   2.23295e-12  -2.36891e-02
```

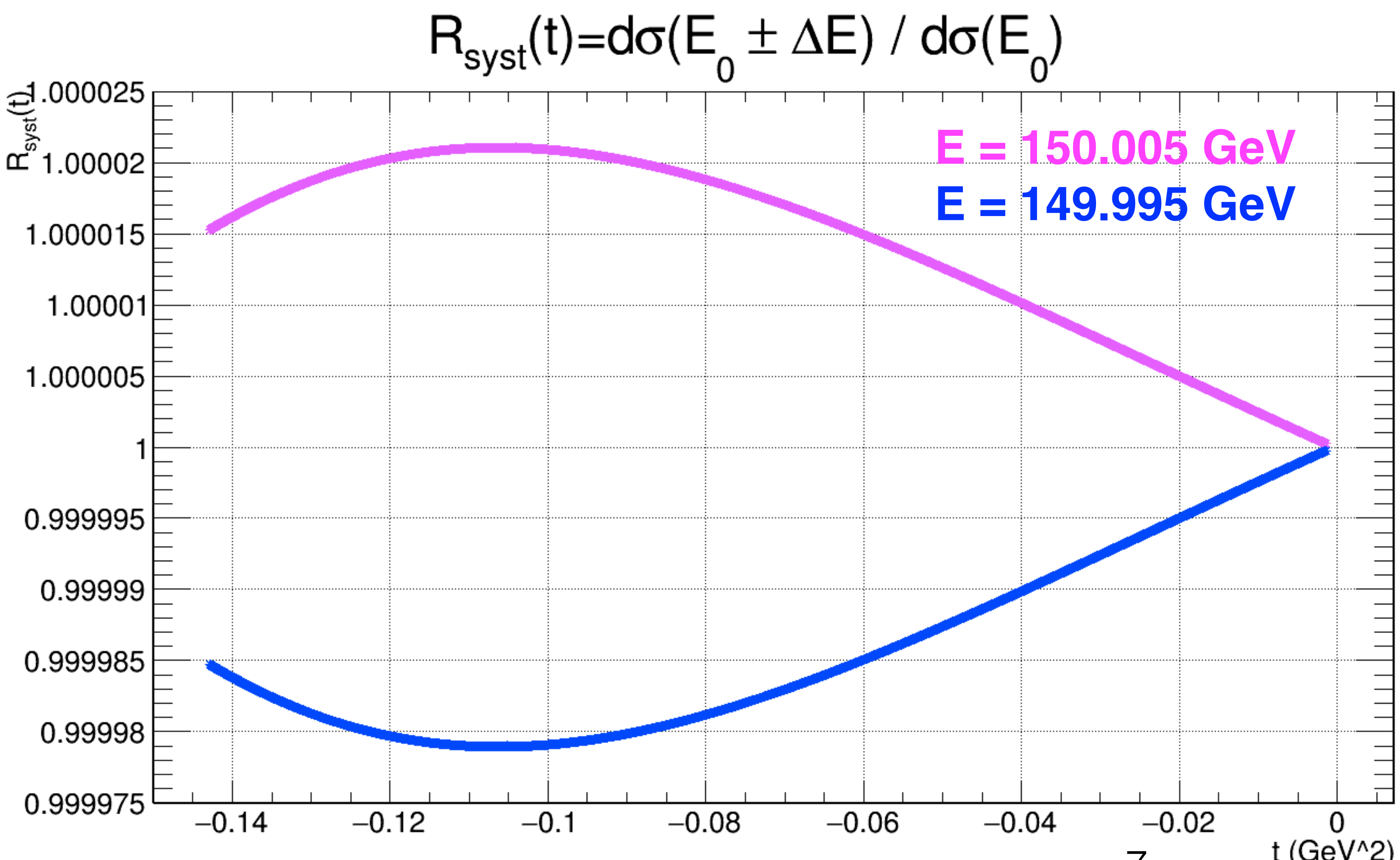
t range [-0.123,-0.00147] GeV^2

Time-like reference in x [0.3, 0.923]: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

From pseudo-fit: $a_\mu^{HLO} = (563.62 \pm 1.69) \cdot 10^{-10}$ -> 0.3% stat precision

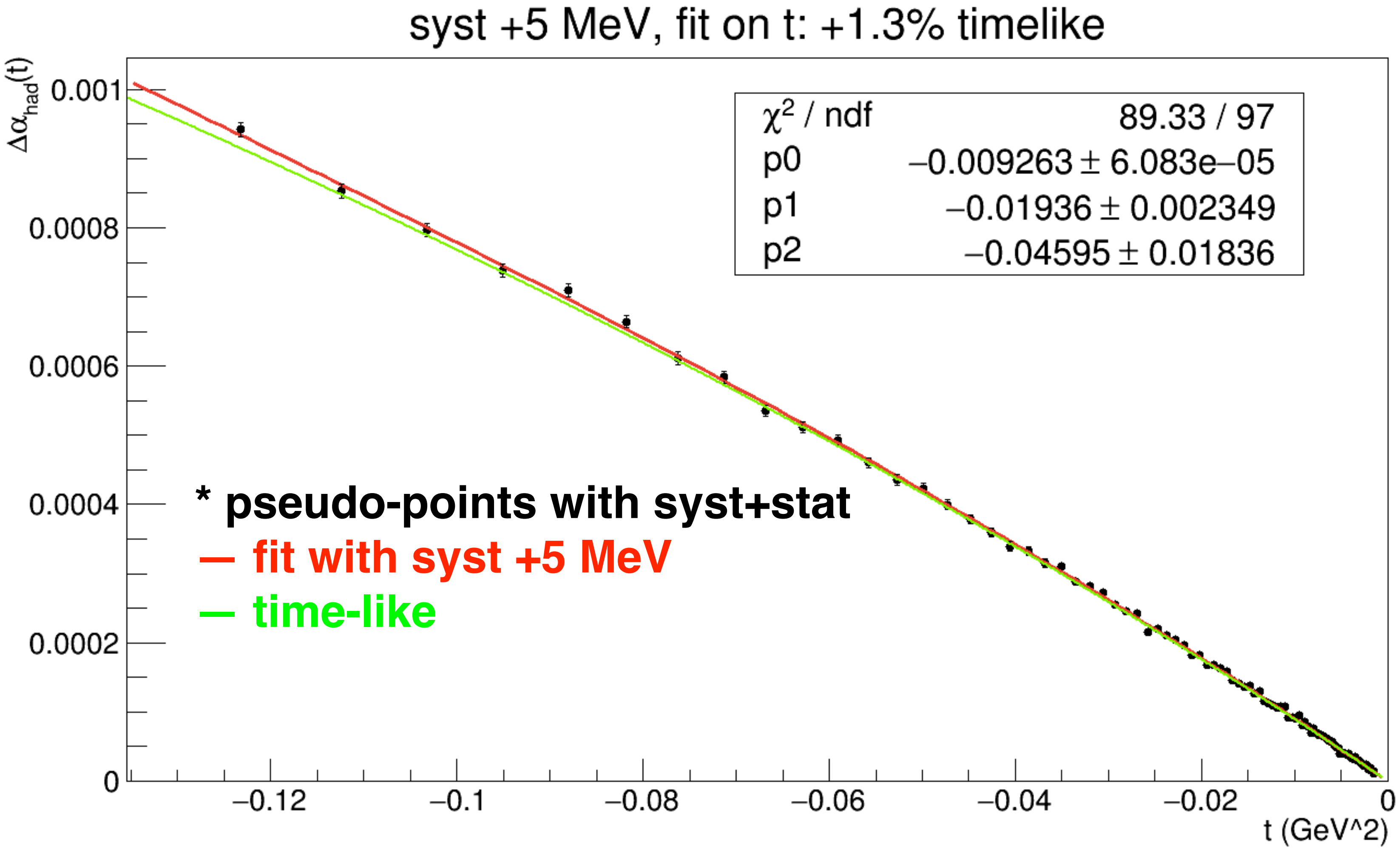
Energy scale systematic:
average value mis-calibration

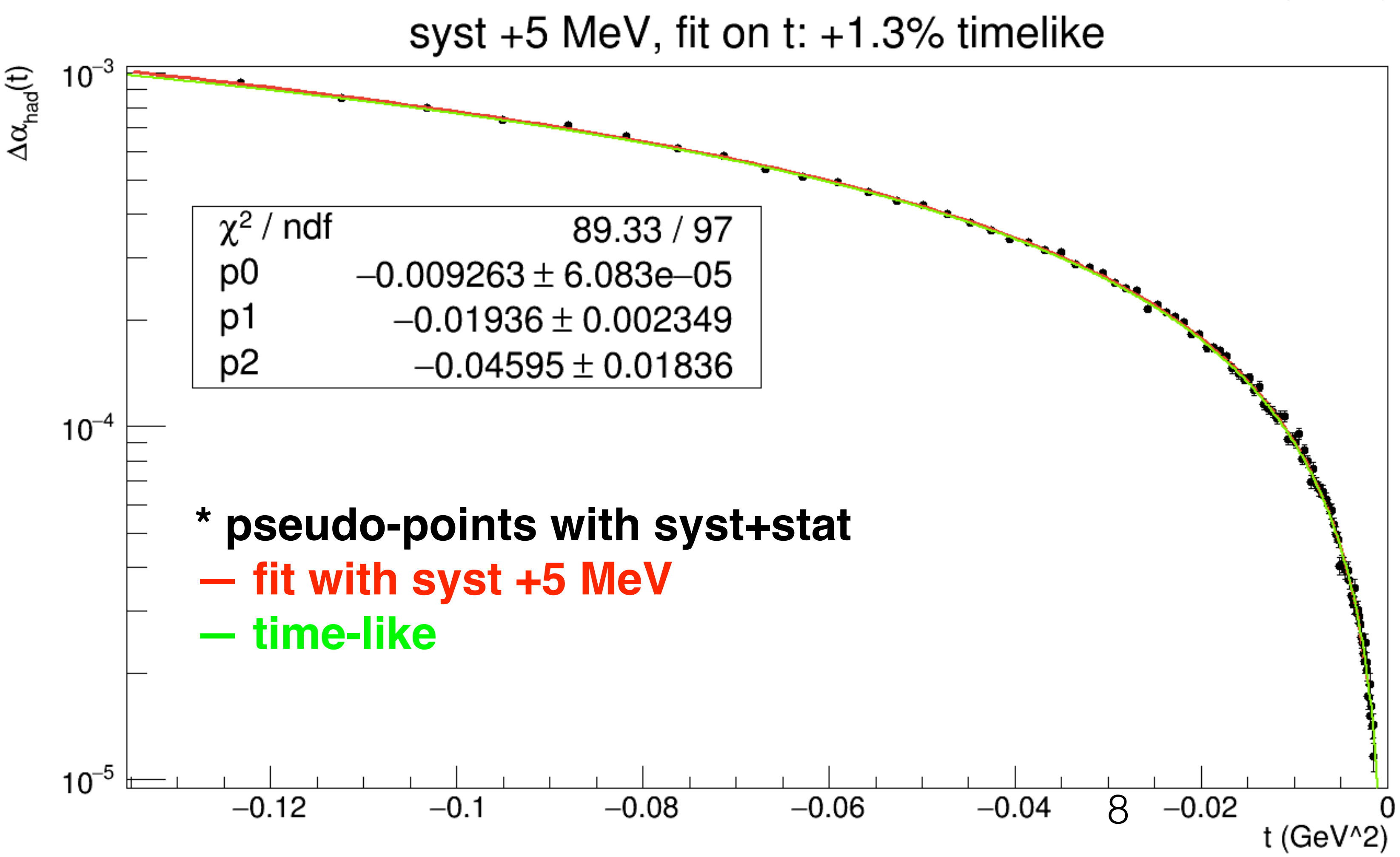




Systematic error due to energy scale mis-calibration on ratios R_{syst}

- ~ 10^-5 at peak, as we known (from "equal angle studies"), but not flat behavior causes a not simply propagation.
- in t, the effect is ~ 2*10^-5 at peak





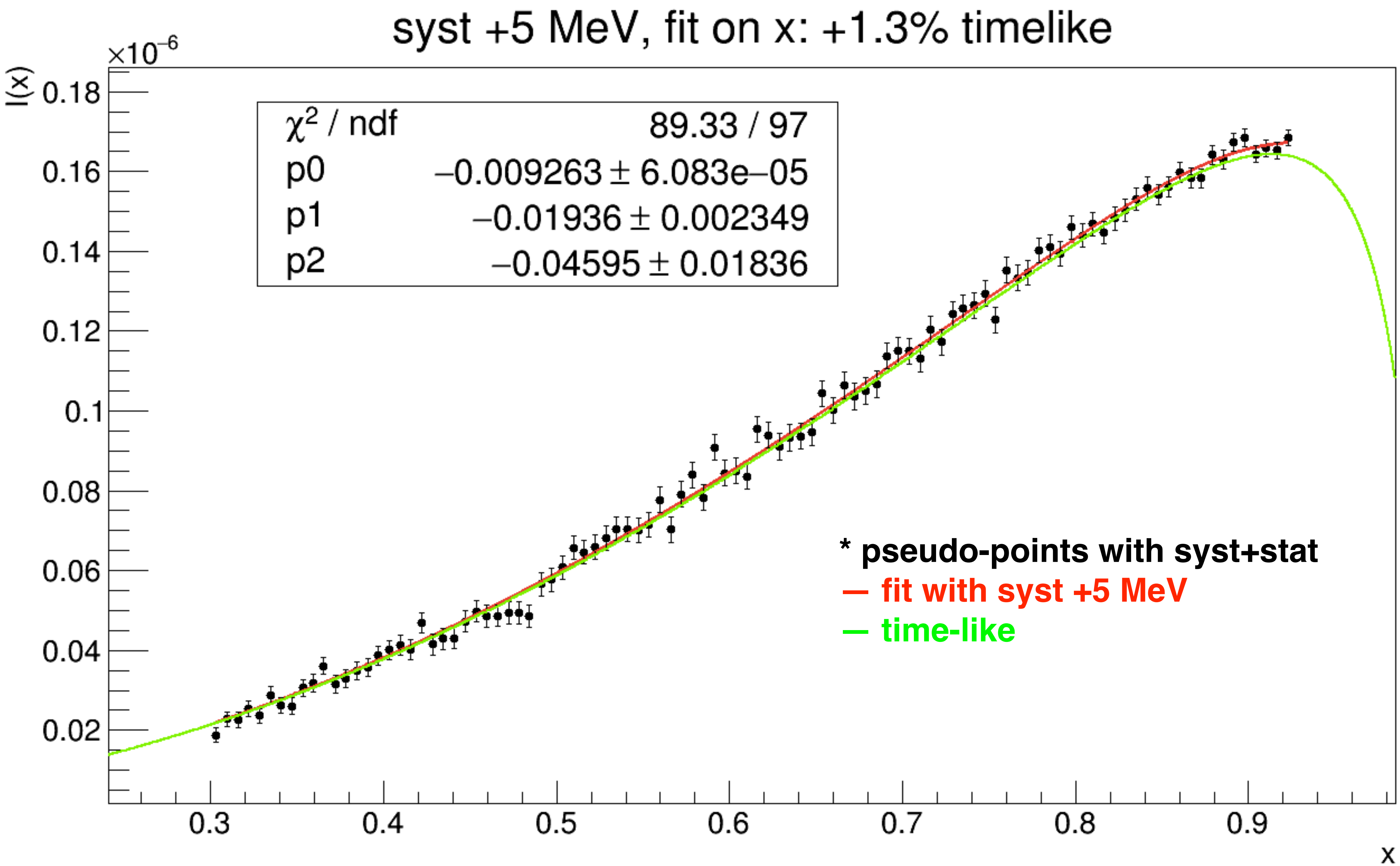
Propagation on $\Delta\alpha_{had}(t)$: syst error of +5 MeV (E_0 = 150 GeV)

Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

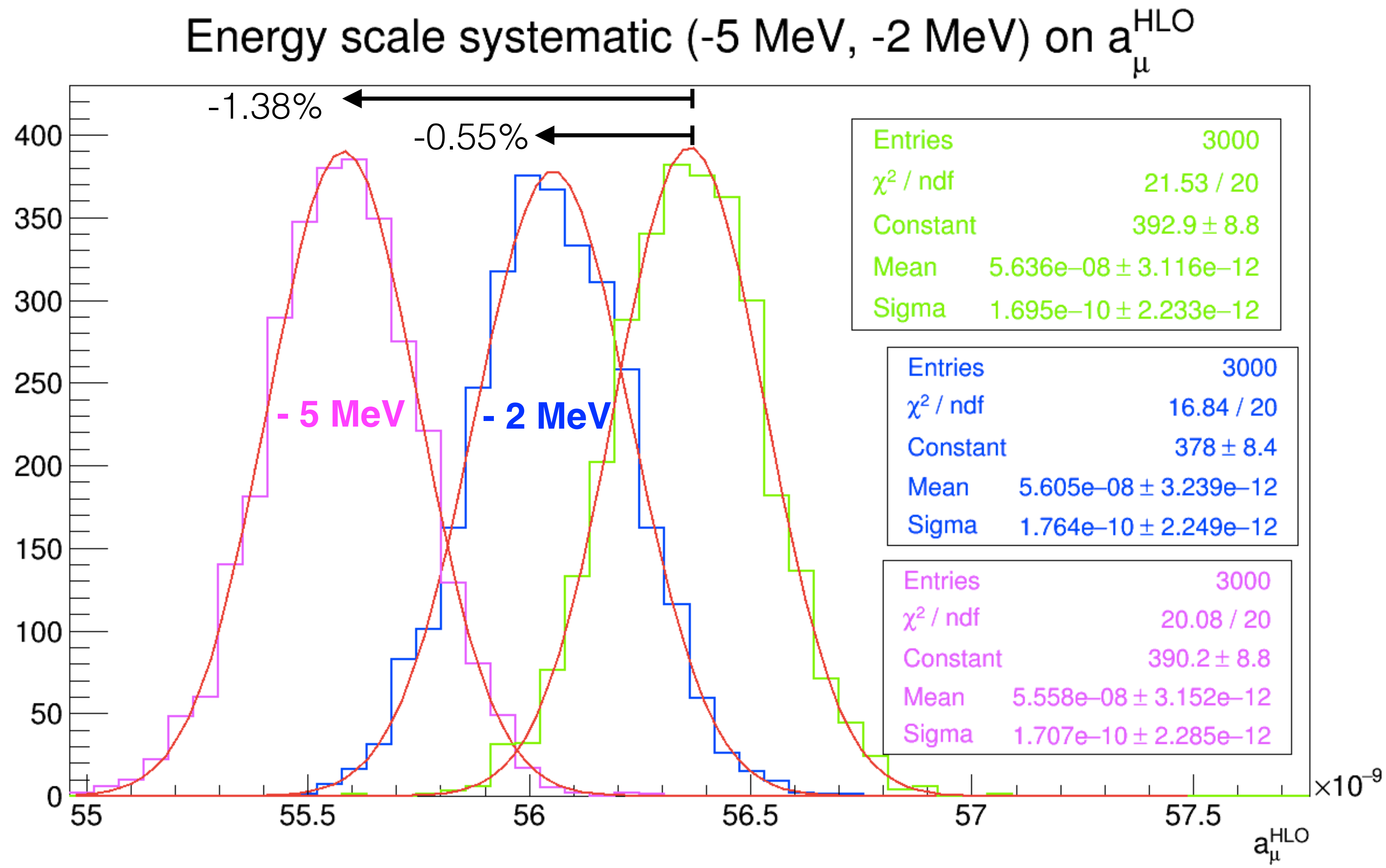
From this single pseudo-fit:
$a_\mu^{HLO} = 571.2 \cdot 10^{-10}$

- As we roughly could expected, an effect of ~10^-5 gives a percent systematic on final value: 10^-3 * 1% = 10^-5.

Propagation on integrand: effect of +5 MeV (E_0 = 150 GeV)



Systematic shifts on final a_μ^{HLO}: effect of -5, -2 MeV



Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

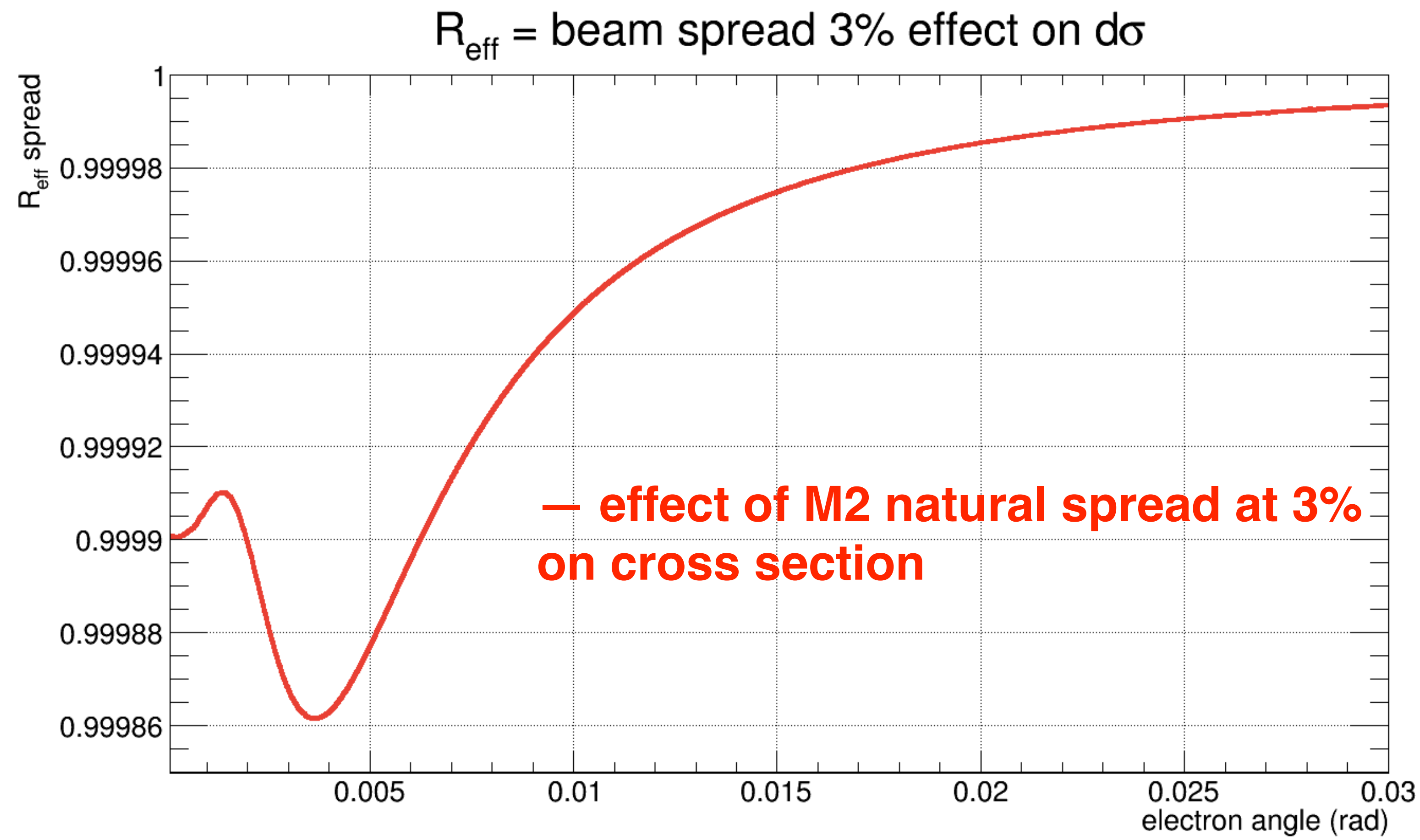
From pseudo-fit: $a_\mu^{HLO} = (563.62 \pm 1.69) \cdot 10^{-10}$ -> Statistical only

From pseudo-fit: $a_\mu^{HLO} = (560.53 \pm 1.76) \cdot 10^{-10}$ -> with syst of -2 MeV on 150 GeV

From pseudo-fit: $a_\mu^{HLO} = (555.80 \pm 1.71) \cdot 10^{-10}$ -> with syst of -5 MeV on 150 GeV

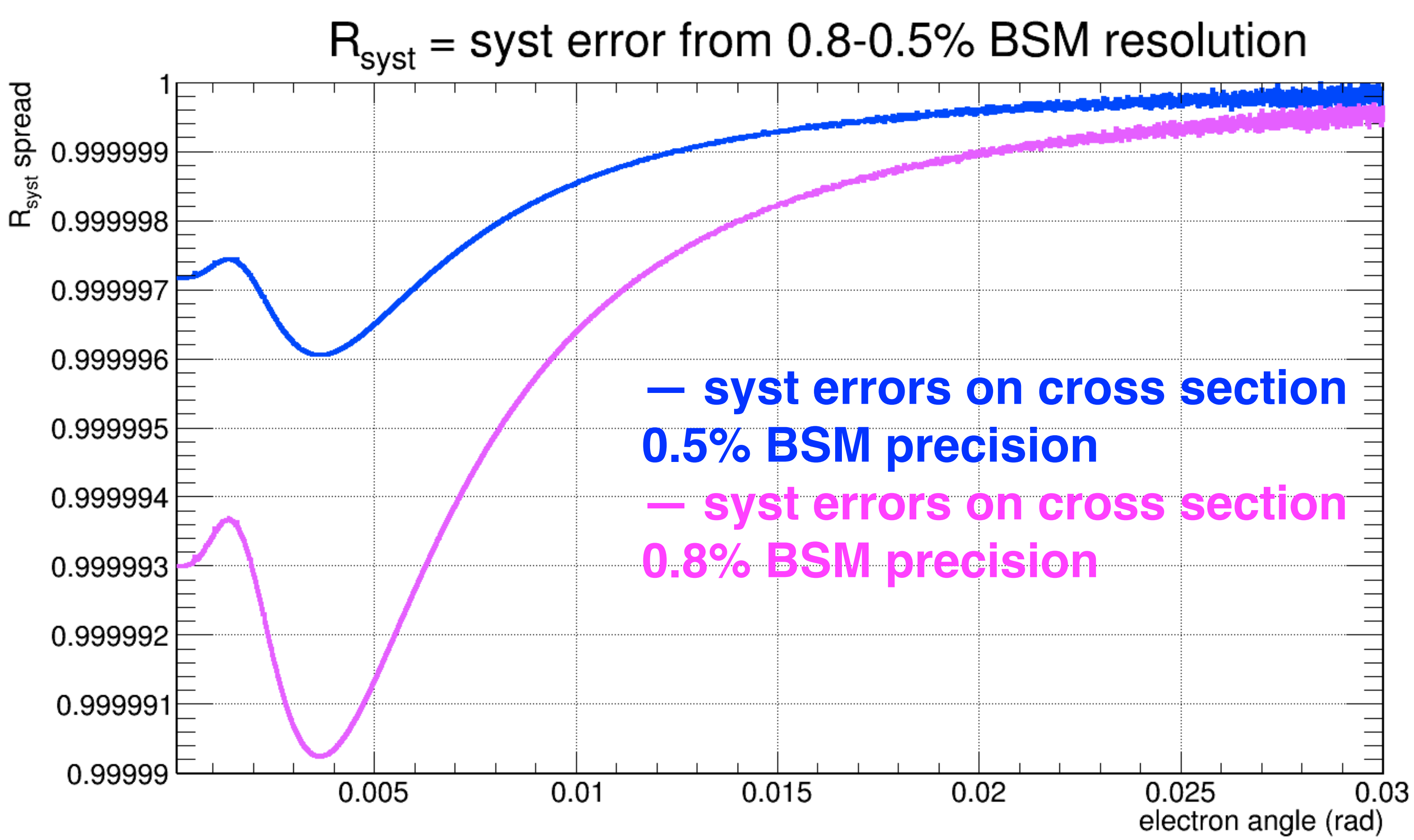
Energy spread systematic:
beam width mis-knowledge

Effect and systematic error due to beam spread on ratios R



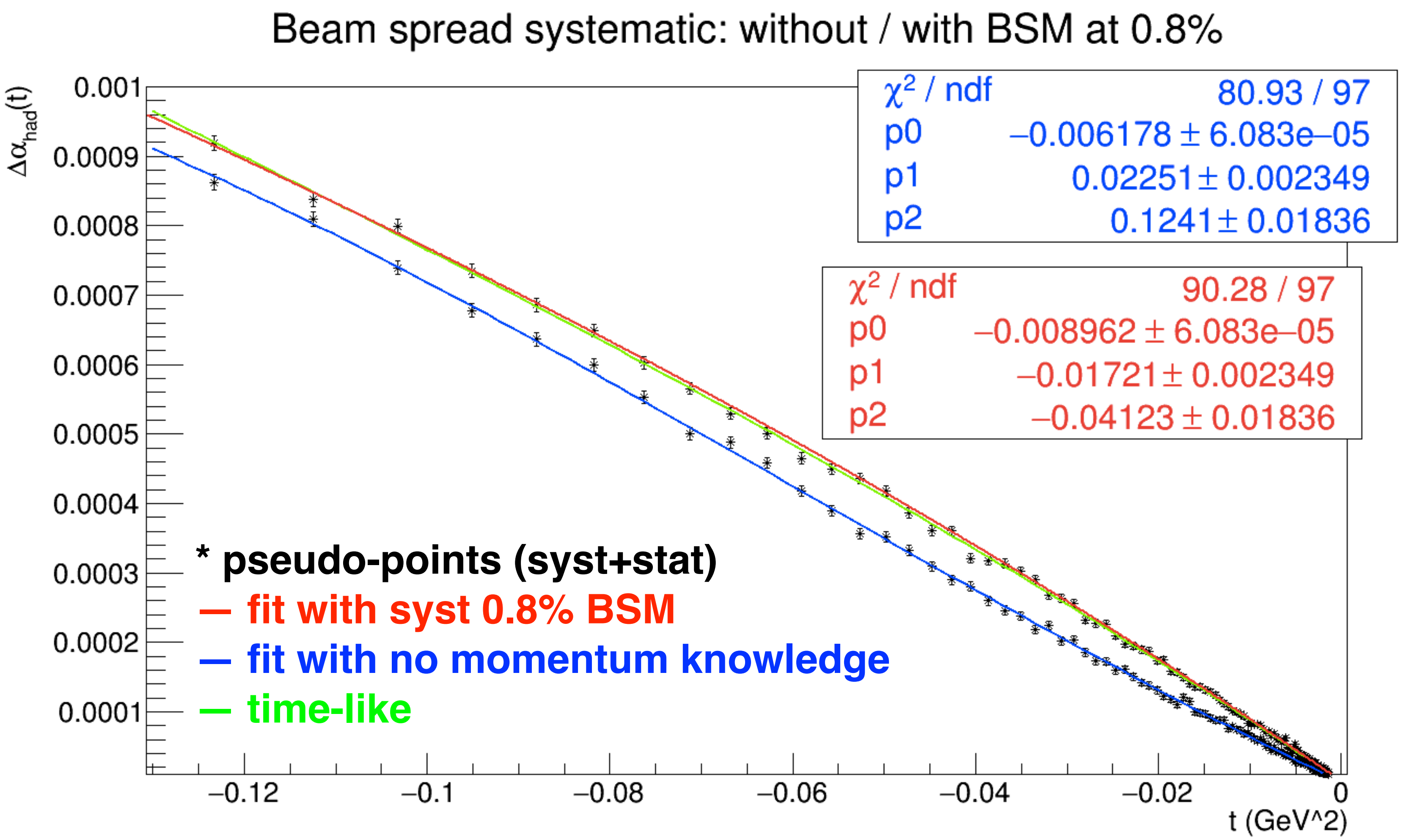
$$R_{\text{effect}}(O) \equiv \frac{d\sigma_{\text{effect}}(O)}{d\sigma_{\text{no effect}}(O)} \Rightarrow \text{knowledge (measurements, models)}$$

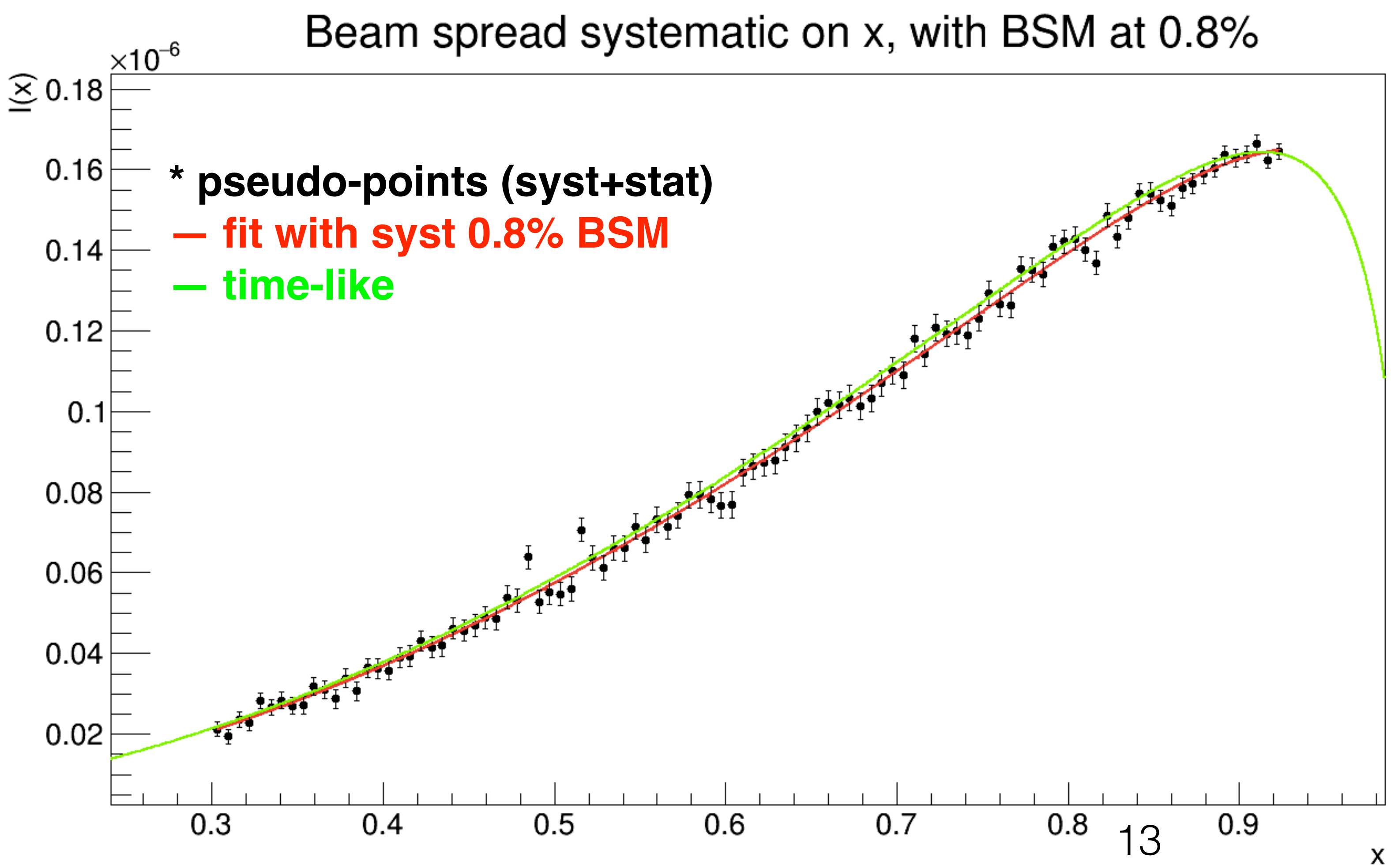
$$\Rightarrow R_{syst}(O) \equiv \frac{d\sigma_{\text{reco effect}}(O)}{d\sigma_{\text{effect}}(O)},$$



- For this estimate, I worked on previous Graziano's study on evaluation of beam spread effect. I used the same ratio in order to evaluate this syst:

$$R_{syst} = \frac{d\sigma(\text{reco width})}{d\sigma(\text{nom width})} = \frac{d\sigma(w_{beam} \oplus w_{BSM})}{d\sigma(w_{beam})} \quad \text{(beam spread)},$$





Propagation on $\Delta\alpha_{had}(t)$: effect of momentum measurement at 0.8%

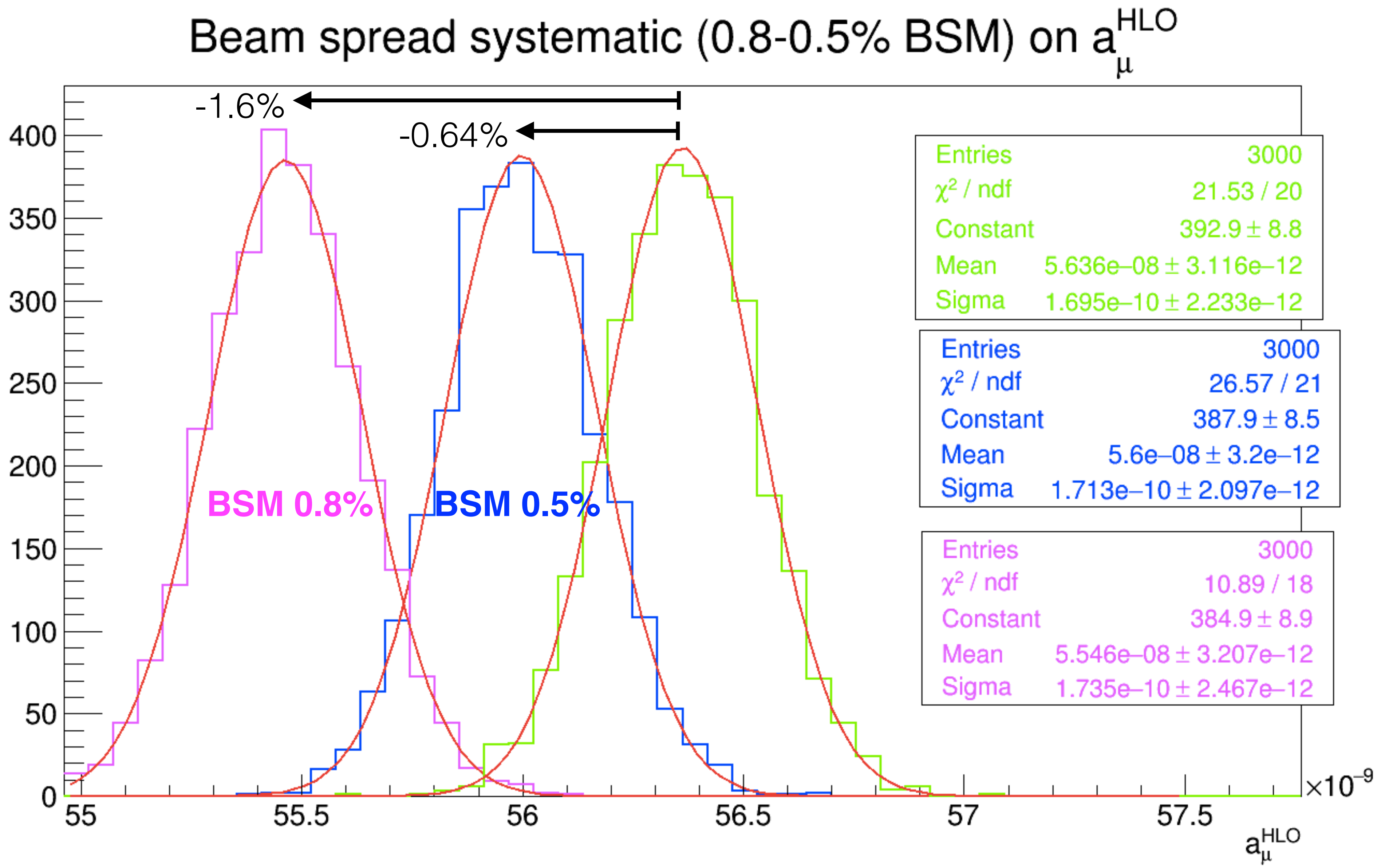
Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

From this <u>single</u> pseudo-fit
with NO momentum knowledge:
$a_\mu^{HLO} = 438.15 \cdot 10^{-10}$

From this <u>single</u> pseudo-fit
with BSM at 0.8% precision on momentum:
$a_\mu^{HLO} = 554.5 \cdot 10^{-10}$

~ -1.6% difference from timelike.

Systematic shifts on final a_μ^{HLO}: effect BSM at 0.8-0.5%



Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

From pseudo-fit: $a_\mu^{HLO} = (563.62 \pm 1.69) \cdot 10^{-10}$ -> Statistical only

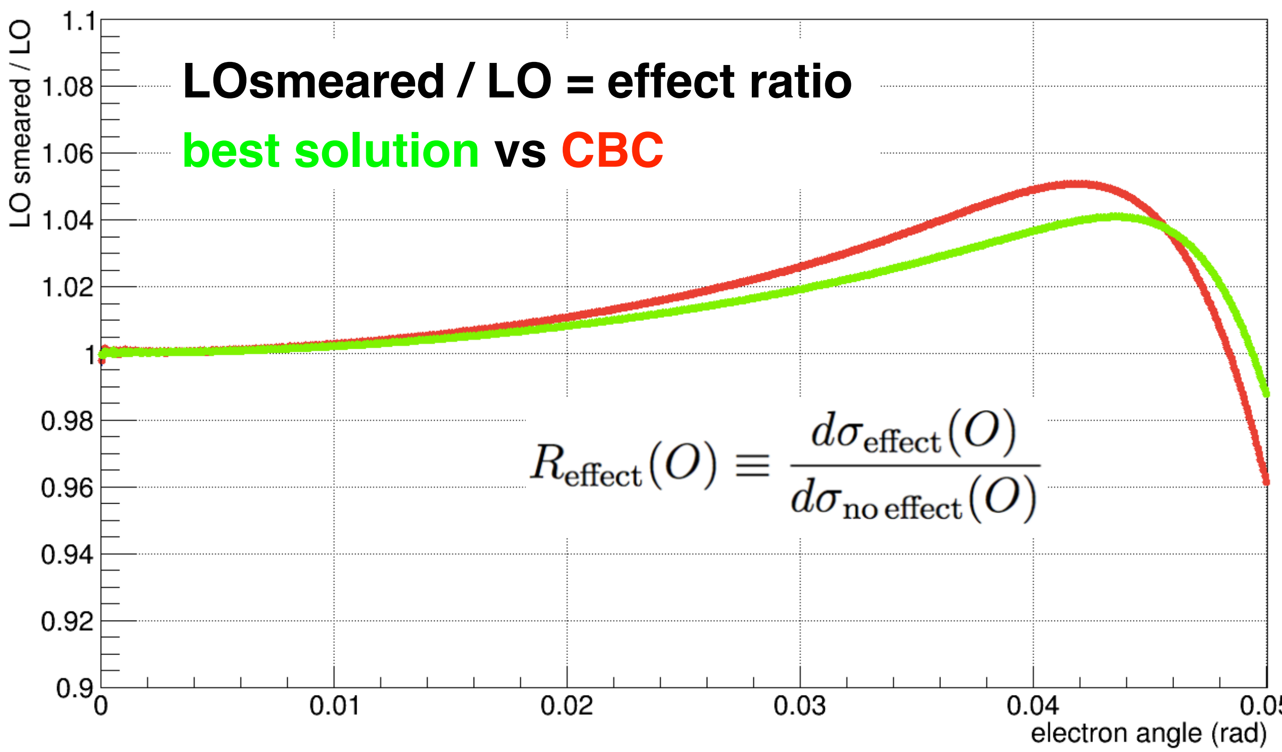
From pseudo-fit: $a_\mu^{HLO} = (559.97 \pm 1.71) \cdot 10^{-10}$ -> with BSM at 0.8% precision

From pseudo-fit: $a_\mu^{HLO} = (554.62 \pm 1.74) \cdot 10^{-10}$ -> with BSM at 0.5% precision

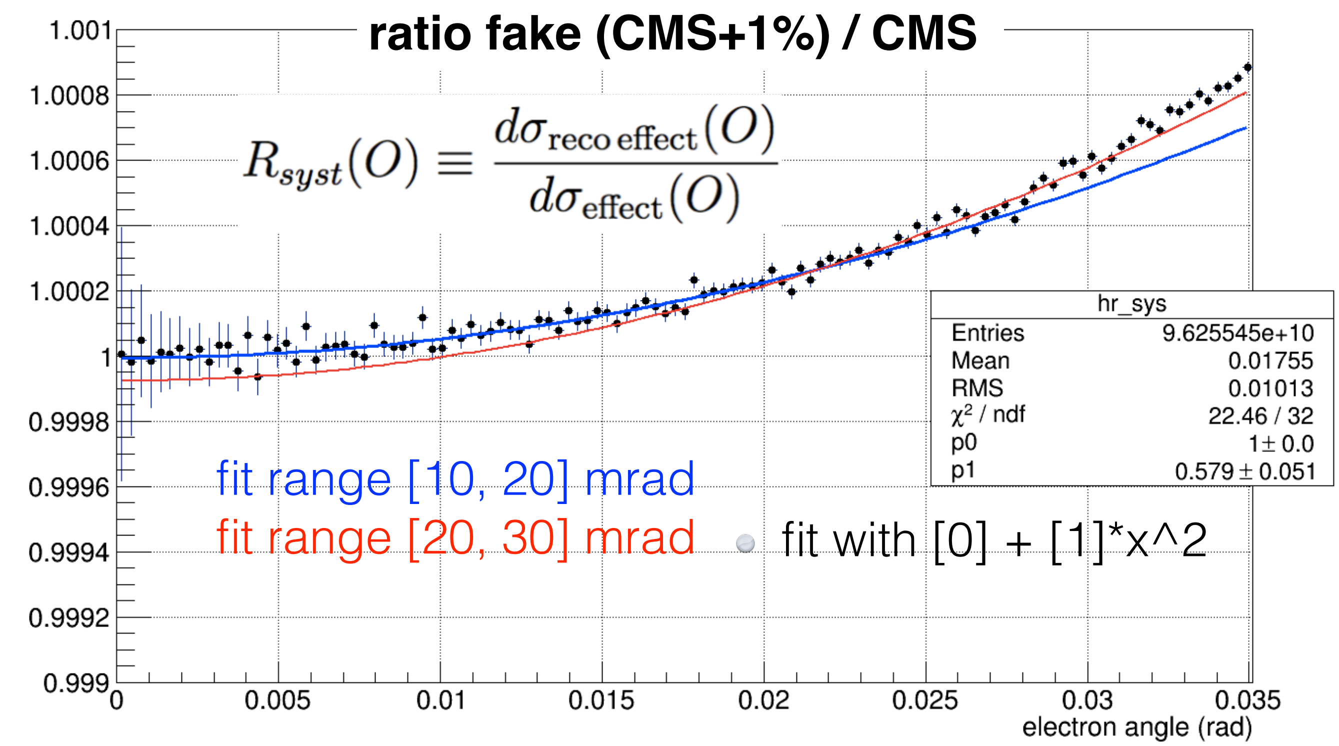
Multiple scattering systematic:
angle smearing mis-knowledge

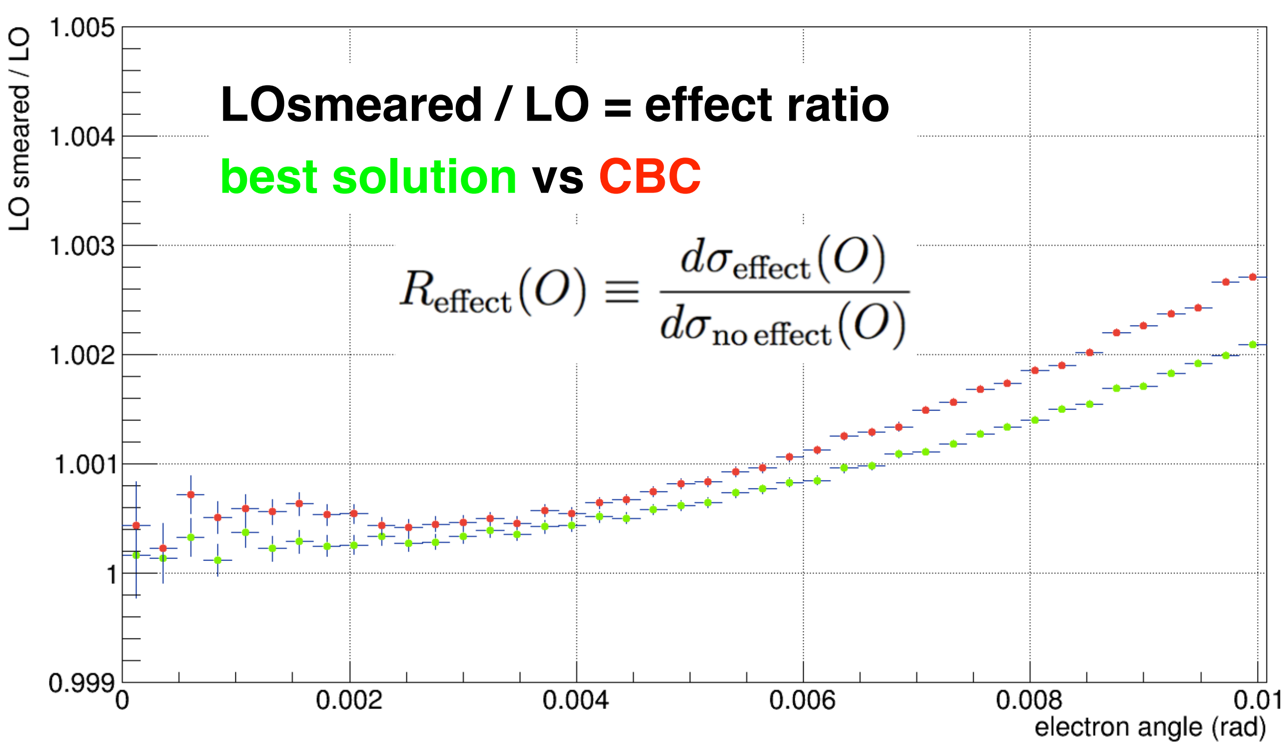
Systematic error due to MS mis-knowledge on ratios

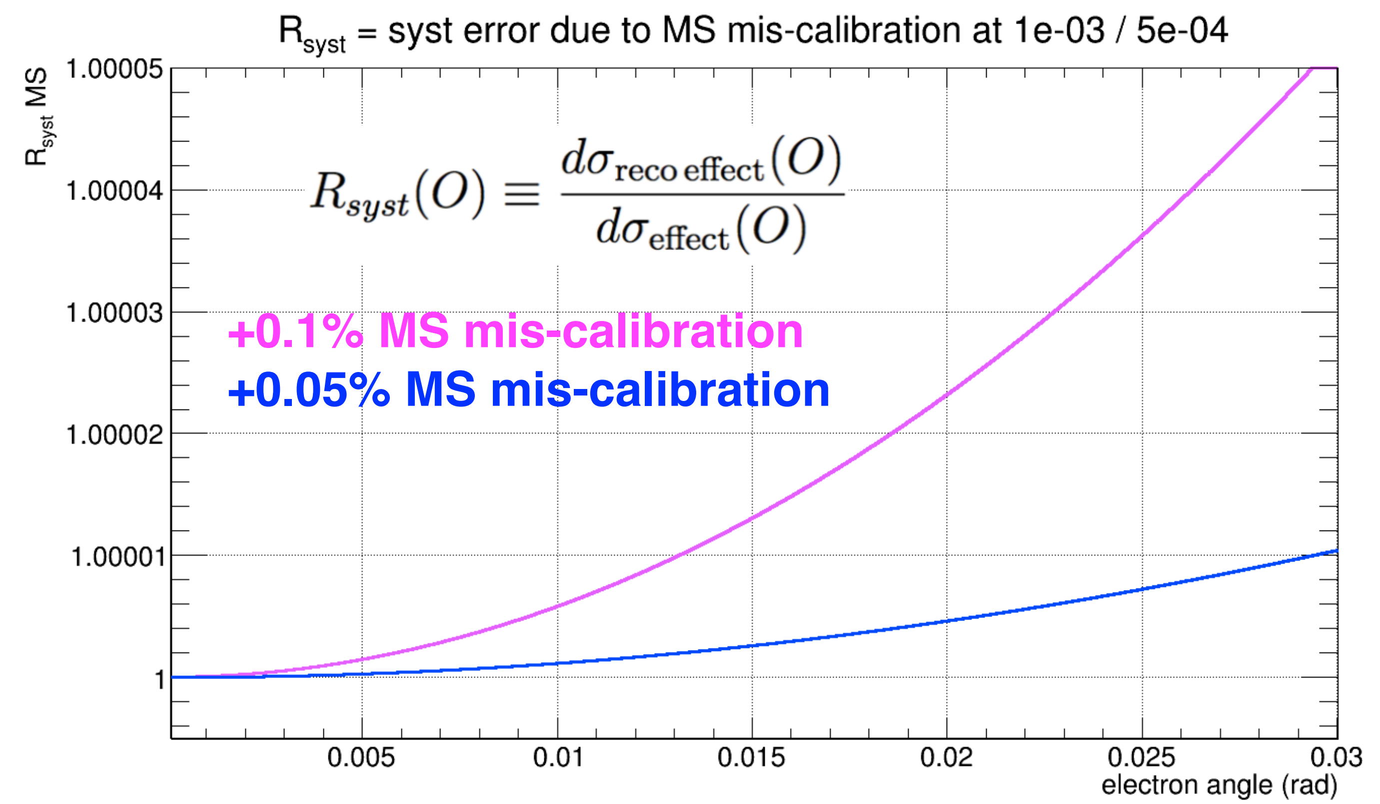
MS effect on cross section



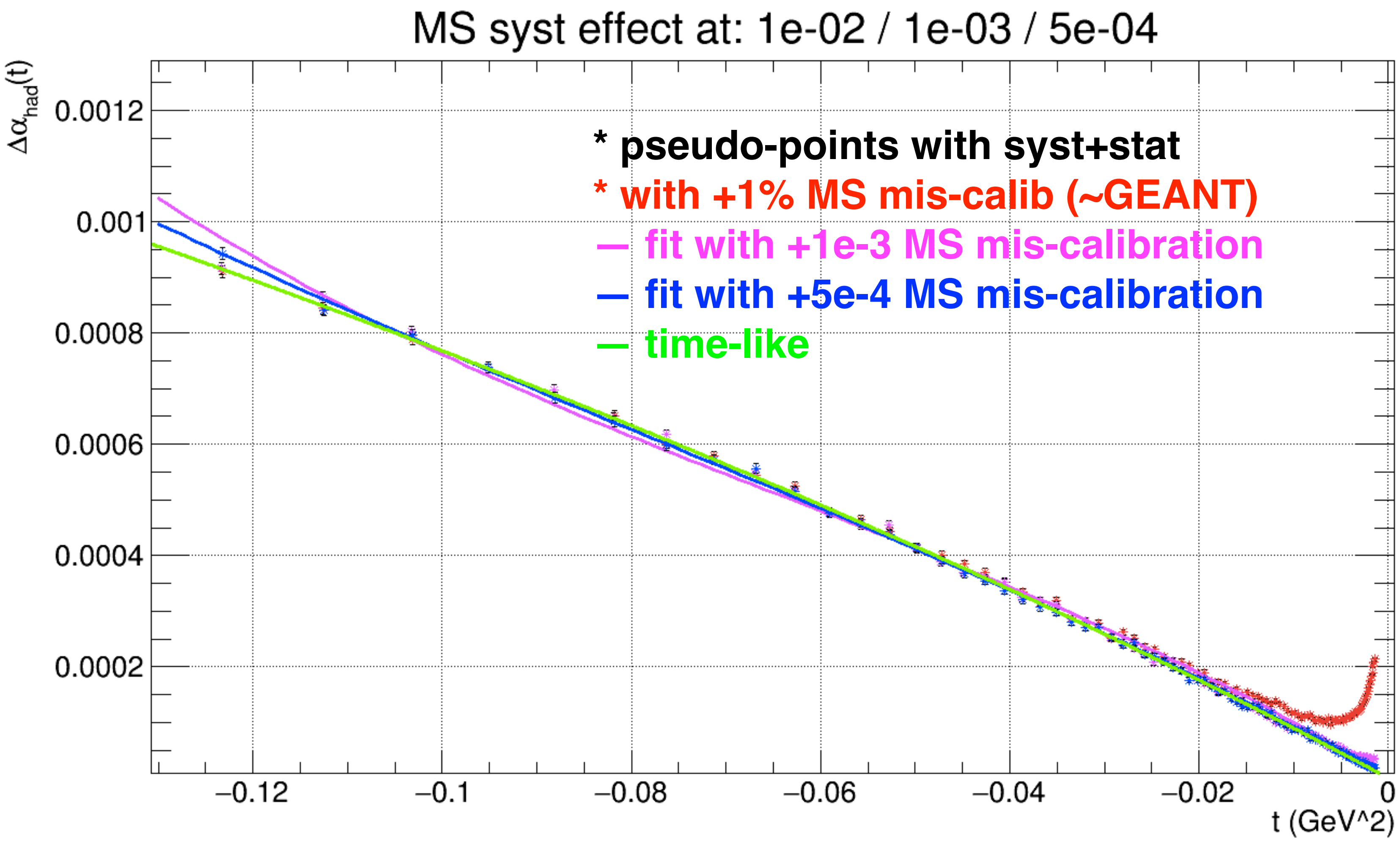
MS syst error on cross section

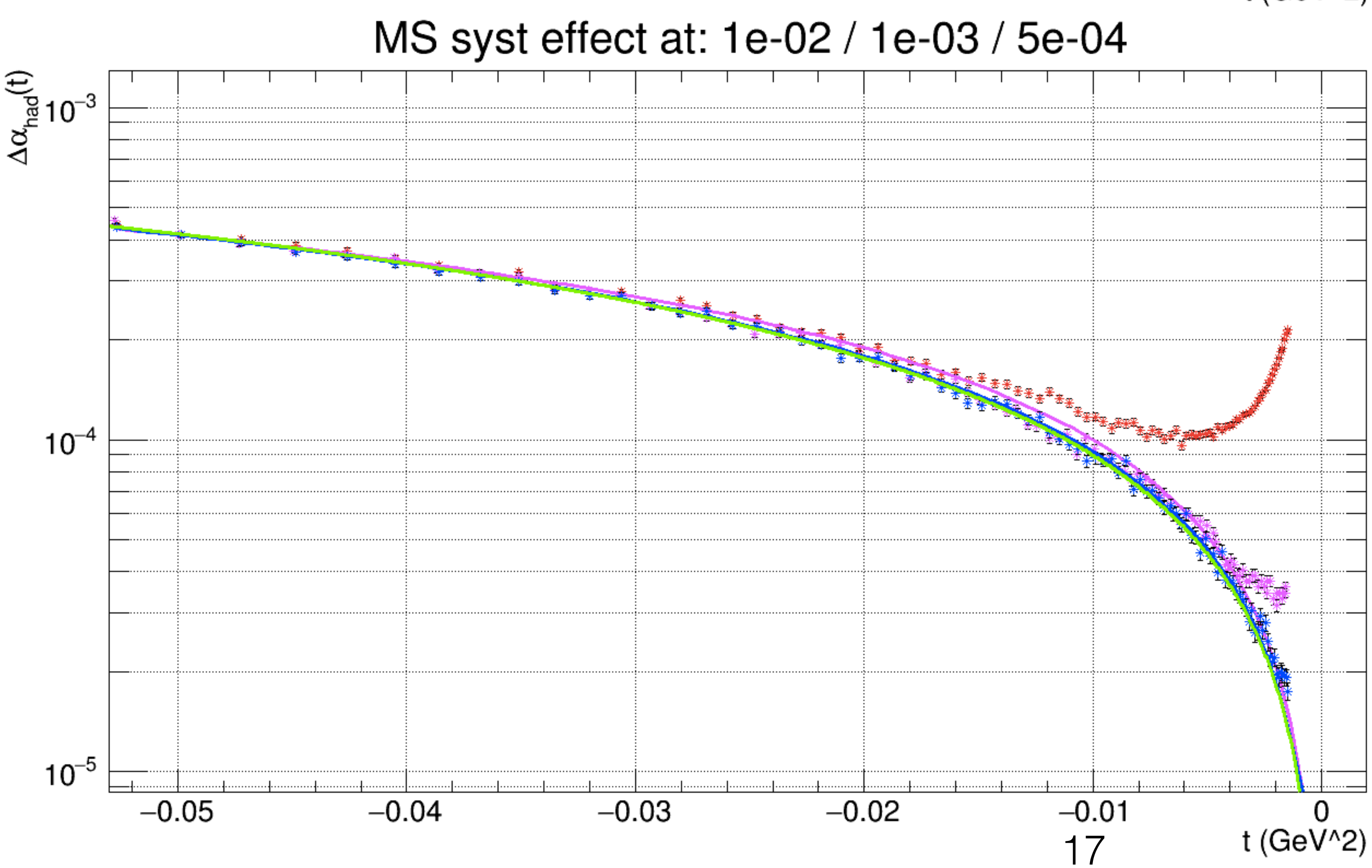






- For MS, I used our previous fast-MC studies and Fedor ones for comparisons.





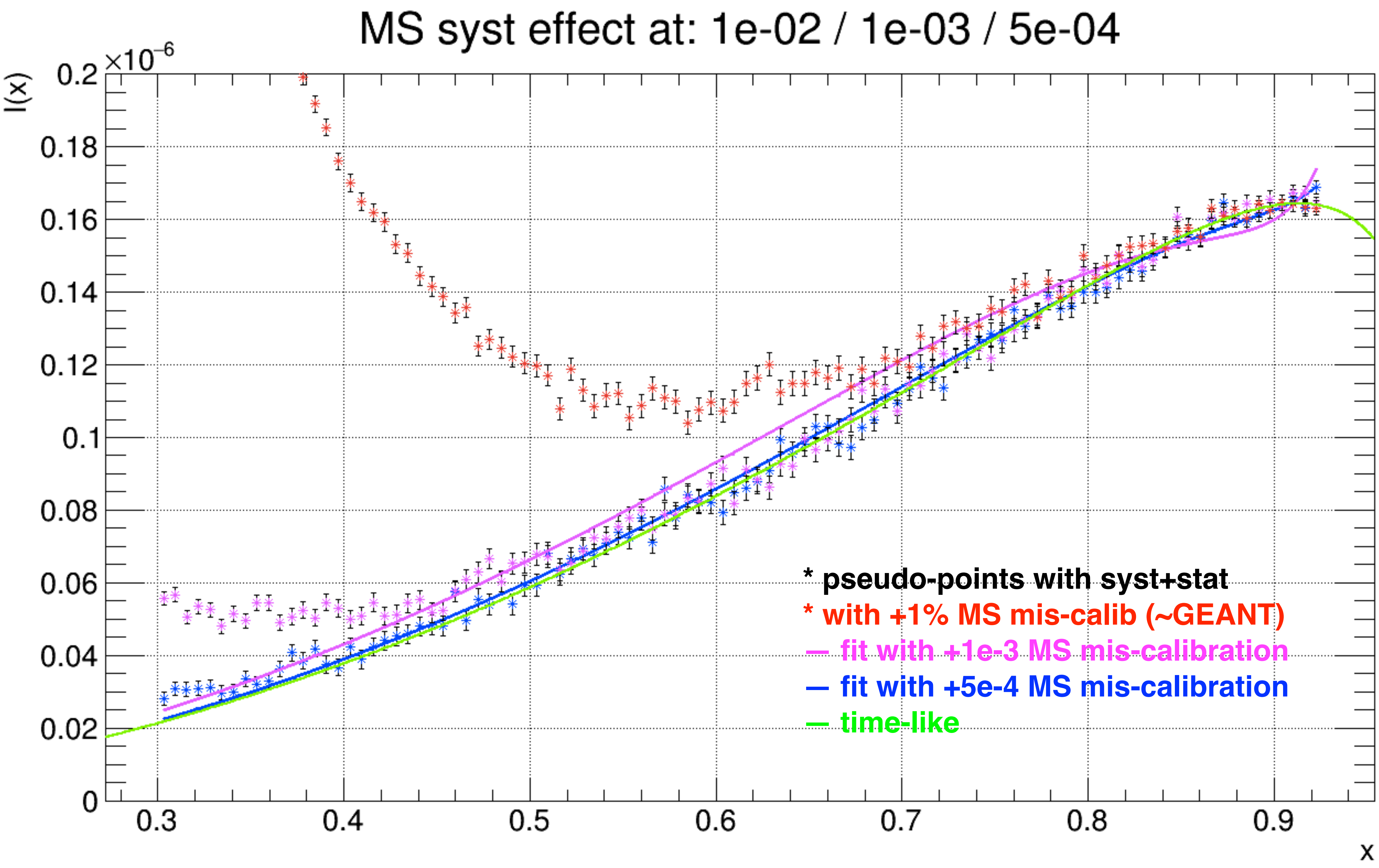
Propagation on $\Delta\alpha_{had}(t)$: effect of MS at various mis-calibration levels

Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

From this single pseudo-fit
with +0.1% MS mis-knowledge:
$a_\mu^{HLO} = 597.1 \cdot 10^{-10}$
~ +6% difference from timelike.

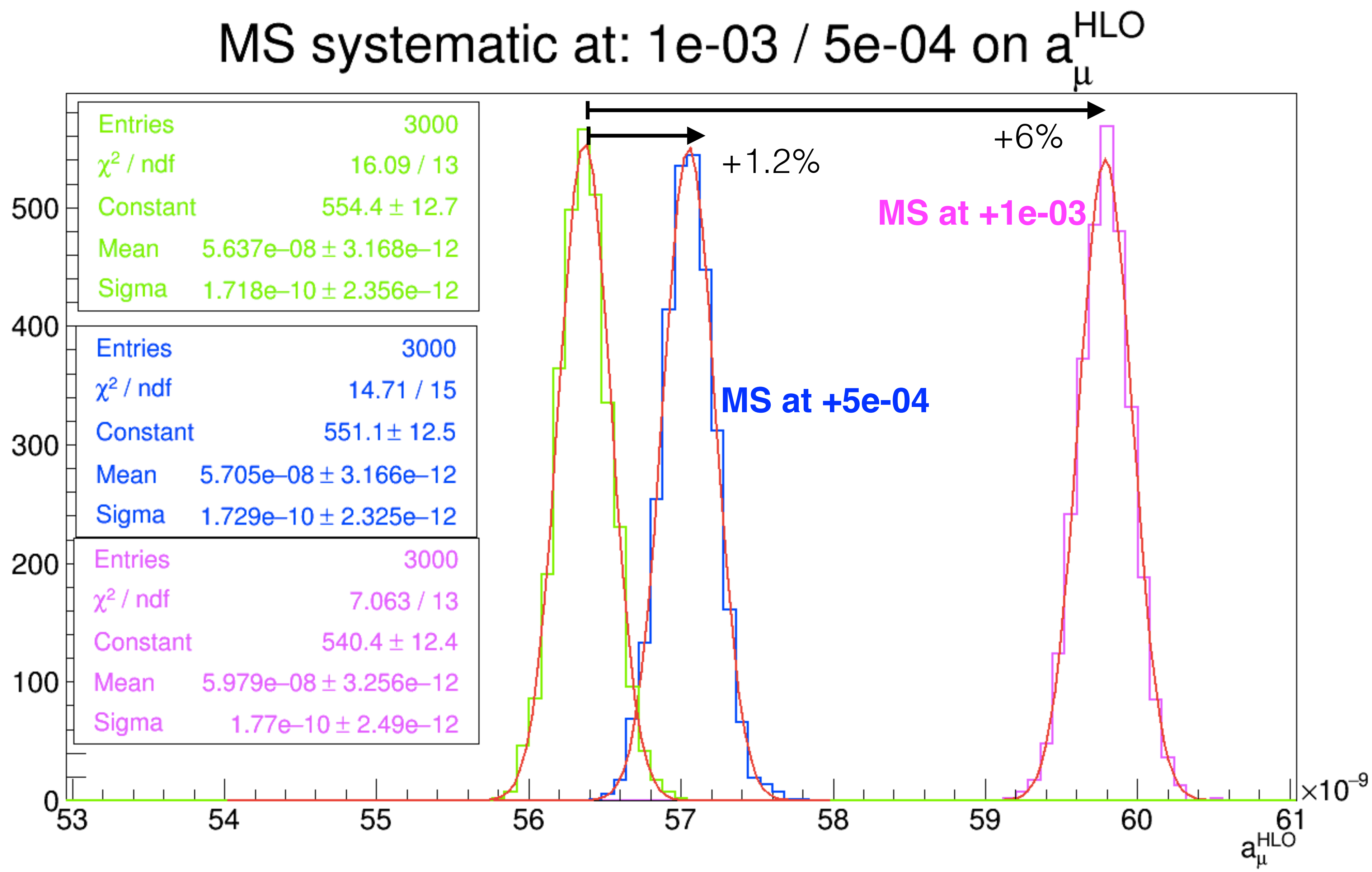
From this single pseudo-fit
with +0.05% MS mis-knowledge:
$a_\mu^{HLO} = 570.2 \cdot 10^{-10}$
~ +1% difference from timelike.

Propagation on integrand: MS effect at various levels



- As we have already known, Geant precision is very far from required one, so we must fit the MS systematic behavior <u>using data</u> and <u>necessarily correct it</u>.

Systematic shifts on final a_μ^{HLO}: MS effect



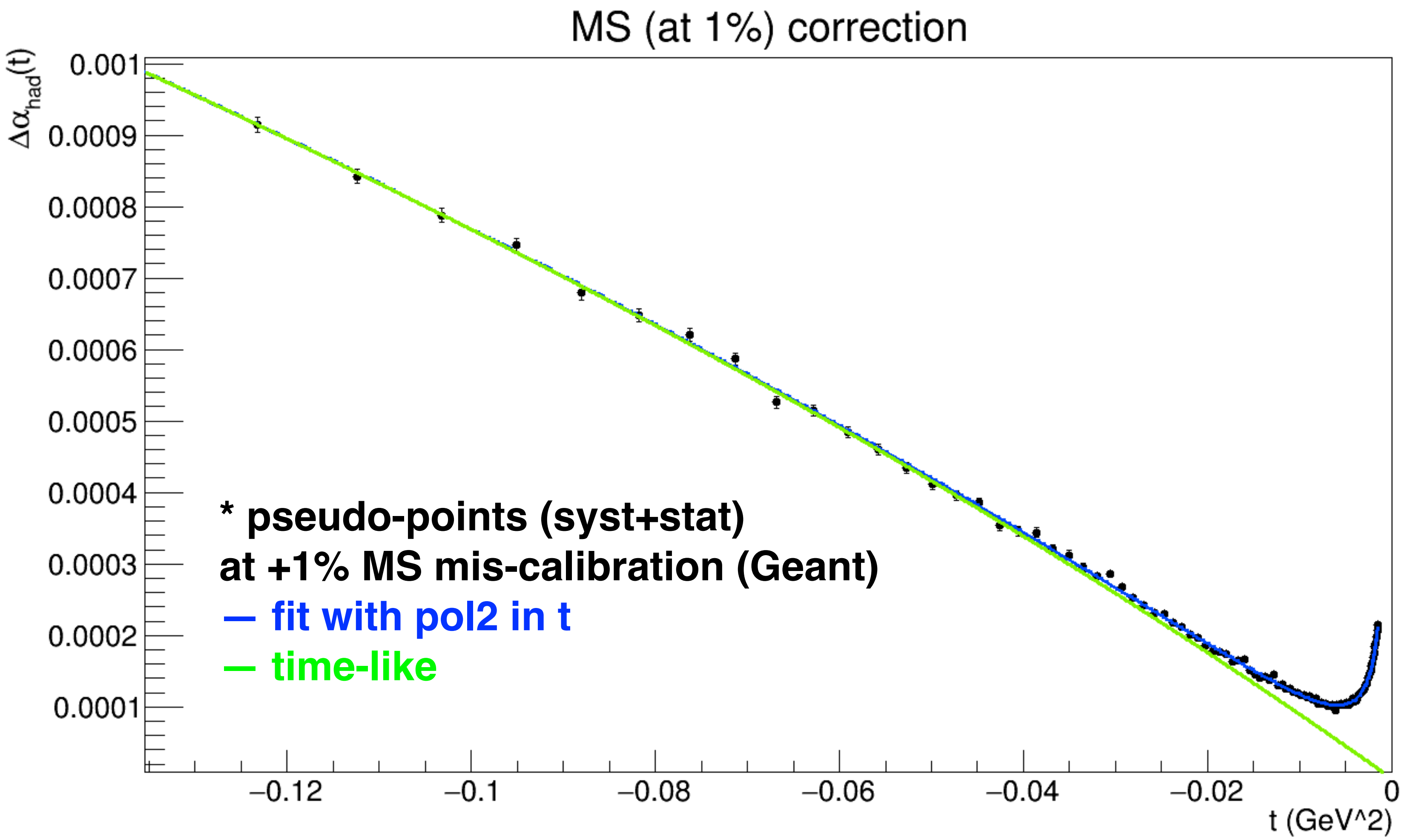
Time-like: $a_\mu^{HLO} = 563.624 \cdot 10^{-10}$

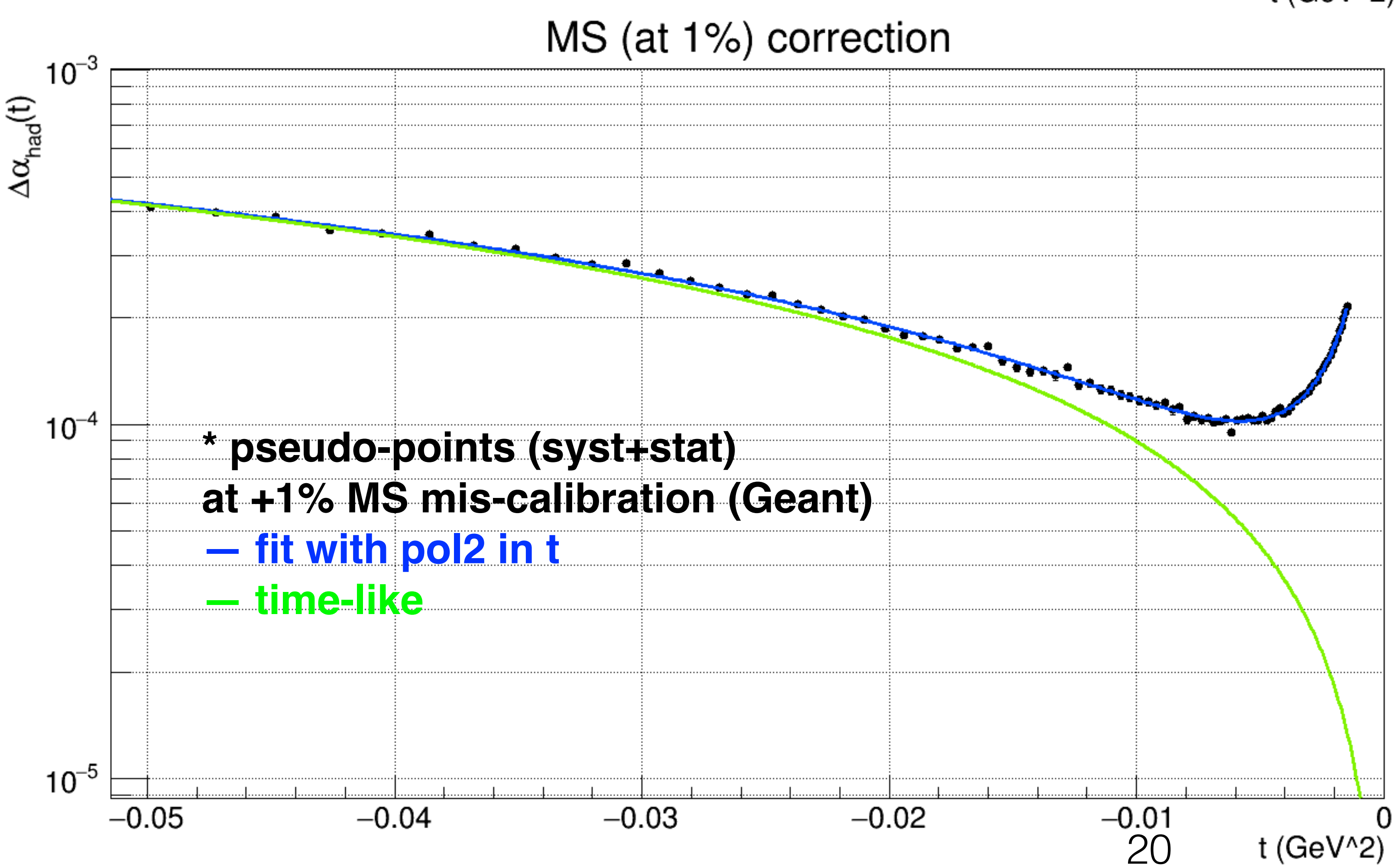
From pseudo-fit: $a_\mu^{HLO} = (563.67 \pm 1.72) \cdot 10^{-10}$ -> Statistical only

From pseudo-fit: $a_\mu^{HLO} = (597.89 \pm 1.77) \cdot 10^{-10}$ -> with 0.1% MS mis-knowledge

From pseudo-fit: $a_\mu^{HLO} = (570.52 \pm 1.73) \cdot 10^{-10}$ -> with 0.05% MS mis-knowledge

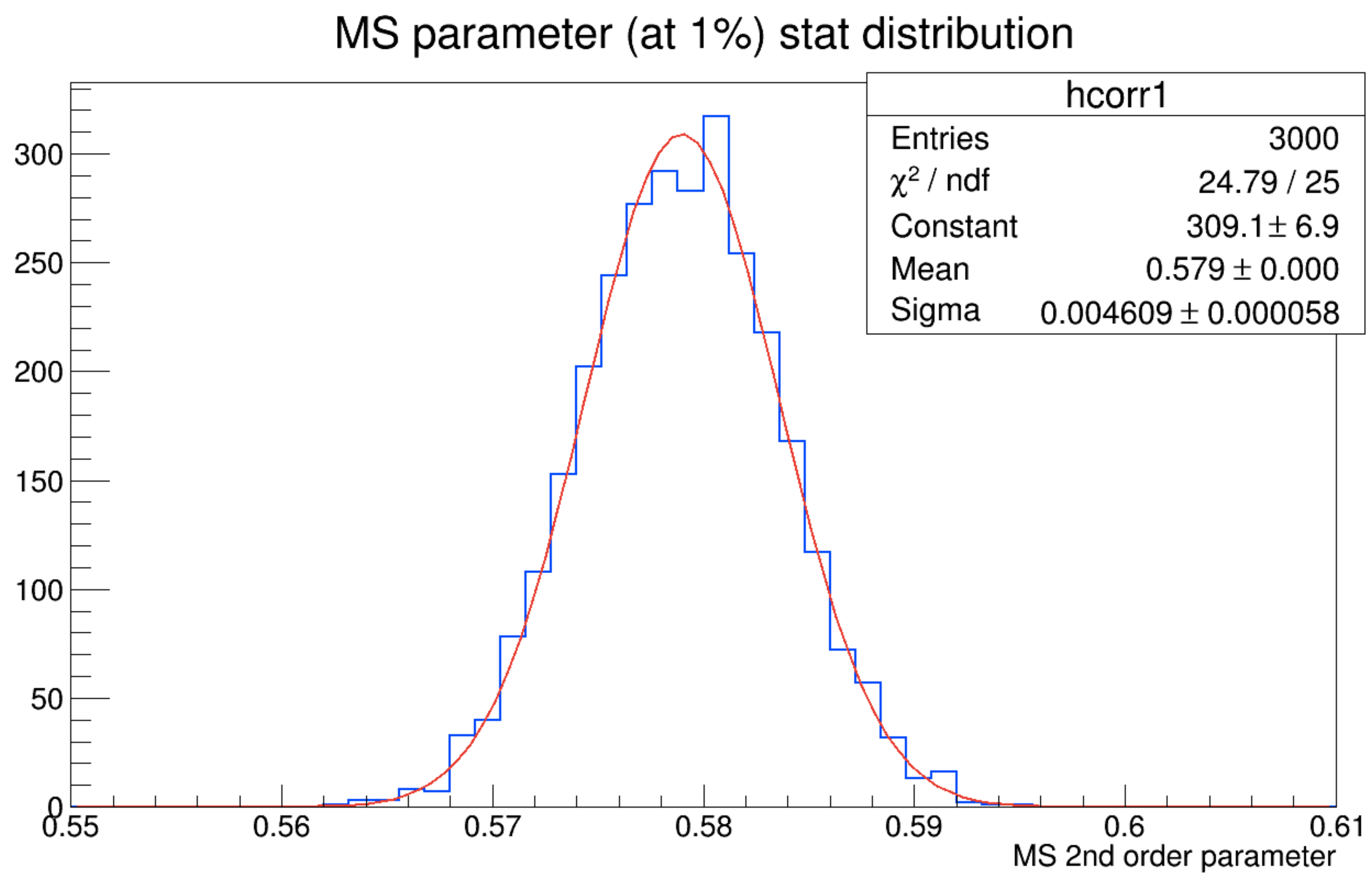
Check of MS correction method: fit of quadratic shape (in t)





- Check of statistical sensitivity to parabolic deformation due to MS systematic (mis-calibration) at 1%.
- Very good agreement with modified pseudo-points: this is an exercise.

Check of MS correction method: 2nd order coefficient



- MS at +1% (see previous slides)
- Quadratic MS function to modify cross section: f(theta) = a + b * theta^2
- a = 5e-06 and b = 0.579 (from CMS tracker simulation)

From 3000 pseudo-exp, <u>from quadratic fit on t</u>: b = (0.5790 ± 0.0046).

- So I figured out that our previous correction studies can keep their validity.

Conclusions

- This study allows to connect experimental systematics to the final fit. Conclusions do not seem to significantly change previous assessments on final precision.

- **Average beam energy** must be known at MeV level (<3e-05 relative to the nominal energy value): this will be possible with the spectrometer (BSM) and by a posteriori methods that rely on the data.

- **Beam energy spread** (natural width) must be known < percent level: such a task can be achieved with BSM, but with a precision closer to ~0.5%.

- **Multiple scattering** must be known (in total) at < 5e-04 to achieve a percent systematic error on a_{μ}^{HLO}: as we already know, it should be possible to correct the 1% accuracy of Geant from the data, identifying related systematic trend on cross section. I stressed this is a delicate and priority question. Multiple scattering correction studies have only been performed at leading order, so they will need to be updated in the light of recent NLO simulations.

- In addition, it must be considered that here I have used the total statistics: given the request for extreme precision, it seems that the individual modules counts are not in fact homogeneous, so their treatment will require special caution.