

# The gravitational S-matrix

Steven B. Giddings

UCSB and CERN

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- 1) Any candidate theory of quantum gravity should describe this regime, at least in principle. (E.g. could put on big computer.)
- 2) Generally high-energy scattering probes the most fundamental structure of a theory.
- 3) Such scattering encounters a deep conceptual paradox, driving at the heart of the conflict between general relativity and quantum mechanics.
- 4) Reasons 2 and 3 suggest that its study may point the way to new principles critical to understanding the quantum mechanics of gravity.
- 5) If we're very lucky, it could be studied at the LHC.

Plan of talk: overview of this and related issues



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The reason:



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Just need:

1) Lorentz invariance

2) very weak notion of locality

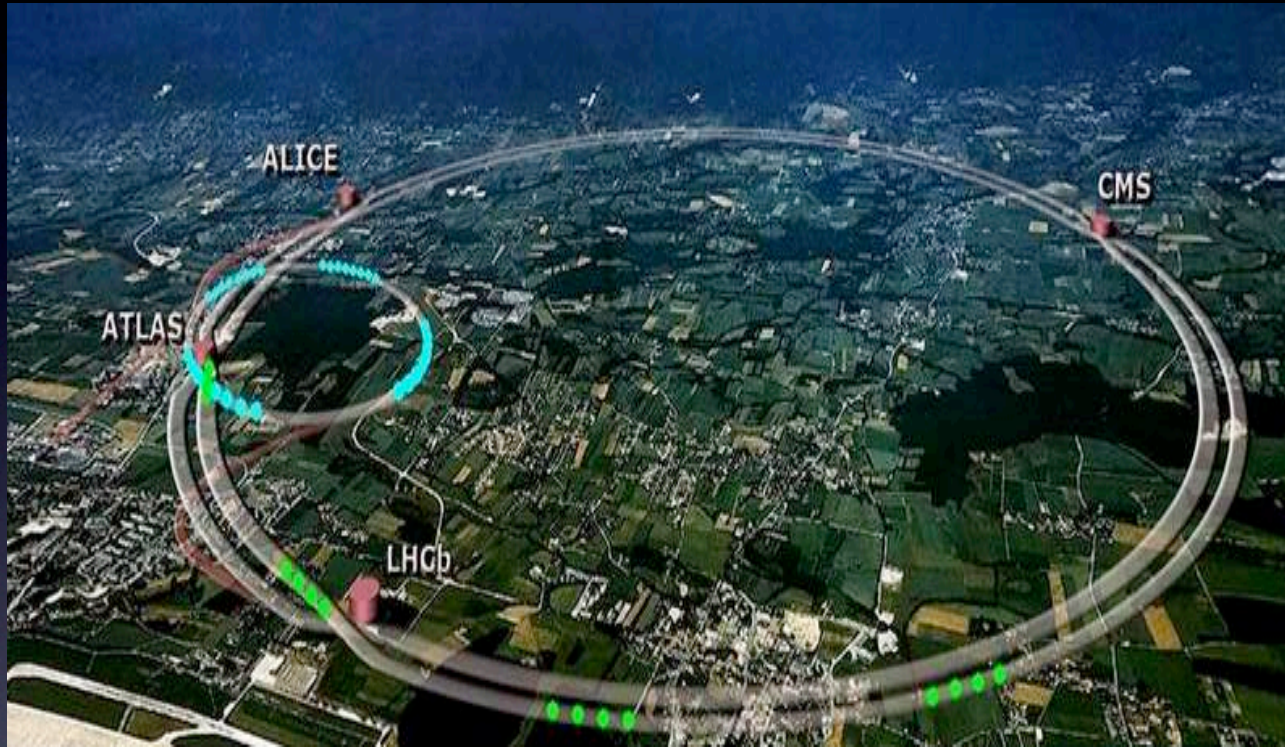
Indeed, nature provides us with observed  
cosmic accelerators (presumably AGN)  
reaching already up to

$$\sim 10^{12} \text{ GeV}$$

Moreover, ...



In extra dimensional scenarios yielding TeV-scale gravity, even



at LHC!

(A review: [arXiv:0709.1107](https://arxiv.org/abs/0709.1107))



# LI violation might alter this story, but:

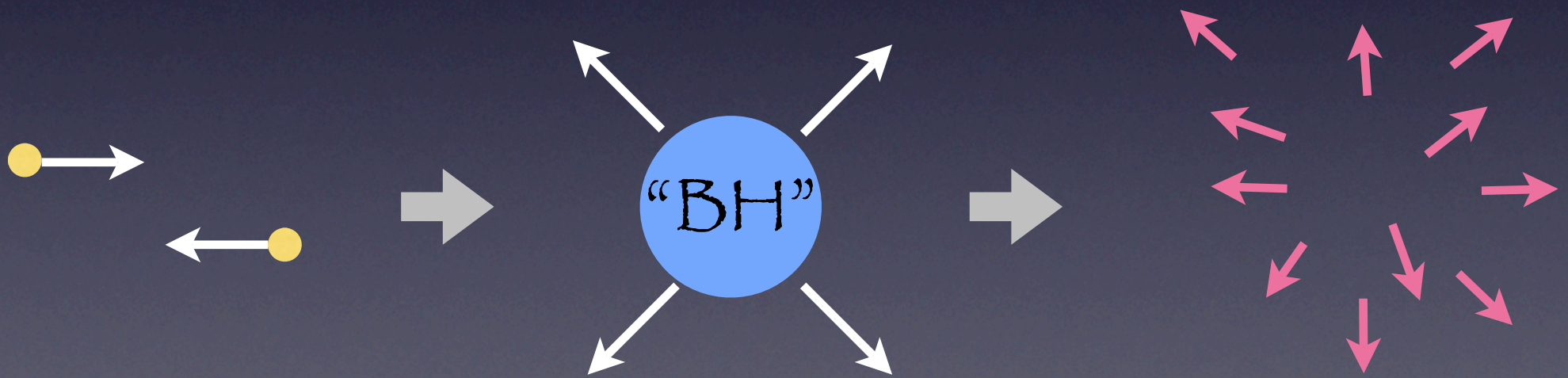
- hard to violate such symmetry a small amount
- stringent constraints
- potentially alters basic properties of black holes
- still find the problem of black holes and evaporation in more complicated contexts

⇒ won't consider



# $E \gg M_p$ : dynamics

- Control impact parameter  $b$  -- wavepackets
- Large  $E$ :  $\sim$  semiclassical picture
- Classically, produce black hole, + radiation
- Quantum corrections: Hawking radiation



(Indeed,  $\cancel{M}$  doesn't avoid, if form BHs other ways)

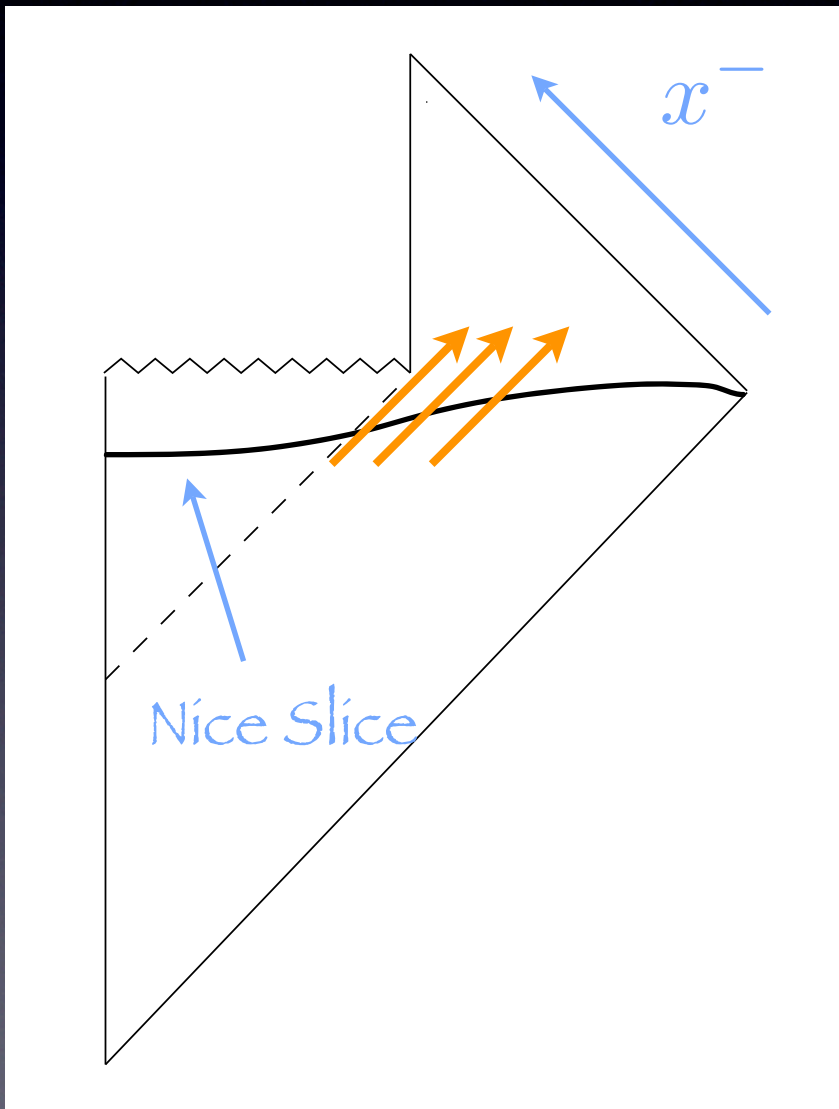
We then confront the “information paradox.”



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Lightening review:

Hawking, updated: nice slice argument



Locality:

$$|\psi_{NS}\rangle \Rightarrow \rho_{HR} \sim \text{Tr}_{in} |\psi_{NS}\rangle \langle \psi_{NS}|$$

$$S_{HR}(x^-) \sim -\text{Tr}(\rho_{HR} \ln \rho_{HR})$$

Increases to  $\sim A_{BH}$

at  $t_{evap}$

$\therefore$  information lost

(Hawking, 1976)

The problem is, QM is remarkably robust:

Banks, Peskin, Susskind (1984)  
-- studied such info loss:



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Basic idea: transmitting info requires energy

$\therefore$  loss of info violates energy conservation

$\therefore$  such virtual effects

$\Rightarrow$  Massive  $E$  nonconservation

$T \sim M_p$ , in this room

So: let's try to keep unitary evolution!

If information isn't lost, maybe it's left  
behind: in remnants?



If information isn't lost, maybe it's left  
behind: in **remnants?**

But: begin w/ arbitrarily large black hole

$\Rightarrow$  infinite species  $M \sim M_p$

$\Rightarrow$  Infinite production instabilities

(See e.g. hep-th/9310101, hep-th/9412159)

# The “paradox:” a conflict between

Lorentz/diff invariance (macroscopic)

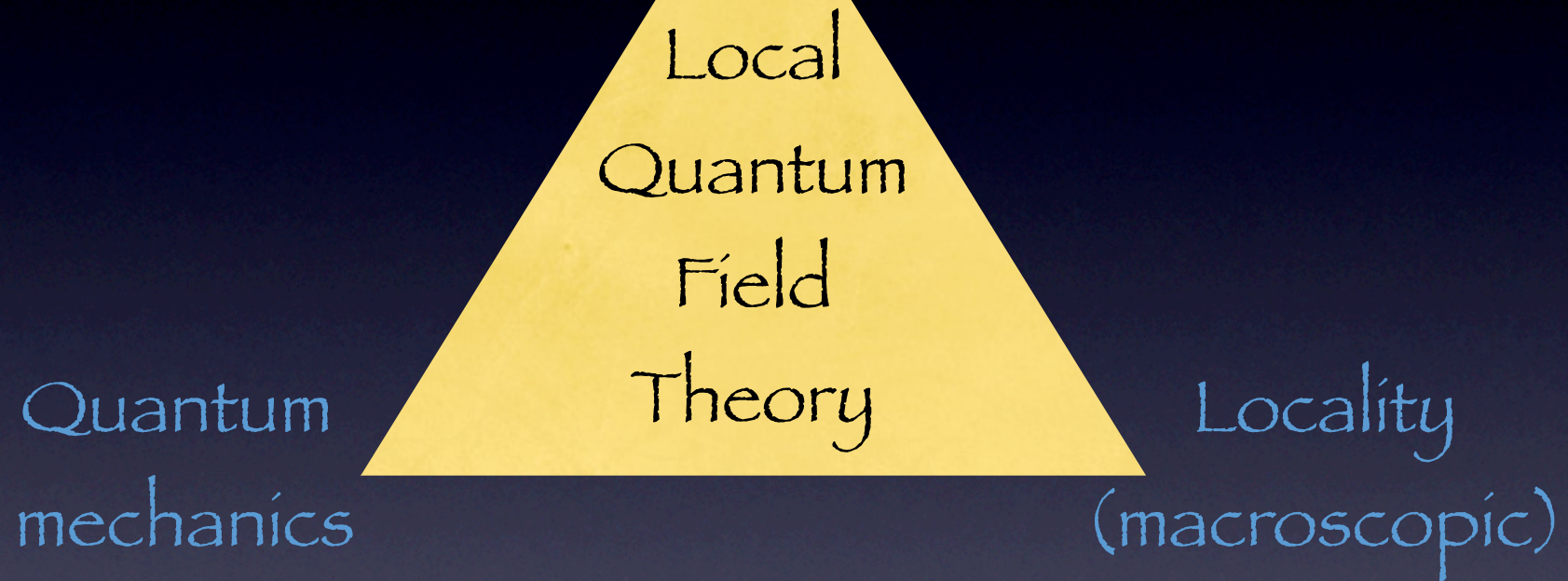
Quantum  
mechanics

Locality  
(macroscopic)



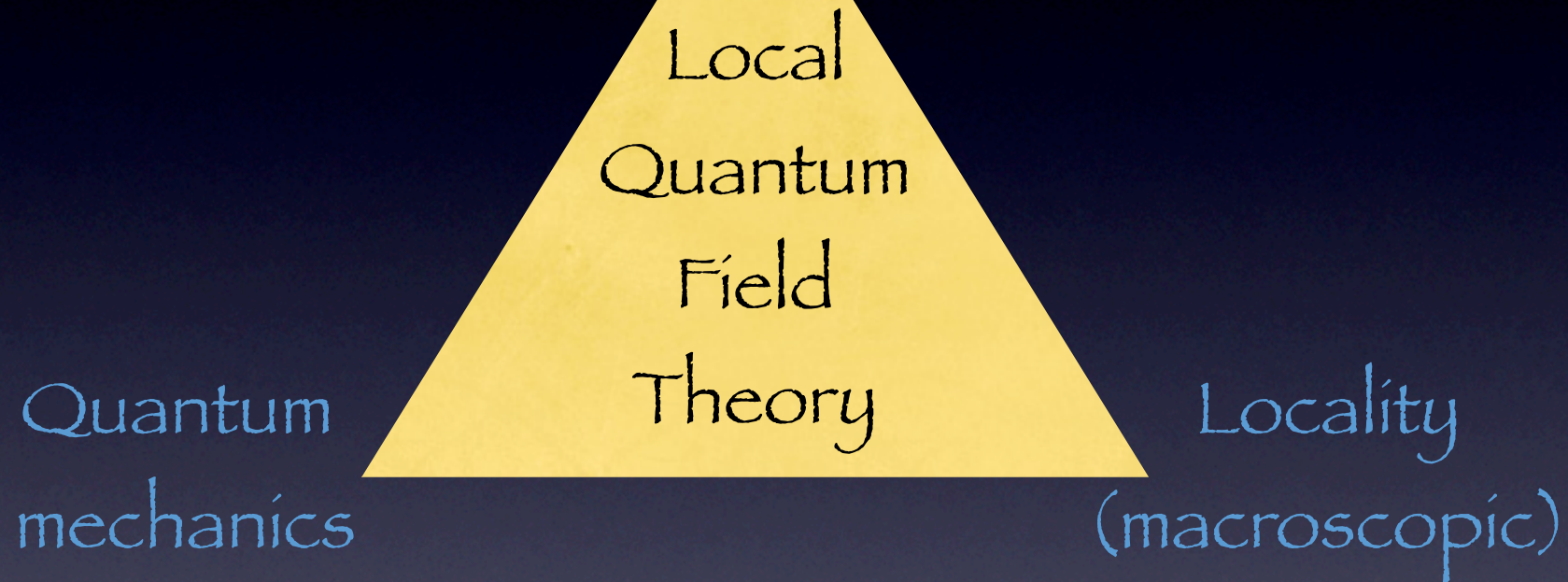
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QM, LI -- can't see how to modify, respecting consistency and observation

A weak point: *locality?*



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# What do the dominant quantum gravity paradigms say?

**LQG:** working to recover the familiar world of ( $\sim$ ) Minkowski space, multi-particle perturbations, and their scattering

(some recent progress; success remains to be seen)



# String theory:

Hints (?) at a solution:

addresses nonrenormalizability  
extendedness/nonlocality  
microstate counting, etc.

Idea: “holography:”

$D$ -dim. grav  $\equiv$   $(D-1)$  non-grav unitary thy  
(AdS/CFT)

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Examine more closely, see what actually says...



# General problem: investigate UHE scattering (D-dimensions)

(Provisional) summary of some of what we know.

(One question: when/how strings relevant?)

(More detail: SBG; SBG, Gross, Maharana; SBG & Srednicki; SBG & Porto)

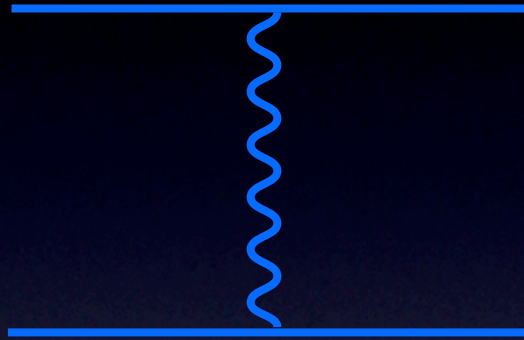
Parameters:

$E = \text{energy}, \gg M_D$

$b = \text{impact parameter} \dots \text{decrease}$

# Regimes:

1) Born  $b \rightarrow \infty$



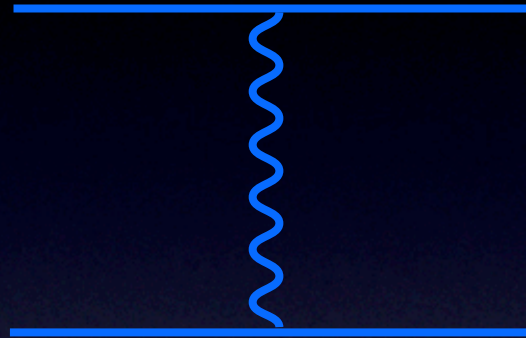
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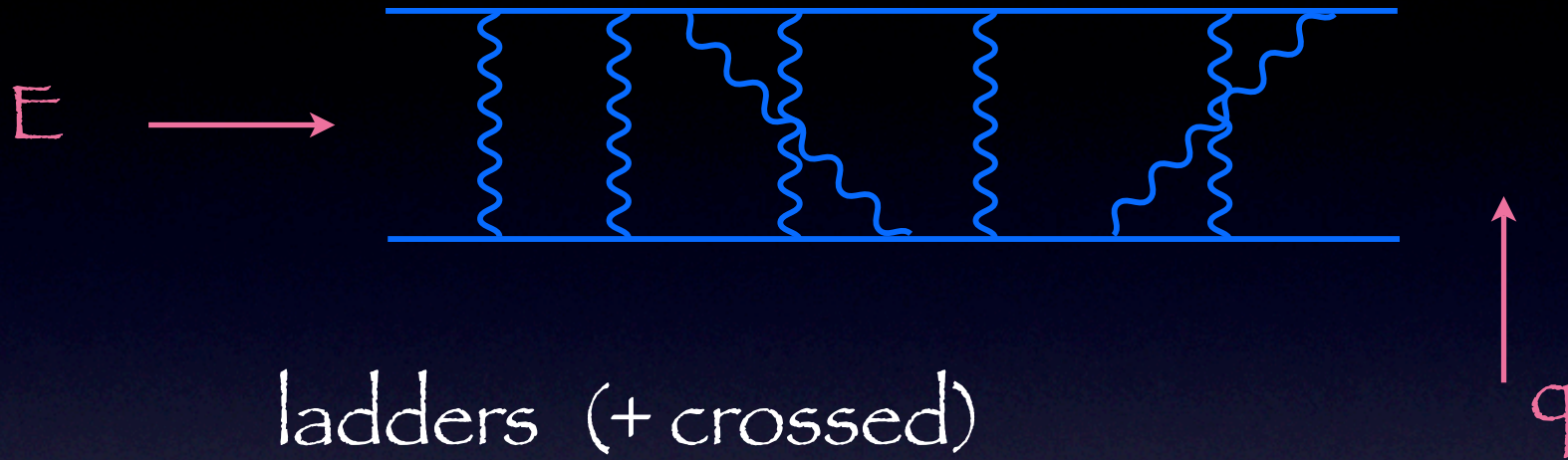


$$E \gg M_p$$

$$T_{tree} = -8\pi G_D s^2 / t$$

Where do strings modify? Naively, might guess  
 $b \sim l_{st}^2 E$  (long strings) but -- tiny corrections  
(will see momentarily)

Instead, leading corrections:



$$s = E^2 ; t = -q^2$$

For  $-t \ll s$ , can write sum over loops in terms of tree amplitude:



$$iT_{\text{eik}}(s, t) = 2s \int d^{D-2}x_{\perp} e^{-iq_{\perp} \cdot x_{\perp}} (e^{i\chi(x_{\perp}, s)} - 1)$$

$q_{\perp}$  = perpendicular to CM momentum

$x_{\perp} \sim$  impact parameter  $b$

$$\begin{aligned} \chi(x_{\perp}, s) &= \frac{1}{2s} \int \frac{d^{D-2}q_{\perp}}{(2\pi)^{D-2}} e^{-iq_{\perp} \cdot x_{\perp}} T_{\text{tree}}(s, -q_{\perp}^2) \\ &= (\text{const.}) \frac{G_D s}{x_{\perp}^{D-4}} \end{aligned}$$

... “eikonal phase”

(here  $T$  is full tree amp.)

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... “eikonal phase” (here  $T$  is full tree amp.)

Eikonal  $\leftrightarrow$  classical approximation

(See, e.g., Amati, Ciafaloni, and Veneziano)



Consider the classical metric of a high energy source:

Schwarzschild, boosted with  $E/m = \gamma \gg 1$

Aichelburg-Sexl solution:

$$ds^2 = -dx^+ dx^- + dx_{\perp}^2 + \Phi(x_{\perp}) \delta(x^-) dx^{-2}$$

$$\Phi = -8G_D E \log(x_{\perp}) , \quad D = 4 ;$$

$$\Phi = (\text{const.}) \frac{G_D E}{x_{\perp}^{D-4}} , \quad D > 4$$

E.g. compare classical scattering angle to  
eikonal saddlepoint

This indicates a second regime:

2) Eikonal  $\sim$  classical

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Born/eikonal transition:

$$\chi \sim 1 \iff b = x_{\perp} \sim (G_D E^2)^{\frac{1}{D-4}} \iff q_{\perp} \sim 1/b$$

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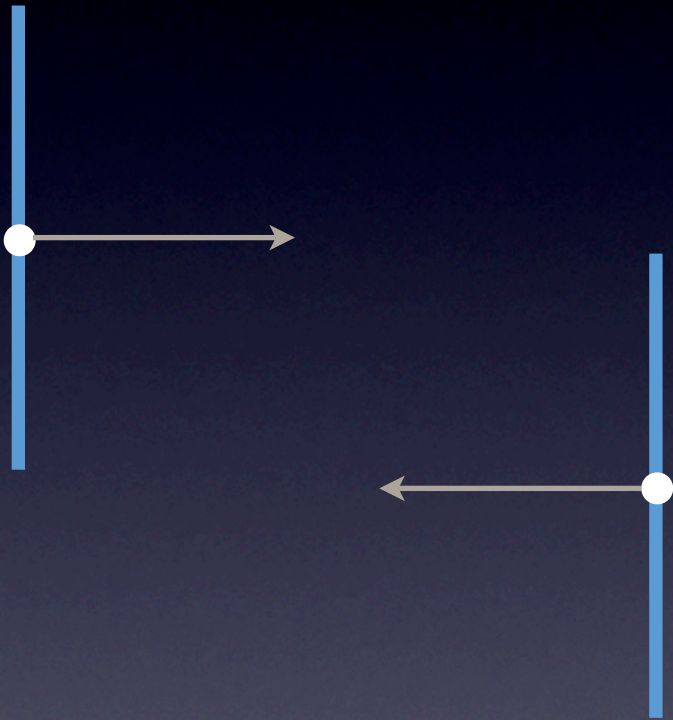
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Where do important corrections to the eikonal picture enter?



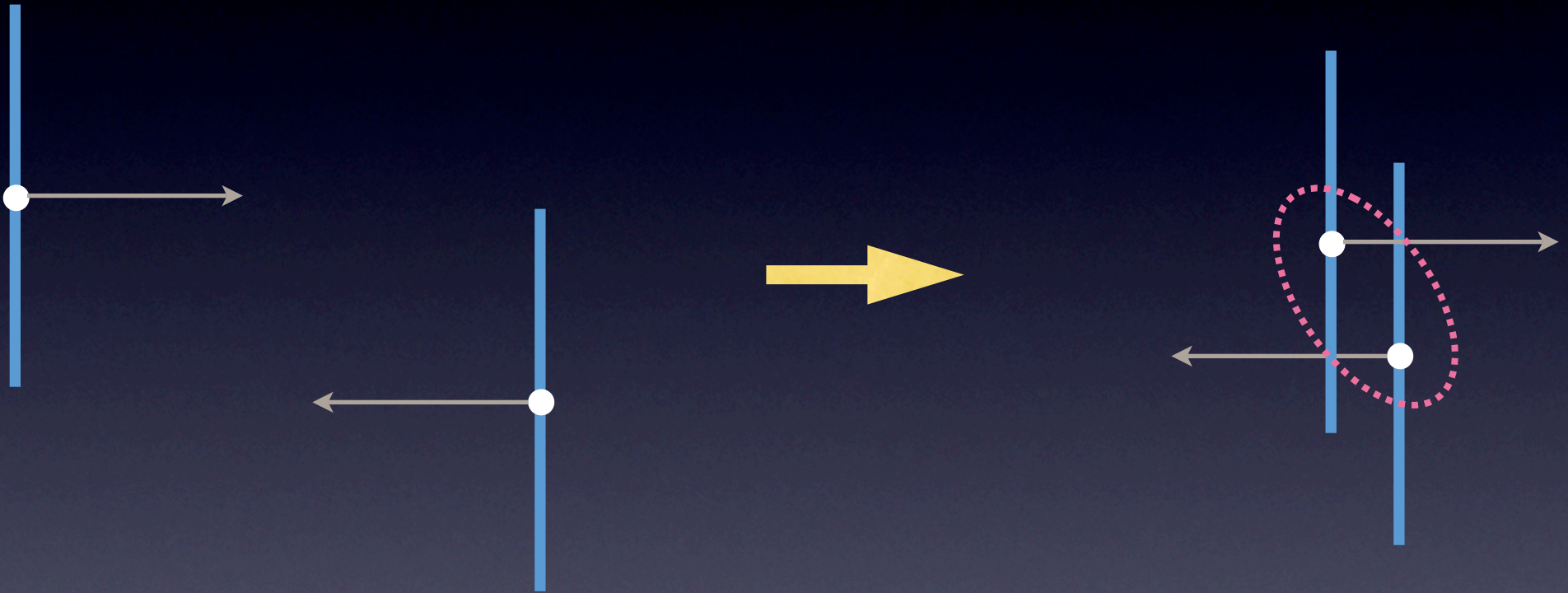
First, consider the classical problem;  
intuitively, form a black hole

Indeed:



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Indeed:



Classically, can show a closed trapped surface forms:

$$b \lesssim R(E) \quad (\text{SBG \& Eardley 2002, extending Penrose})$$



But: what important corrections?

- stringy

- quantum (e.g. other loops)

First, let's systematically look at string corrections

Begin w/tree-level amplitude: high E

$$T_{tree}^{string}(s, t) \propto g_s^2 \frac{\Gamma(-t/8)}{\Gamma(1+t/8)} s^{2+t/4} e^{2-t/4}$$

vs.

$$T_{tree}^{grav}(s, t) \propto G_D \frac{s^2}{t} \quad (D \text{ noncompact dims})$$



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- Agree for  $-t \ll 1$

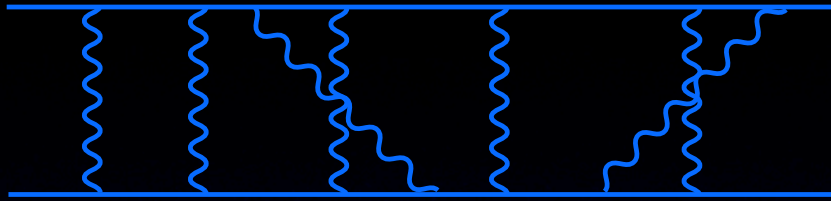
- Here see no evidence for long string effects:

$$b \sim E \leftrightarrow t \sim E^{-2(D-5)}$$

- But significant modifications for

$$t \sim -1$$

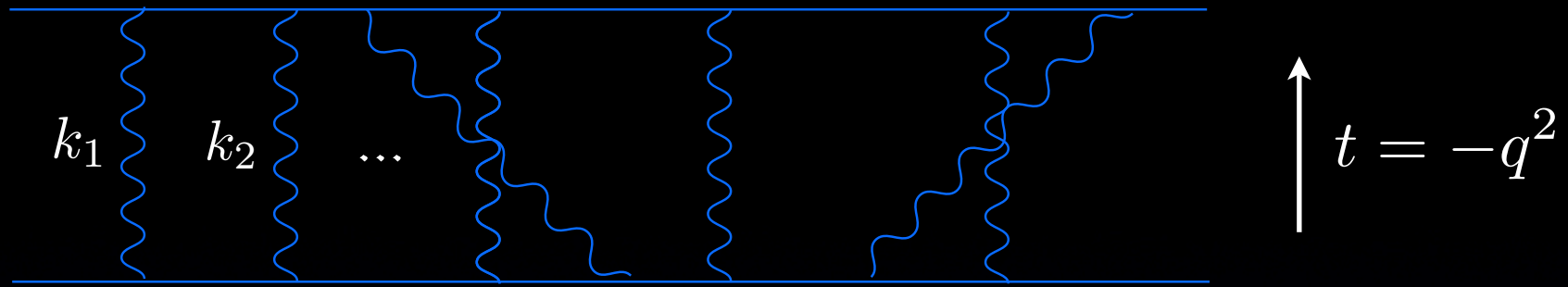
- However, as noted, diagrams



compete for  $t = -q^2 \gtrsim -\frac{1}{b^2}$   
( $\ll 1$ )

Suppose, for example, decrease  $b$ /increase  $-t$ :

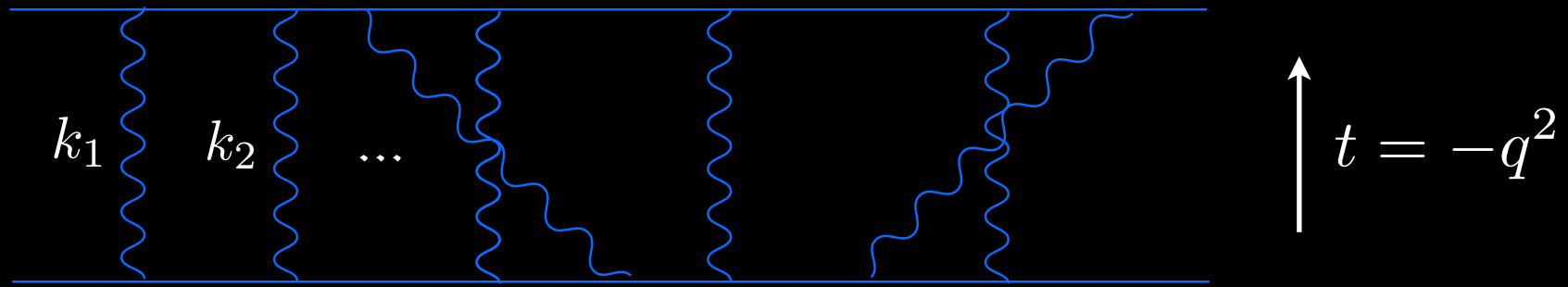




Dominant N:  $N \sim \chi \sim \frac{G_D E^2}{b^{D-4}} :$

At  $t \sim -1$  :  $N \sim (G_D E^2)^{\frac{1}{D-3}}$

$\Rightarrow$  Large loop order dominates.



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At given loop order, N, expect:

1)  $k_j \approx q/(N+1)$

2)  $E^{-\alpha' q^2/(N+1)}$

Thus at large N, string corrections small

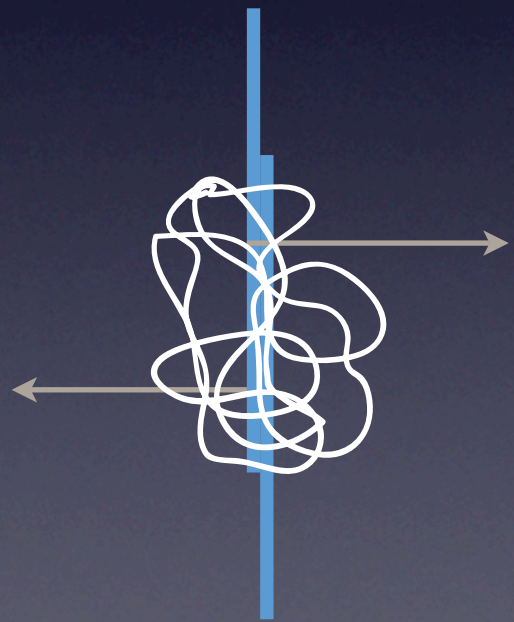


But - another effect: can excite strings through accumulated effect of grav exchange- “diffractive excitation” (ACV)

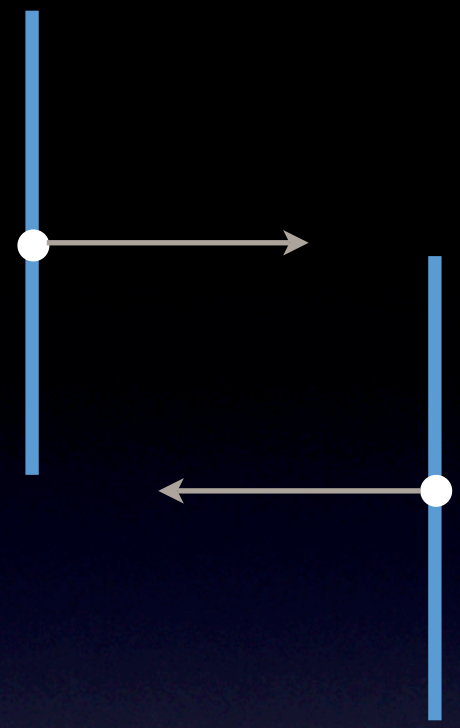
Indeed, unexcited (elastic) amplitude, near Schwarzschild impact parameter:

$$\mathcal{A}_{el} \sim \exp \left\{ -E^{(D-4)/(D-3)} \right\} \quad !!$$

So:

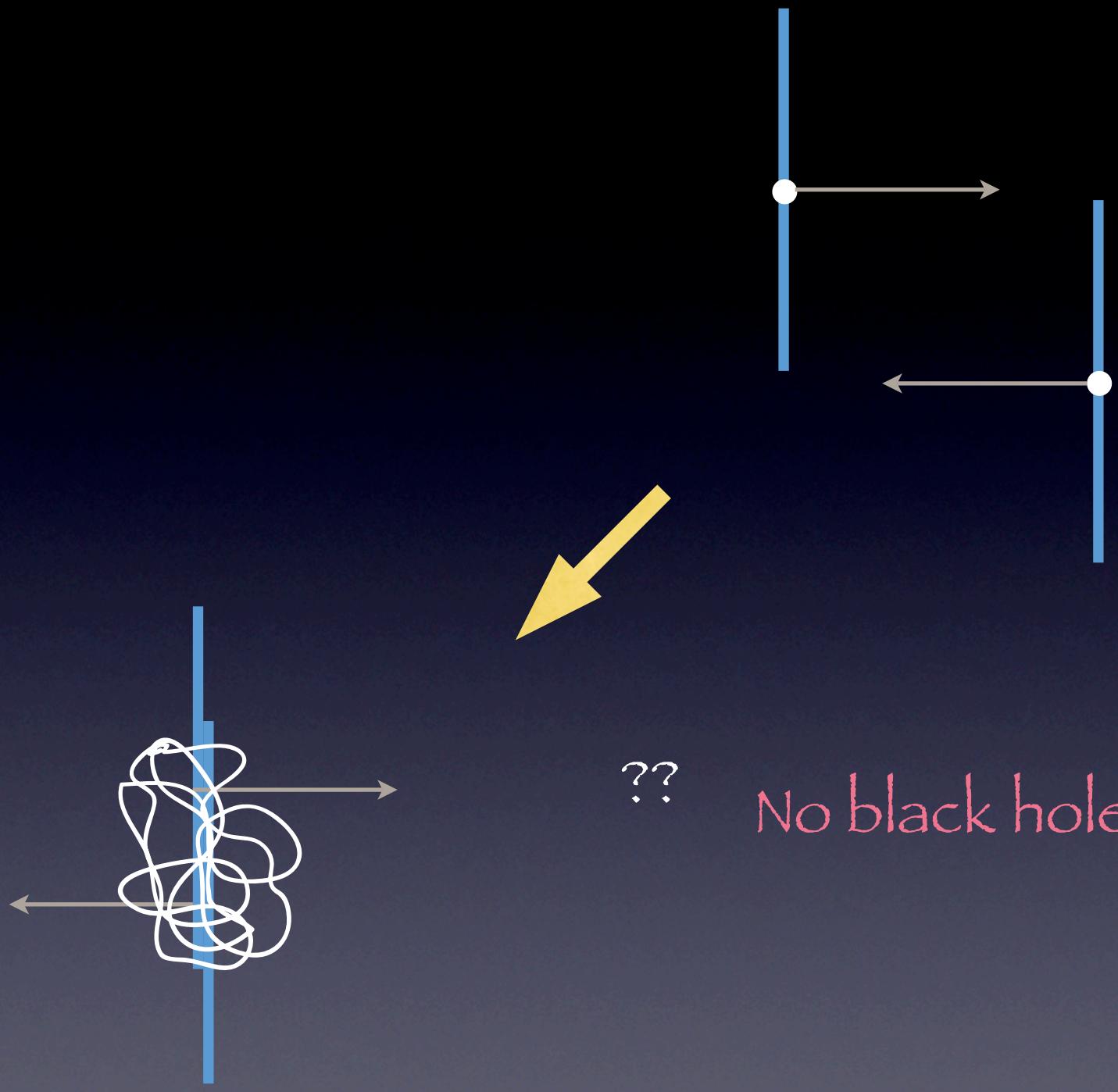


??





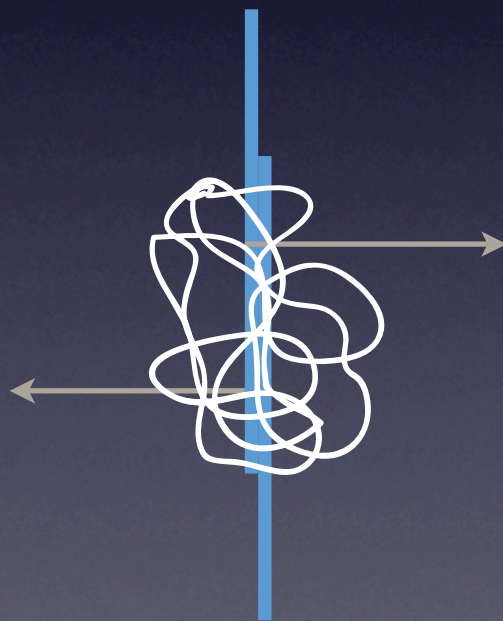
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??

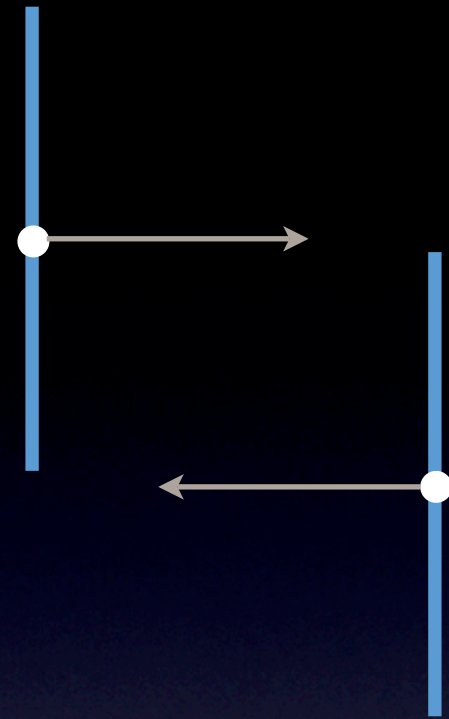
No black hole??

So:



??

No black hole??  
Info carried away?  
(Veneziano, 2004)





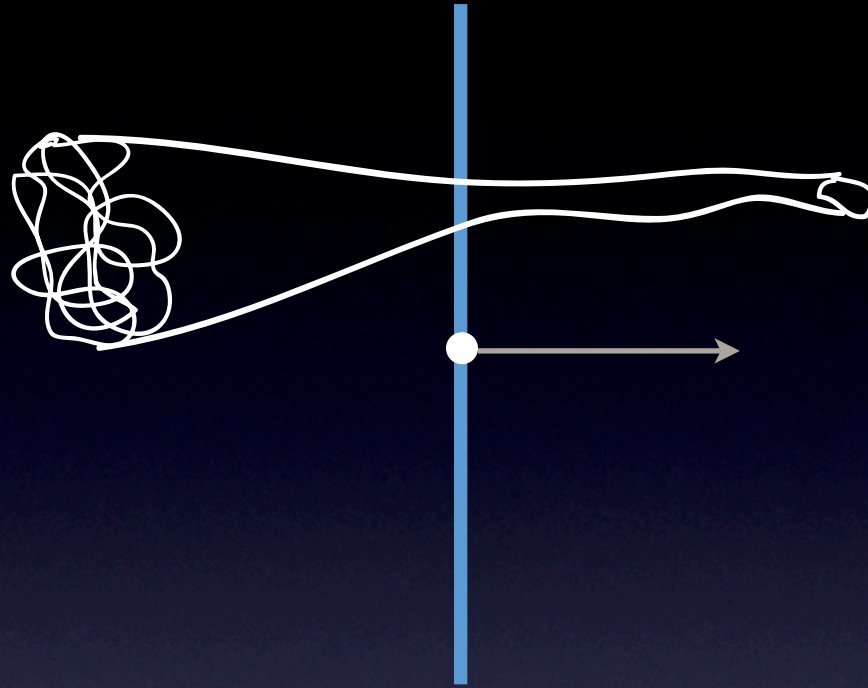
But there is a contrary intuition: string only  
“spreads out” “after” collision??

String spreading is a notoriously fuzzy  
concept, and requires some care

Depends on process in question, and its  
“resolving power”

Find:

SBG, Gross, Maharana, arXiv:0705.1816

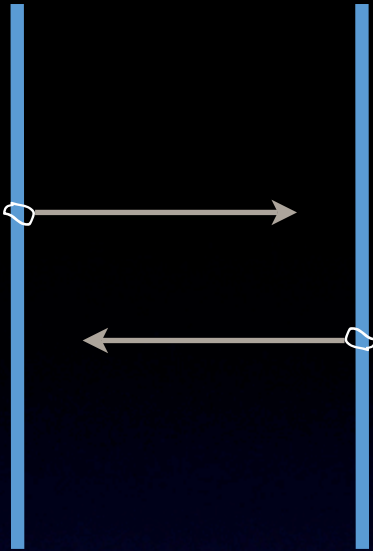


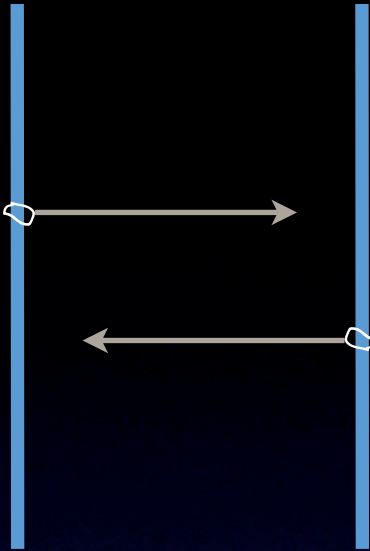
Indeed, origin of effect is “tidal string excitation”

$$(\Delta X)^2 \sim |\ln \epsilon| + \left[ \frac{G_D E^2}{b^{D-2}} \tau \right]^2 |\ln \tau| \quad \epsilon \ll \tau$$

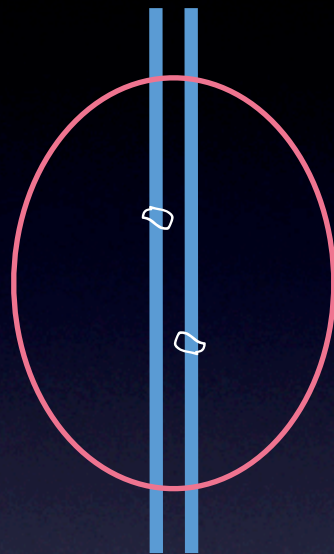
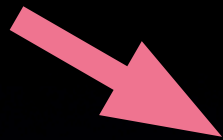
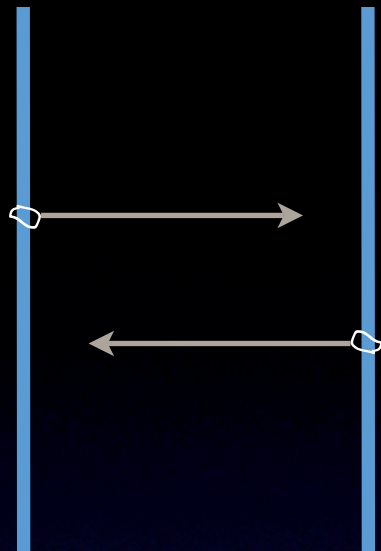
For small tau: inside trapped surface:



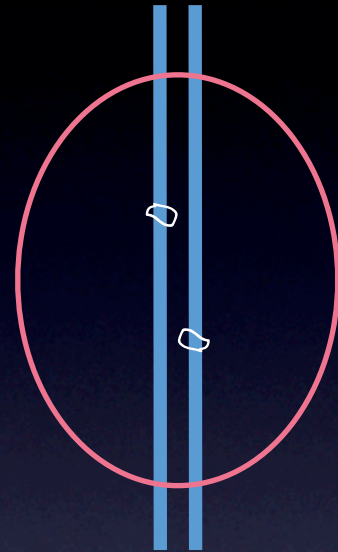
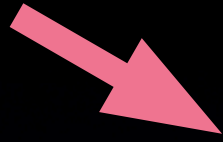
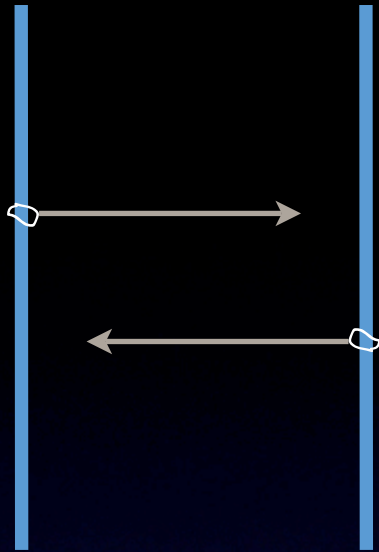








Trapped  
surface



Trapped  
surface



Black hole

$\therefore$  No apparent role for string  
extendedness

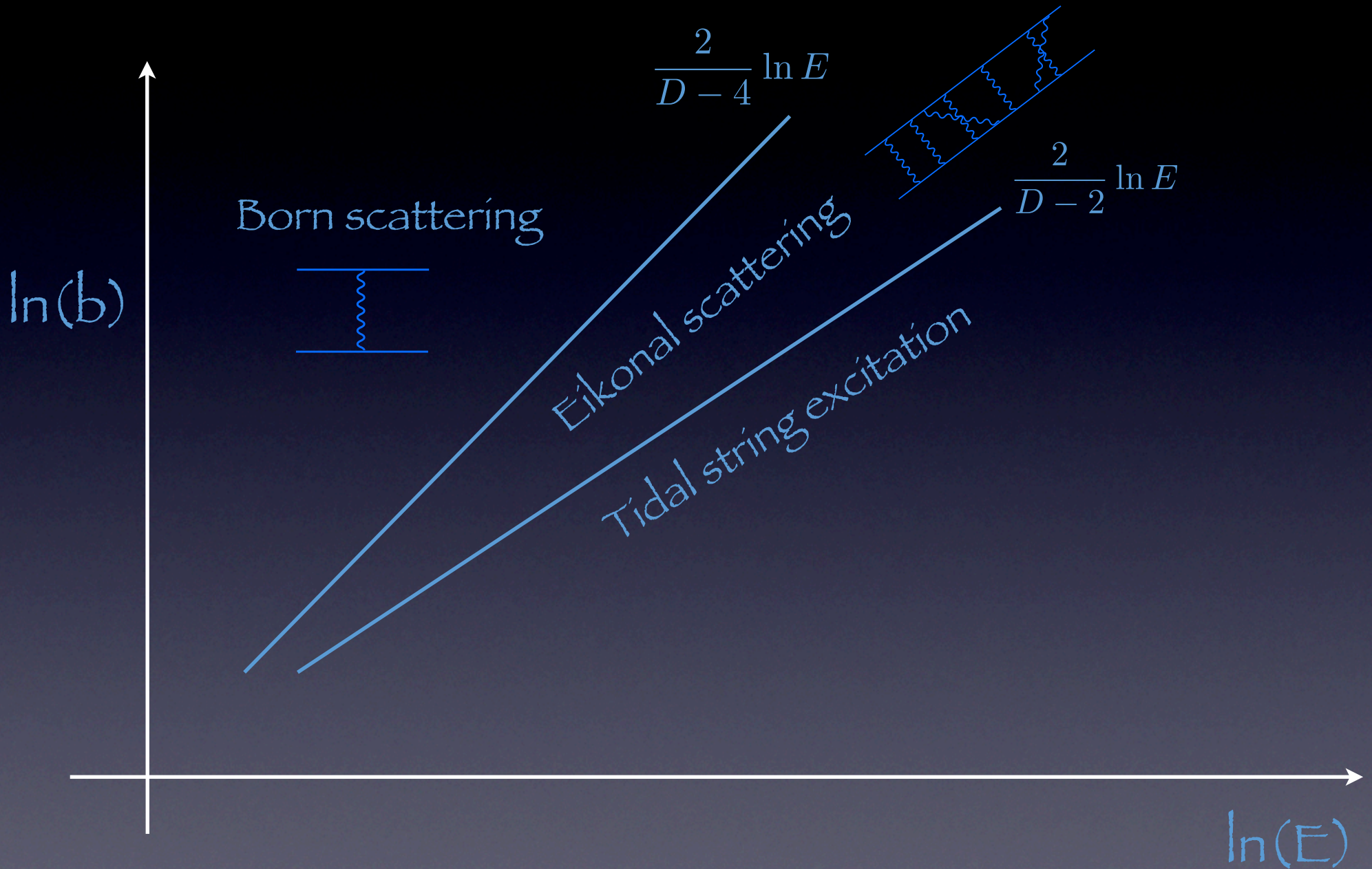
SBG, hep-th/0604072

SBG, Gross, Maharana, arXiv:0705.1816

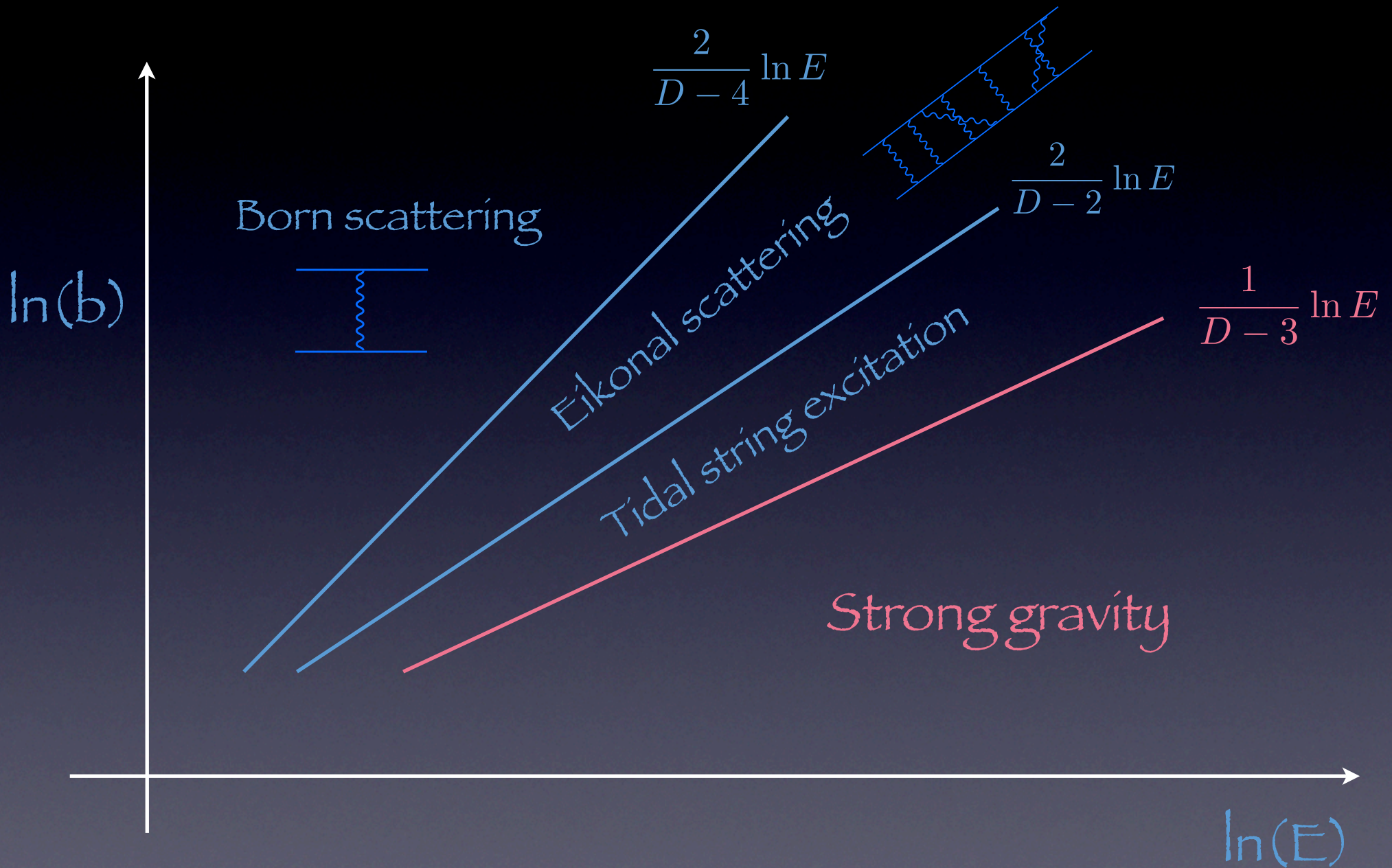
“different time scales”



Summarize story in a proposed “phase diagram:”

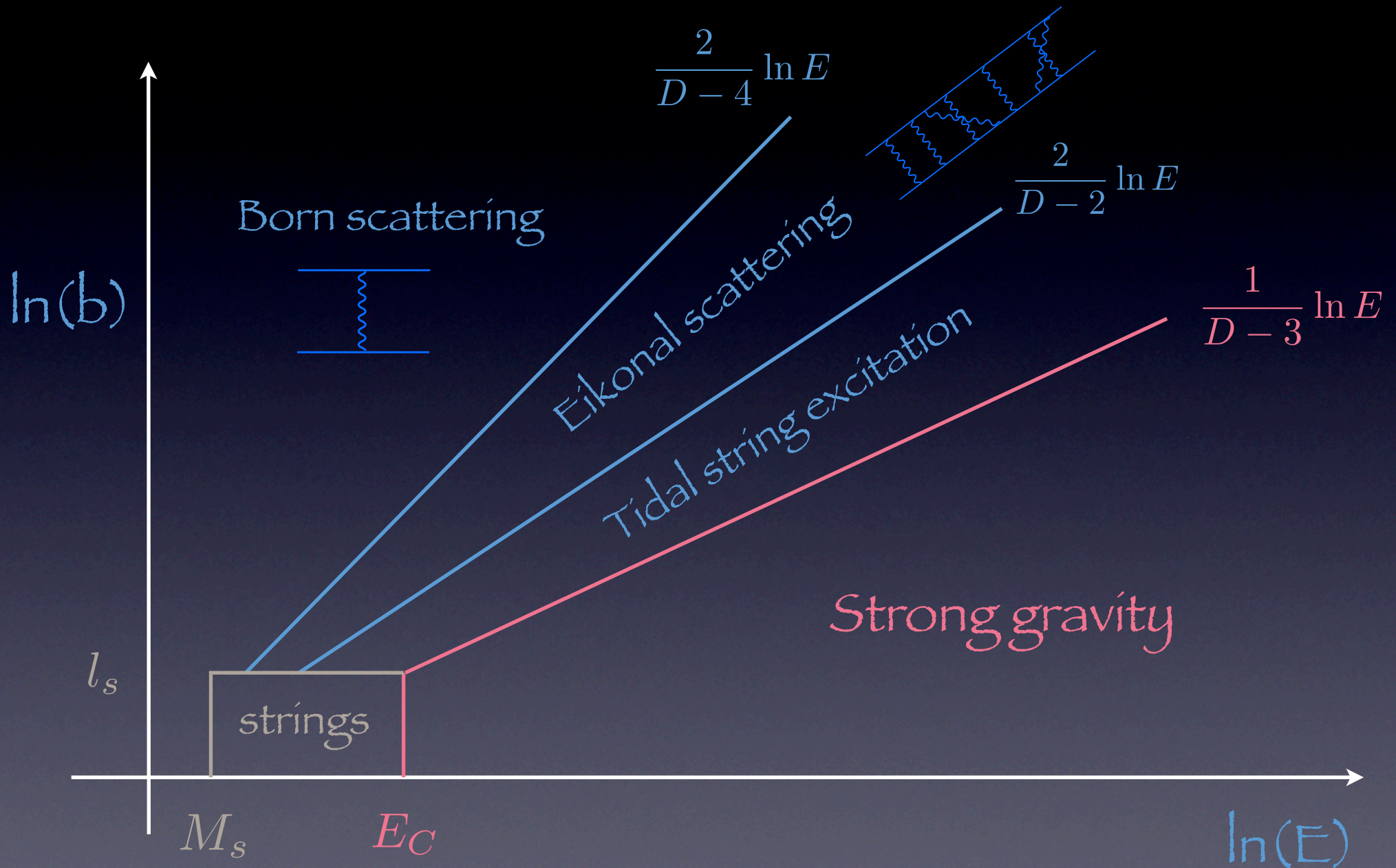


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Strong gravity region: an important mystery.

Important aspect? :

The problem appears intrinsically nonperturbative



$$1 + \mathcal{O} \left[ \left( \frac{R_S(E)}{b} \right)^{2(D-3)} \right]$$

(series not even asymptotic)

(unitarity a more critical issue than renormalizability?)



String perturbative finiteness, extendedness not  
clearly relevant

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What about state counting; duality/holography?

- Microstate counting: not far from BPS,  $\sim$  solitonic  
(not Schwarzschild)

- Holographic duals: nonperturbative  
do they answer our questions?



Holographic duals: AdS/CFT;  $\sim$  matrix theory

- do they address the “paradox”?

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- nonetheless, can investigate whether, e.g., they  
reproduce a unitary  $S$ -matrix with the correct  
features

- for clear interpretation, want to reproduce  $S$ -matrix in  
flat space limit, that is, on scales  $r \ll R$   
(then, can take  $R \rightarrow \infty$ )



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An important open problem!

Polchinski, hep-th/9903048

Susskind, hep-th/9901079

SBG, hep-th/9907129

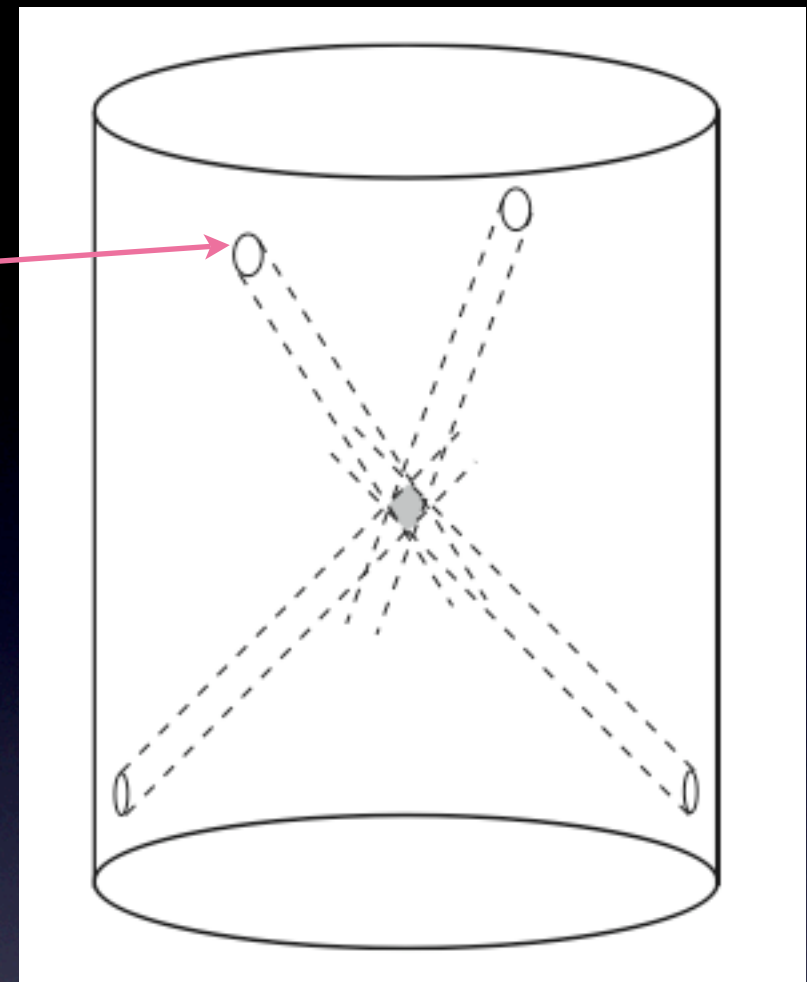
Gary, SBG, and Penedones, arXiv:0903.4437

Gary, SBG, arXiv:0904.3544

Heemskerk, Penedones, Polchinski, Sully, arXiv:0907.0151

An issue:

control sources at boundary



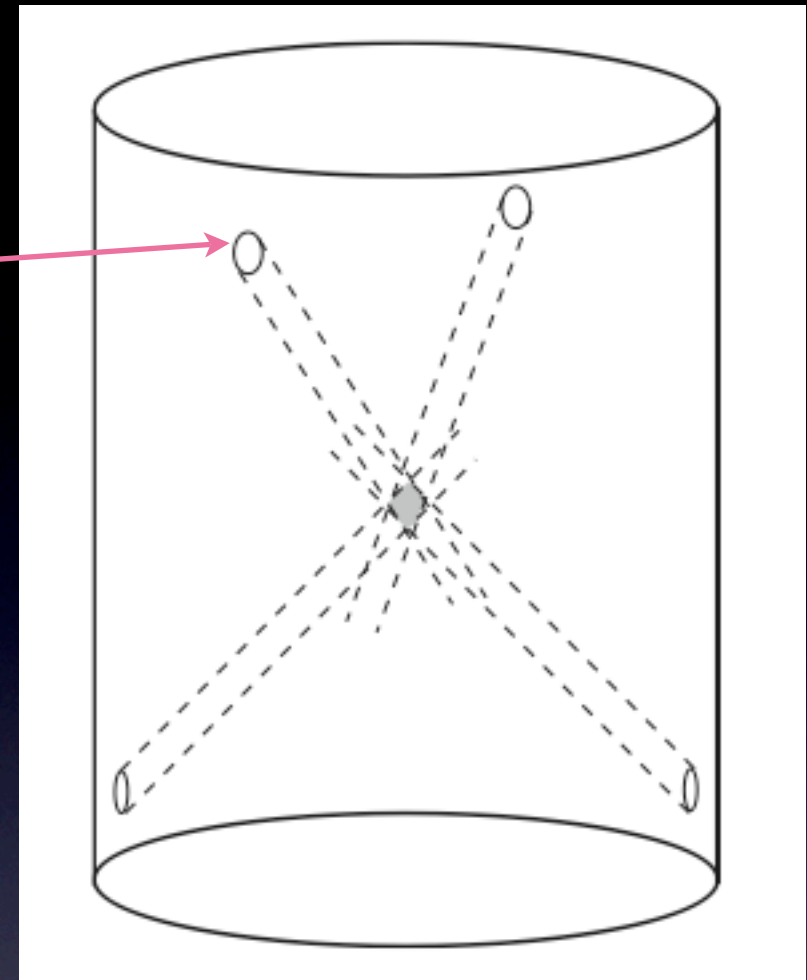


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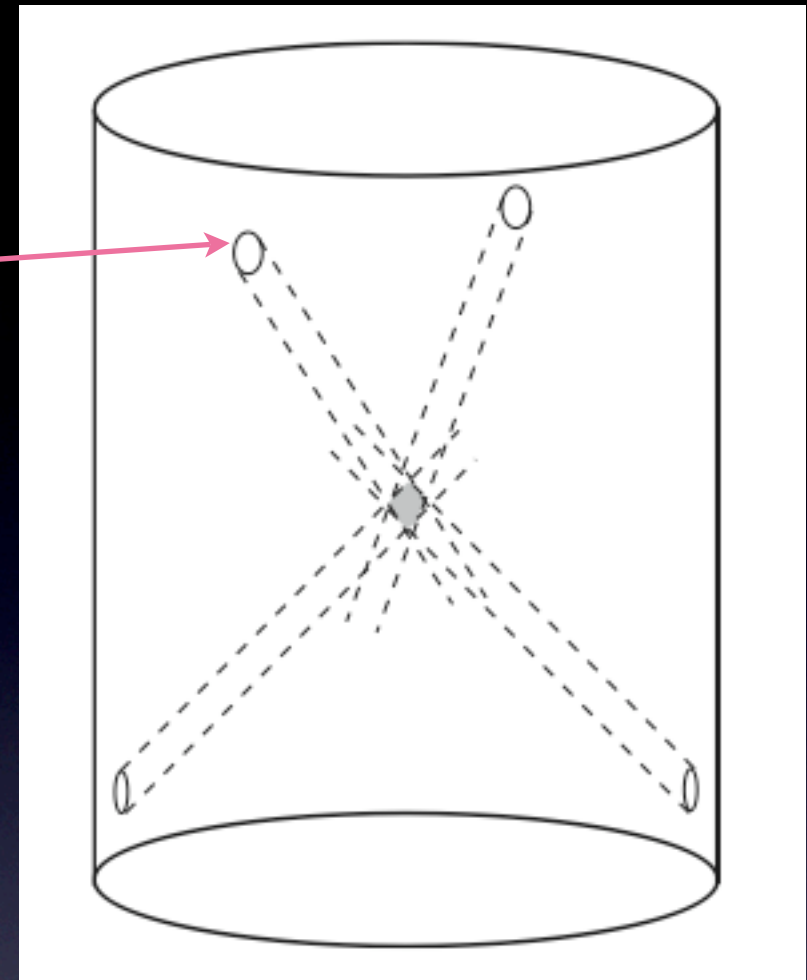


can they be “focussed”  
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structure at scales  $r \ll R$ ?



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control sources at boundary



can they be “focussed”  
sufficiently to resolve  
structure at scales  $r \ll R$ ?

Or, might the boundary theory only summarize  
some version of the bulk theory theory averaged  
over scales  $< R$ ?

(thus, holography only in “coarse-grained” sense?)



Let's understand more carefully:

- AdS/CFT:  $\phi(x) \leftrightarrow \mathcal{O}(b)$

( $b$ =boundary point;  
 $x$ =bulk point)

- consider boundary sources  $f_i(b)$

- produce bulk wavepacket as  $\int db f_i(b) \mathcal{O}(b)$

- scattering amplitude:  $\mathcal{A} = \int \prod_{i=1}^4 [db_i f_i(b_i)] \langle \mathcal{O}(b_1) \cdots \mathcal{O}(b_4) \rangle$

Can we choose  $f_i(b)$  so that we produce the flat space  
S-matrix, at scales  $r \ll R$ ?

## A test (SBG, 1999)

For  $E \gg 1/R$        $q \gg 1/R$

$$q \ll 1/(G_D E^2)^{1/(D-4)}, < 1/l_{st}$$

Should be able to reproduce Born amplitudes:

$$S = 1 + i(2\pi)^D \delta\left(\sum_i p_i\right) T \quad T \propto \frac{G_D s^2}{t}$$

i.e.  $A \approx \int dp_i \psi_i(p_i) S(p_i)$

for a basis of “healthy” wavepackets  $\psi_i(p_i)$



(Since we don't know how to compute correlators in the boundary gauge theory, a warm-up test: if we use a bulk theory to define the boundary correlators, can we recover the  $S$ -matrix of that bulk theory?)

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Immediate problem:  $f(b) \leftrightarrow \psi_{NN}$


$$\sim \int dx \psi_{NN} \psi_{NN} G_{Bulk}$$

this integral dominated near the boundary

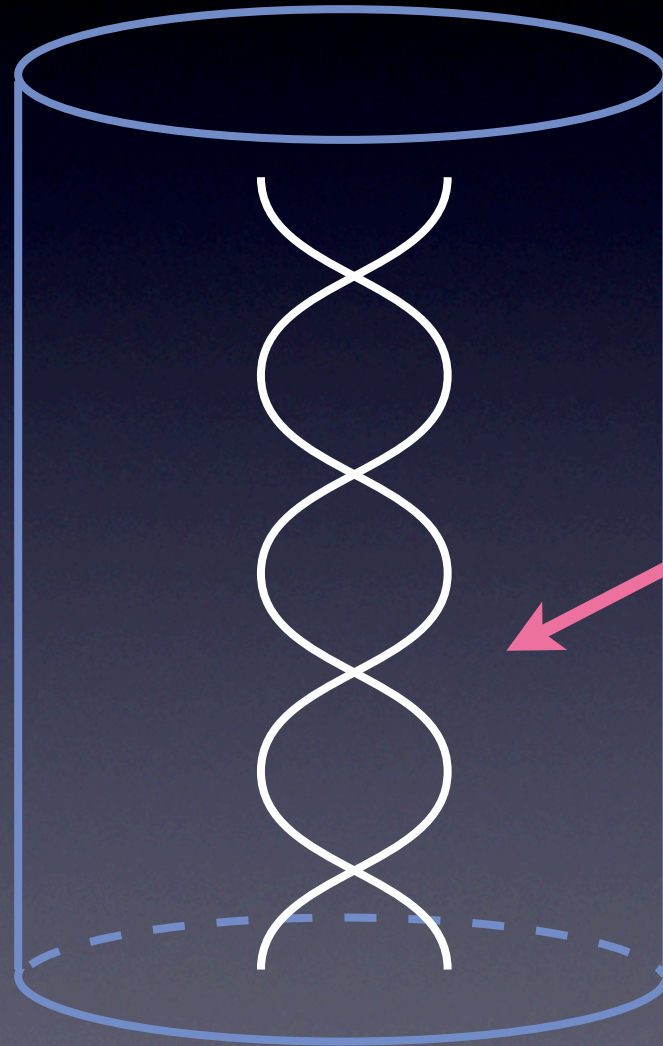
lack of focus



So, use normalizable solutions?

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problem:

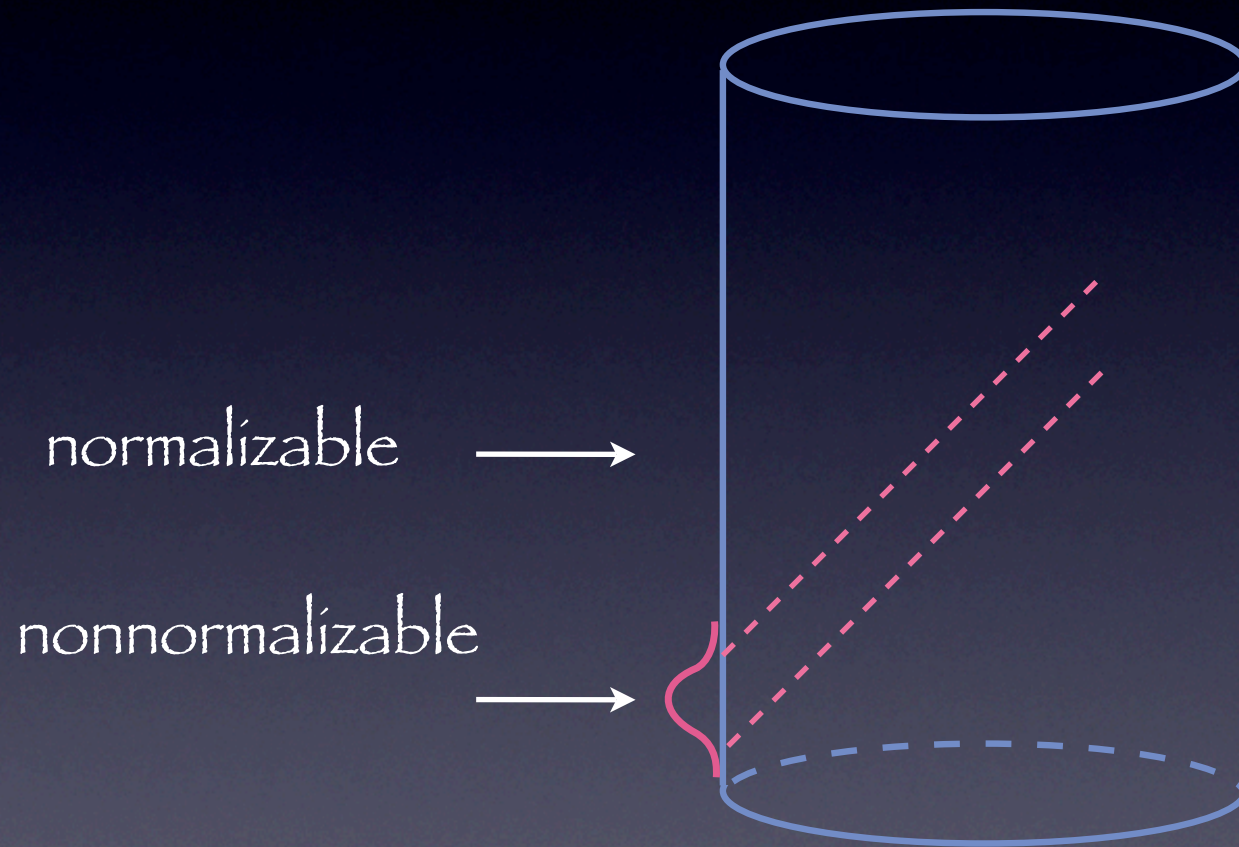


infinite #  
collisions



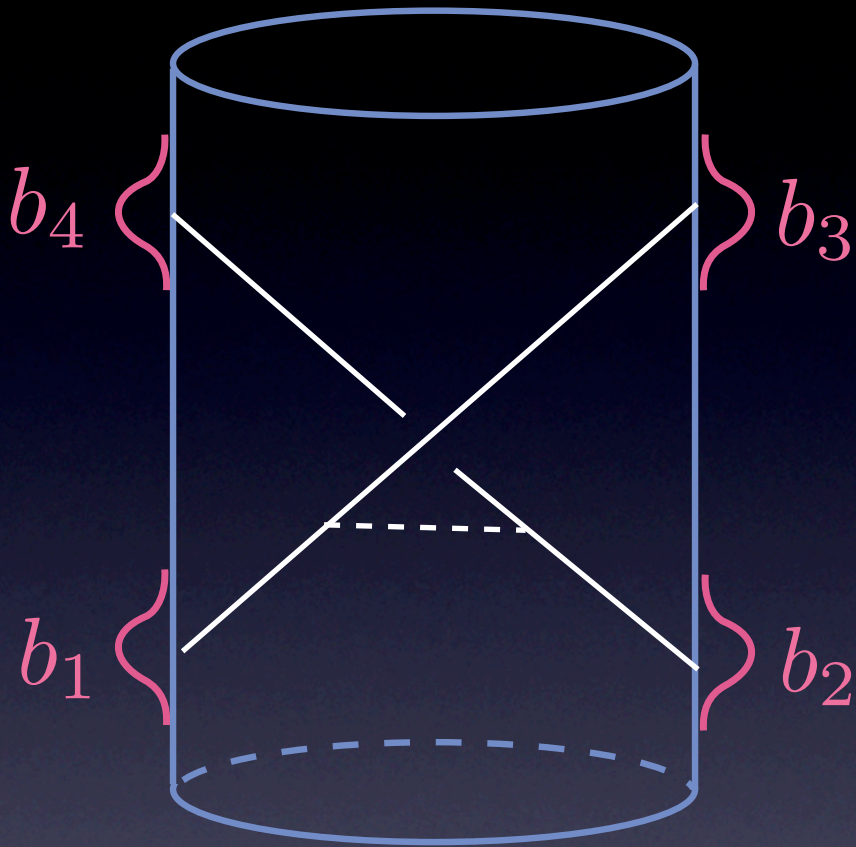


Only obvious way to proceed: compromise  
compact sources -- “boundary compact wavefunctions”



(Gary, SBG, and Penedones, arXiv:0903.4437)

Indeed, consider:

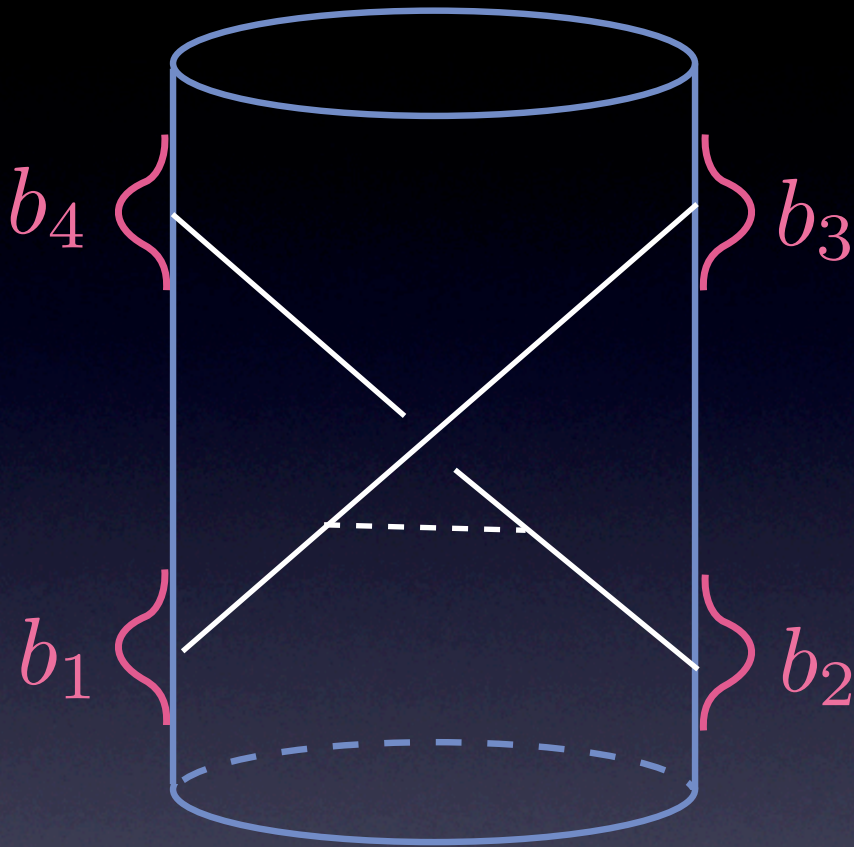


$$f_i(b) \sim L(b - b_i)e^{i\omega_i t}$$

↑  
cpct support



Indeed, consider:



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There is a limit:

(~Polchinski,  
Susskind)

$$\eta \rightarrow \infty$$

$$R = \eta^2 \hat{R}$$

$$\omega = \text{fixed}$$

$$\Delta t = \eta \hat{\Delta t}$$

$$\Delta \theta = \hat{\Delta \theta} / \eta$$

giving plane waves in  
flat space

Recall the target:

$$S = 1 + i(2\pi)^D \delta\left(\sum_i p_i\right) T$$

$$T \propto \frac{G_D s^2}{t}$$

If isolate “by hand”

$$\langle \mathcal{O}(b_1) \cdots \mathcal{O}(b_4) \rangle_{scatt}$$

can show

$$\mathcal{A}_{scatt} = \int \prod_{i=1}^4 [db_i f_i(b_i)] \langle \mathcal{O}(b_1) \cdots \mathcal{O}(b_4) \rangle_{scatt}$$



$$i(2\pi)^D \delta\left(\sum_i p_i\right) T$$

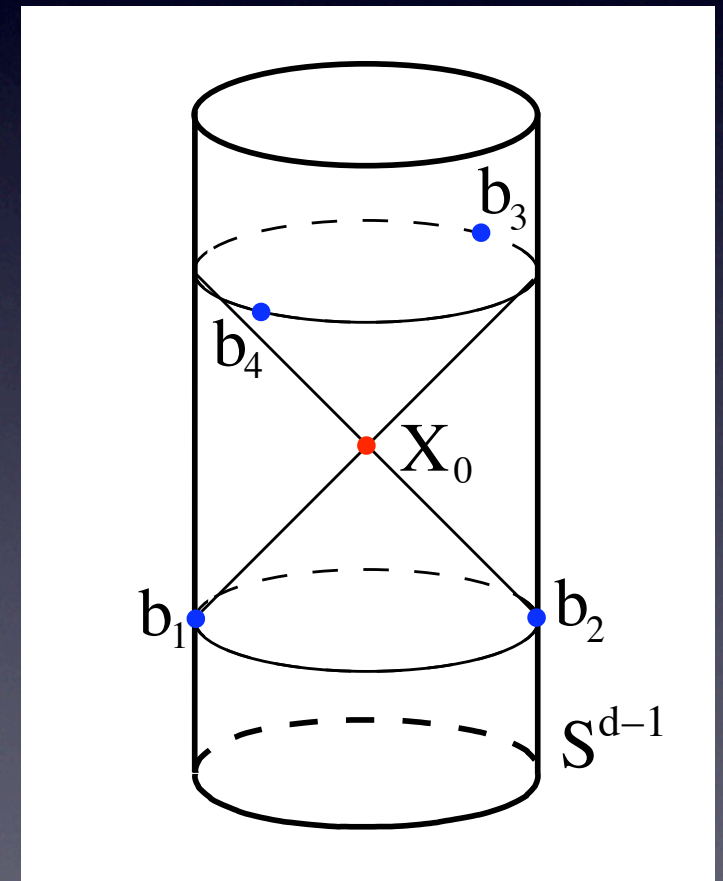
if the correlator has a certain  
singularity structure  
( $\sim$  delta function)



$$\langle \mathcal{O}(b_1) \cdots \mathcal{O}(b_4) \rangle \propto \frac{\mathcal{A}(z, \bar{z})}{b_{13}^{\Delta_1} b_{24}^{\Delta_2}}$$

cross ratios:  $z\bar{z} = \frac{b_{13}b_{24}}{b_{12}b_{34}} \quad (1-z)(1-\bar{z}) = \frac{b_{14}b_{23}}{b_{12}b_{34}}$

singularity:  $z = \bar{z}$



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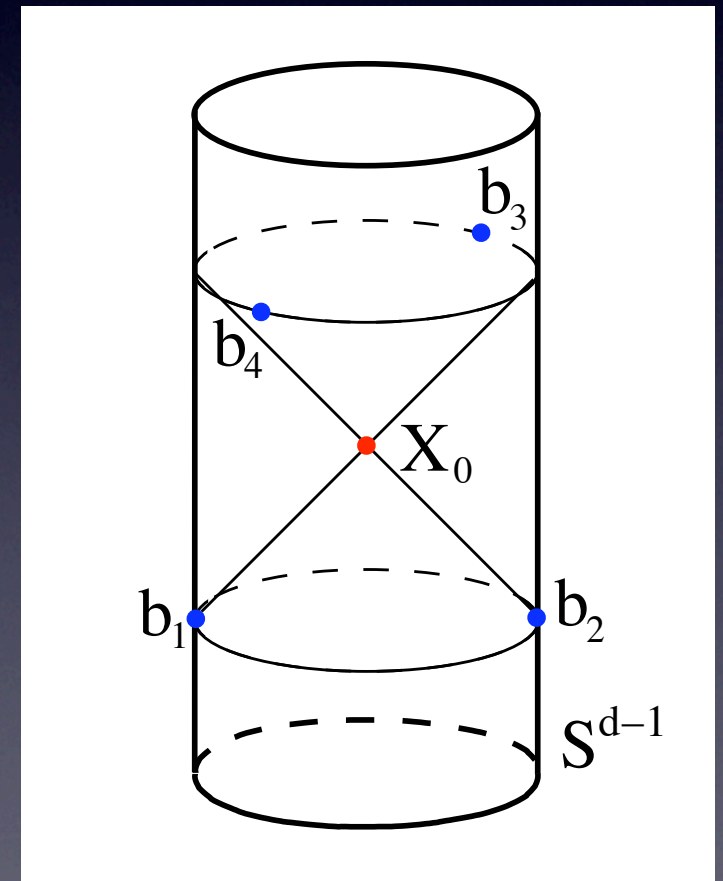
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Can extract (quite explicit, and nontriv.)  $\mathcal{T}$  from coeff of singularity in

$$\mathcal{A}_{scatt} = \int \prod_{i=1}^4 [db_i f_i(b_i)] \langle \mathcal{O}(b_1) \cdots \mathcal{O}(b_4) \rangle_{scatt}$$

(See 0903.4437)





- this is very suggestive.
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Heemskerk, Penedones, Polchinski, Sully:

Conjecture/prelim. arguments: any CFT that has a large- $N$  expansion, and in which all single-trace operators of spin greater than two have parametrically large dimensions, exhibits such behavior



- it's certainly important to investigate whether this is true.
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Gary, SBG, arXiv:0904.3544:

- plane-wave limit is rather singular
- ordinarily control by using well-defined (“regular”) wavepackets
- for finite but large  $R$ , can we reproduce these from boundary-compact wavepackets?

# Not necessarily!

boundary compact  $\Rightarrow$  low-energy tails

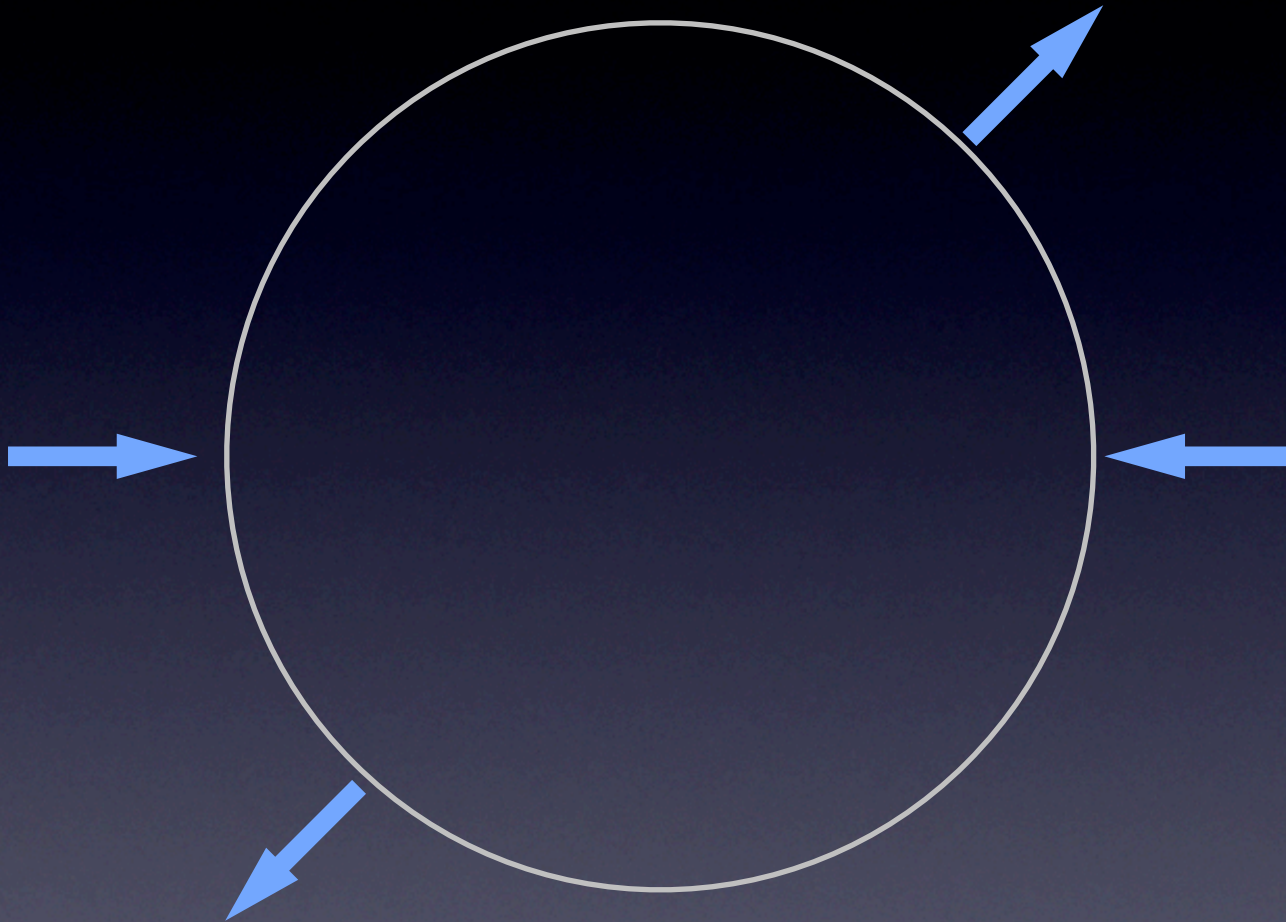
become power law tails, in position space;  
don't vanish in  $R \rightarrow \infty$  limit

thus, one doesn't have an argument that well-localized  
(regular) wavepackets can be produced from well-  
defined (boundary compact) boundary data



Example of possible effect:

AdS, top down view

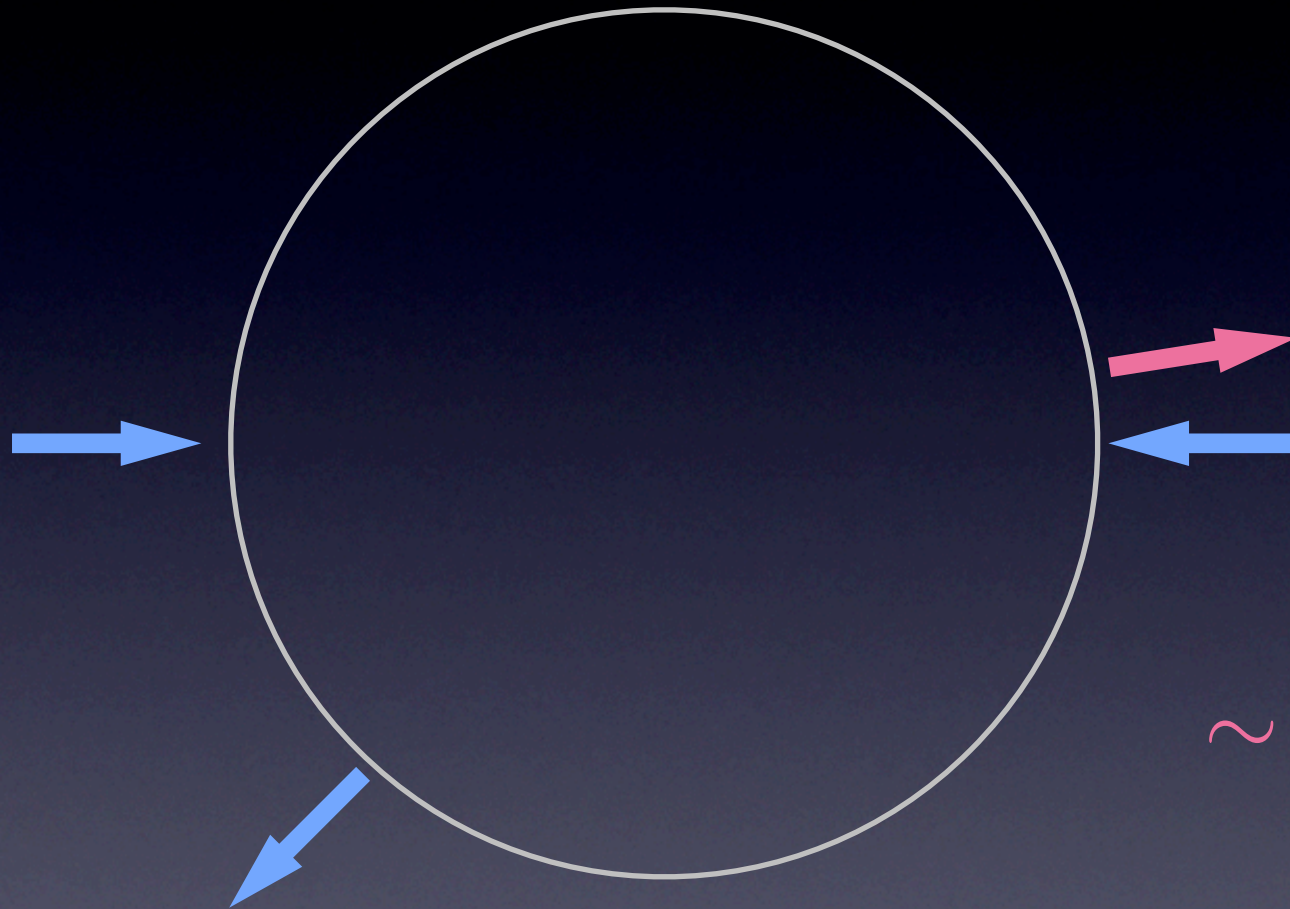


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$\sim$  Rutherford  
experiment

Example of possible effect:

AdS, top down view



$$A \sim \frac{1}{\theta^2}$$

$\sim$  Rutherford  
experiment

$\sim$  bad LHC detector:  
mis-ID dynamics

(S-matrix)



- Part of the issue: separating

$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{scatt}$  from  $\langle \mathcal{O} \cdots \mathcal{O} \rangle_{direct}$

- possible indication: need to excite  $N^2$  matrix degrees of freedom? (Some indications all along)

- but why should these produce local amplitudes on scales  $\ll R$  ??

To summarize the AdS/CFT discussion:

We have found some nontrivial tests for whether the CFT produces local dynamics on scales  $\ll R$

1. Presence of certain singularities  $\mathcal{A} \sim \frac{T}{(z - \bar{z})^{2\beta}} h(s, t, u)$   
with  $T \sim T_{bulk}$

is this structure present in the CFT?

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These are nontrivial; it is a very interesting question how (and whether) the CFT can produce fine-grained bulk dynamics

To summarize the broader string discussion:

- perturbative string theory (and its “finiteness”) doesn’t obviously address our set of questions
- there is so far no substantial indication of a role for extendedness of strings (or branes ...)
- it is very non-trivial to show that non-perturbative duals sharply capture complete bulk dynamics (do they?)



Whether strings (or LQG) ultimately answer these questions, can we see outlines of the answers?

Whether strings (or LQG) ultimately answer these questions, can we see outlines of the answers?

- We see strong indications for new effects at scales  $\sim R_S(E)$

- Nonperturbative gravity (distinct from, e.g. string extendedness?)

- Good indications: breakdown of locality, as conventionally formulated

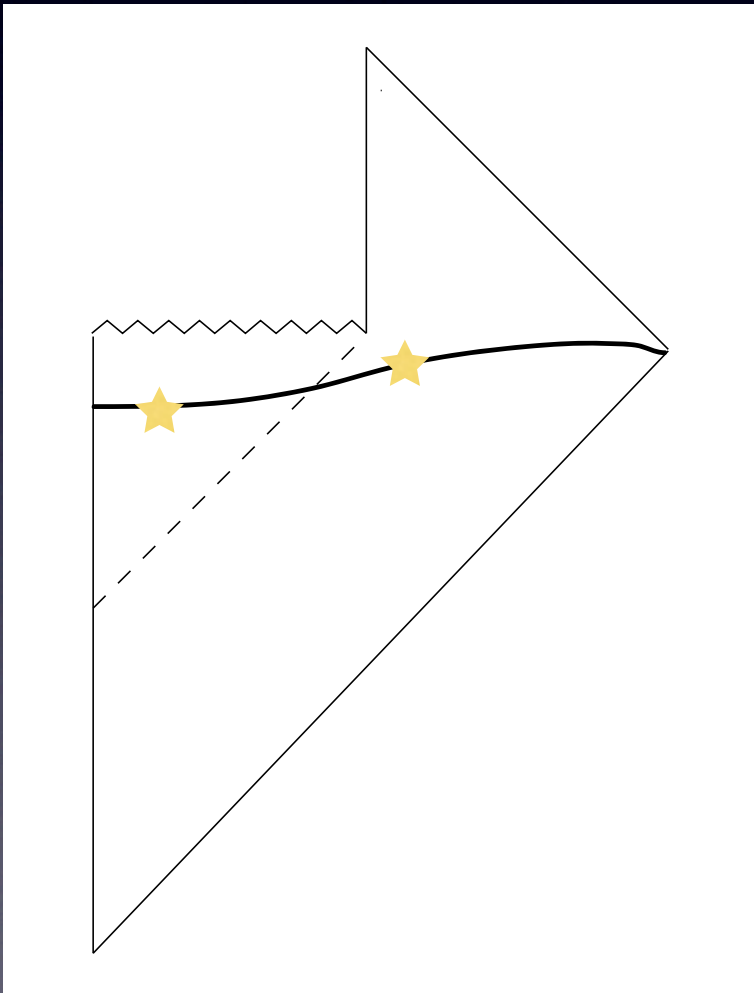


Reasons to question locality, at  $\sim R_S(E)$  :

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1) information paradox

if keep Lorentz invariance and QM:



On scale :

$$R_S \propto (G_D M)^{1/(D-3)}$$

$$\gg \gg \gg l_p$$



## 2) growth of size in scattering

$$\theta_c \sim \left[ \frac{R_S(E)}{b} \right]^{D-3}$$

indicates gravitational growth of object  
(though not nonperturbative regime)

black holes: 2 body  $b \sim R_S(E)$

(connection to “nonpolynomiality” - momentarily)

## 3) lack of local observables

approximately local observables fail in same regime

Want to better understand physics - a basic set of questions:

1) Where does local QFT fail?

Correspondence boundary

2) What is the mechanism?

3) What physical/mathematical framework replaces QFT, and how might locality emerge from it in familiar contexts?



# Some previous proposals for a correspondence boundary for gravity:

planckian curvature:

$$\mathcal{R} < M_P^2$$

string uncertainty principle:  
(Veneziano/Gross)

$$\Delta X \geq \frac{1}{\Delta p} + \alpha' \Delta p$$

modified dispersion:

$$p < M_p$$

} 1 particle

holographic (information)  
bounds:

$$S \leq A/4G_N$$

multiparticle

# Compare CM/QM

dynamical descript.

validity

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$x(t), p(t)$

$$\Delta x \Delta p > 1$$



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“locality bound”

(generalizations: N-particle; dS)

SBG & Lippert;  
hep-th/0605196;  
hep-th/0606146

Correspondingly, mechanism:

“delocalization w.r.t. semiclassical geometry,  
intrinsic to unitary dynamics of nonperturbative  
gravity”

~ “nonlocality principle”

contrast with: extended strings (or branes)

(correspondingly, clear distinction between “string  
uncertainty principle” and the locality bound)



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- local observables

- polynomial behavior of HE scattering

Indeed, independently interesting problem:

## The gravitational S-matrix

- conjecture: well defined in “the” theory of gravity
- what are its general properties, consistent with unitary quantum evolution + basic features of gravity?
- can its study provide information about the principles of the underlying theory?

(remember the Veneziano amplitude ...)

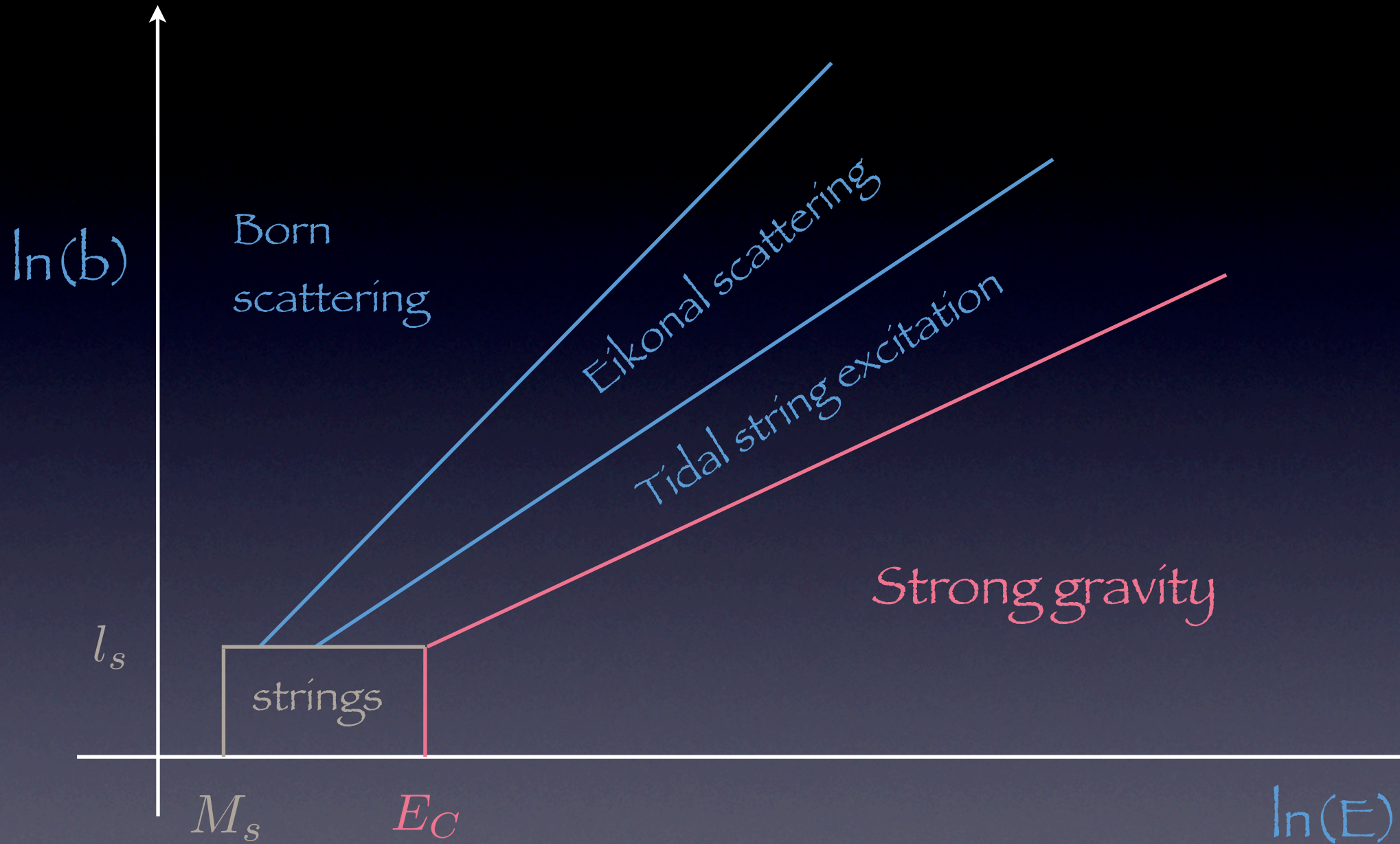
locality  $\longleftrightarrow$  polynomiality?

SBG and Srednicki, arXiv:0711.5012

SBG and Porto, arXiv:0908.0004



# Some basic features:



- different characteristic behavior in different regimes

2 → 2 scattering:

PW expansion:

$$T(s, t) = (\text{const}) E^{4-D} \sum_{l=0}^{\infty} (l + \nu) C_l^\nu(\cos \theta) \left[ e^{2i\delta_l(s) - 2\beta_l(s)} - 1 \right]$$

$$\nu = \frac{D-3}{2}$$



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$$\nu = \frac{D-3}{2}$$

Born, eikonal regions: “weak gravity” regime; can infer features of  $\delta_l$ ,  $\beta_l$  from preceding considerations

E.g.:  $\delta_l^{eik} \sim \frac{E^{D-2}}{l^{D-4}}$

$$\beta_l^{br} \sim \frac{E^{3D-6}}{l^{3D-10}}$$

... soft gravitons

+ other “model dependent” effects  
(string excitation, etc.)

Strong gravity/black hole region:  $l \lesssim ER_S(E) = L$

1. Black hole Ansatz:

$$\beta_l \approx \frac{S(E, l)}{4} \quad (\text{Bekenstein-Hawking entropy - approx. thermal description})$$

2. Black holes ~ resonances

$$\frac{\Gamma}{M} \sim \frac{1}{RM} \sim \frac{1}{S} \quad N_{\text{accessible}}(M, M + 1/R) \sim S(M)$$

$\Rightarrow$  information about  $\delta_l$  ( $\lesssim \pi S(E)$ )

( $\sim$  Levinson's thm, but multichannel)



## Features:

- significant indications, amplitudes not polynomial:

$$T(s, t) \sim e^{s^\alpha t^\beta}$$

plausibly associated w/ lack of usual locality?

related: viol. of Froissart, eg  $\sigma_{BH} \sim [R_S(E)]^{D-2}$

... growth of range of gravity w/ energy

- interesting constraints from crossing

crossing more nontrivial than in massive th;

provides constraints on nonpoly behavior

$\sim$  bdd. in physical region, e.g.  $t < 0$ ,  $s$  cplx

(not “too” nonlocal)

This is “outside” (asymptotic) viewpoint.  
To discuss “inside” (cosmology, black hole)  
need ~ local observables

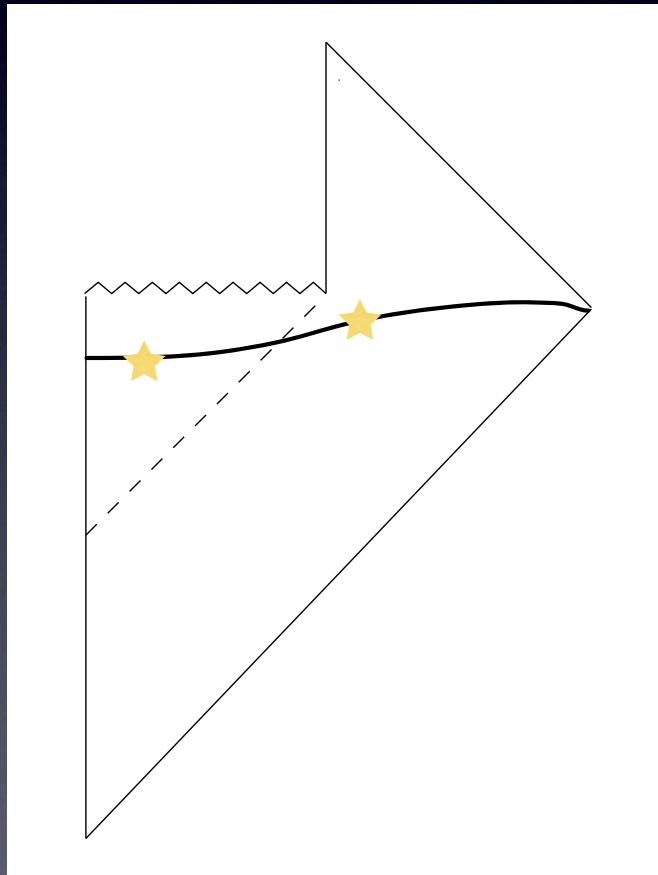
Indeed, locality ~ QFT:

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0, \quad (x - y)^2 > 0$$

Diff invariance  $\Rightarrow$  None in gravity!



For example, to properly **formulate** the information paradox, need to discuss inside, approximately local description:



Possible resolution: Relational approach:

“proto-local observables”

see: SBG, Marolf, Hartle;

Gary & SBG: 2d, concrete

Basic idea:

$$\mathcal{O} = \int d^4x \sqrt{-g} B(x) O(x)$$
$$\langle B(x) \rangle = b(x)$$

for appropriate background:  $\langle \mathcal{O} \rangle \approx O(x_0)$

localization relative to background

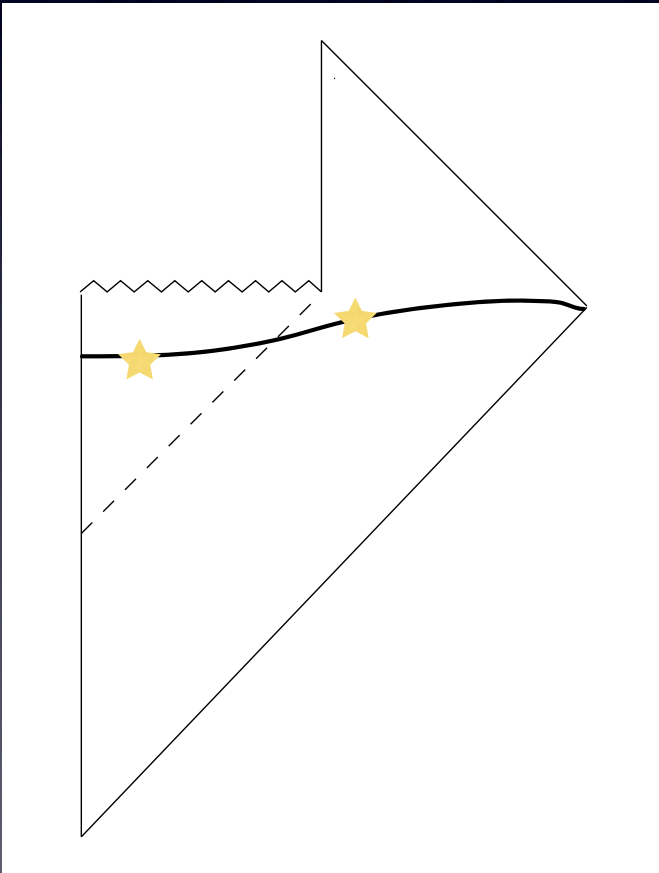
- But:
- localization only approximate
  - must include background/observer



Can we find a flaw in nice slice argument, and see where Hawking went wrong?

Some thoughts:  
hep-th/0606146

Sharp computation of  $S_{HR}$   
requires fine-grained, local  $|\psi\rangle_{NS}$



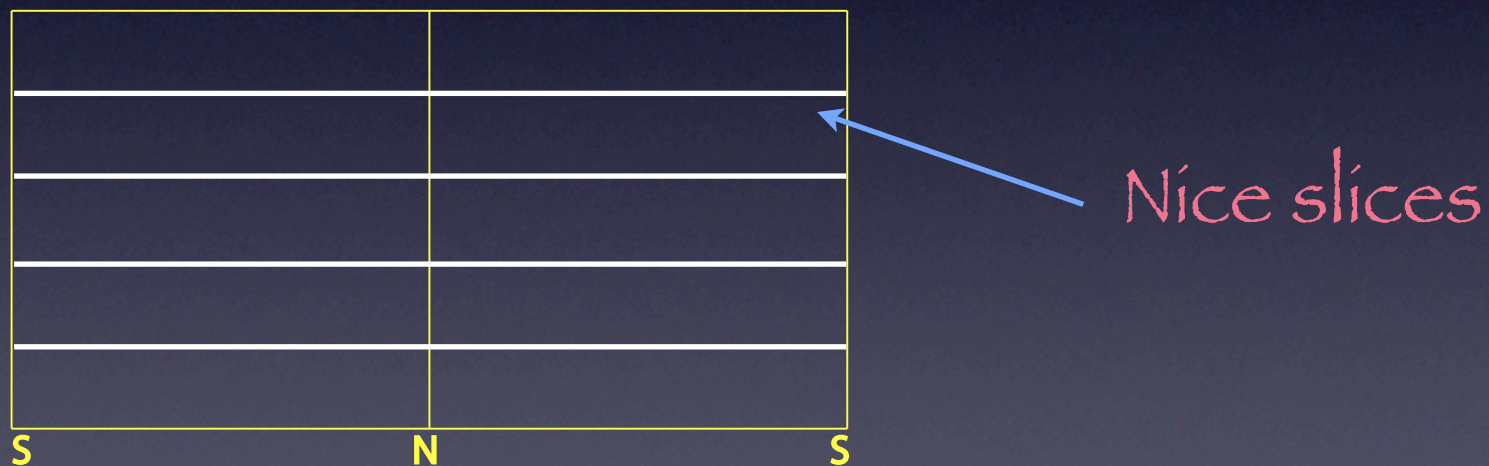
- Two potential obstacles:
- 1) observing background  $\Rightarrow$   
large mods. to  $|\psi\rangle_{NS}$
  - 2) backreaction of fluctuations  $\Rightarrow$   
large mods. to  $|\psi\rangle_{NS}$

Both by  $\tau_{Page} \sim R_S S_{BH}$

(literal CM/QM analogy may be another out...)

- Apparent signals of pert. thy. breakdown; proposed resolution of information paradox
- Non-pert. completion would be required to describe information “escape” / restore unitarity  
but, a clue ...

- Interestingly, there are parallel arguments in dS,



suggesting LQFT incomplete after  $\tau \sim R_{dS} S_{dS}$

(Likely related argument: Arkani-Hamed ... Villadoro: arXiv:0704.1814)



In general, expect this set of considerations to be important in cosmology

Work w/ Marolf on dS, etc. arXiv:0705.1178, and WIP x2

- More general limitations on local QFT for volumes  $> R_{dS}^4 e^{S_{dS}}$

- Investigation of proto-local observables in dS  
deal w/ constraints, linearization stability

- Measurement for protolocal observables

To sum up, should be probing limits of local quantum field theory description, likely on scales  $\gg l_P$ , in certain circumstances

“unitarity restored at price of locality”



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(~ How to invent QM w/out experiment?)

To sum up, should be probing limits of local quantum field theory description, likely on scales  $\gg l_P$ , in certain circumstances

“unitarity restored at price of locality”

How to make more concrete progress?

(~ How to invent QM w/out experiment?)

- investigate properties of  $S$ -matrix
- approximately local observables, and limitations
- Another ingredient: what is a general enough quantum-mechanical framework to incorporate these ideas?

More general than Hartle’s “generalized QM”

arXiv:0711.0757



How can we have a theory w/ features of gravity,

1) Consistent (~causal)

2) Quantum mechanical

3) Nonlocal

4) Nearly-local

} essential tension

(i.e. behaves locally in usual low-energy circumstances)

... a highly non-trivial set of conditions to satisfy!

This, plus relevant gedanken experiments:

guides to such a “Non-Local (but Nearly-Local) Mechanics”?