Gravitational waves from compact binaries

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Lectures outline

- The two body problem in General Relativity and its implications for GW emission
- The LIGO/Virgo detections (physics & astrophysics)
- Supermassive BH binaries (LISA and pulsar timing arrays)
- Exotic science with GWs

BHs in GR (or BHs have no hair)

$$G^{\mu\nu} = 0$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

Schwarzschild BH (parametrized by mass alone)

$$\begin{aligned} ds^2 &= -\frac{\left(\Delta - a^2\sin^2\theta\right)}{\Sigma} dt^2 - 2a\sin^2\theta \frac{\left(r^2 + a^2 - \Delta\right)}{\Sigma} dt\, d\phi \\ &+ \left(\frac{\left(r^2 + a^2\right)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right) \sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \end{aligned}$$

$$\begin{aligned} \Sigma &= r^2 + a^2\cos^2\theta \\ \Delta &= r^2 - 2Mr + a^2 \end{aligned}$$

$$a = \frac{J}{M}$$

Kerr BH (parametrized by mass and spin)

Electrically charged BHs (Reissner-Nordström, Kerr-Newman) probably irrelevant astrophysical

Geodesics in Schwarzschild

Metric is static and spherically symmetric

Conserved energy and orbital angular momentum (per unit particle mass) E and L

$$\theta = \frac{\pi}{2}$$

$$\epsilon = -g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

$$\left(1 - \frac{2GM}{r}\right)\frac{dt}{d\lambda} = E\ ,$$

$$r^2\frac{d\phi}{d\lambda} = L\ .$$

$$\frac{1}{2}\left(\frac{dr}{d\lambda}\right)^2 + V(r) = \frac{1}{2}E^2\ ,$$

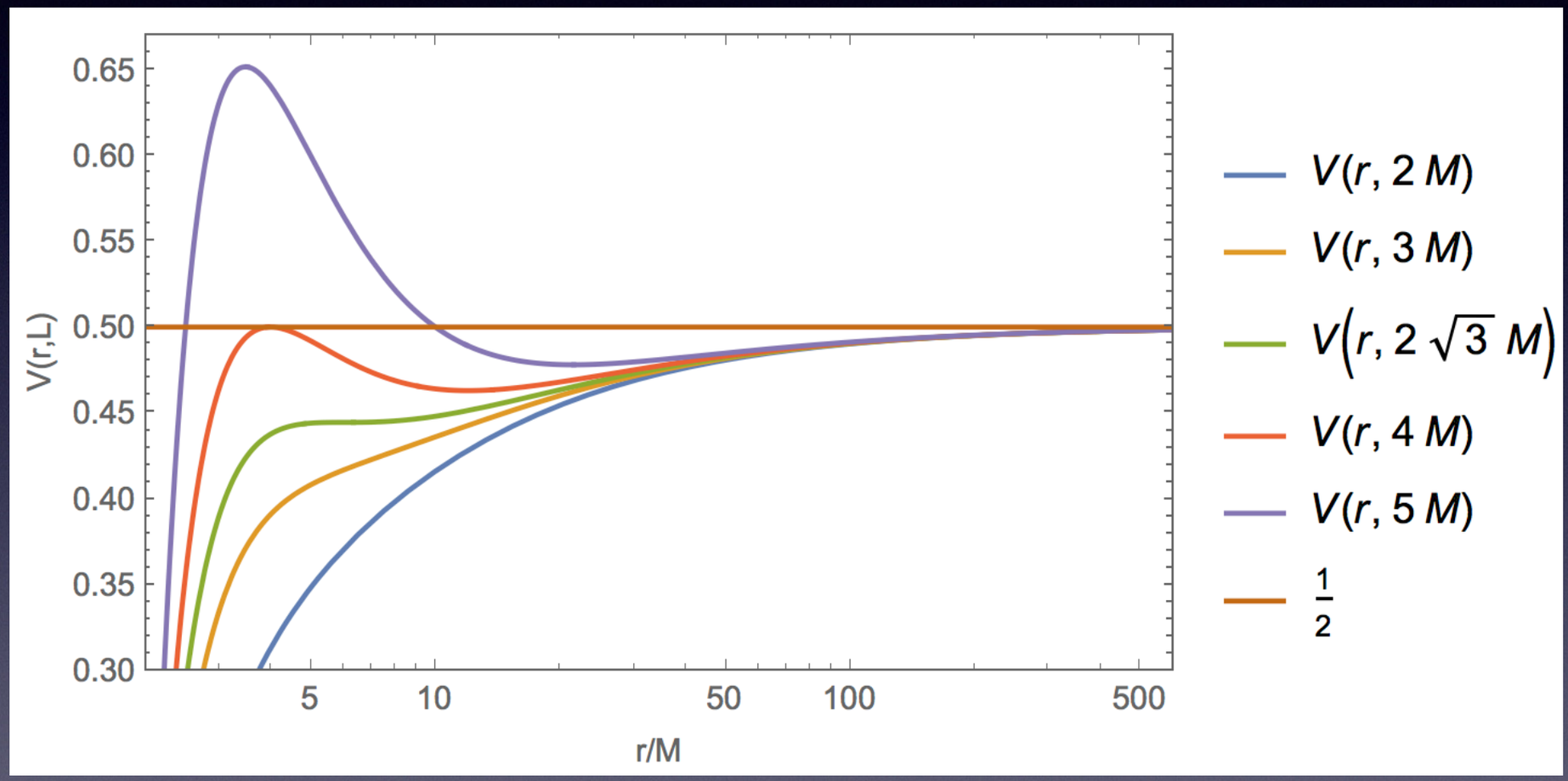
$$V(r) = \frac{1}{2}\epsilon - \epsilon\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{r^3}$$

Motion in 1D potential (Newtonian + corrections!)

Geodesics in Schwarzschild

Massive particles

$$\frac{1}{2}\left(\frac{dr}{d\lambda}\right)^2 + V(r) = \frac{1}{2}E^2 \ ,$$



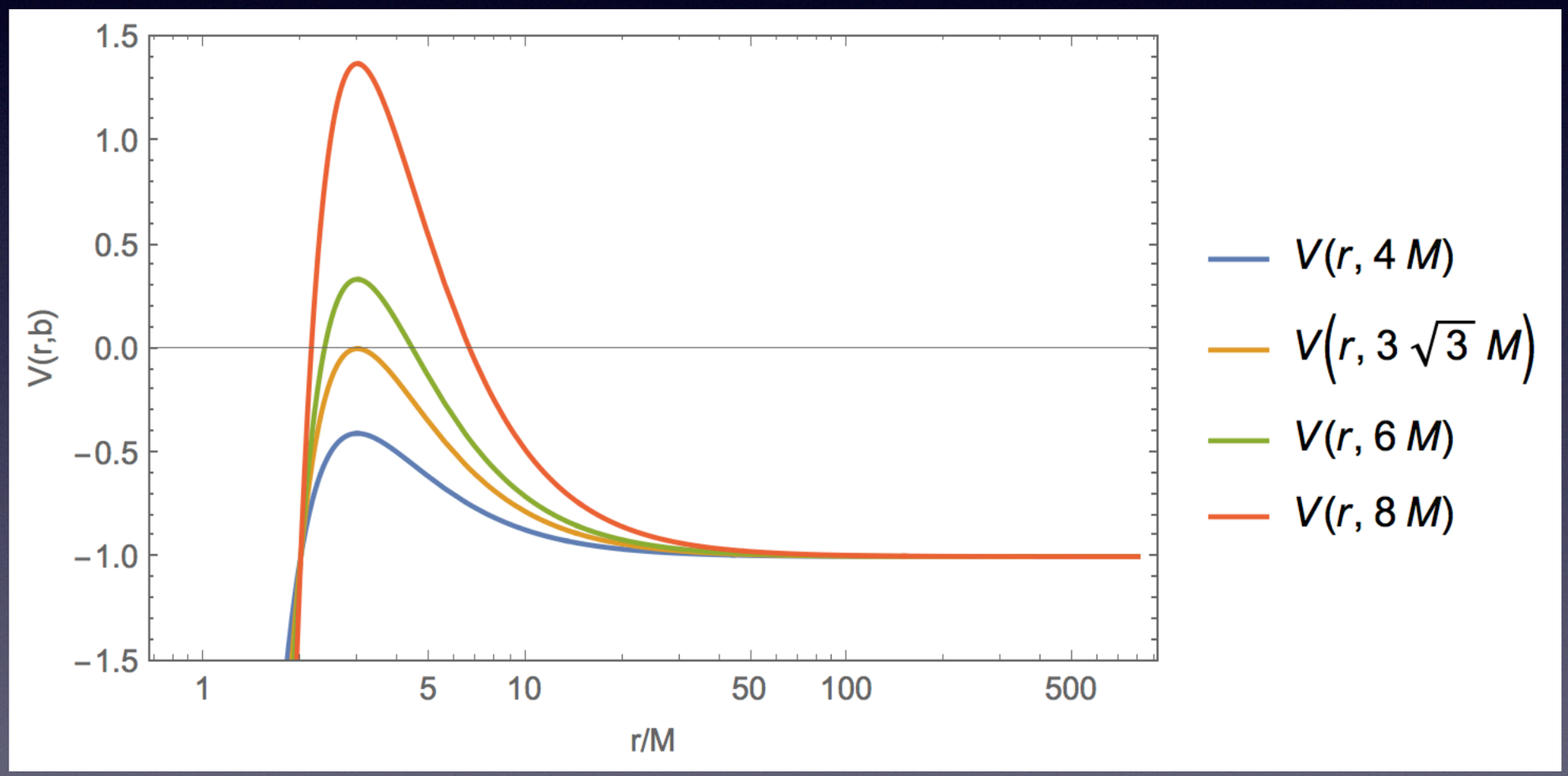
- Innermost stable circular orbit (ISCO) at r=6M
- Marginally bound orbit at r=4M
- Different than in Newtonian gravity

(circular orbits all the way down to r=0)

Geodesics in Schwarzschild

Dynamics of massless particles only depends on b=L/E

$$\frac{1}{E^2}\left(\frac{dr}{d\lambda}\right)^2 = -V = 1 - \frac{b^2}{r^2}\left(1 - \frac{2GM}{r}\right)$$

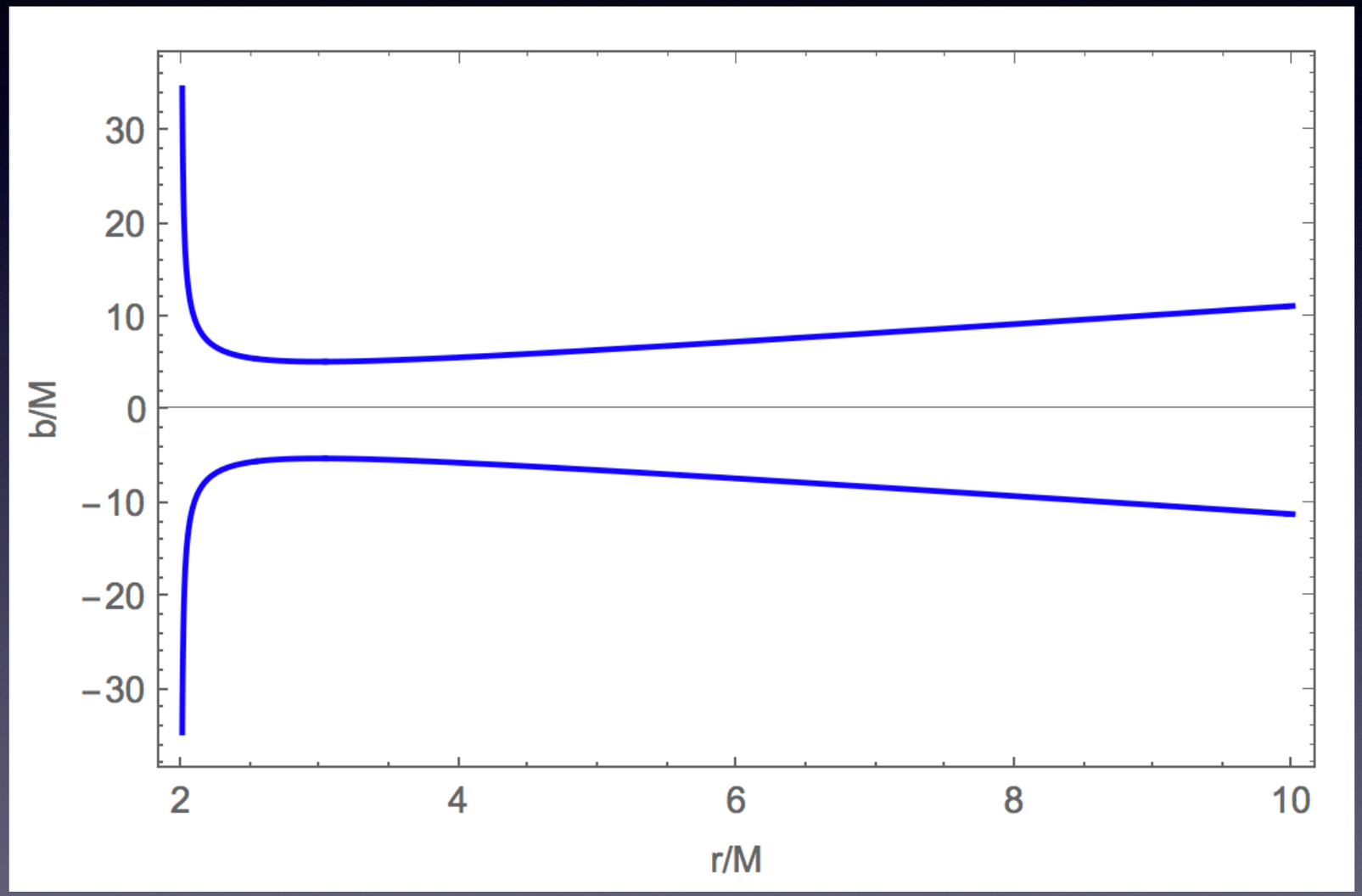


(Unstable) circular photon orbit (“light ring”) at r=3M

Peak of “potential barrier” at r=3M

Geodesics in Schwarzschild

Dynamics of massless particles only depends on b=L/E



Below critical impact parameter $b = 3\sqrt{3}M$ photons fall into BH

Geodesics in Kerr

- "Separability" of geodesics equations not trivial, but possible due to presence of "hidden symmetry" that gives Carter constant Q

$$\left(\mu\frac{dr}{d\lambda}\right)^2 = V_r(r), \qquad \mu\frac{dt}{d\lambda} = V_t(r,\theta),$$
$$\left(\mu\frac{d\theta}{d\lambda}\right)^2 = V_\theta(\theta), \qquad \mu\frac{d\phi}{d\lambda} = V_\phi(r,\theta)\ ,$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta, \qquad \Delta \equiv r^2 - 2Mr + a^2$$

$$\varpi^2 \equiv r^2 + a^2$$

$$V_t(r,\theta) \equiv E\left(\frac{\varpi^4}{\Delta} - a^2\sin^2\theta\right) + aL_z\left(1 - \frac{\varpi^2}{\Delta}\right),$$

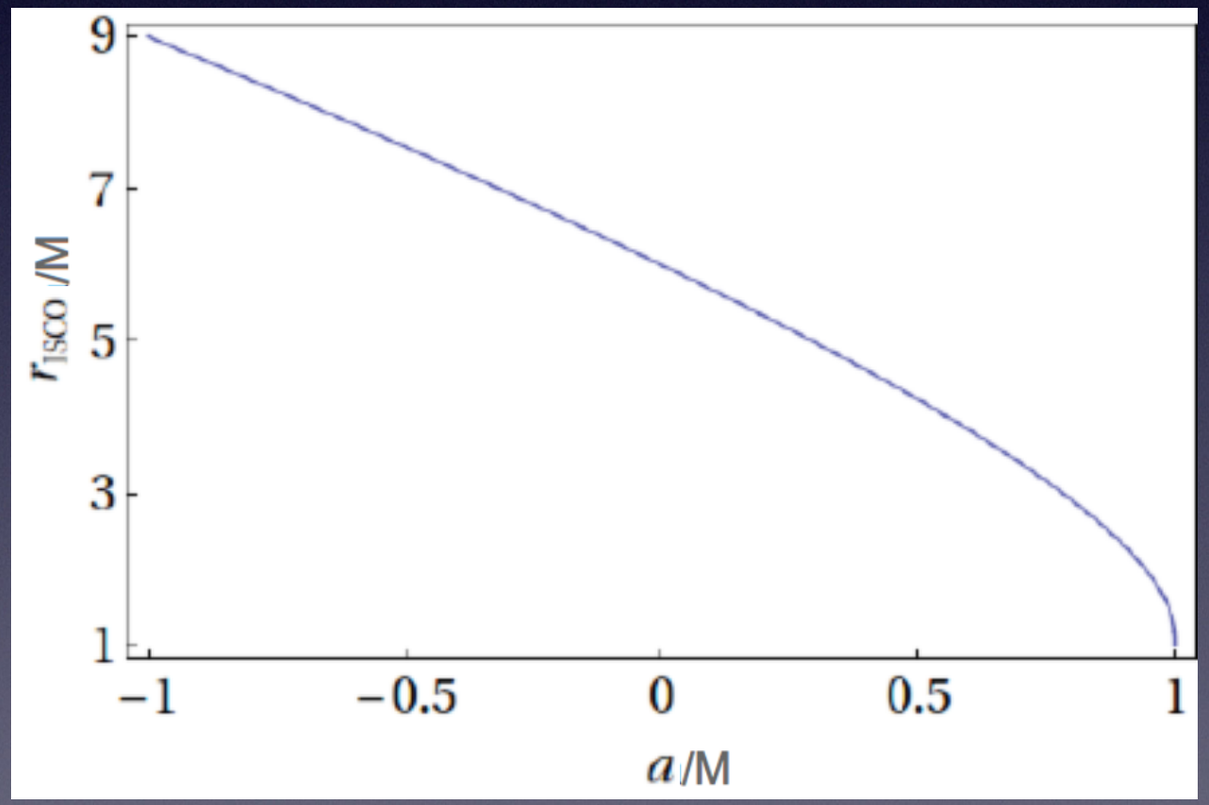
$$V_r(r) \equiv \left(E\varpi^2 - aL_z\right)^2 - \Delta\left[\mu^2 r^2 + (L_z - aE)^2 + Q\right]$$

$$V_\theta(\theta) \equiv Q - L_z^2\cot^2\theta - a^2(\mu^2 - E^2)\cos^2\theta,$$
$$V_\phi(r,\theta) \equiv L_z\csc^2\theta + aE\left(\frac{\varpi^2}{\Delta} - 1\right) - \frac{a^2 L_z}{\Delta},$$

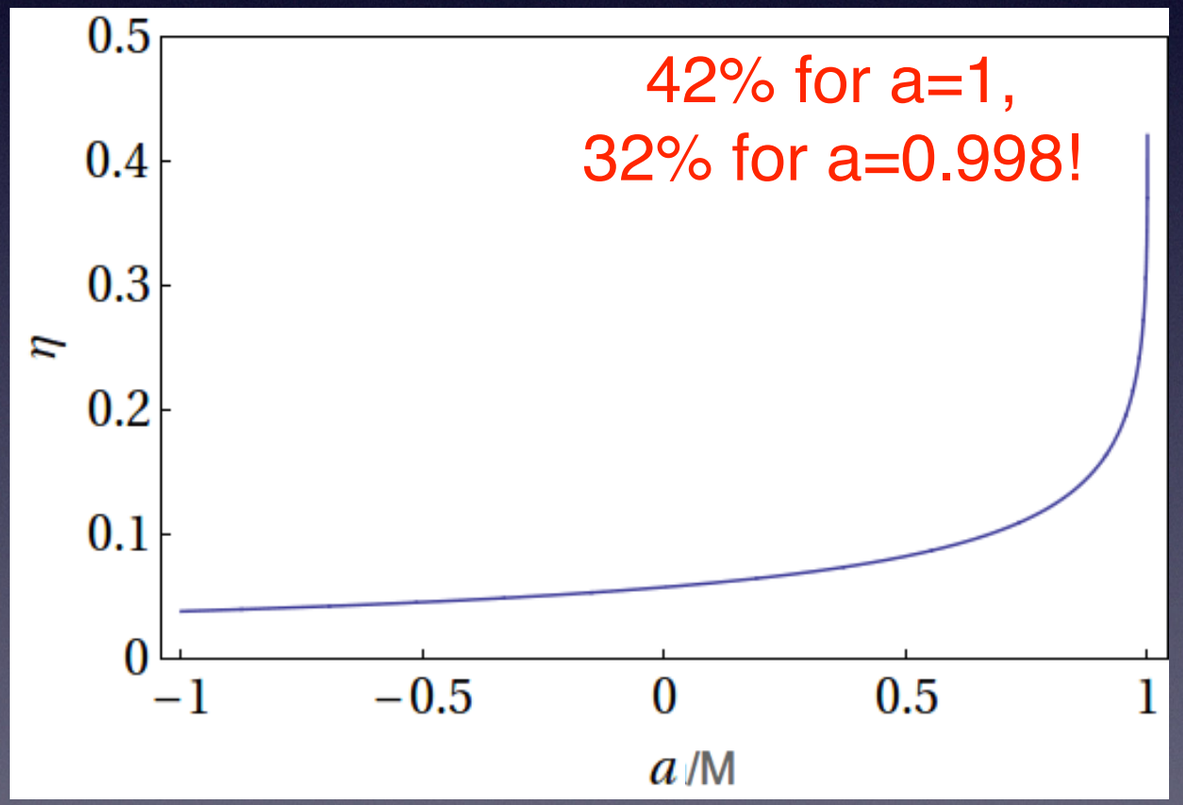
- Qualitatively same dynamics as in Schwarzschild ("light ring", ISCO, marginally bound orbits), but details depend on whether motion is prograde/retrograde

The effect of BH spins: frame-dragging in isolated BHs

Spin affects motion around BHs ("frame dragging")



Innermost Stable Circular Orbit
(i.e. inner edge of thin disks)



Efficiency of EM
emission from thin disks

EM BH spin measurements

Continuum fitting/iron-Kα lines

Binary System	$M/M_\odot$	a	Reference
4U 1543-47	9.4 ± 1.0	$0.75 - 0.85$	Shafee et al. (2006)
GRO J1655-40	6.30 ± 0.27	$0.65 - 0.75$	Shafee et al. (2006)
GRS 1915+105	14.0 ± 4.4	> 0.98	McClintock et al. (2006)
LMC X-3	$5 - 11$	< 0.26	Davis et al. (2006)
M33 X-7	15.65 ± 1.45	0.84 ± 0.05	Liu et al. (2008, 2010)
LMC X-1	10.91 ± 1.41	$0.92^{+0.05}_{-0.07}$	Gou et al. (2009)
XTE J1550-564	9.10 ± 0.61	$0.34^{+0.20}_{-0.28}$	Steiner et al. (2010b)

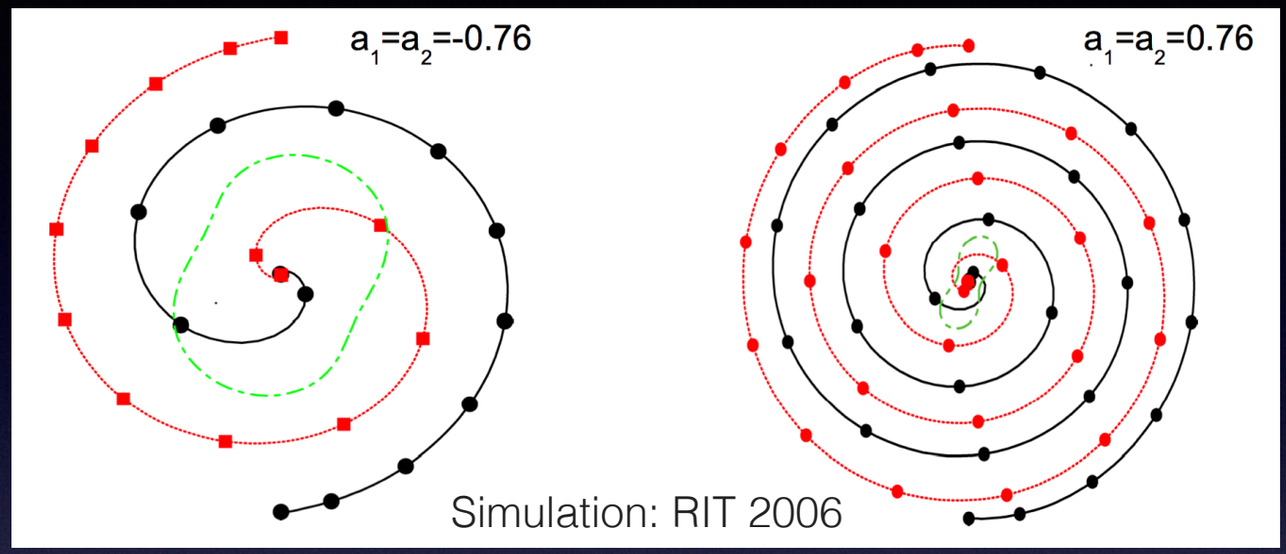
Stellar-mass BH spins

Object name	Galaxy type	z	L_X[erg s^{-1}]	$f_{\rm Edd}$	$\log(M_{\rm bh}[M_\odot])$	spin
1H0707-495	–	0.0411	3.7×10^{43}	1.0	6.70 ± 0.4	> 0.97
Mrk1018	S0	0.043	9.0×10^{43}	0.01	8.15	$0.58^{+0.36}_{-0.74}$
NGC4051	SAB(rs)bc	0.0023	3.0×10^{42}	0.03	6.28	> 0.99
NGC3783	SB(r)ab	0.0097	1.8×10^{44}	0.06	7.47 ± 0.08	> 0.88
1H0419-577	–	0.104	1.8×10^{44}	0.04	8.18 ± 0.05	> 0.89
3C120	S0	0.033	2.0×10^{44}	0.31	$7.74^{+0.20}_{-0.22}$	> 0.95
MCG-6-30-15	E/S0	0.008	1.0×10^{43}	0.4	6.65 ± 0.17	> 0.98
Ark564	SB	0.0247	1.4×10^{44}	0.11	< 6.90	$0.96^{+0.01}_{-0.06}$
TonS180	–	0.062	3.0×10^{44}	2.15	$7.30^{+0.60}_{-0.40}$	$0.91^{+0.02}_{-0.09}$
RBS1124	–	0.208	1.0×10^{45}	0.15	8.26	> 0.97
Mrk110	–	0.0355	1.8×10^{44}	0.16	7.40 ± 0.09	> 0.89
Mrk841	E	0.0365	8.0×10^{43}	0.44	7.90	> 0.52
Fairall9	Sc	0.047	3.0×10^{44}	0.05	8.41 ± 0.11	$0.52^{+0.19}_{-0.15}$
SWIFTJ2127.4+5654	SB0/a(s)	0.0147	1.2×10^{43}	0.18	7.18 ± 0.07	0.6 ± 0.2
Mrk79	SBb	0.0022	4.7×10^{43}	0.05	7.72 ± 0.14	0.7 ± 0.1
Mrk335	S0a	0.026	5.0×10^{43}	0.25	7.15 ± 0.13	$0.83^{+0.09}_{-0.13}$
Ark120	Sb/pec	0.0327	3.0×10^{45}	1.27	8.18 ± 0.12	$0.64^{+0.19}_{-0.11}$
Mrk359	pec	0.0174	6.0×10^{42}	0.25	6.04	$0.66^{+0.30}_{-0.54}$
IRAS13224-3809	–	0.0667	7.0×10^{43}	0.71	7.00	> 0.987
NGC1365	SB(s)b	0.0054	2.7×10^{42}	0.06	$6.60^{+1.40}_{-0.30}$	$0.97^{+0.01}_{-0.04}$

Compilations (Reynolds, Brenneman,...) of massive BH spins

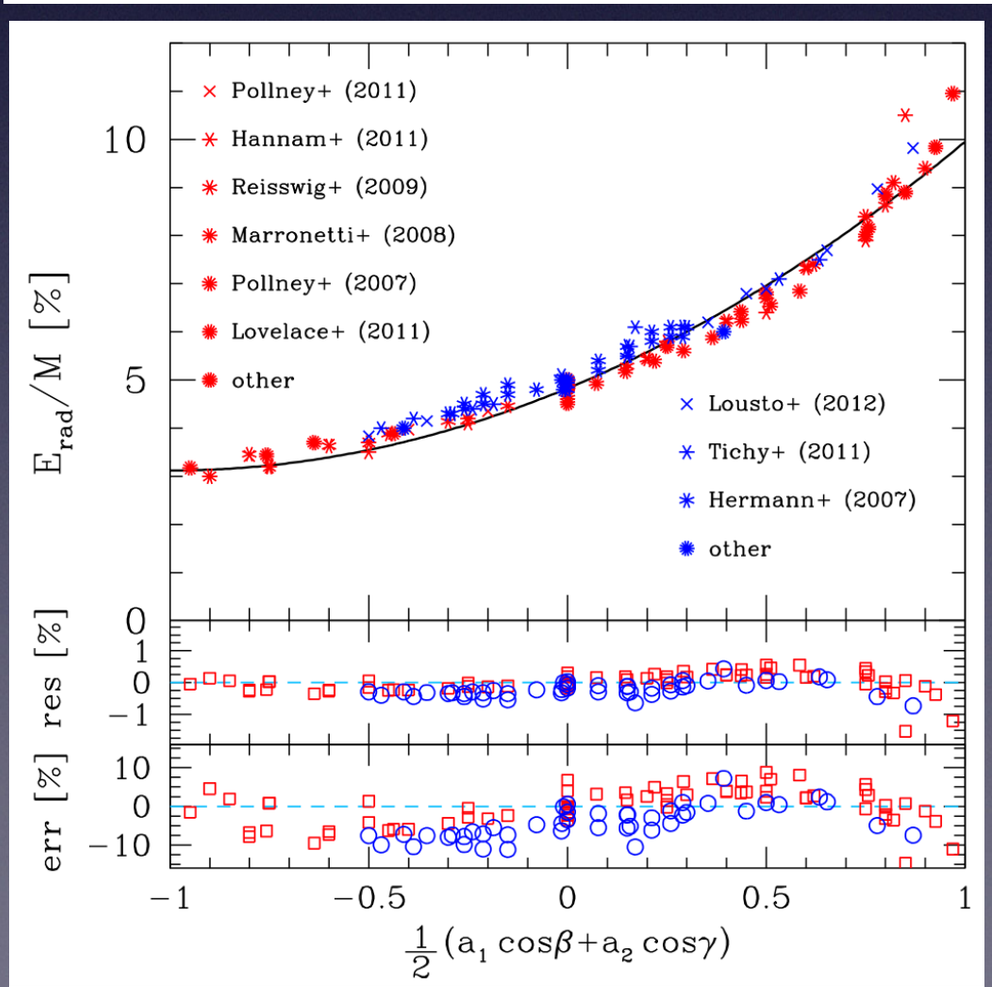
The effect of BH spins: frame-dragging in binaries

- For large spins aligned with L, effective ISCO moves inward …



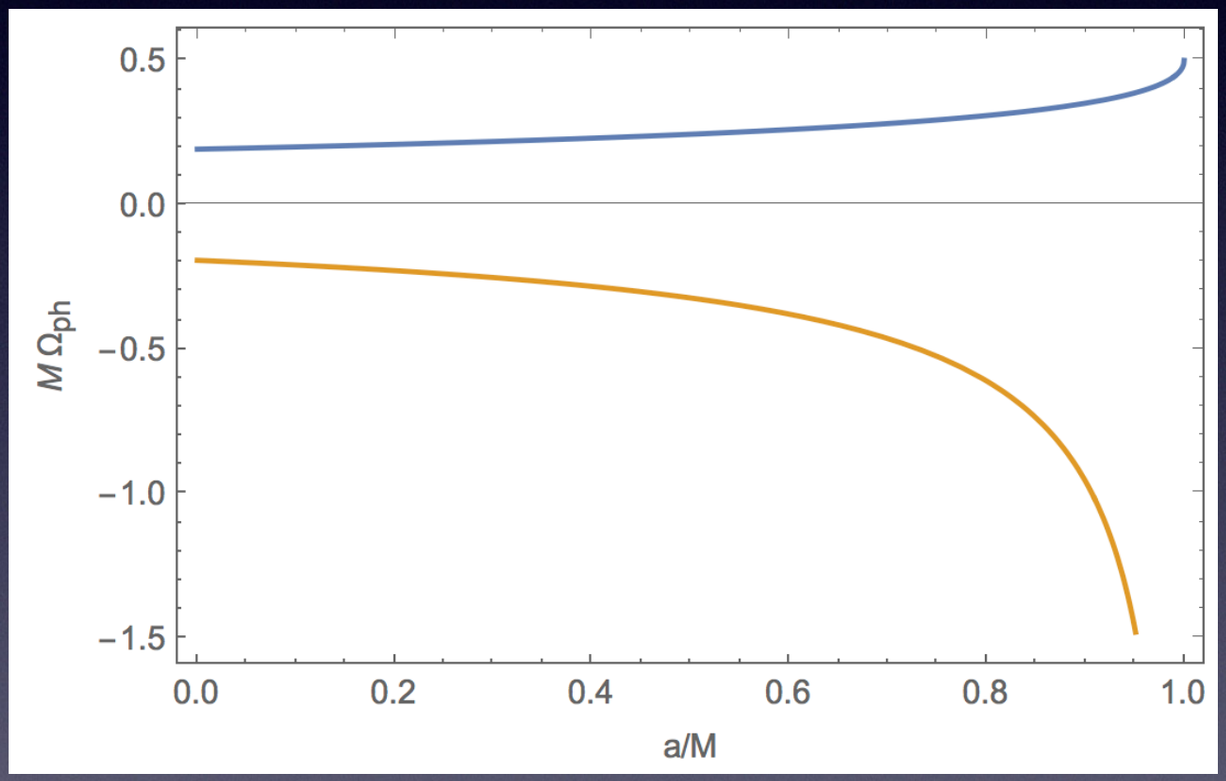
- ... and GW “efficiency” gets larger

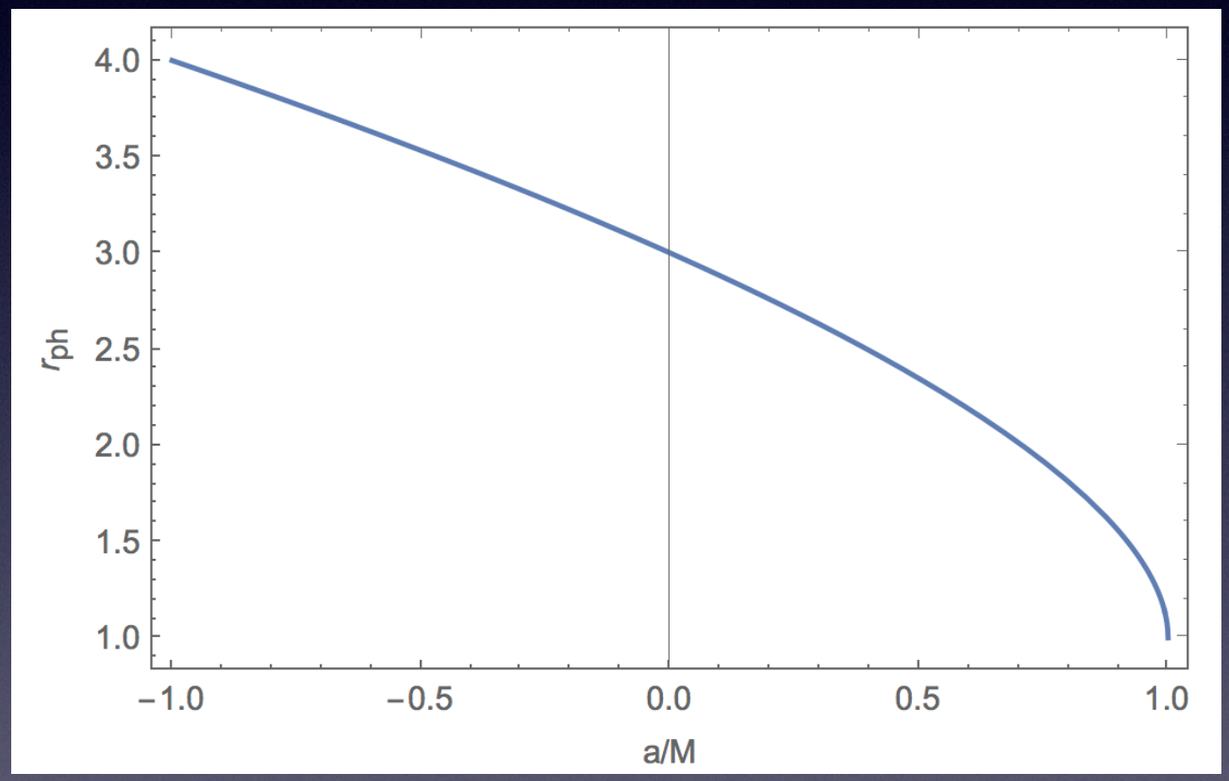
Spins increase GW amplitudes



EB, Morozova & Rezzolla (2012)

The effect of BH spins: frame-dragging in isolated BHs

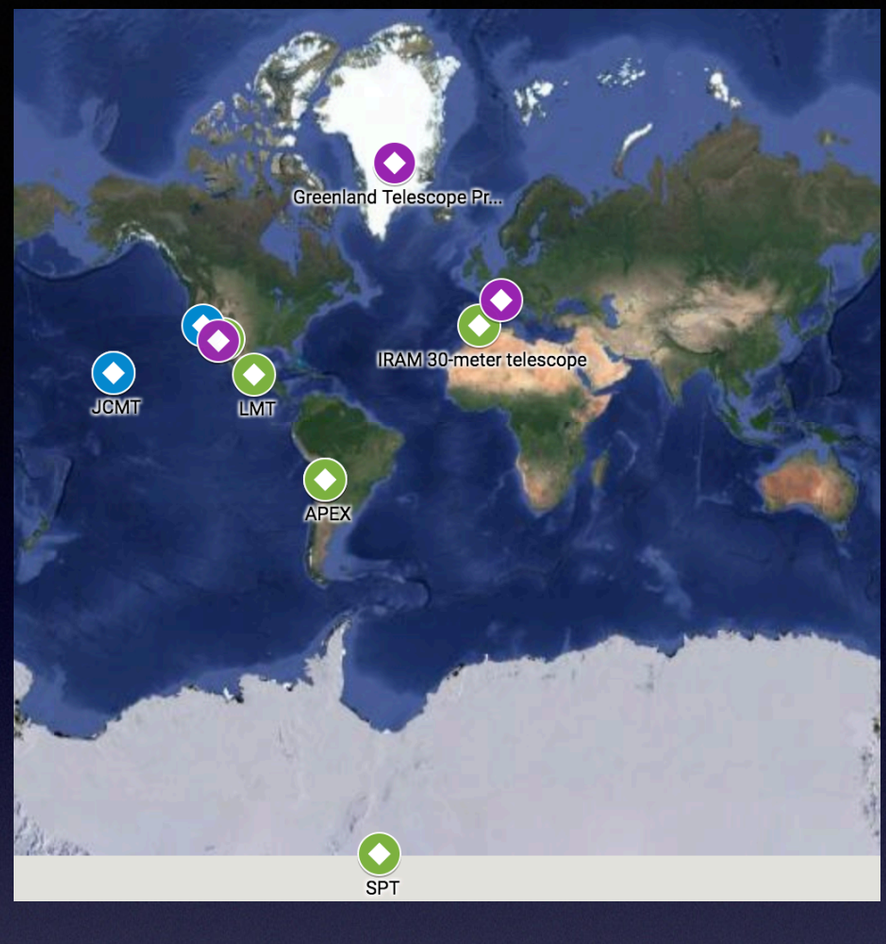




Orbital frequency of prograde circular photon orbit matches horizon's when a=M

BH shadows

Event Horizon Telescope
will image SgrA* and M87
via VLBI radio (mm wavelength)
observations



EHT collaboration 2019

BH binary dynamics

- We can rewrite geodesic motion in Schwarzschild/Kerr as Hamiltonian/Lagrangian

$$H = \beta^i P_i + \alpha \sqrt{m^2 + \gamma^{ij} P_i P_j} \,,$$

$$\begin{aligned} \alpha &= \frac{1}{\sqrt{-g^{tt}}} \,, \\ \beta^i &= \frac{g^{ti}}{g^{tt}} \,, \\ \gamma^{ij} &= g^{ij} - \frac{g^{ti} g^{tj}}{g^{tt}} \,, \end{aligned}$$

- How to go from test-particle limit to BH binary?
- In Newtonian gravity, one can go to center of mass frame and replace test-particle mass by binary's reduced mass
- At post-Newtonian orders ($O(v/c)^{2n}$ beyond Newton) things are more involved

The post-Newtonian Hamiltonian

$$H = m_1c^2 + m_2c^2 + H_N + H_{1PN} + H_{2PN} + H_{3PN} + \dots$$

$$\hat{H} = (H - Mc^2)/\mu \qquad M = m_1 + m_2$$

$$\mu = m_1m_2/M \qquad \nu = \mu/M$$

$$p_r = (\mathbf{n}\cdot\mathbf{p}),\ \mathbf{q} = (\mathbf{x}_1 - \mathbf{x}_2)/GM,$$

$$\mathbf{p} = \mathbf{p}_1/\mu,\ r = r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|,$$

$$\mathbf{n} = \mathbf{q}/|\mathbf{q}| \qquad \mathbf{p}_1 + \mathbf{p}_2 = 0$$

$$H^{1PN}_{S_1S_2} = \frac{G}{c^2}\sum_a\sum_{b\neq a}\frac{1}{2r^3_{ab}}\left[3(\mathbf{S}_a\cdot\mathbf{n}_{ab})(\mathbf{S}_b\cdot\mathbf{n}_{ab}) - (\mathbf{S}_a\cdot\mathbf{S}_b)\right]$$

$$H^{1PN}_{SO} = \frac{G}{c^2}\sum_a\sum_{b\neq a}\frac{1}{r^2_{ab}}(\mathbf{S}_a\times\mathbf{n}_{ab})\cdot\left[\frac{3m_b}{2m_a}\mathbf{p}_a - 2\mathbf{p}_b\right]$$

$$\hat{H}_N = \frac{p^2}{2} - \frac{1}{q},$$

$$c^2\hat{H}_{1PN} = \frac{1}{8}(3\nu-1)p^4 - \frac{1}{2}[(3+\nu)p^2 + \nu p_r^2]\frac{1}{q} + \frac{1}{2q^2},$$

$$c^4\hat{H}_{2PN} = \frac{1}{16}(1-5\nu+5\nu^2)p^6 + \frac{1}{8}[(5-20\nu-3\nu^2)p^4 - 2\nu^2p_r^2p^2 - 3\nu^2p_r^4]\frac{1}{q} + \frac{1}{2}[(5+8\nu)p^2 + 3\nu p_r^2]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3},$$

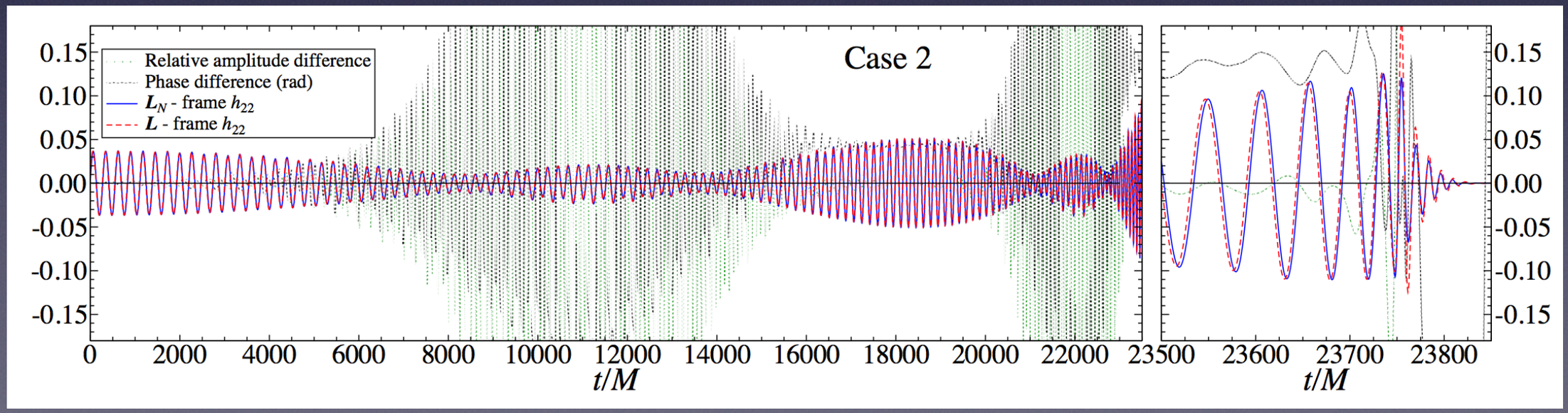
$$c^6\hat{H}_{3PN} = \frac{1}{128}(-5+35\nu-70\nu^2+35\nu^3)p^8 + \frac{1}{16}\left[(-7+42\nu-53\nu^2-5\nu^3)p^6 + (2-3\nu)\nu^2p_r^2p^4 + 3(1-\nu)\nu^2p_r^4p^2 - 5\nu^3p_r^6\right]\frac{1}{q} + \left[\frac{1}{16}(-27+136\nu+109\nu^2)p^4 + \frac{1}{16}(17+30\nu)\nu p_r^2p^2 + \frac{1}{12}(5+43\nu)\nu p_r^4\right]\frac{1}{q^2} + \left[\left(-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right)p^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu p_r^2\right]\frac{1}{q^3} + \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2\right)\nu\right]\frac{1}{q^4}.$$

BH spin precession

- Spin precesses around total angular momentum $J=L+S_1+S_2$
- Precession-induced modulations observable with GW detectors:

  - Increase SNR and improve measurements of binary parameters (e.g. luminosity distance and sky localization)

  - Allow measurements of angle between spins



EOB waveforms for BH binary with mass ratio 1:6 and spins 0.6 and 0.8, from Pan et al (2013) [using spin-EOB model of EB & Buonanno 2010, 2011]

The PN formalism

Main idea: expand dynamics in powers of 1/c [i.e. of v/c, ∂_t/c, GM/(r c^2)]:

$$g_{00} = -\left(1 + 2\frac{\phi}{c^2}\right),$$
$$g_{0i} = \frac{\hat{\omega}_i}{c^3},$$
$$g_{ij} = \left(1 - 2\frac{\psi}{c^2}\right)\delta_{ij} + \frac{\hat{\chi}_{ij}}{c^2},$$

$$\hat{\omega}_i = \partial_i \omega + \omega_i,$$

$$\hat{\chi}_{ij} = \left(\partial_{ij} - \frac{1}{3}\delta_{ij}\nabla^2\right)\chi + \partial_{(i}\chi_{j)} + \chi_{ij},$$

$$\partial_i \omega^i = \partial_i \chi^i = \partial_i \chi^{ij} = \chi^i_i = 0.$$

$$\omega = \chi = \chi_i = 0.$$

$$\partial_i \hat{\omega}^i = \partial_i \hat{\chi}^{ij} = 0$$

"Poisson gauge"

$$g_{00} = -\left(1 + 2\frac{\phi}{c^2}\right),$$
$$g_{0i} = \frac{\omega_i}{c^3},$$
$$g_{ij} = \left(1 - 2\frac{\psi}{c^2}\right)\delta_{ij} + \frac{\chi_{ij}}{c^2},$$

Einstein equations

$$\nabla^2\phi = \ldots$$
$$\nabla^2\phi = \ldots$$
$$\nabla^2\omega_i = \ldots$$

$$\Box\chi_{ij} = \ldots$$

The PN formalism

Expand Einstein eqs+perfect fluid in 1/c, over flat space:

$$\psi \equiv \phi + \frac{\delta\psi}{c^2}$$

$$\nabla^2\phi = 4\pi\left(3\frac{p}{c^2}+\rho\right) + \frac{2}{c^2}\phi_{,i}\phi_{,i} + 8\pi\rho\left(\frac{v}{c}\right)^2 - \frac{3}{c^2}\phi_{,tt}$$
$$+\frac{1}{c^4}\Big[-16\pi\rho\phi^2 - 8\pi\rho\delta\psi + \phi_{,i}\delta\psi_{,i} + \phi_{,i}\omega^i{}_{,t} - \frac{1}{2}\omega^i{}_{,j}\omega^i{}_{,j}$$
$$+\frac{1}{2}\omega^i{}_{,j}\omega^j{}_{,i} + 8\pi(p-4\rho\phi)v^2 + 8\pi\rho v^4 + \phi_{,ij}\chi^{ij}$$
$$-3\delta\psi_{,tt}\Big] + \mathcal{O}\left(\frac{1}{c^6}\right),$$

$$\nabla^2\delta\psi = -12p\pi - 16\pi\rho\phi - \frac{7}{2}\phi_{,j}\phi_{,j}$$
$$- 4\pi\rho v^2 + 3\phi_{,tt} + \mathcal{O}\left(\frac{1}{c^2}\right).$$

$$\nabla^2\omega^i = 4(4\pi\rho v^i + \phi_{,ti}) + \frac{2}{c^2}\Big[\phi_{,j}\omega^j{}_{,i} - \phi_{,ij}\omega^j$$
$$+2\left(\phi_{,t}\phi_{,i} + \delta\psi_{,it} + 4p\pi v^i - 16\pi\rho\phi v^i + 4\pi\rho v^2 v^i + 2\pi\rho\omega^i\right)\Big]$$
$$+\mathcal{O}\left(\frac{1}{c^4}\right).$$

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$$+\frac{1}{2}\omega^i{}_{,j}\omega^j{}_{,i} + 8\pi(p - 4\rho\phi)v^2 + 8\pi\rho v^4 + \phi_{,ij}\chi^{ij}$$
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$$- 4\pi\rho v^2 + 3\phi_{,tt} + \mathcal{O}\left(\frac{1}{c^2}\right).$$

$a=a_N$

$$\nabla^2\omega^i = 4(4\pi\rho v^i + \phi_{,ti}) + \frac{2}{c^2}\Big[\phi_{,j}\omega^j{}_{,i} - \phi_{,ij}\omega^j$$
$$+2\left(\phi_{,t}\phi_{,i} + \delta\psi_{,it} + 4p\pi v^i - 16\pi\rho\phi v^i + 4\pi\rho v^2 v^i + 2\pi\rho\omega^i\right)\Big]$$
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$$+\frac{1}{c^4}\Big[-16\pi\rho\phi^2-8\pi\rho\delta\psi+\phi_{,i}\delta\psi_{,i}+\phi_{,i}\omega^i{}_{,t}-\frac{1}{2}\omega^i{}_{,j}\omega^i{}_{,j}$$
$$+\frac{1}{2}\omega^i{}_{,j}\omega^j{}_{,i}+8\pi(p-4\rho\phi)v^2+8\pi\rho v^4+\phi_{,ij}\chi^{ij}$$
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$$-4\pi\rho v^2+3\phi_{,tt}+\mathcal{O}\left(\frac{1}{c^2}\right).$$

$$\nabla^2\omega^i = 4(4\pi\rho v^i+\phi_{,ti})+\frac{2}{c^2}\Big[\phi_{,j}\omega^j{}_{,i}-\phi_{,ij}\omega^j$$
$$+2\left(\phi_{,t}\phi_{,i}+\delta\psi_{,it}+4p\pi v^i-16\pi\rho\phi v^i+4\pi\rho v^2 v^i+2\pi\rho\omega^i\right)\Big]$$
$$+\mathcal{O}\left(\frac{1}{c^4}\right).$$

$a=a_N(1+1PN/c^2$

The PN formalism

Expand Einstein eqs+perfect fluid in 1/c, over flat space:

$$\psi \equiv \phi + \frac{\delta\psi}{c^2}$$

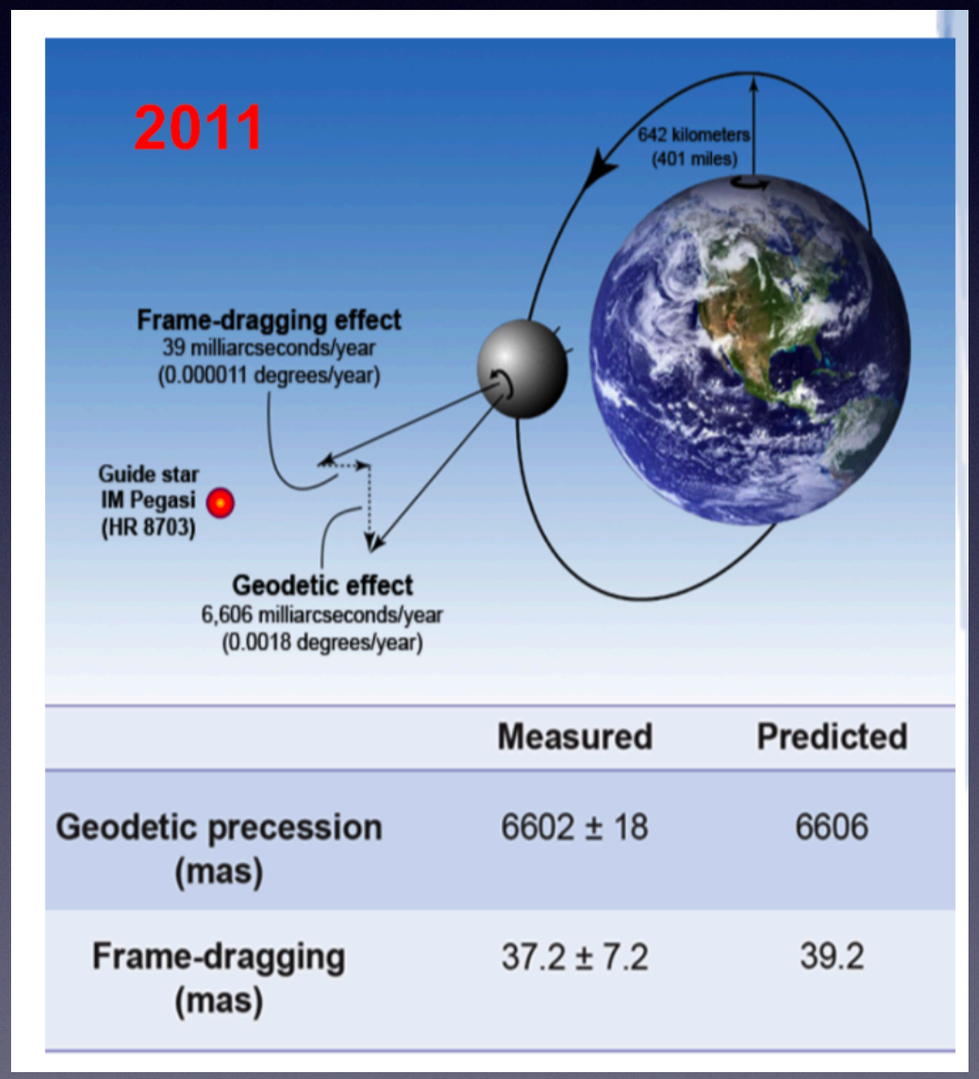
$$\nabla^2\phi = 4\pi\left(3\frac{p}{c^2}+\rho\right)+\frac{2}{c^2}\phi_{,i}\phi_{,i}+8\pi\rho\left(\frac{v}{c}\right)^2-\frac{3}{c^2}\phi_{,tt}$$
$$+\frac{1}{c^4}\Big[-16\pi\rho\phi^2-8\pi\rho\delta\psi+\phi_{,i}\delta\psi_{,i}+\phi_{,i}\omega^i{}_{,t}-\frac{1}{2}\omega^i{}_{,j}\omega^i{}_{,j}$$
$$+\frac{1}{2}\omega^i{}_{,j}\omega^j{}_{,i}+8\pi(p-4\rho\phi)v^2+8\pi\rho v^4+\phi_{,ij}\chi^{ij}$$
$$-3\delta\psi_{,tt}\Big]+\mathcal{O}\left(\frac{1}{c^6}\right),$$

$$\nabla^2\delta\psi = -12p\pi-16\pi\rho\phi-\frac{7}{2}\phi_{,j}\phi_{,j}$$
$$-4\pi\rho v^2+3\phi_{,tt}+\mathcal{O}\left(\frac{1}{c^2}\right).$$

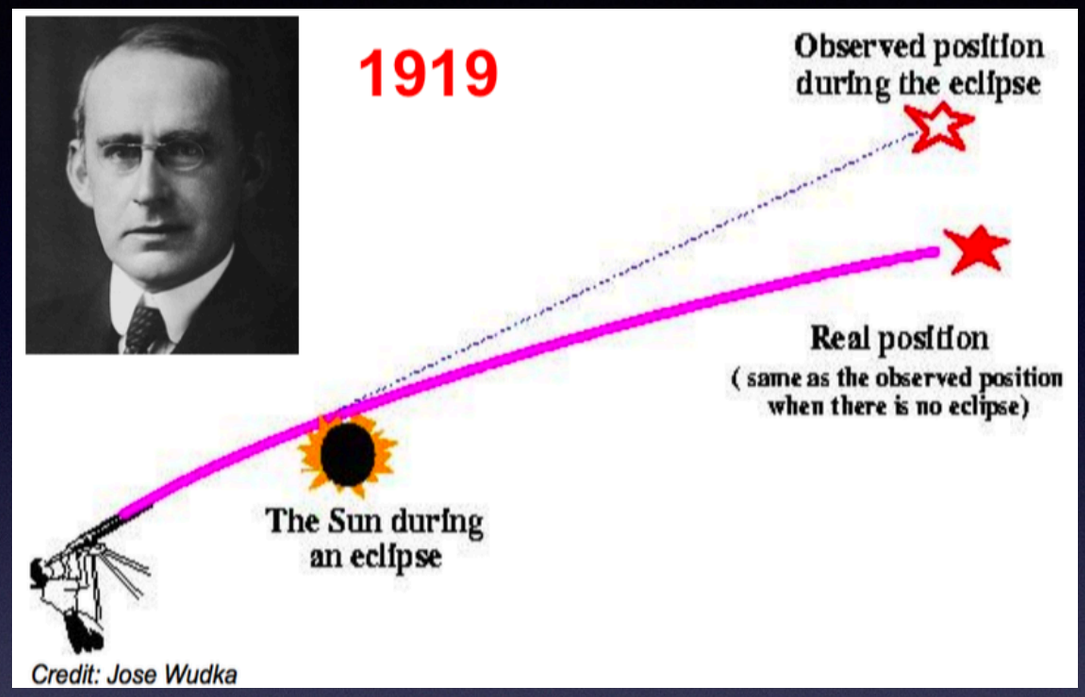
$$\nabla^2\omega^i = 4(4\pi\rho v^i+\phi_{,ti})+\frac{2}{c^2}\Big[\phi_{,j}\omega^j{}_{,i}-\phi_{,ij}\omega^j$$
$$+2\left(\phi_{,t}\phi_{,i}+\delta\psi_{,it}+4p\pi v^i-16\pi\rho\phi v^i+4\pi\rho v^2 v^i+2\pi\rho\omega^i\right)\Big]$$
$$+\mathcal{O}\left(\frac{1}{c^4}\right).$$

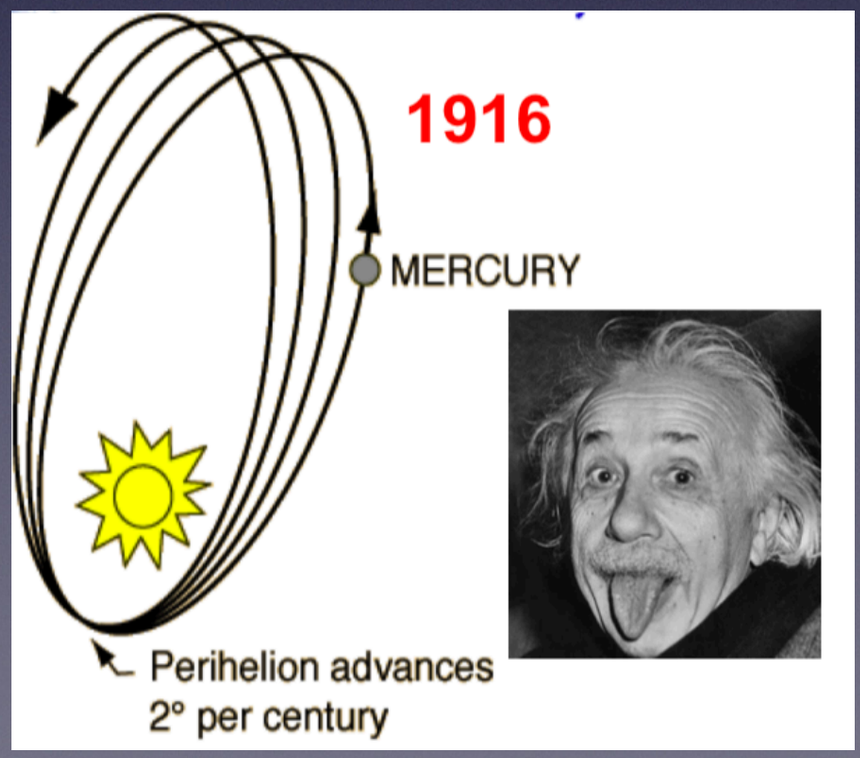
$a=a_N(1+1PN/c^2+2PN/c^4+\ldots)$

PN effects are not only relevant for BH binaries



	Measured	Predicted
Geodetic precession (mas)	6602 ± 18	6606
Frame-dragging (mas)	37.2 ± 7.2	39.2





How about the TT term?

Keep time derivatives even though
they carry factor 1/c, because for GWs $\partial_t/c \sim \partial_x$)

$$\Box\chi_{ij} \approx -16\pi\sigma_{ij}$$

$$\sigma_{ij} = P_i^k P_j^l T_{kl} - P_{ij}P^{kl}T_{kl}/2$$

$$P_{ij} = \delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}$$

$$\chi_{ij} = -16\pi\Box^{-1}\sigma_{ij} = -16\pi\Box^{-1}\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)T_{kl}$$

$$= -16\pi\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)\Box^{-1}T_{kl}$$

$$= 4\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)\int\frac{T_{ij}\left(t-|\boldsymbol{x}-\boldsymbol{x}'|,\boldsymbol{x}'\right)}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x'$$

$$\approx \frac{4}{r}\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)\int T_{ij}(t-r,\boldsymbol{x}')\mathrm{d}^3x'$$

The quadrupole formula

$$\partial_t^2 \left(T^{tt} x^i x^j\right) = \partial_k \partial_l \left(T^{kl} x^i x^j\right) - 2\partial_k \left(T^{ik} x^j + T^{kj} x^i\right) + 2T^{ij}$$

$$\frac{4}{r}\int d^3x'\, T_{ij} = \frac{4}{r}\int d^3x' \left[\frac{1}{2}\partial_t^2 \left(T^{tt} x'^i x'^j\right) + \partial_k \left(T^{ik} x'^j + T^{kj} x'^i\right) - \frac{1}{2}\partial_k \partial_l \left(T^{kl} x'^i x'^j\right)\right]$$

$$= \frac{2}{r}\int d^3x'\, \partial_t^2 \left(T^{tt} x'^i x'^j\right) = \frac{2}{r}\frac{\partial^2}{\partial t^2}\int d^3x'\, \rho\, x'^i x'^j = \frac{2}{r}\frac{d^2 I_{ij}(t-r)}{dt^2}$$

$$I_{ij}(t) = \int d^3x'\, \rho(t, \mathbf{x}')\, x'^i x'^j$$

$$\chi_{ij} \approx \frac{4}{r}\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)\int T_{ij}(t-r, \boldsymbol{x}')\mathrm{d}^3x'$$

$$= \frac{2}{r}\frac{d^2\mathcal{I}_{kl}(t-r)}{dt^2}\left[P_{ik}(\mathbf{n})P_{jl}(\mathbf{n}) - \frac{1}{2}P_{kl}(\mathbf{n})P_{ij}(\mathbf{n})\right]\frac{G}{c^4}$$

$$P_{ij} = \delta_{ij} - n_i n_j \qquad \mathcal{I}_{ij} = I_{ij} - \frac{1}{3}\delta_{ij} I,$$

Quadrupole tensor

small number!

The quadrupole formula

Binary moving on circular orbit on x,y plane with orbital frequency Ω, GW traveling along z

$$\chi_{ij} = h \times \begin{bmatrix} \cos 2\Omega t & \sin 2\Omega t & 0 \\ \sin 2\Omega t & -\cos 2\Omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h = \frac{4\mu\Omega^2 R^2}{r} = \frac{4\mu(M\Omega)^{2/3}}{r} = \frac{4M_c^{5/3}\Omega^{2/3}}{r} \qquad M_c = M\nu^{3/5} \quad \nu = \frac{\mu}{M}$$

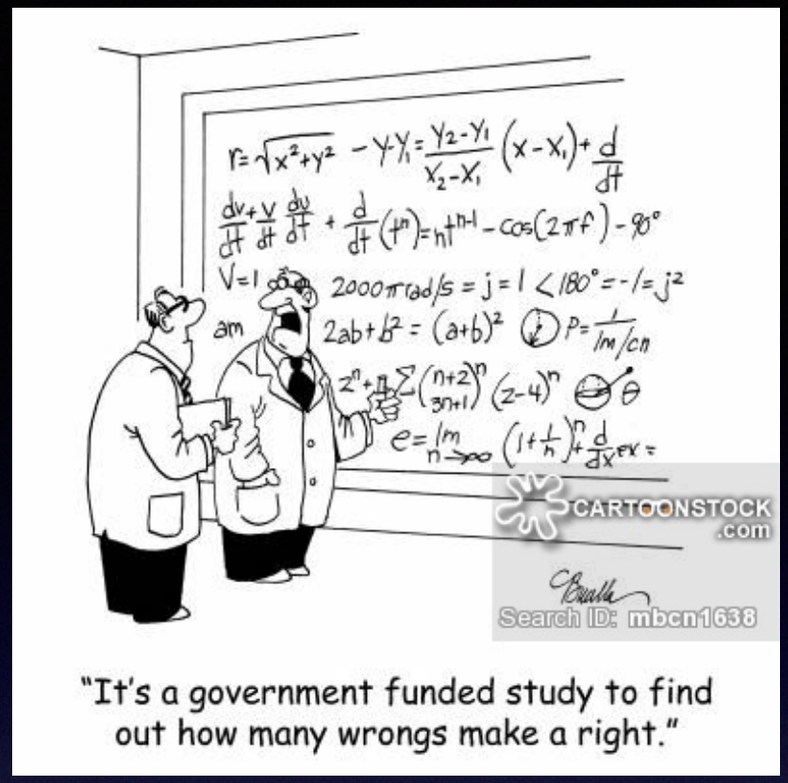
$$h \approx 1.9 \times 10^{-21} \left(\frac{f_{\rm gw}}{100Hz}\right)^{2/3} \left(\frac{M_{c,z}}{30M_\odot}\right)^{5/3} \left(\frac{400{\rm Mpc}}{d_L}\right)$$

$$\approx 3.5 \times 10^{-18} \left(\frac{f_{\rm gw}}{10{\rm mHz}}\right)^{2/3} \left(\frac{M_{c,z}}{10^6 M_\odot}\right)^{5/3} \left(\frac{16{\rm Gpc}}{d_L}\right)$$

$h_{Sun} \sim G\, M_{sun}/R_{sun}\, c^2$
$\sim 2 \times 10^{-6}$

$$M_{c,z} = M_c(1+z) \qquad f_{\rm gw} = \frac{2f_{\rm orb}}{1+z} = \frac{\Omega}{\pi(1+z)} \qquad d_L(z=2) \approx 16\ {\rm Gpc}$$

Wrong+wrong=right



- We have started from linearized theory over Minkowski
- This implies that stress energy tensor is conserved wrt to Minkowski metric ...
- … and that is used to go from ”Green formula” to ”quadrupole formula”
- This is inconsistent as a binary system in GW-dominated regimes does NOT move on Minkowski geodesics (i.e. straight lines)
- Exercise: compute GWs from Green formula for a system of two unequal masses on Keplerian orbits one around the other and verify that the GW amplitudes differ by a factor 2 (assume propagation along z axis)
- Which one is correct? Quadrupole or Green?
- One would expect Green, but actually the quadrupole formula is the correct one

A (more) correct derivation

$$\bar{H}^{\mu\nu} \equiv \eta^{\mu\nu} - (-g)^{1/2} g^{\mu\nu} \qquad \partial_\beta \bar{H}^{\alpha\beta} = 0 \qquad \text{harmonic gauge}$$

$$\Longrightarrow \quad \Box_{\rm flat} \bar{H}^{\alpha\beta} = -16\pi\tau^{\alpha\beta} \qquad \tau^{\alpha\beta} = (-g)T^{\alpha\beta} + (16\pi)^{-1}\Lambda^{\alpha\beta}$$

Full Einstein equations!

$$\Lambda^{\alpha\beta} = 16\pi(-g)t_{\rm LL}^{\alpha\beta} + (\bar{H}^{\alpha\mu}{}_{,\nu}\,\bar{H}^{\beta\nu}{}_{,\mu} - \bar{H}^{\alpha\beta}{}_{,\mu\nu}\,\bar{H}^{\mu\nu})$$

$$16\pi(-g)t_{\rm LL}^{\alpha\beta} \equiv g_{\lambda\mu}g^{\nu\rho}\bar{H}^{\alpha\lambda}_{,\nu}\bar{H}^{\beta\mu}_{,\rho}$$
$$+\frac{1}{2}g_{\lambda\mu}g^{\alpha\beta}\bar{H}^{\lambda\nu}_{,\rho}\bar{H}^{\rho\mu}_{,\nu} - 2g_{\mu\nu}g^{\lambda(\alpha}\bar{H}^{\beta)\nu}_{,\rho}\bar{H}^{\rho\mu}_{,\lambda}$$
$$+\frac{1}{8}(2g^{\alpha\lambda}g^{\beta\mu} - g^{\alpha\beta}g^{\lambda\mu})(2g_{\nu\rho}g_{\sigma\tau} - g_{\rho\sigma}g_{\nu\tau})\bar{H}^{\nu\tau}_{,\lambda}\bar{H}^{\rho\sigma}_{,\mu}$$

From gauge condition,

$$\tau^{\alpha\beta}{}_{,\beta} = 0$$

= geodesic motion in curved metric *g*

$$\bar{H}^{\mu\nu} \approx \bar{h}^{\mu\nu} \equiv h^{\mu\nu} - \frac{1}{2}h\eta^{\mu\nu}$$

Proceed as before but with T replaced by τ

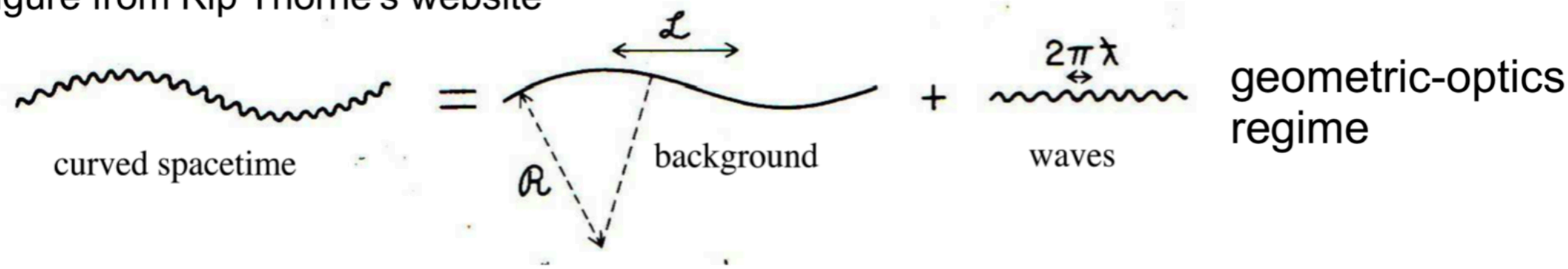
A (more) correct derivation

$$
\begin{aligned}
g_{00} &= -1 - 2\frac{\phi}{c^2} + O(1/c^4) & \tau^{00} &= T^{00} + O(1/c^0) \\
g_{0i} &= O(1/c^3) & \tau^{0i} &= T^{0i} + O(1/c^1) \\
g_{ij} &= \left(1 - 2\frac{\phi}{c^2}\right)\delta_{ij} + O(1/c^4) & \tau^{ij} &= T^{ij} + \frac{1}{4\pi G}\left(\partial^i\phi\partial^j\phi - \frac{1}{2}\delta^{ij}\partial_k\phi\partial^k\phi\right) + O(1/c^2),
\end{aligned}
$$

- Non-linear terms are important in τ_{ij} but not τ_{00}
- Non-linear terms in Green formula account for discrepancy with quadrupole formula for circular Keplerian binaries
- Quadrupole formula is correct, Green's is not

GW "backreact" on geometry at second order

Figure from Kip Thorne's website



geometric-optics regime

$$g^{\rm B}_{\alpha\beta} \equiv \langle g_{\alpha\beta}\rangle \qquad g_{\alpha\beta} = g^{\rm B}_{\alpha\beta} + \varepsilon h_{\alpha\beta} + \varepsilon^2 j_{\alpha\beta} + O(\varepsilon^3)$$

$$\begin{aligned} 0 &= G_{\alpha\beta} \\ &= G_{\alpha\beta}[g^{\rm B}_{cd}] + \varepsilon G^{(1)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] + \varepsilon^2 G^{(1)}_{\alpha\beta}[j_{cd}; g^{\rm B}_{ef}] + \varepsilon^2 G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] \\ &\quad + O(\varepsilon^3)\ . \end{aligned}$$

$$G_{\alpha\beta}[g^{\rm B}_{cd}] = 0, \qquad G^{(1)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] = 0\ , \qquad G^{(1)}_{\alpha\beta}[j_{cd}; g^{\rm B}_{ef}] = -G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}]$$

GWs carry energy and momentum

Average Einstein equations on scale >> λ and << L

$$\Delta j_{\alpha\beta} = j_{\alpha\beta} - \langle j_{\alpha\beta} \rangle$$

$$G^{(1)}_{\alpha\beta}[\langle j_{cd} \rangle; g^{\rm B}_{ef}] = -\langle G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] \rangle$$

$$G^{(1)}_{\alpha\beta}[\Delta j_{cd}] = -G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] + \langle G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] \rangle$$

$$G_{\alpha\beta}[g^{\rm B}_{cd} + \varepsilon^2 \langle j_{cd} \rangle] = 8\pi G\, T^{\rm GW,eff}_{\alpha\beta} + O(\varepsilon^3) \qquad T^{\rm GW,eff}_{\alpha\beta} = -\frac{1}{8\pi G} \langle G^{(2)}_{\alpha\beta}[h_{cd}; g^{\rm B}_{ef}] \rangle$$

Using gauge freedom and integrating by parts:

$$T^{\rm GW,eff}_{\alpha\beta} = \frac{1}{32\pi G} \left\langle \nabla^{\rm B}_{\alpha} h^{\rm TT}_{\rho\sigma}\, \nabla^{\rm B}_{\beta} h^{\rho\sigma}_{\rm TT} \right\rangle$$

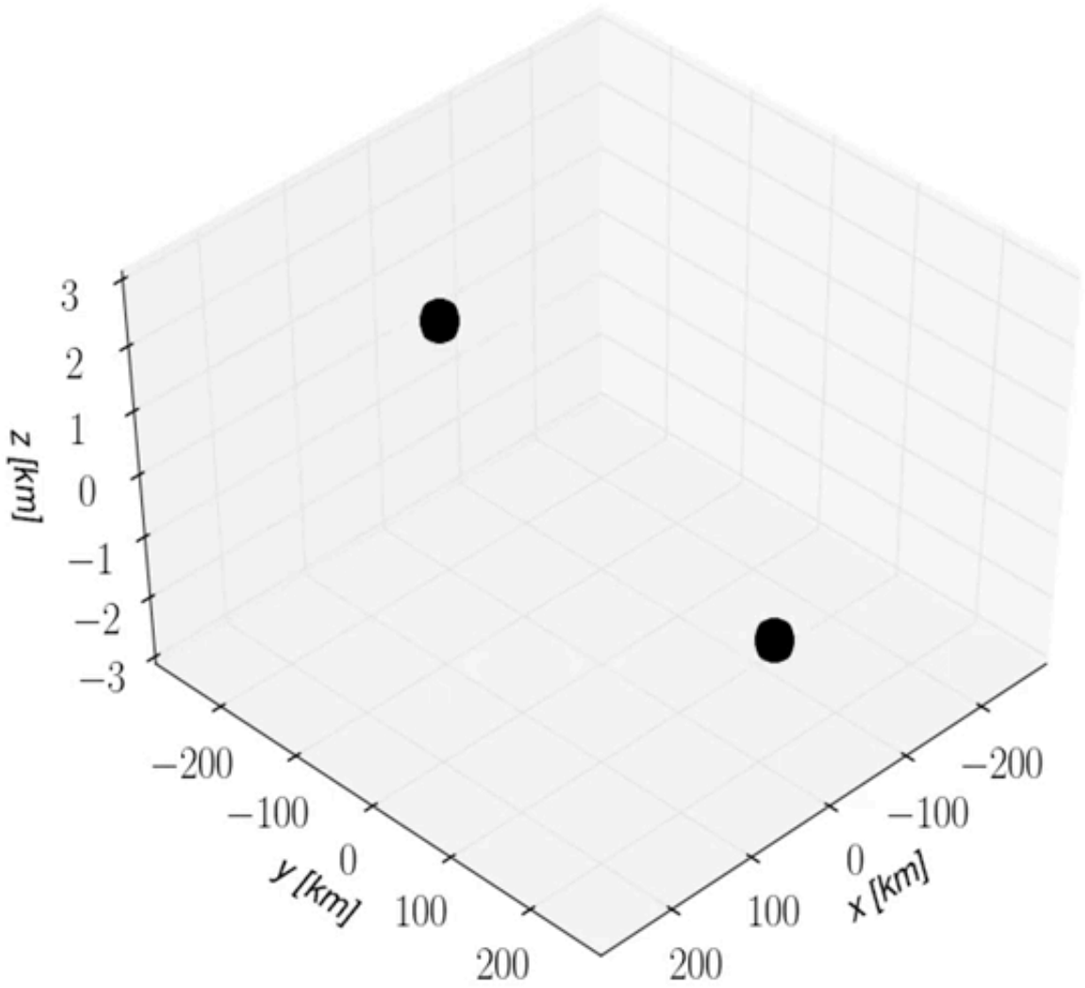
$$L_{\text{mass quadrupole}} \equiv \frac{1}{5} \frac{G}{c^5} \langle \dddot{I} \rangle^2 = \frac{1}{5} \frac{G}{c^5} \langle \dddot{\mathcal{I}}_{jk} \dddot{\mathcal{I}}_{jk} \rangle^2$$

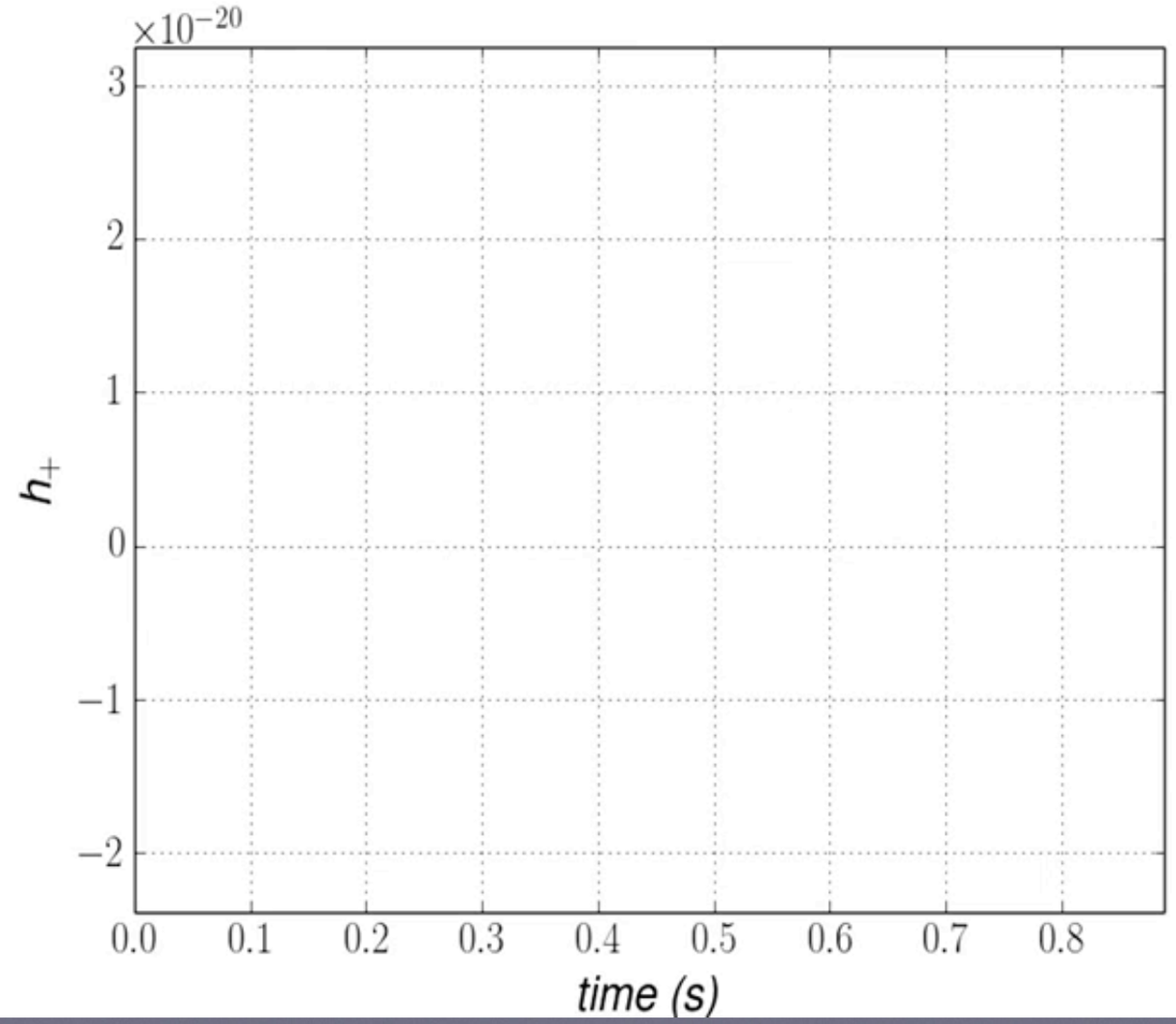
$$\dddot{\mathcal{I}}_{jk} \sim \frac{(\text{mass of the system in motion}) \times (\text{size of the system})^2}{(\text{time scale})^3} \sim \frac{MR^2}{\tau^3} \sim \frac{Mv^2}{\tau}$$

BH binaries inspiral till (effective) ISCO

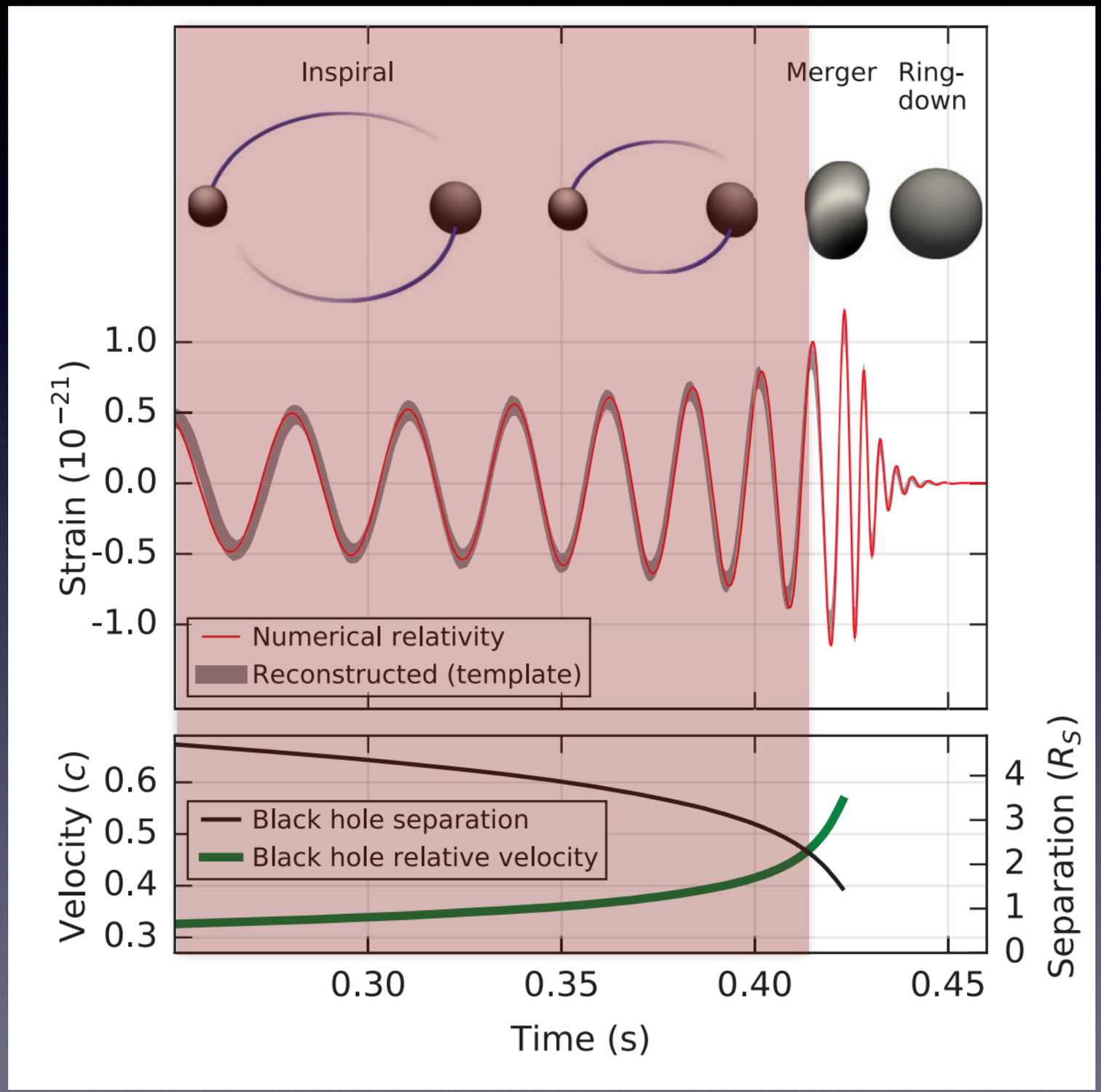
From energy balance:

$$\frac{\dot{P}_b}{P_b} = -\frac{3}{2}\frac{\dot{E}_b}{E_b}$$





GWs from binary BHs



LSC collaboration 2015

Extracting the BH masses

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

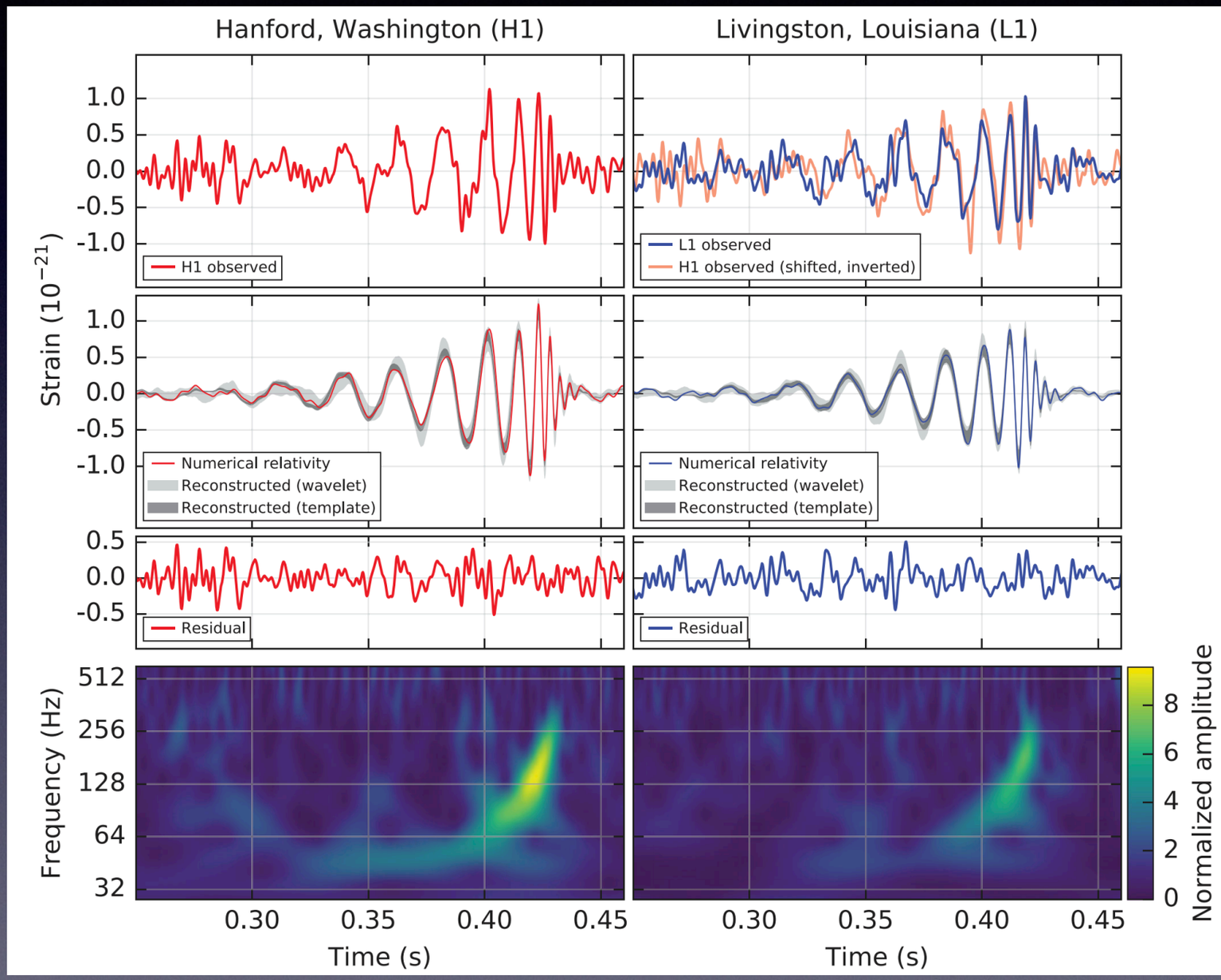
$$\mathcal{M} \simeq 30 M_\odot$$

$$M = m_1 + m_2 \gtrsim 70 M_\odot$$

$$2GM/c^2 \gtrsim 210 \text{ km}$$

f_{gw}~ 75 Hz corresponds to r_{12}~350 km

Objects in GW150914 must be BHs (not WDs or NSs)



LSC collaboration 2015

Extracting the BH masses

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	$\chi_{\rm eff}$	$M_{\rm f}/M_\odot$	$a_{\rm f}$	$E_{\rm rad}/(M_\odot c^2)$	$\ell_{\rm peak}/({\rm erg\,s^{-1}})$	$d_L/{\rm Mpc}$	z	$\Delta\Omega/{\rm deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	179
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9}\times10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9}\times10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5}\times10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1\times10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7}\times10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9}\times10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

LSC 2018, O1+O2 detections

Last week's update

Cornell University

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the Simons

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GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4 M_\odot$

The LIGO Scientific Collaboration, the Virgo Collaboration: B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R. X. Adhikari, V.

	Low-spin prior ($\chi < 0.05$)	High-spin prior ($\chi < 0.89$)
Primary mass m_1	$1.62-1.88\,M_\odot$	$1.61-2.52\,M_\odot$
Secondary mass m_2	$1.45-1.69\,M_\odot$	$1.12-1.68\,M_\odot$
Chirp mass $\mathcal{M}$	$1.44^{+0.02}_{-0.02}\,M_\odot$	$1.44^{+0.02}_{-0.02}\,M_\odot$
Detector-frame chirp mass	$1.4868^{+0.0003}_{-0.0003}\,M_\odot$	$1.4873^{+0.0008}_{-0.0006}\,M_\odot$
Mass ratio m_2/m_1	$0.8 - 1.0$	$0.4 - 1.0$
Total mass $m_{\rm tot}$	$3.3^{+0.1}_{-0.1}\,M_\odot$	$3.4^{+0.3}_{-0.1}\,M_\odot$
Effective inspiral spin parameter $\chi_{\rm eff}$	$0.013^{+0.01}_{-0.01}$	$0.058^{+0.11}_{-0.05}$
Luminosity distance $D_{\rm L}$	$161^{+67}_{-73}\,{\rm Mpc}$	$159^{+69}_{-71}\,{\rm Mpc}$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 600	≤ 1100

Extracting the BH masses

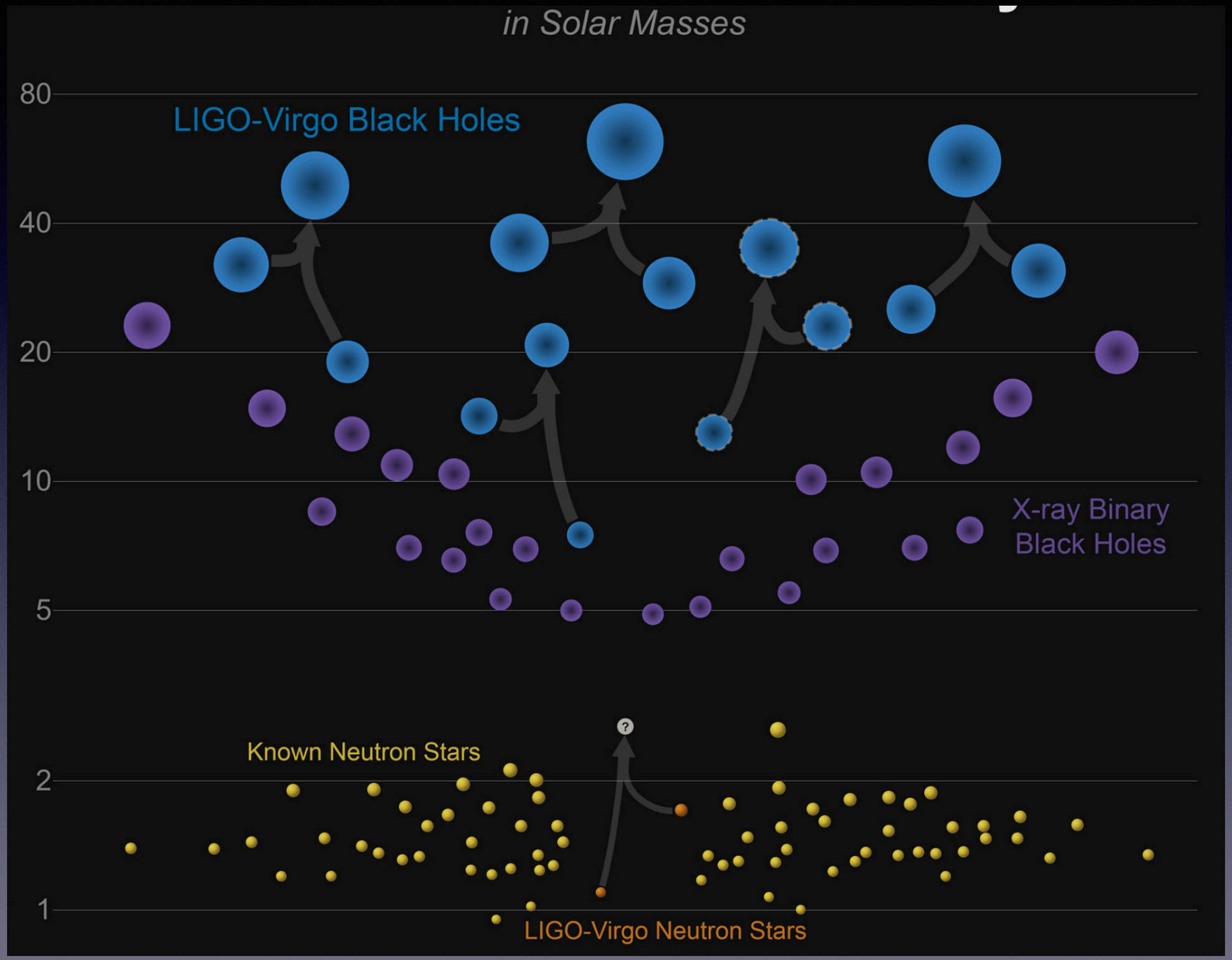
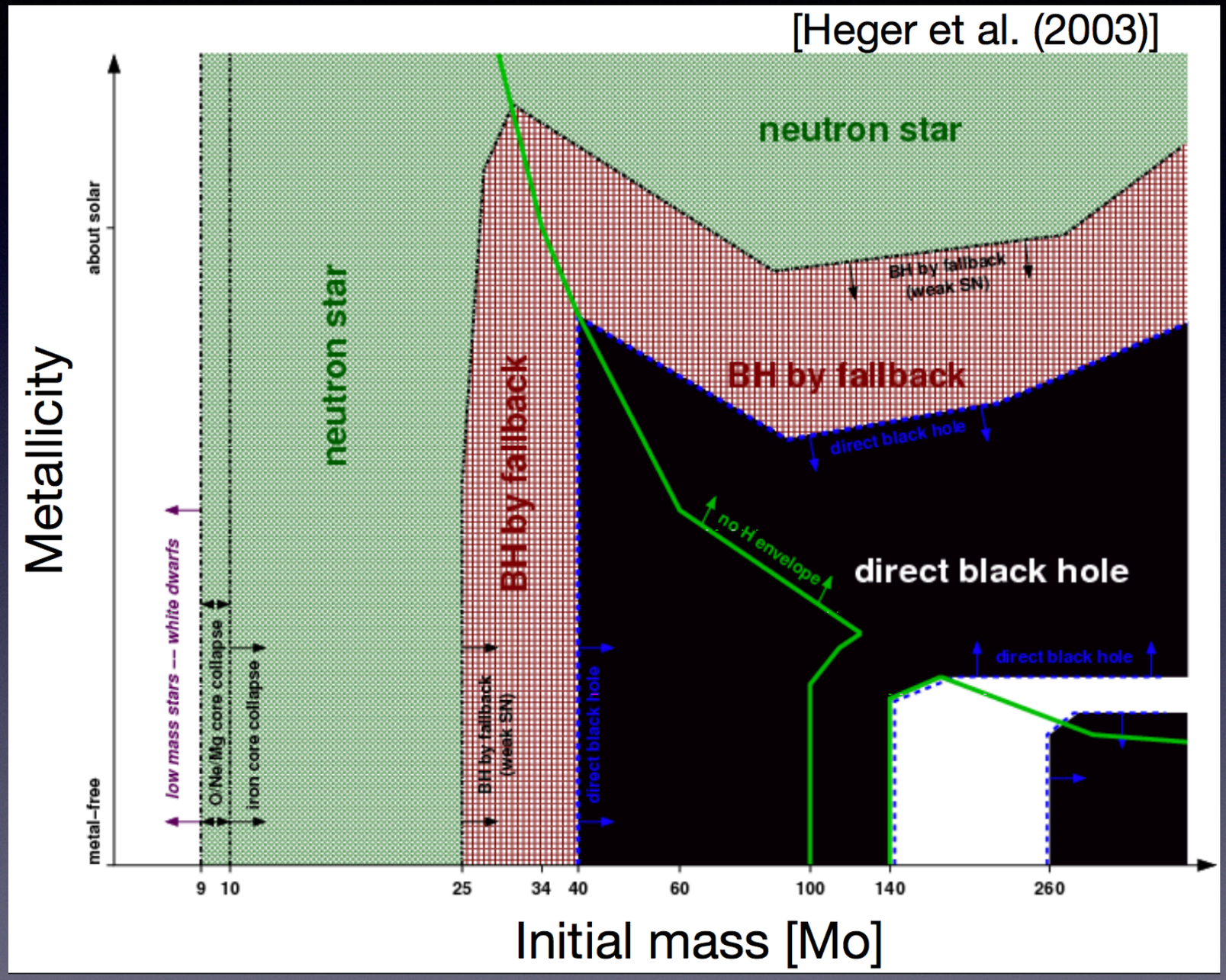


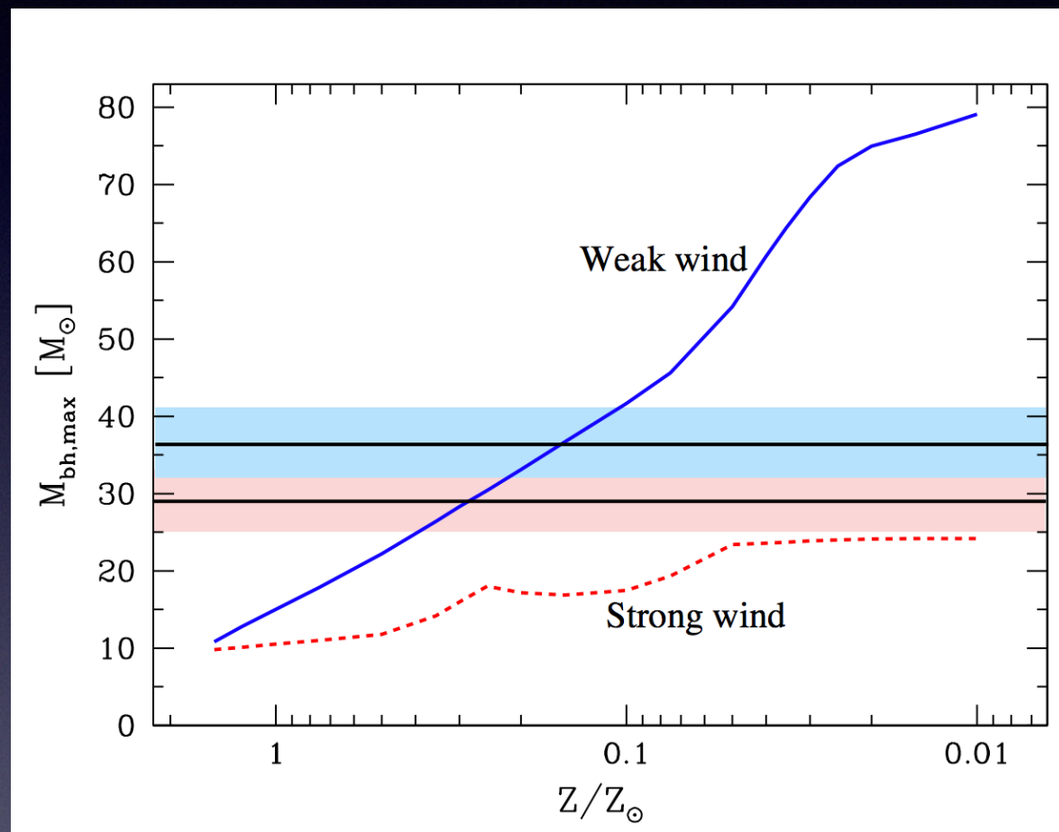
Figure: LSC collaboration 2018

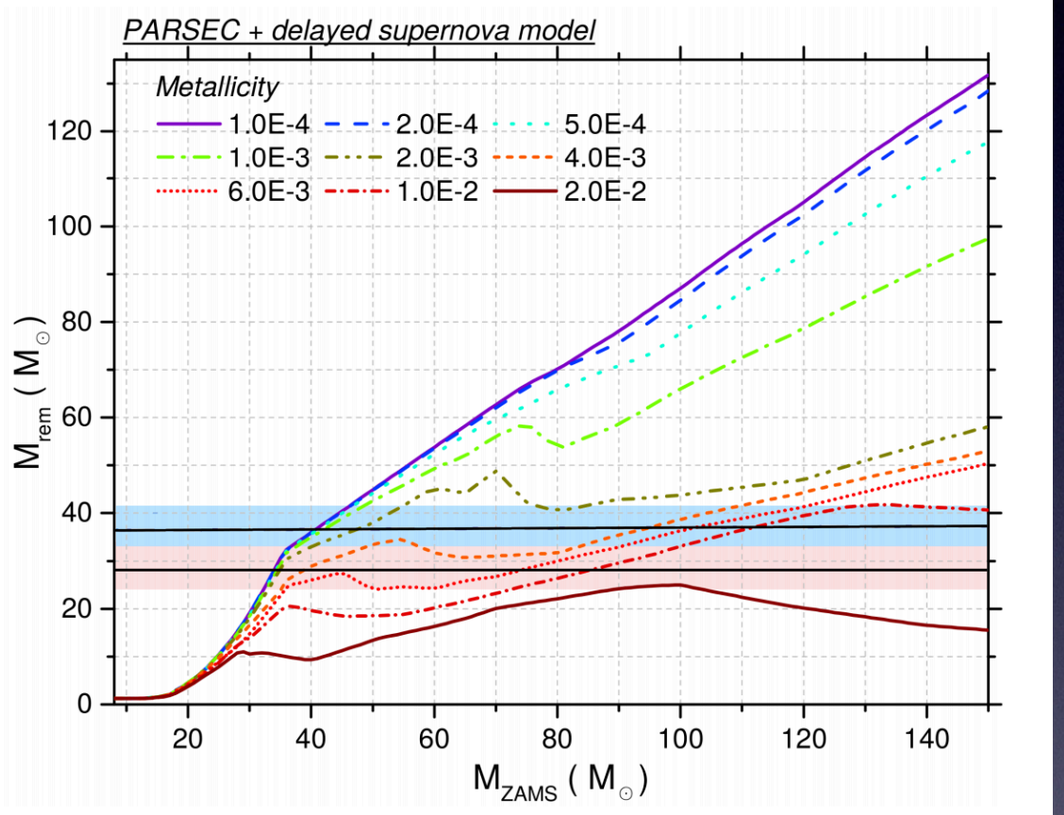
The formation of stellar-mass BHs

- Stellar-mass BH form from massive stars
- Difficult problem: stellar evolution needed to understand mass loss from stellar winds, and explosion mechanism (core collapse SN, direct collapse to BH)
- Evolution depends on mass, metallicity, rotation



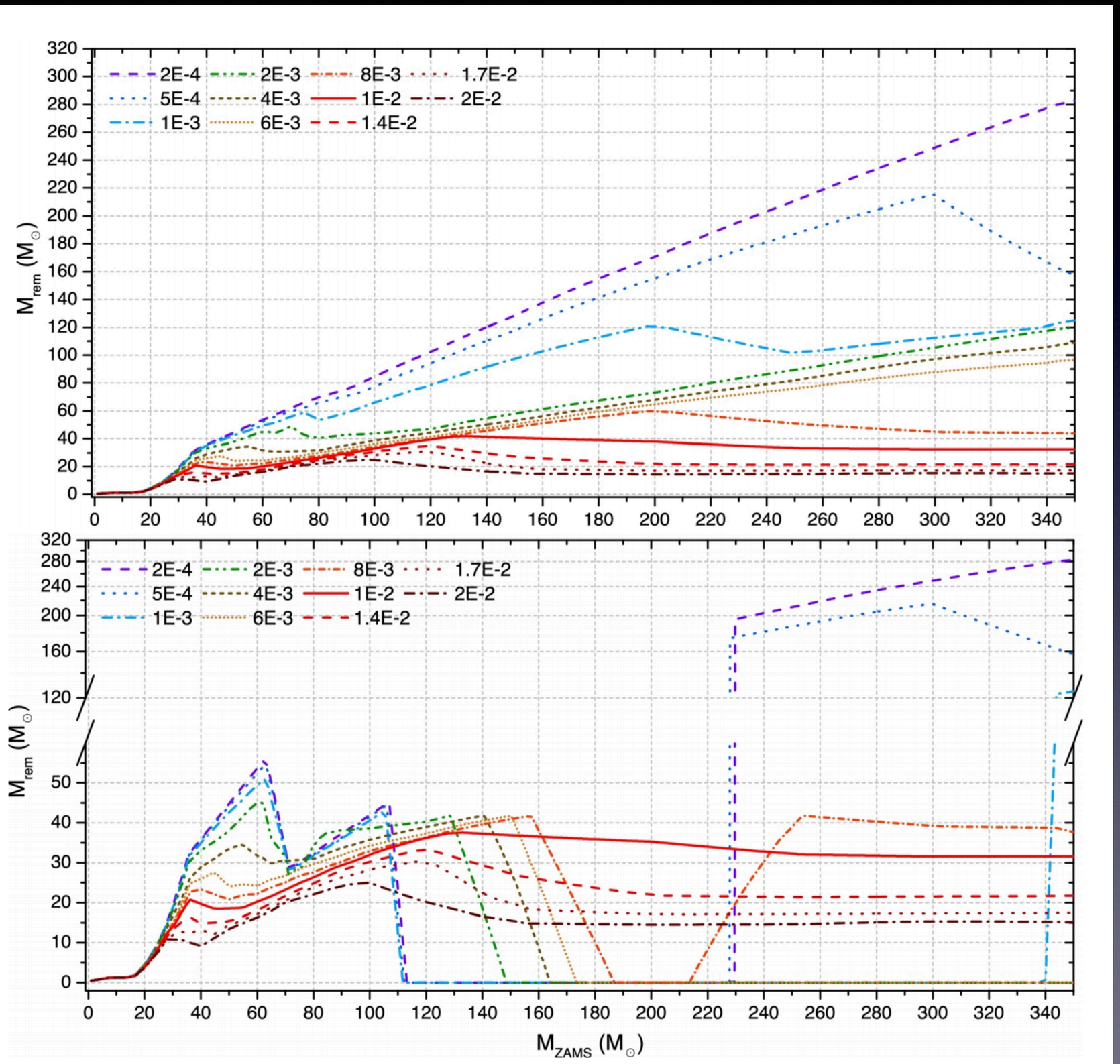
The role of metallicity and stellar winds





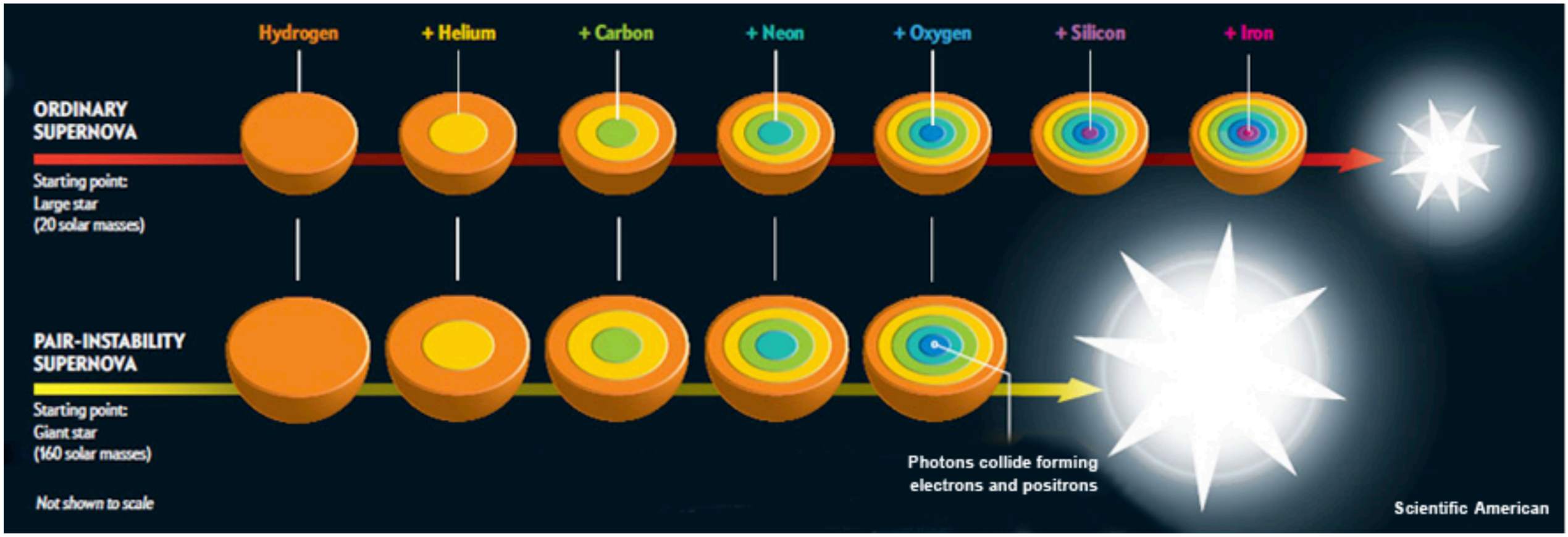
LSC 2015; Belczynski et al 2010; Spera et al 2015

The role of metallicity and stellar winds



Mapelli 2018;
Spera & Mapelli 2017

Pair instability SN



Mass at birth (solar masses)	Helium core mass (solar masses)	Compact remnant	Event
10–95	2–40	Neutron star, black hole	Ordinary supernova
95–130	40–60	Neutron star, black hole	Pulsational pair-instability supernova
130–260	60–137	Explosion, no remnant	Pair-instability supernova
>260	>137	Black hole	?

Woosley, Blinnikov, Heger (2007)

A cutoff at 40 Msun?

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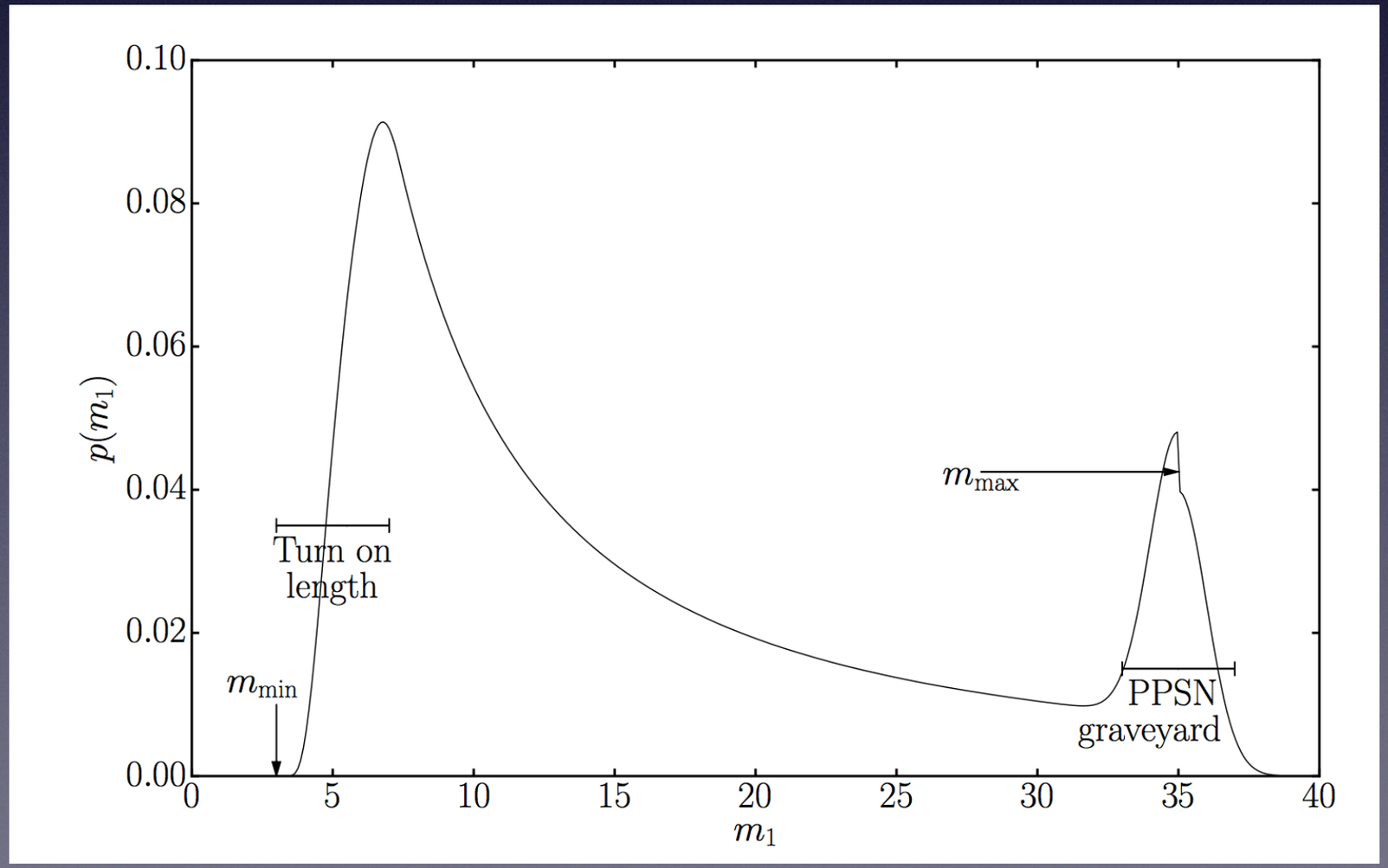
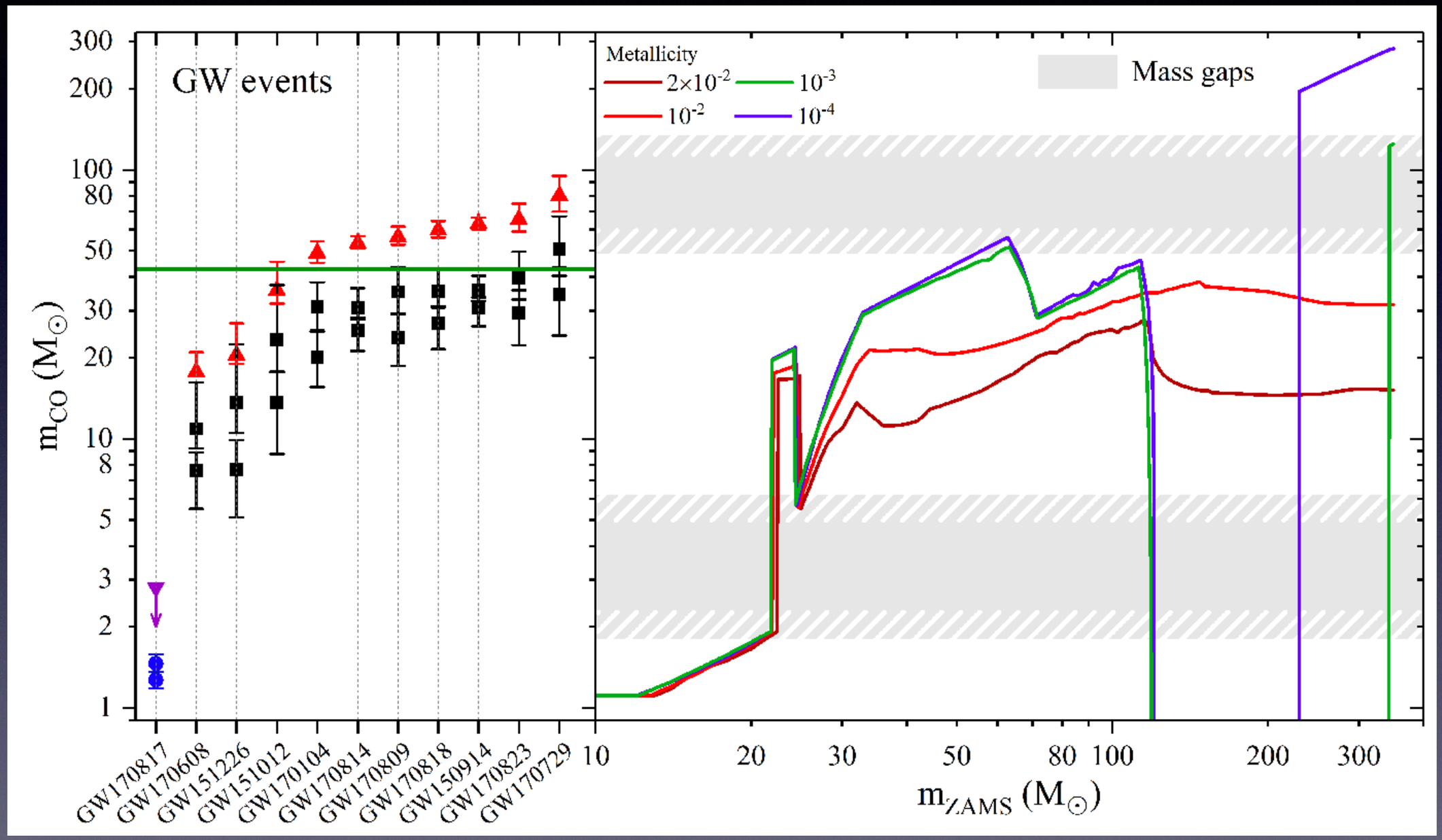


Talbot & Thrane 2018

O2 (and soon O3) updates



LSC 2018

How do stellar-mass BH *binaries* form?

- In the field (plausible because ~70% of massive stars have companion, c.f. Sana et al 2012)
- In dense environments (globular clusters/nuclear star clusters) via dynamical mechanisms
- Primordial BHs? But problems with CMB/absence of enough MW candidates in radio/X-rays if one wants to explain all of Dark Matter. Formation mechanism also unclear (clustering vs lack of clustering), conflicting predictions for spins

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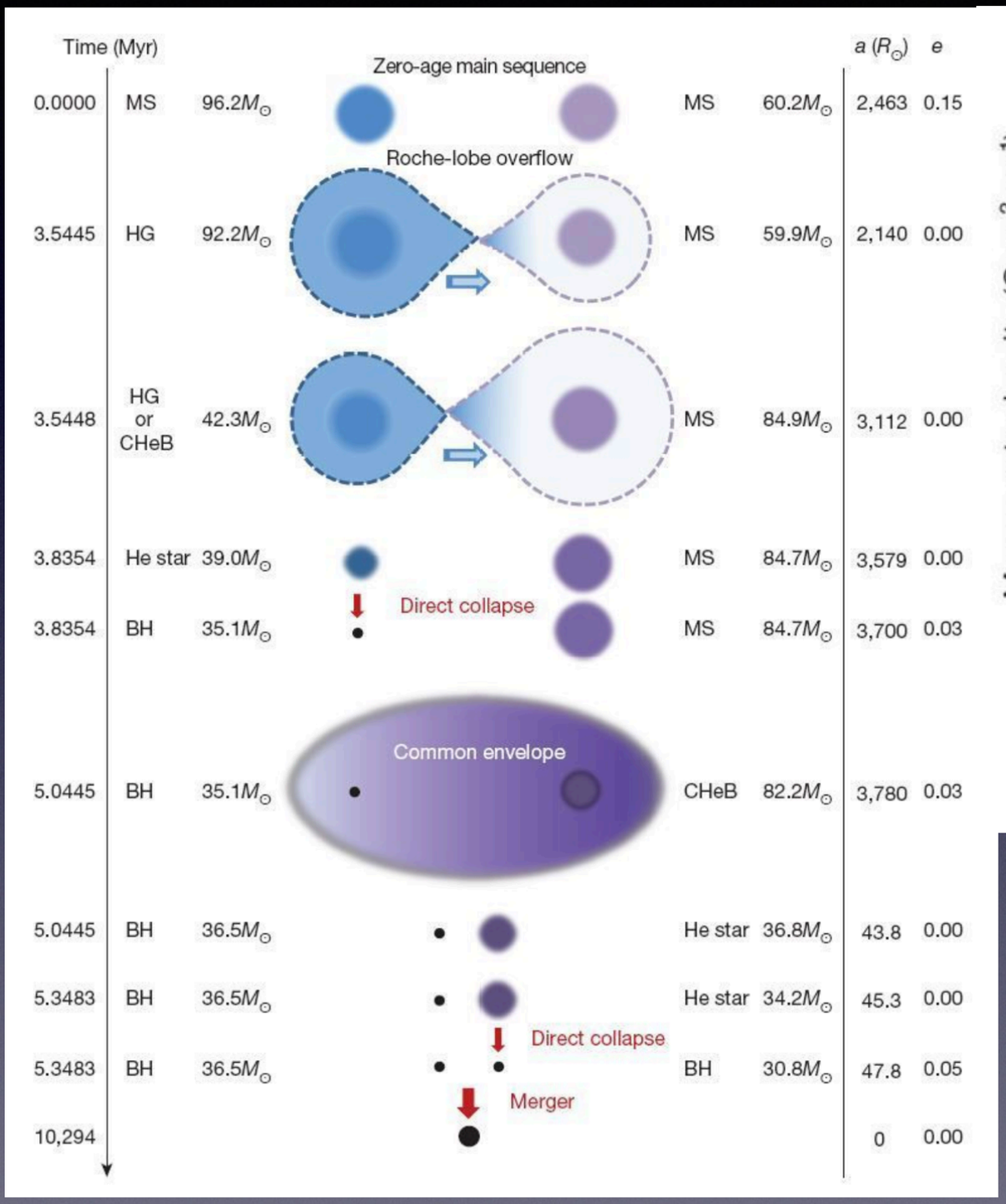
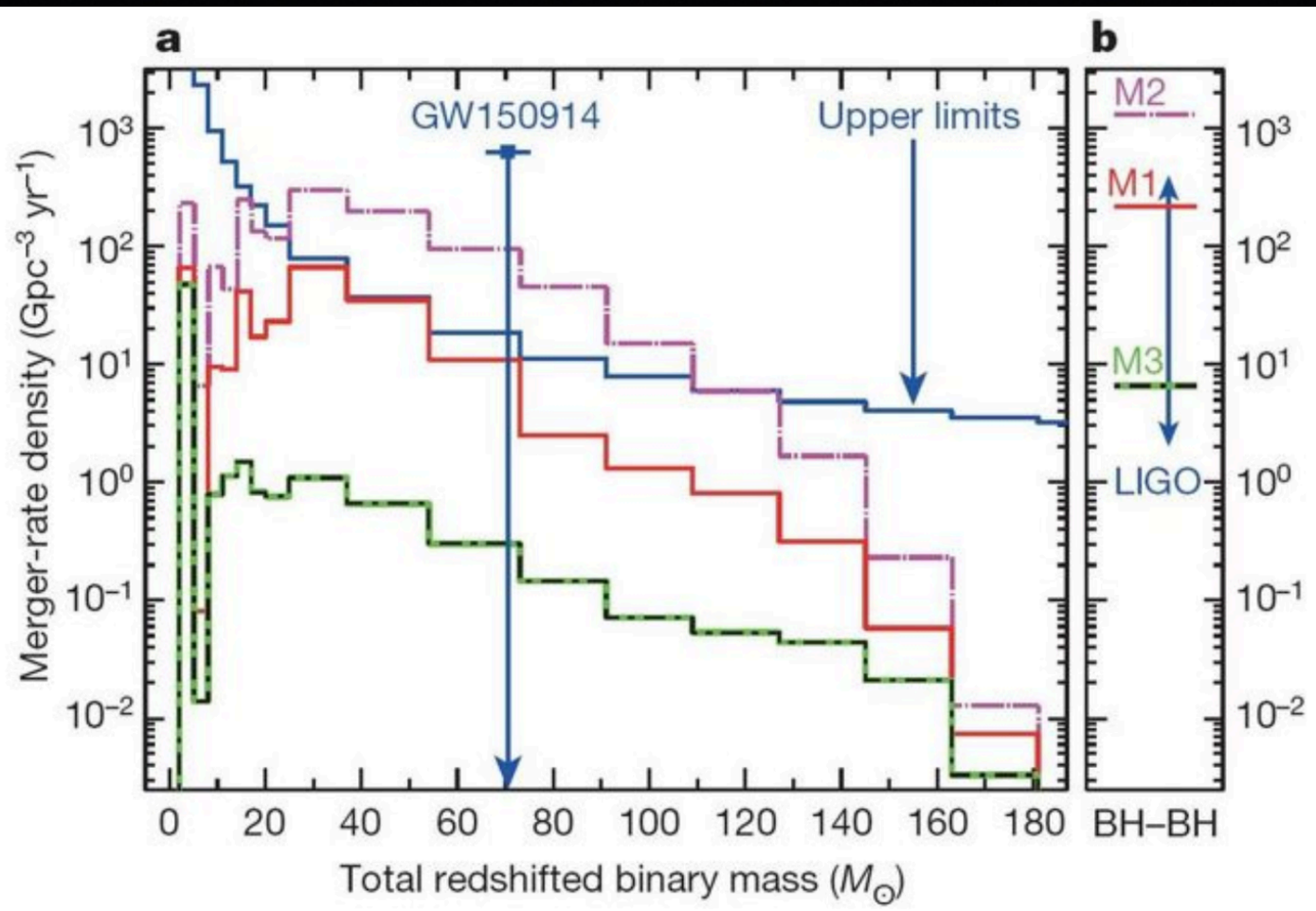
Field BH binaries

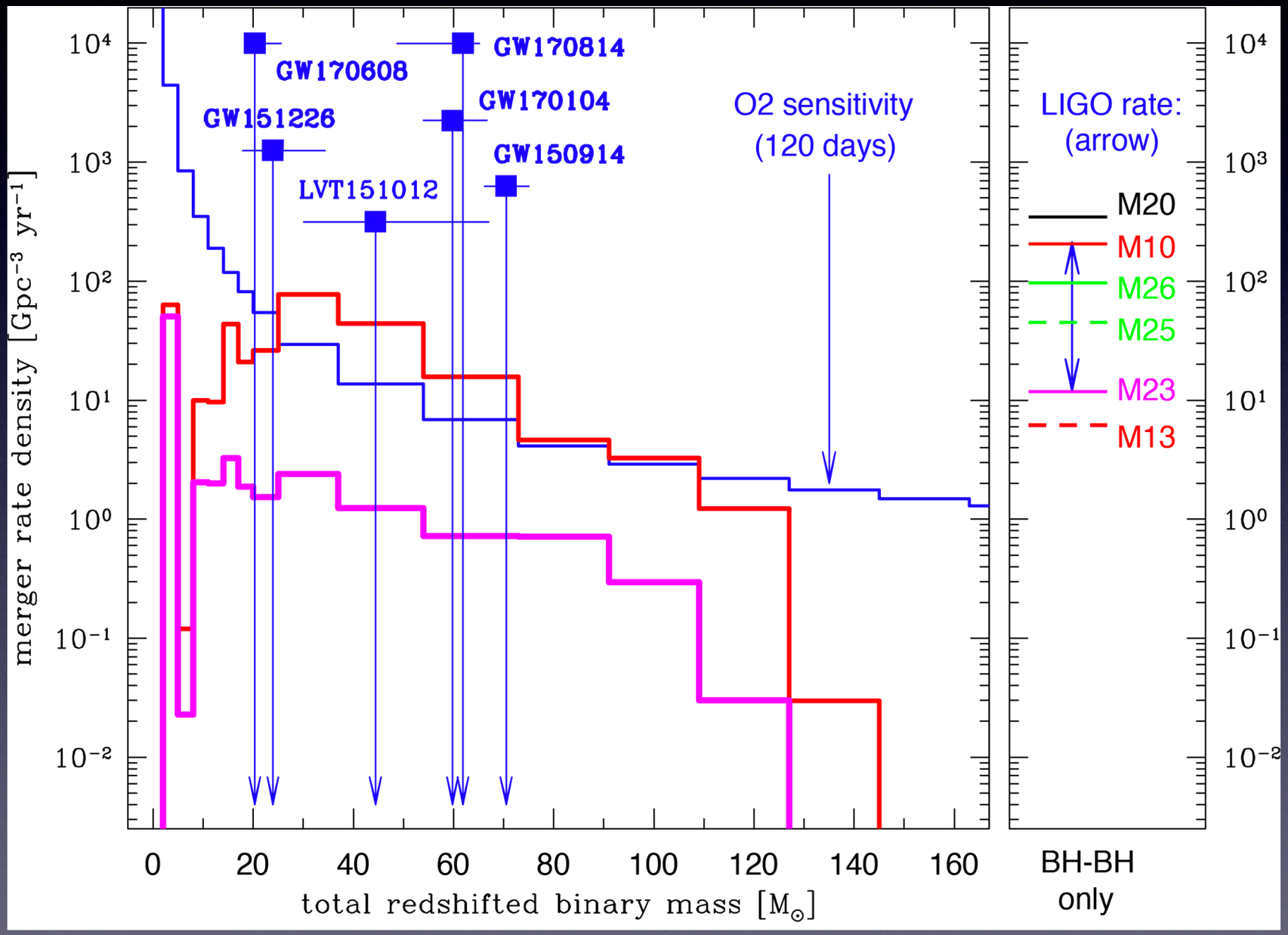




Merger rates for standard model (M1; red); optimistic common-envelope phase (M2; pink); and pessimistic large black-hole kicks (M3; green/black)

Belczynski et al 2016

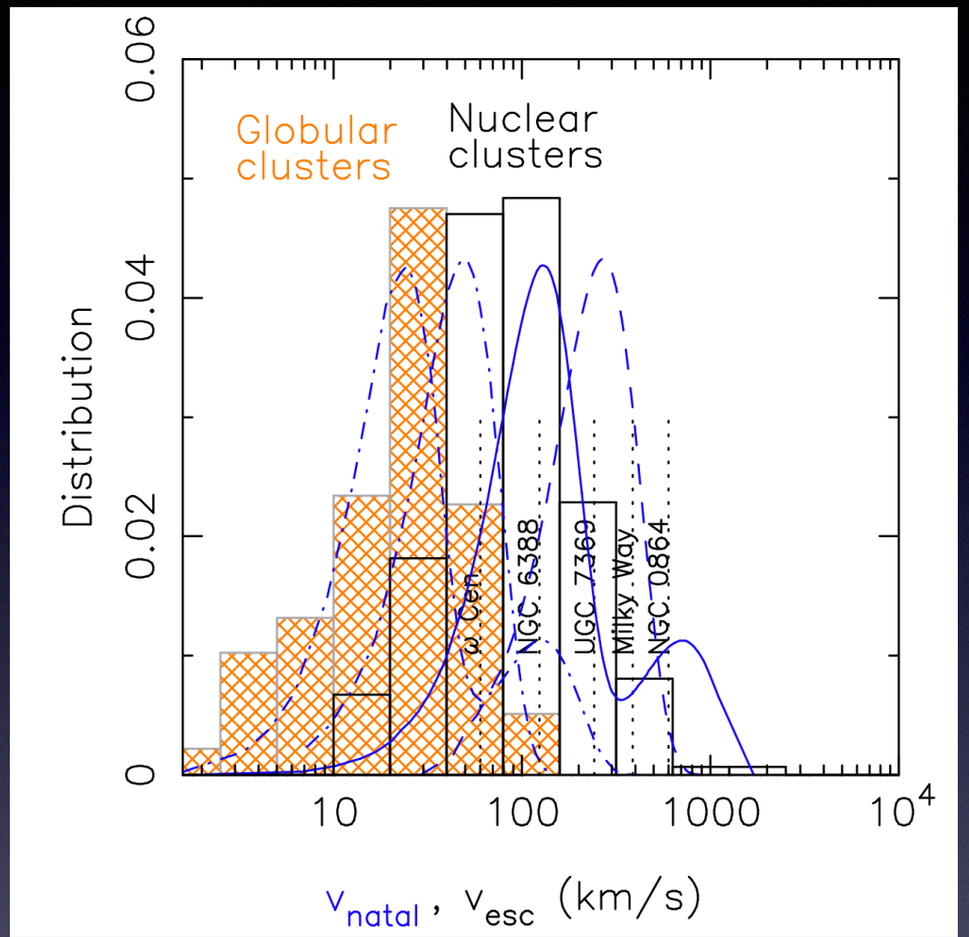
Field BH binaries



Decreasing natal kicks

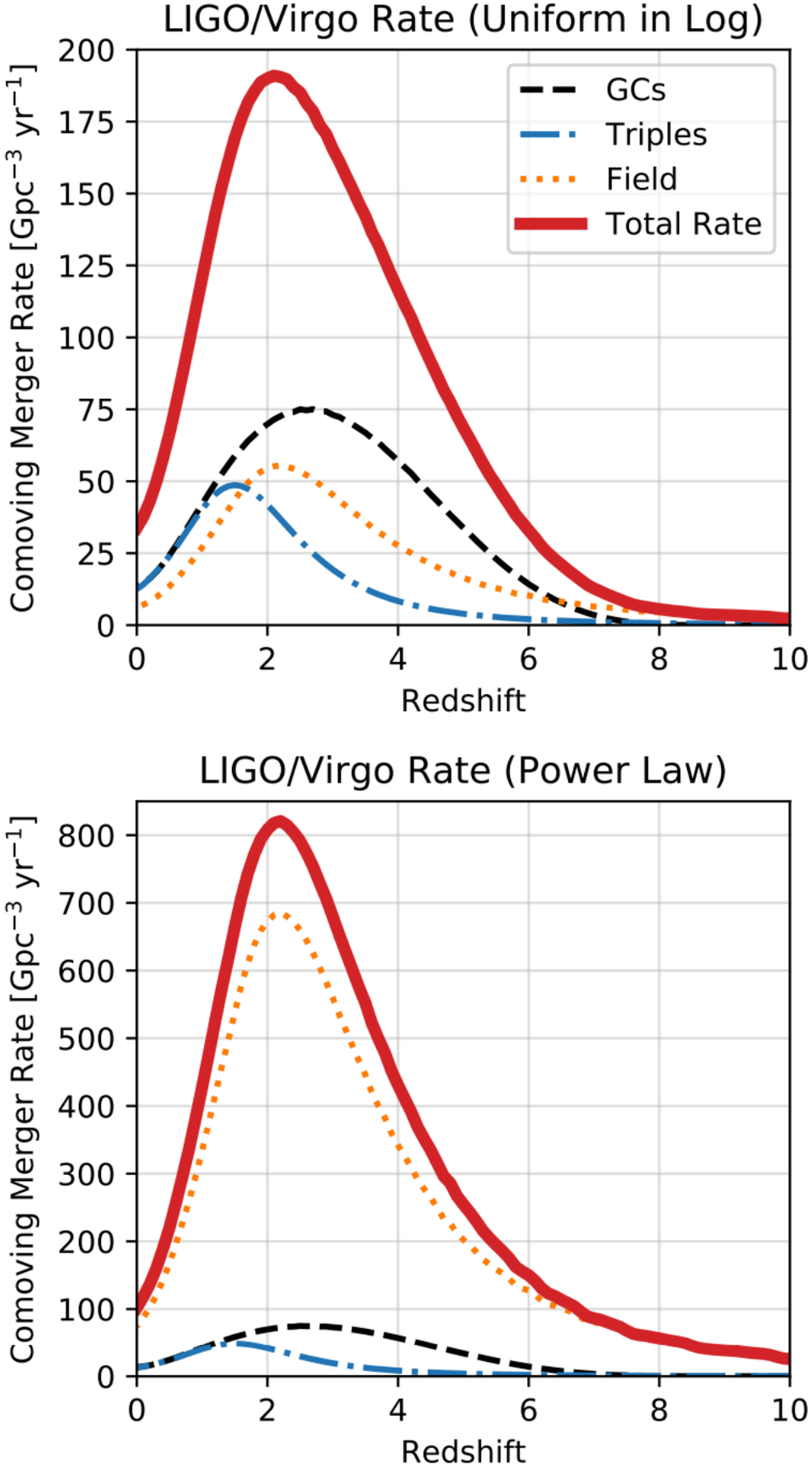
From www.syntheticuniverse.org

Dynamical channel



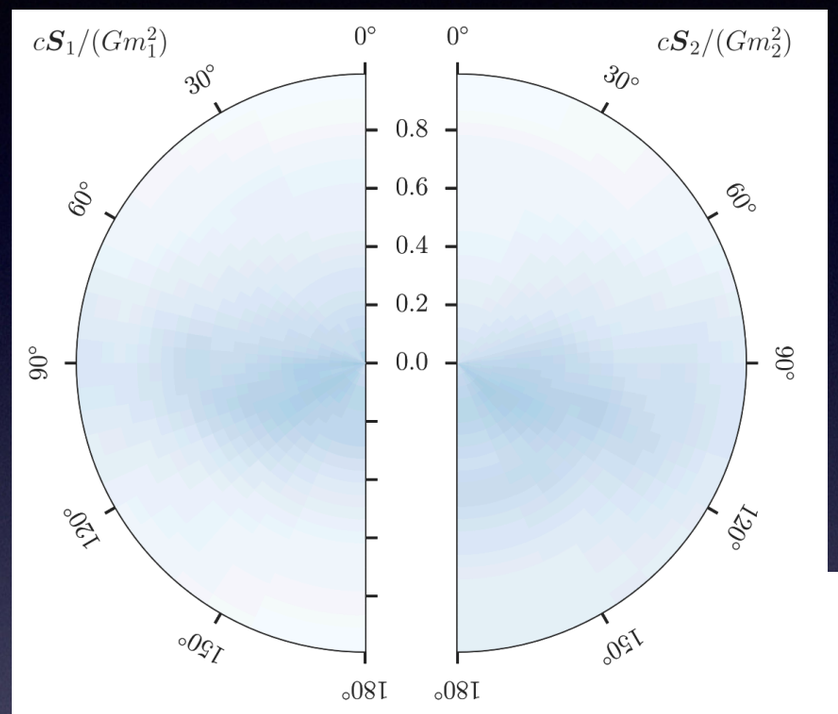
Antonini & Radio 2016

- Similar uncertainties (natal kicks)
- Possible in globular clusters and nuclear star clusters, or even in the field (field triples)
- May be as important as field channel

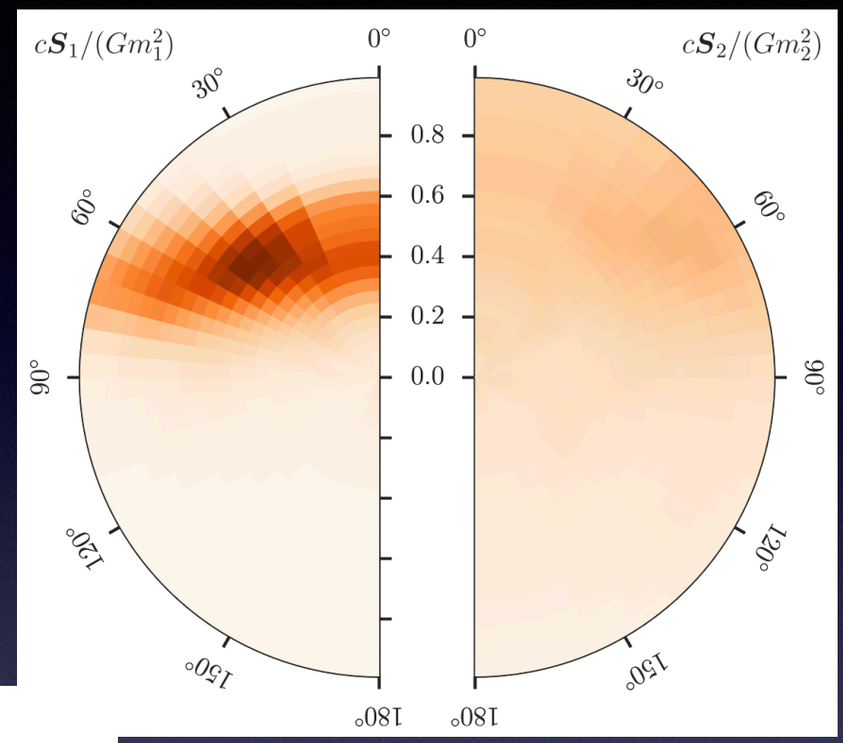


Rodriguez & Loeb 2018

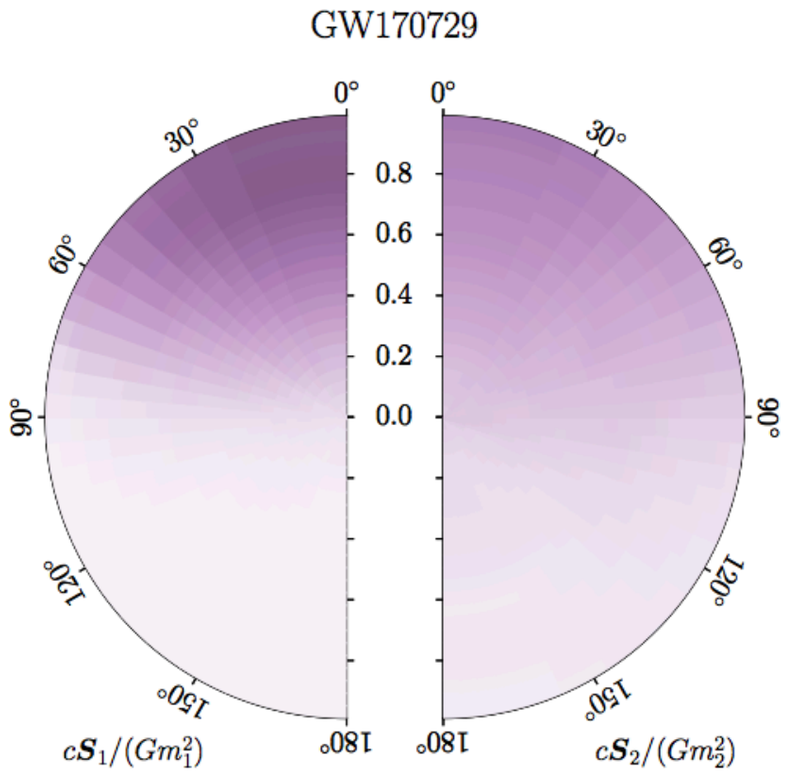
Measuring BH spins



GW150914

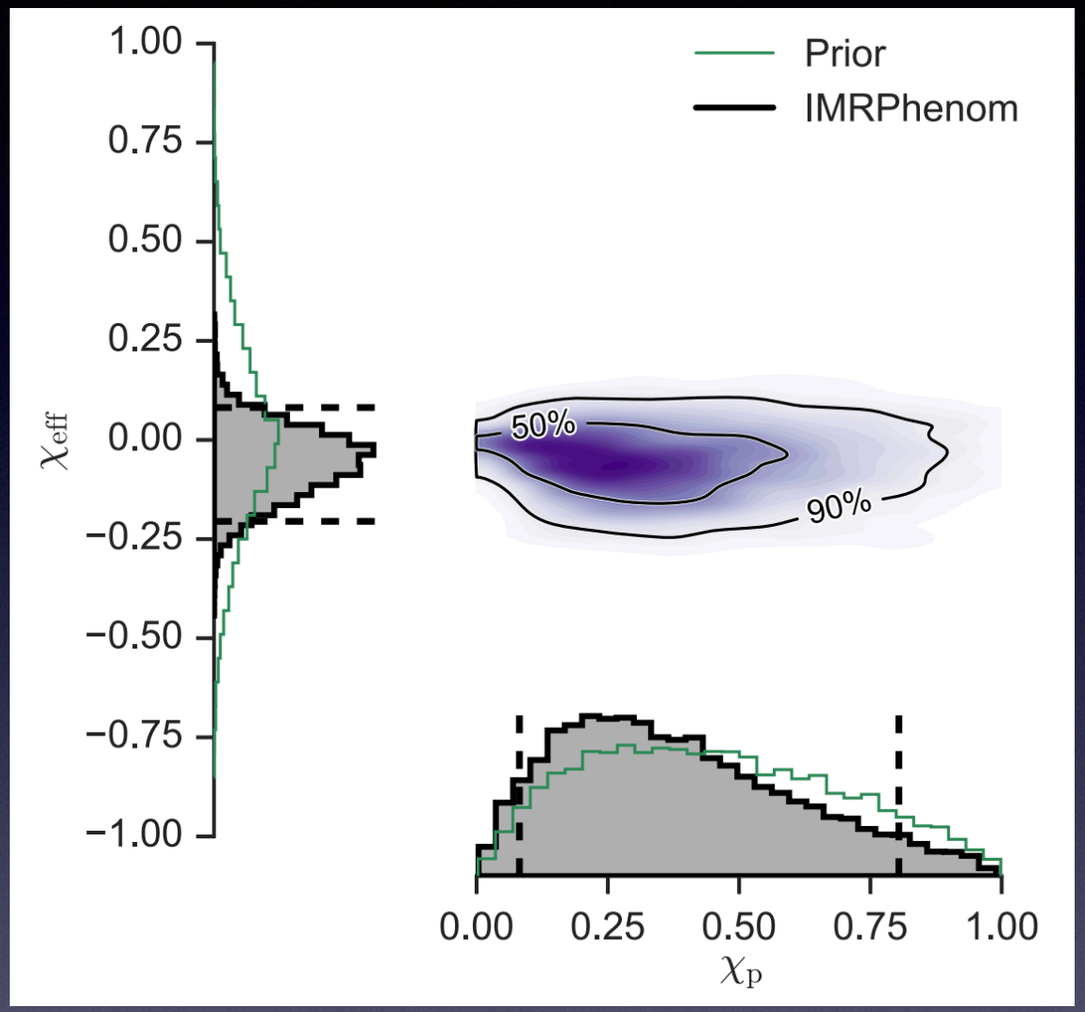


GW151226



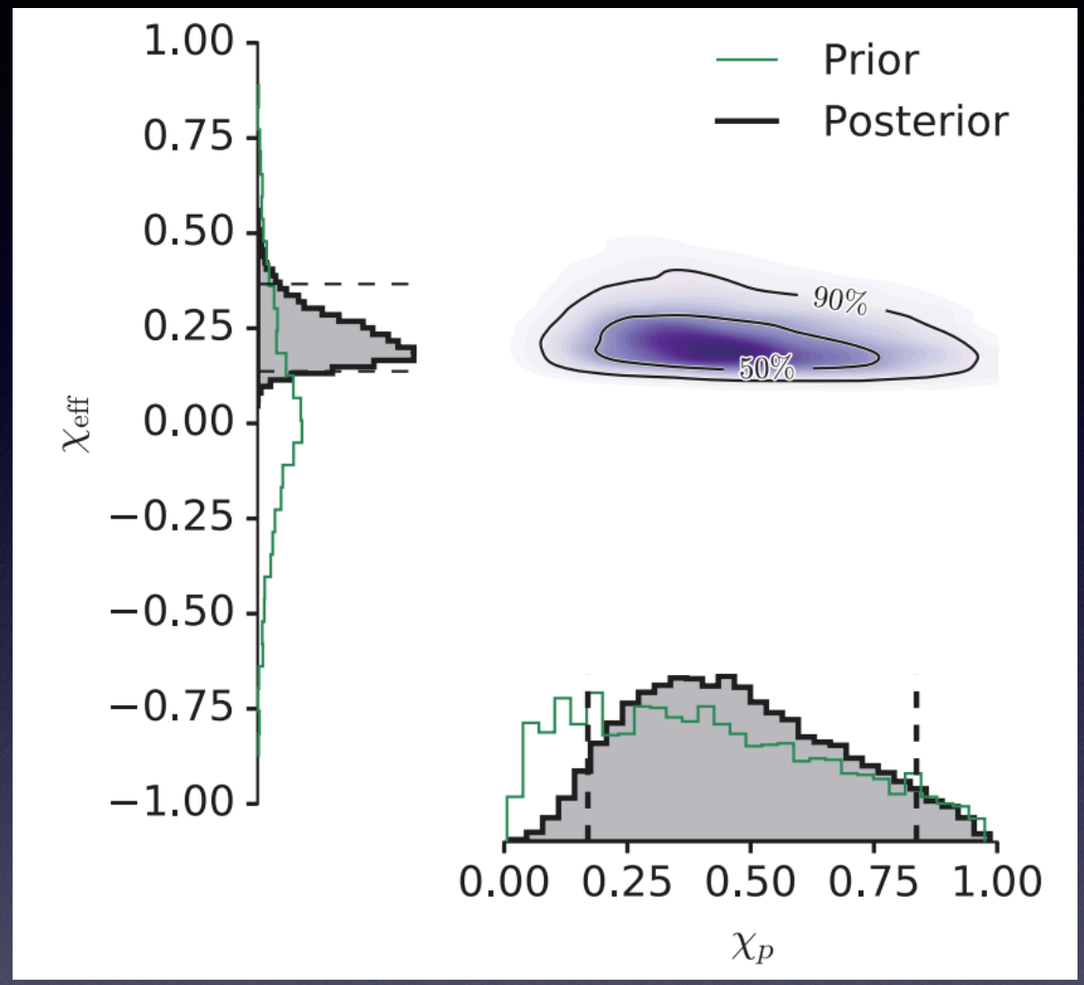
Figures: LSC collaboration 2016, 2018

Measuring BH spins



GW150914

$$\chi_{\rm eff} = \frac{c}{GM}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \frac{\boldsymbol{L}}{|\mathbf{L}|}$$



GW151226

Spins either small or precessing?

Figures: LSC collaboration 2016

Comparison to models

Misaligned spins possible in field channel if large kicks, natural in dynamical channel

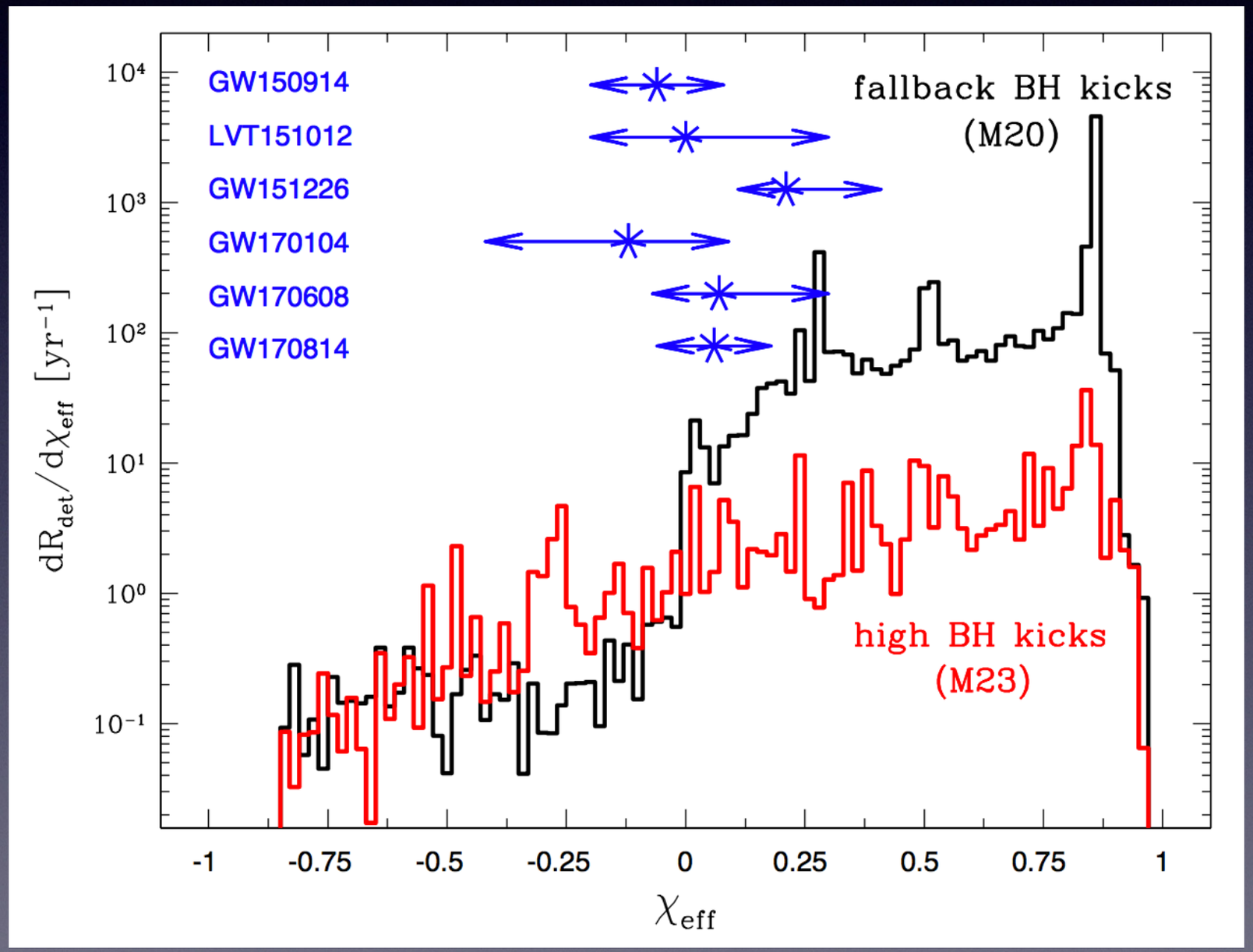


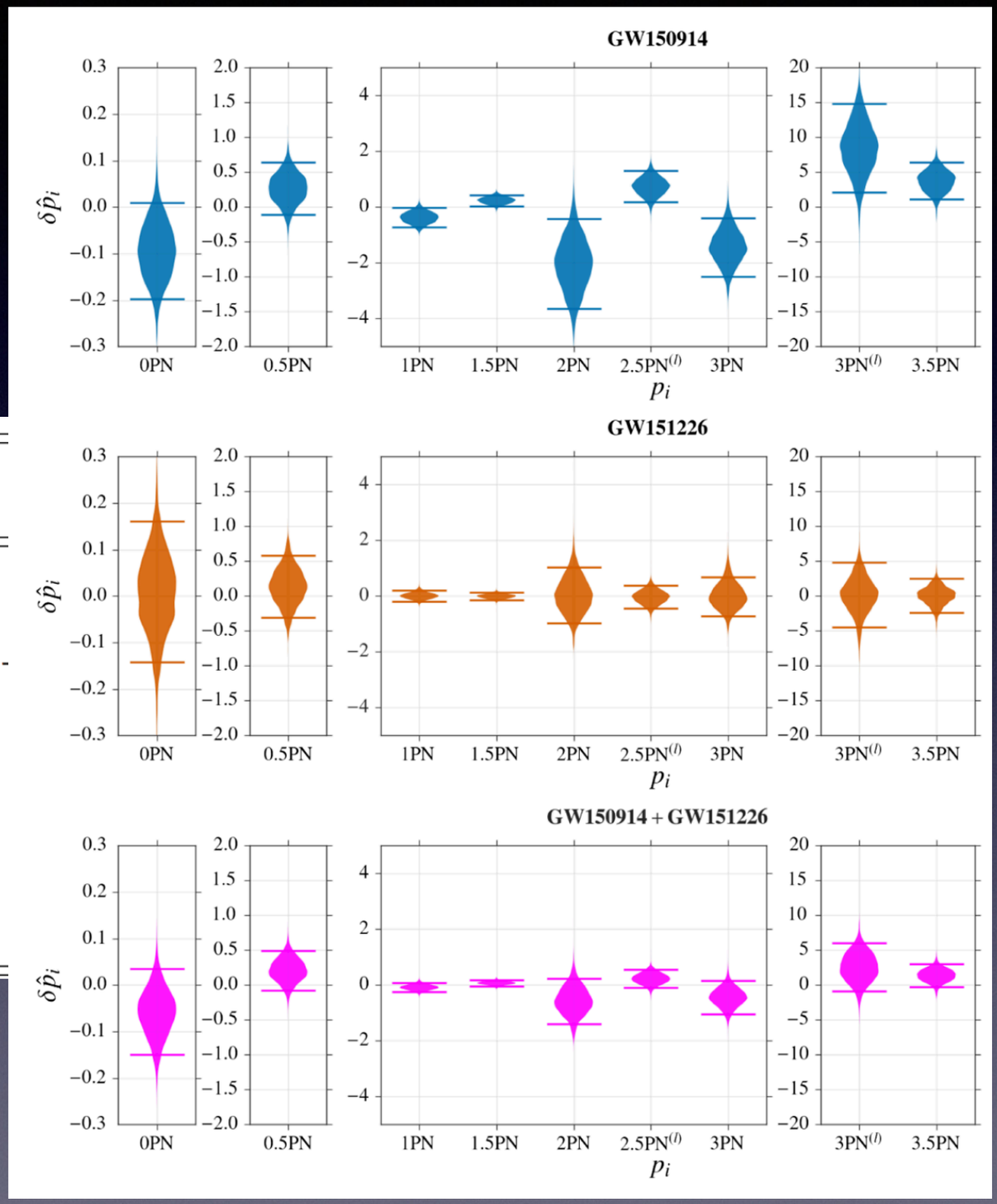
Figure from Belczynski et al 2017

Parametrized inspiral tests of GR

$$h_{ppE}(f) = h_{GR}(f)(1 + \alpha u^a)e^{i\beta u^b}$$

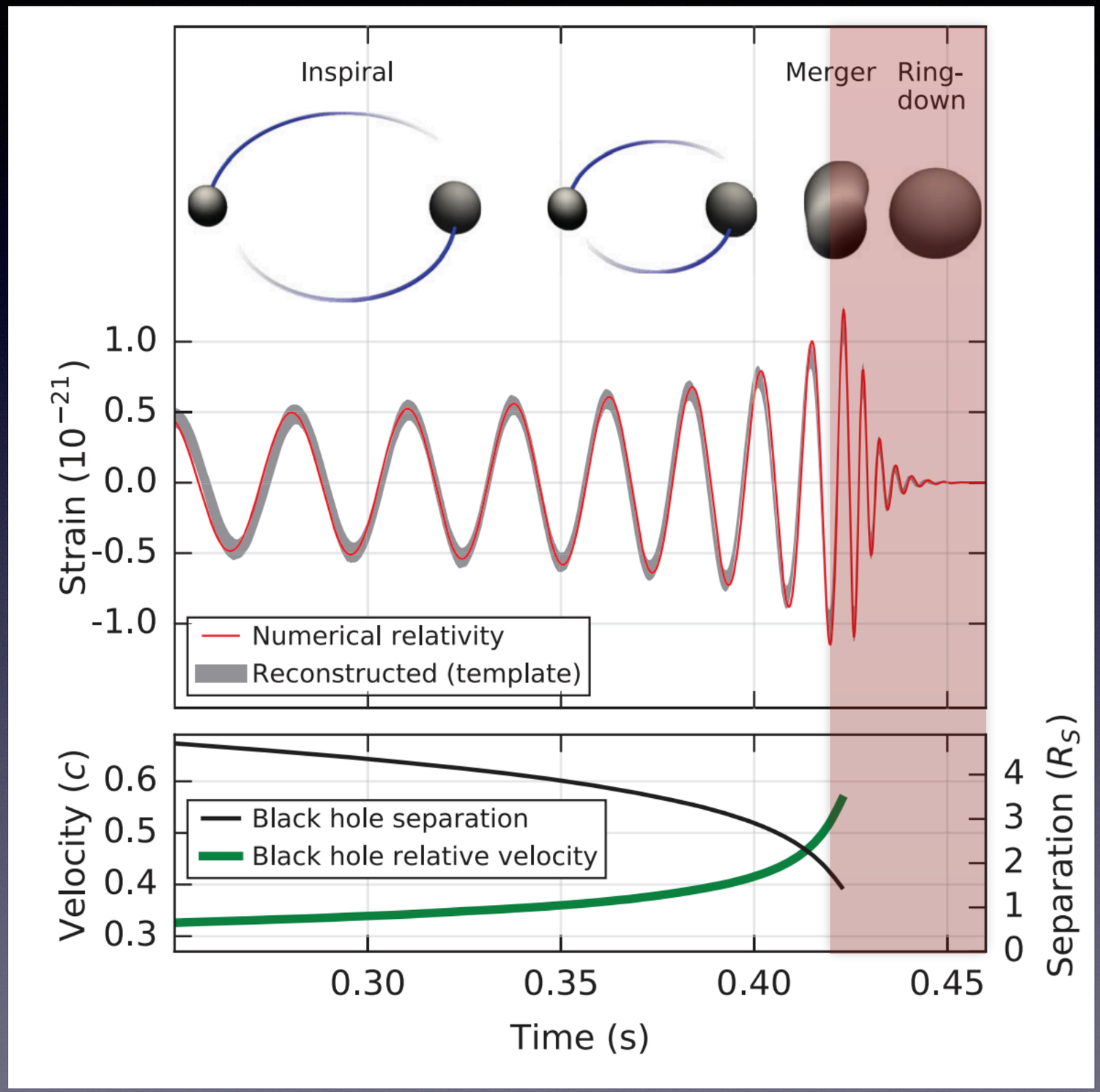
$$u = (\pi \mathcal{M} f)^{1/3}$$

waveform regime	parameter	$f-$dependence
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$
	$\delta\hat{\varphi}_1$	$f^{-4/3}$
	$\delta\hat{\varphi}_2$	f^{-1}
	$\delta\hat{\varphi}_3$	$f^{-2/3}$
	$\delta\hat{\varphi}_4$	$f^{-1/3}$
	$\delta\hat{\varphi}_{5l}$	$\log(f)$
	$\delta\hat{\varphi}_6$	$f^{1/3}$
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$
	$\delta\hat{\varphi}_7$	$f^{2/3}$



Caveat: ppE parameters may depend on sources (should be viewed as BH charges), so stacking may not be physically meaningful!

GWs from binary BHs



LSC collaboration 2015

Perturbations of non-spinning BHs

- Consider scalar field toy model first $g^{\mu\nu}\nabla_\mu\nabla_\nu\varphi = 0$
- On Schwarzschild, decompose in spherical harmonics

$$\varphi = \sum_{\ell,m} \frac{R_{\ell m}(r)}{r} Y_{\ell m}(\theta,\phi) e^{-i\omega t}$$

- Because of symmetry, equations “separate”:

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r_*^2} + (\omega^2 - V)R = 0$$

$$r_* \equiv r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

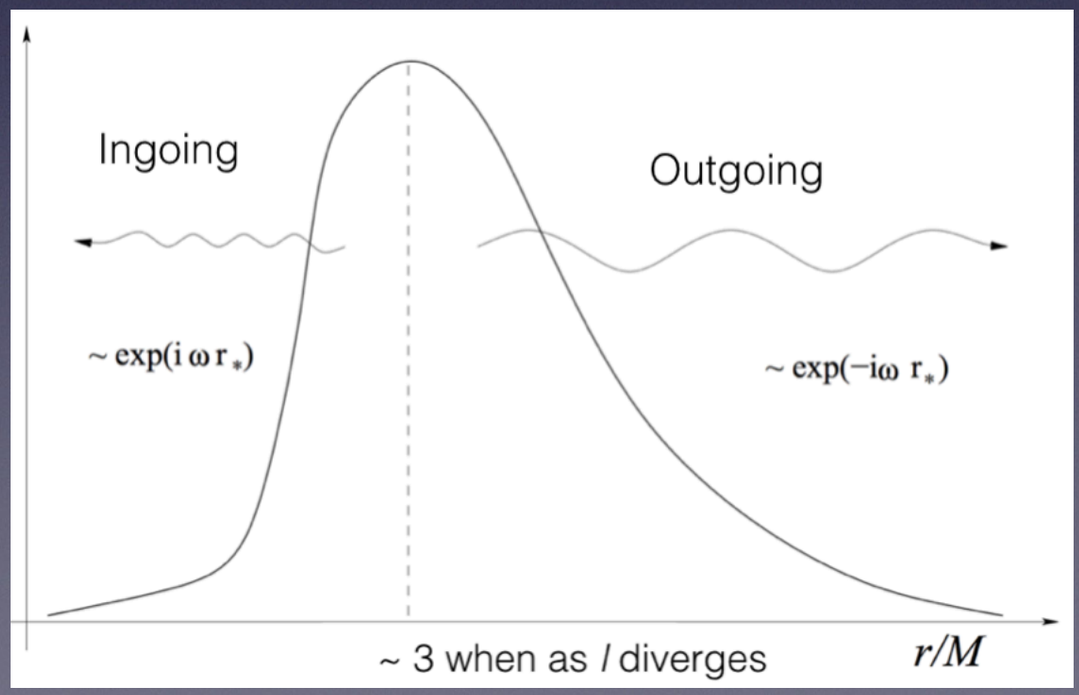
Tortoise coordinates ranging +/- ∞

$$V(r) \equiv \left(1 - \frac{2M}{r}\right)\left[\frac{l\,(l+1)}{r^2} + p\right]$$

$$p = 2M/r^3$$

Perturbations of non-spinning BHs

- Ingoing/Outgoing boundary conditions at event horizon/infinity
- Akin to solving Schrodinger equation in 1D in quantum mechanics 101
- Counting of degrees of freedom + continuity = discrete complex quasinormal mode frequencies
- Imaginary part of frequency shows linear stability
- Peak at ~ 3M as *l* diverges (because geometric optics limit of Klein-Gordon equation is geodesics equation)



Perturbations of non-spinning BHs

- Separability on Schwarzschild extend to vector and tensor perturbations
- Expand in vector and tensor harmonics (of even/odd parity, c.f. E/B modes of CMB)

$$\psi_{L^M,\mu} = \text{const} \frac{\partial}{\partial x^\mu} Y_{L^M}(x_2 x_3), \quad \text{parity } (-)^L;$$

$$\phi_{L^M,\mu} = \text{const}\, \epsilon_\mu{}^\nu \frac{\partial}{\partial x_\nu} Y_{L^M}(x_2 x_3), \quad \text{parity } (-)^{L+1}.$$

Basis for vectors on 2-sphere

$$\psi_{L^M\mu\nu} = \text{const}\, Y_{L^M;\mu\nu} (\text{covariant derivatives}), \quad \text{parity } (-)^L;$$

$$\phi_{L^M\mu\nu} = \text{const}\, \gamma_{\mu\nu} Y_{L^M}, \quad \text{parity } (-)^L;$$

$$\chi_{L^M\mu\nu} = \tfrac{1}{2}\, \text{const} [\epsilon_\mu{}^\lambda \psi_{L^M\lambda\nu} + \epsilon_\nu{}^\lambda \psi_{L^M\lambda\mu}], \quad \text{parity } (-)^{L+1}.$$

Basis for tensors on 2-sphere

Perturbations of non-spinning BHs

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & -h_0(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_0(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ 0 & 0 & -h_1(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_1(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ \mathrm{Sym} & \mathrm{Sym} & h_2(T,r)(\partial^2/\sin\theta\partial\theta\partial\varphi-\cos\theta\partial/\sin^2\theta\partial\varphi)Y_L{}^M & \mathrm{Sym} \\ \mathrm{Sym} & \mathrm{Sym} & \tfrac{1}{2}h_2(T,r)(\partial^2/\sin\theta\partial\varphi\partial\varphi+\cos\theta\partial/\partial\theta-\sin\theta\partial^2/\partial\theta\partial\theta)Y_L{}^M & -h_2(T,r)(\sin\theta\partial^2/\partial\theta\partial\varphi-\cos\theta\partial/\partial\varphi)Y_L{}^M \end{vmatrix}$$

Odd-parity metric perturbations (Regge-Wheeler 1957)

$$h_{\mu\nu}=\begin{vmatrix} (1-2m^*/r)H_0(T,r)Y_L{}^M & H_1(T,r)Y_L{}^M & h_0(T,r)(\partial/\partial\theta)Y_L{}^M & h_0(T,r)(\partial/\partial\varphi)Y_L{}^M \\ H_1(T,r)Y_L{}^M & (1-2m^*/r)^{-1}H_2(T,r)Y_L{}^M & h_1(T,r)(\partial/\partial\theta)Y_L{}^M & h_1(T,r)(\partial/\partial\varphi)Y_L{}^M \\ \mathrm{Sym} & \mathrm{Sym} & r^2[K(T,r) & \mathrm{Sym} \\ & & \quad +G(T,r)(\partial^2/\partial\theta^2)]Y_L{}^M & \\ \mathrm{Sym} & \mathrm{Sym} & r^2G(T,r)(\partial^2/\partial\theta\partial\varphi & r^2[K(T,r)\sin^2\theta \\ & & \quad -\cos\theta\partial/\sin\theta\partial\varphi)Y_L{}^M & \quad +G(T,r)(\partial^2/\partial\varphi\partial\varphi \\ & & & \quad +\sin\theta\cos\theta\partial/\partial\theta)]Y_L{}^M \end{vmatrix}.$$

Even-parity metric perturbations (Zerilli 1970)

Perturbations of non-spinning BHs

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & -h_0(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_0(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ 0 & 0 & -h_1(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_1(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ \text{Sym} & \text{Sym} & h_2(T,r)(\partial^2/\sin\theta\partial\theta\partial\varphi-\cos\theta\partial/\sin^2\theta\partial\varphi)Y_L{}^M & \text{Sym} \\ \text{Sym} & \text{Sym} & \tfrac{1}{2}h_2(T,r)(\partial^2/\sin\theta\partial\varphi\partial\varphi+\cos\theta\partial/\partial\theta-\sin\theta\partial^2/\partial\theta\partial\theta)Y_L{}^M & h_2(T,r)(\sin\theta\partial^2/\partial\theta\partial\varphi-\cos\theta\partial/\partial\varphi)Y_L{}^M \end{vmatrix}$$

Odd-parity metric perturbations (Regge-Wheeler 1957):
2 free radial functions

$$h_{\mu\nu}=\begin{vmatrix} (1-2m^*/r)H_0(T,r)Y_L{}^M & H_1(T,r)Y_L{}^M & h_0(T,r)(\partial/\partial\theta)Y_L{}^M & h_0(T,r)(\partial/\partial\varphi)Y_L{}^M \\ H_1(T,r)Y_L{}^M & (1-2m^*/r)^{-1}H_2(T,r)Y_L{}^M & h_1(T,r)(\partial/\partial\theta)Y_L{}^M & h_1(T,r)(\partial/\partial\varphi)Y_L{}^M \\ \text{Sym} & \text{Sym} & r^2[K(T,r) & \text{Sym} \\ & & \quad +G(T,r)(\partial^2/\partial\theta^2)]Y_L{}^M & \\ \text{Sym} & \text{Sym} & r^2G(T,r)(\partial^2/\partial\theta\partial\varphi & r^2[K(T,r)\sin^2\theta \\ & & \quad -\cos\theta\partial/\sin\theta\partial\varphi)Y_L{}^M & \quad +G(T,r)(\partial^2/\partial\varphi\partial\varphi \\ & & & \quad +\sin\theta\cos\theta\partial/\partial\theta)]Y_L{}^M \end{vmatrix}.$$

Even-parity metric perturbations (Regge-Wheeler 1957, Zerilli 1970):
4 free radial functions, but 2 algebraic relations from Einstein eqs

Perturbations of non-spinning BHs

Construct complex variables out of each pair of free radial functions

$$\frac{d^2\Psi_s}{dr_*^2} + \left(\omega^2 - V_s\right)\Psi_s = 0\,.$$

$$V^{-}_{s=2} = f(r)\left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right]$$

Odd-parity

$$V^{+}_{s=2} = \frac{2f(r)}{r^3}\frac{9M^3 + 3\lambda^2 M r^2 + \lambda^2(1+\lambda)r^3 + 9M^2\lambda r}{(3M+\lambda r)^2}$$

Even-parity

$$\lambda \equiv (l-1)(l+2)/2 \qquad f(r) = 1 - 2M/r$$

Effective potentials peak at r=3M in the large l limit
(i.e. in the geodesics limit)

Perturbations of spinning BHs

- Separability of equations not at all obvious in Kerr, but possible due to “hidden symmetry” (c.f. Carter’s constant)

- Use Newman-Penrose scalars (projections of Weyl curvature on null tetrad) to get Teukolsky equation

$$\Psi_0 = -C_{1313} = -C_{\mu\nu\lambda\sigma} l^\mu m^\nu l^\lambda m^\sigma ,$$
$$\Psi_4 = -C_{2424} = -C_{\mu\nu\lambda\sigma} n^\mu m^{*\nu} n^\lambda m^{*\sigma}$$

$$\psi(t\,,r\,,\theta\,,\phi) = \frac{1}{2\pi}\int e^{-i\omega t}\sum_{l=|s|}^{\infty}\sum_{m=-l}^{l} e^{im\phi}\,{}_sS_{lm}(\theta)R_{lm}(r)d\omega$$

$$\left[\frac{\partial}{\partial u}(1-u^2)\frac{\partial}{\partial u}\right]{}_sS_{lm}$$ Spin-weighted spheroidal harmonics

$$+\left[a^2\omega^2u^2 - 2a\omega su + s + {}_sA_{lm} - \frac{(m+su)^2}{1-u^2}\right]{}_sS_{lm} = 0$$

$$\Delta\partial_r^2 R_{lm} + (s+1)(2r-2M)\partial_r R_{lm} + V R_{lm} = 0\,.$$

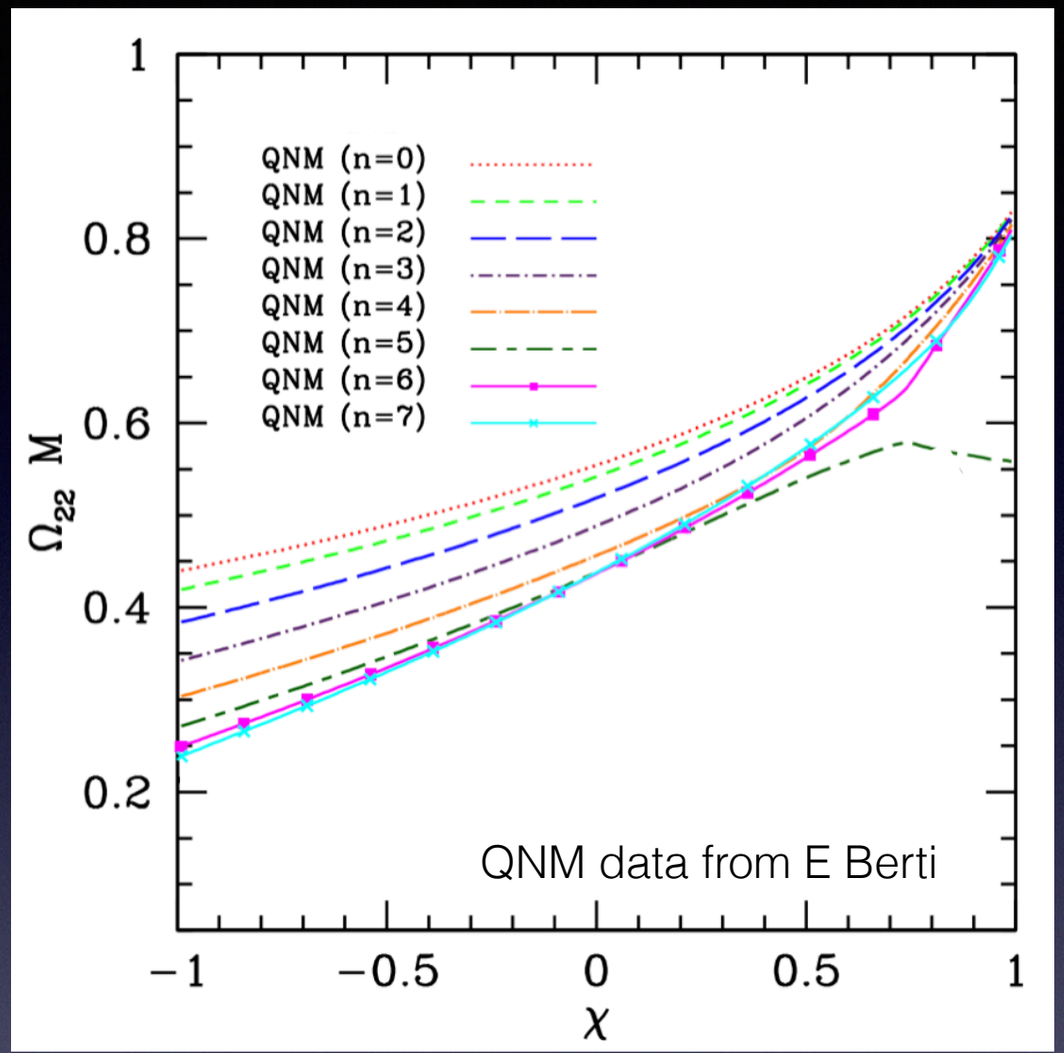
s	$(+2,-2)$
ψ	$(\Psi_0, \rho^{-4}\Psi_4)$

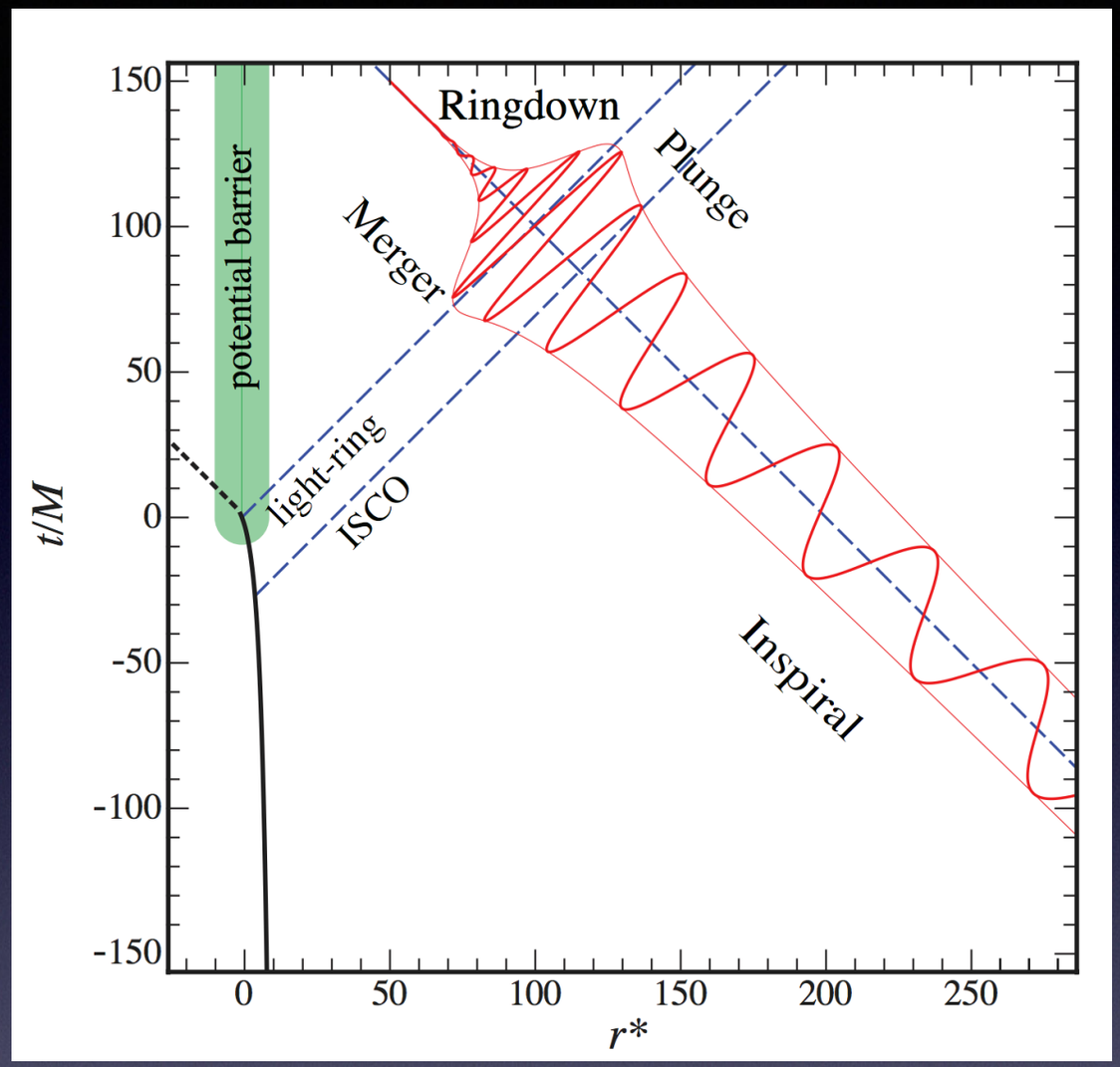
$$V = 2is\omega r - a^2\omega^2 - {}_sA_{lm} + \frac{1}{\Delta}\left[(r^2+a^2)^2\omega^2 - 4Mam\omega r + a^2m^2 \;+ is\left(am(2r-2M) - 2M\omega(r^2-a^2)\right)\right]$$

$u \equiv \cos\theta,\ \Delta = (r-r_-)(r-r_+)$

${}_sA_{lm}(a=0) = l(l+1) - s(s+1)$ Separation constant

Perturbations of spinning BHs





- Connection to circular photon orbit frequency ω and Lyapunov coefficient λ (i.e. curvature of geodesics effective potential) in geometric optics limit!

$$\omega_{ln}^{m=l} \approx l\omega_{+} - i\lambda_{+}(n+1/2)$$

- Amplitude of modes depends on merger physics/initial conditions; set by “continuity” near circular photon orbit in phenomenological waveform models (e.g. EOB)

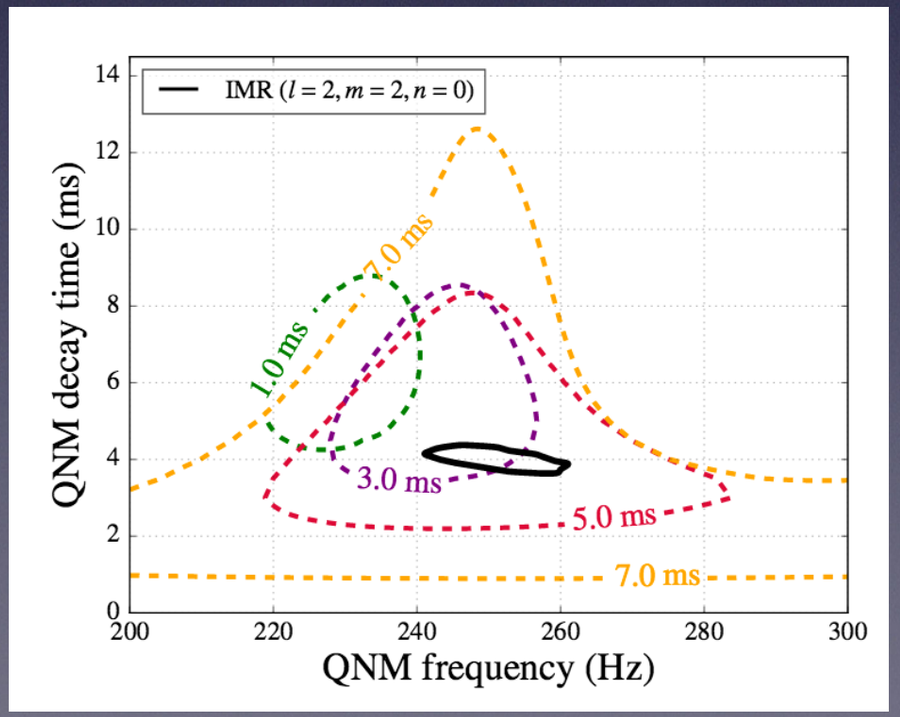
Ringdown tests of the no-hair theorem

$$\omega_{\ell m} = \omega^{GR}_{\ell m}(M, J)(1 + \delta\omega_{\ell m}) \quad \tau_{\ell m} = \tau^{GR}_{\ell m}(M, J)(1 + \delta\tau_{\ell m})$$

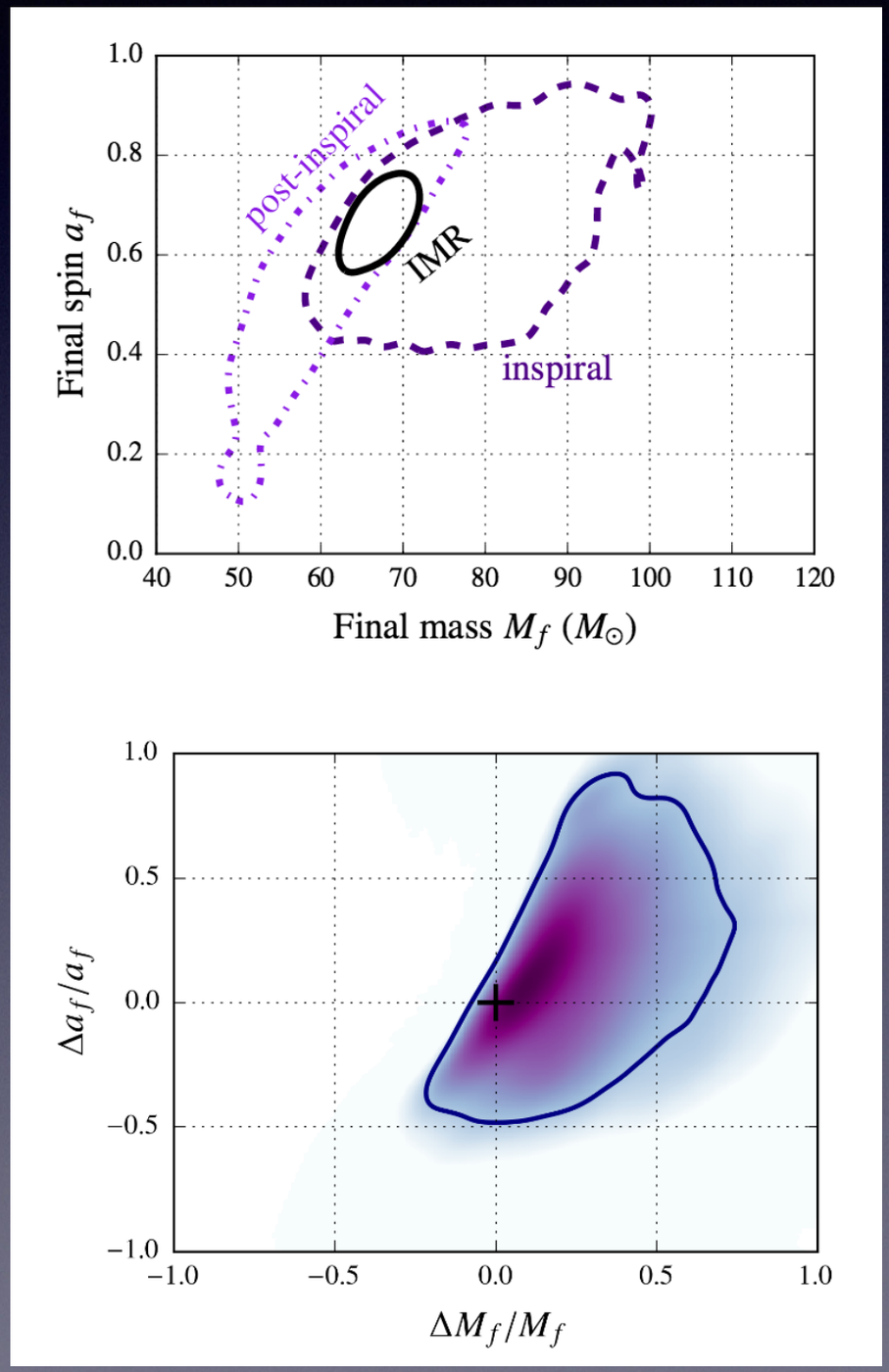
- Difficult with 2nd generation detectors because little SNR in ringdown

- Can perform consistency tests between merger/ringdown

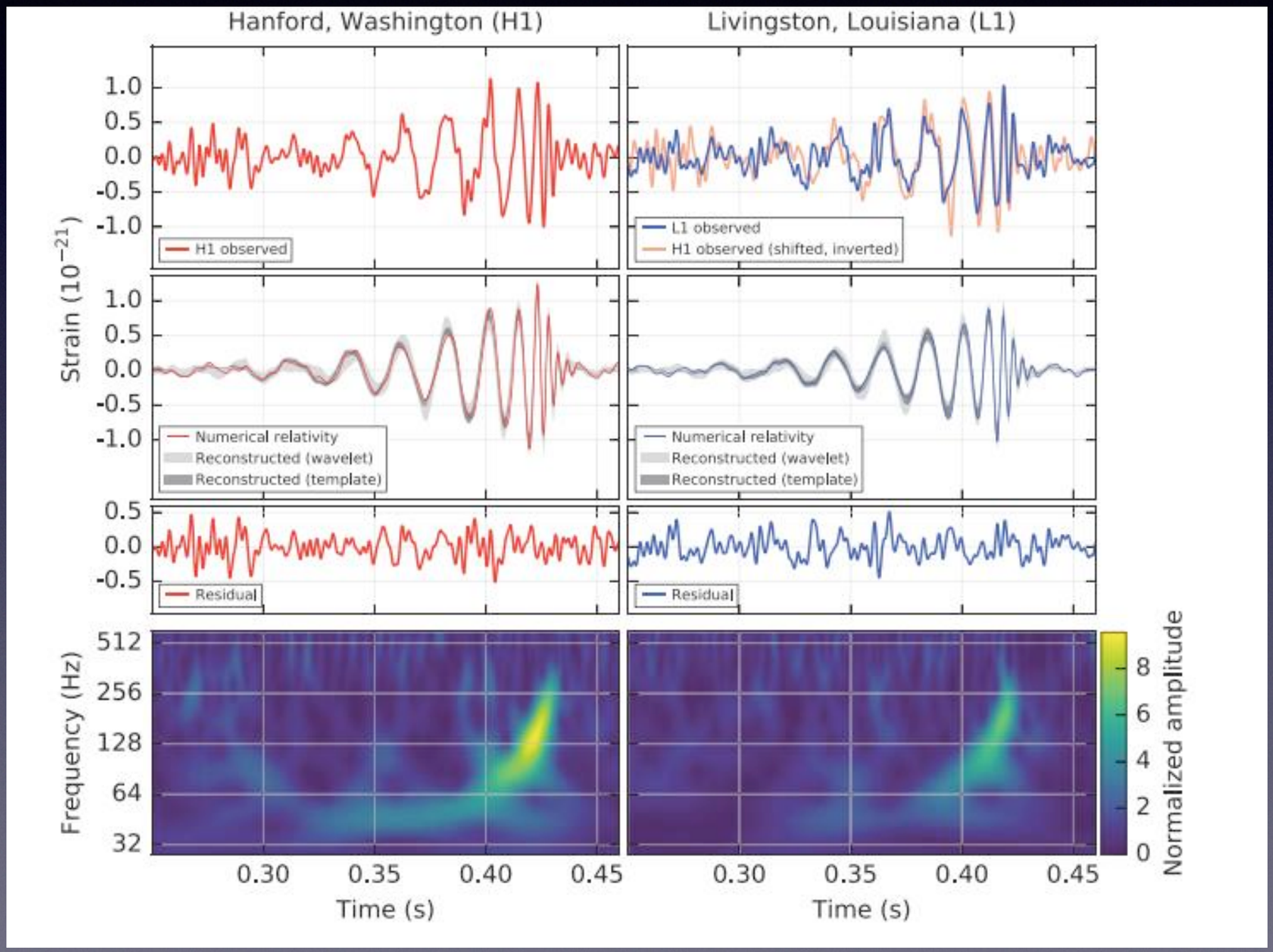
- Overtones of 22 mode may help (Giesler+2019)



From the LSC paper on tests of GR



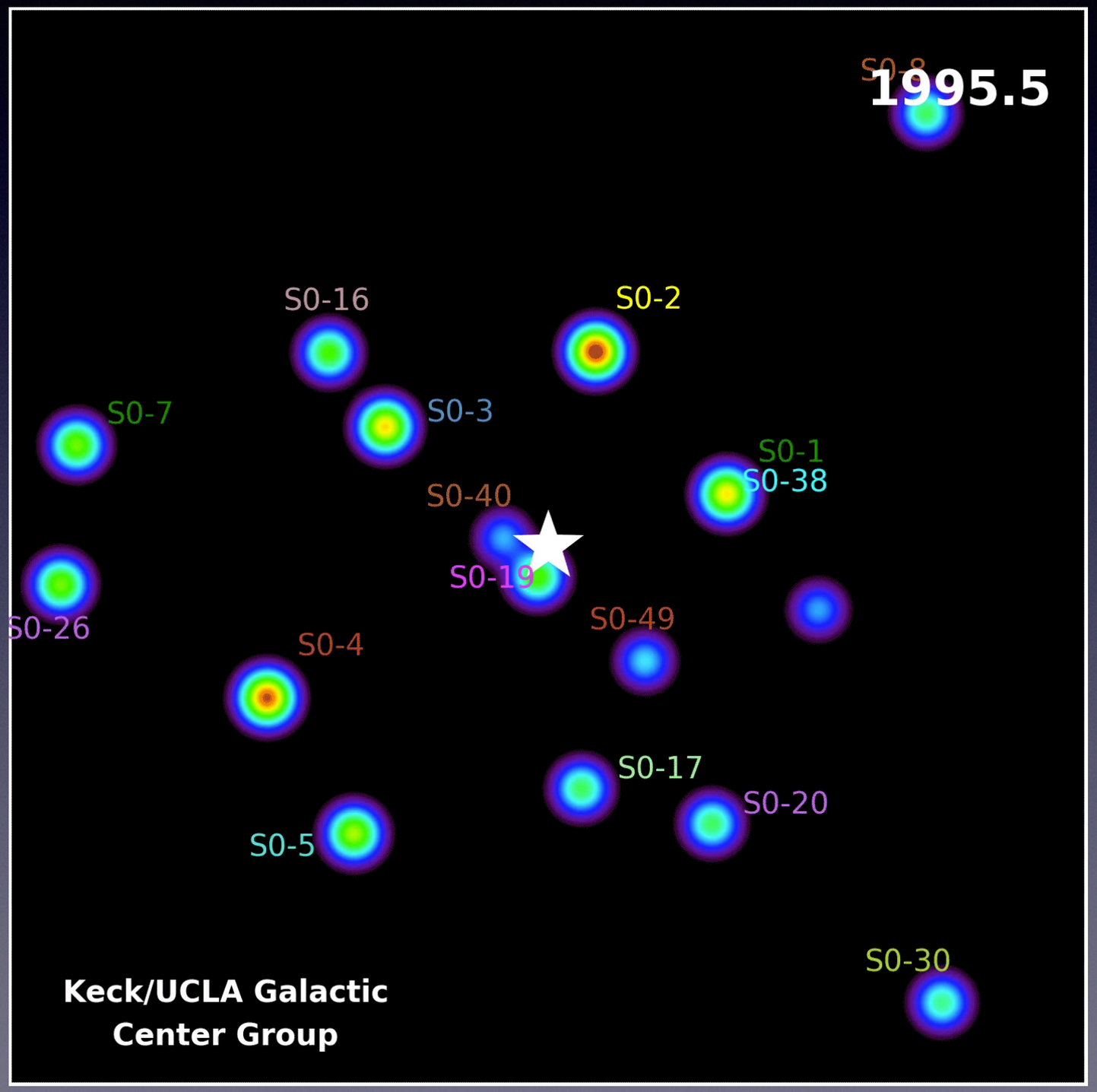
The first direct observation of GWs and … BHs!



LSC collaboration 2015

Not the biggest BHs in the Universe!

A monster of 4.5 million solar masses in the centre of our Galaxy!



Galaxies merge…

… so massive BHs must merge too!

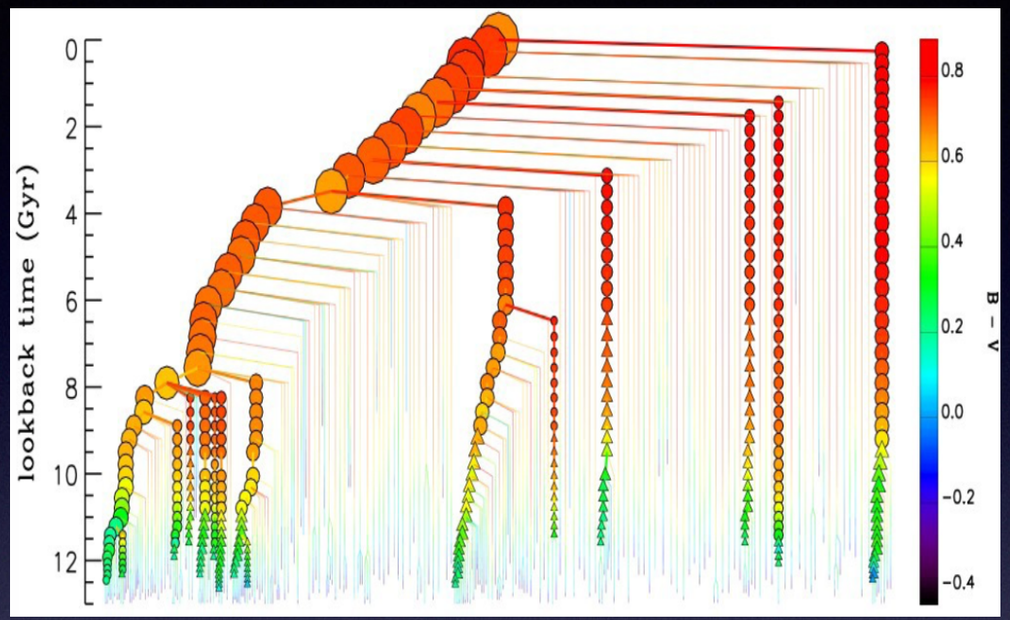


Figure from De Lucia & Blaizot 2007

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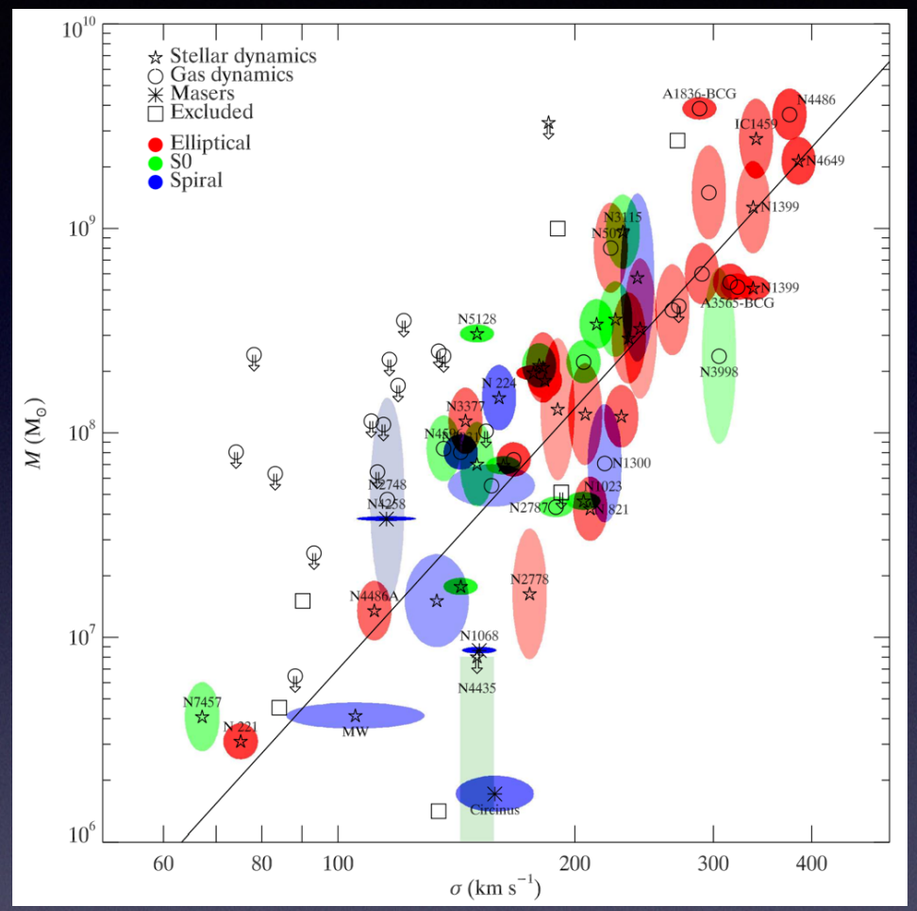

Ferrarese & Merritt 2000
Gebhardt et al. 2000,
Gültekin et al (2009)

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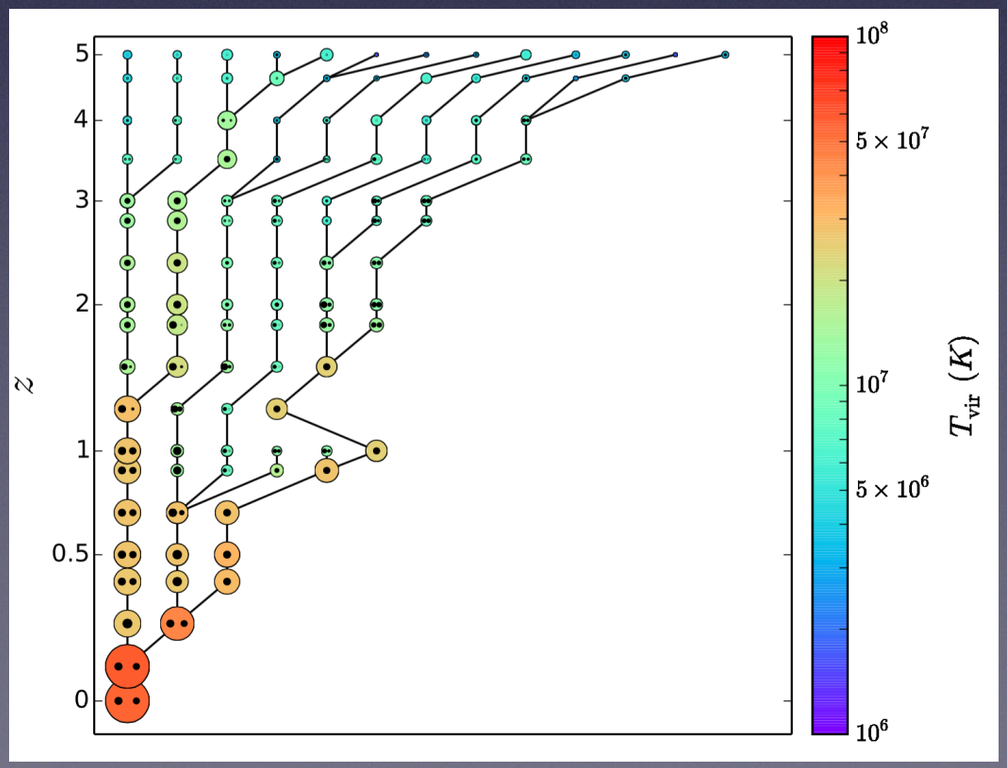
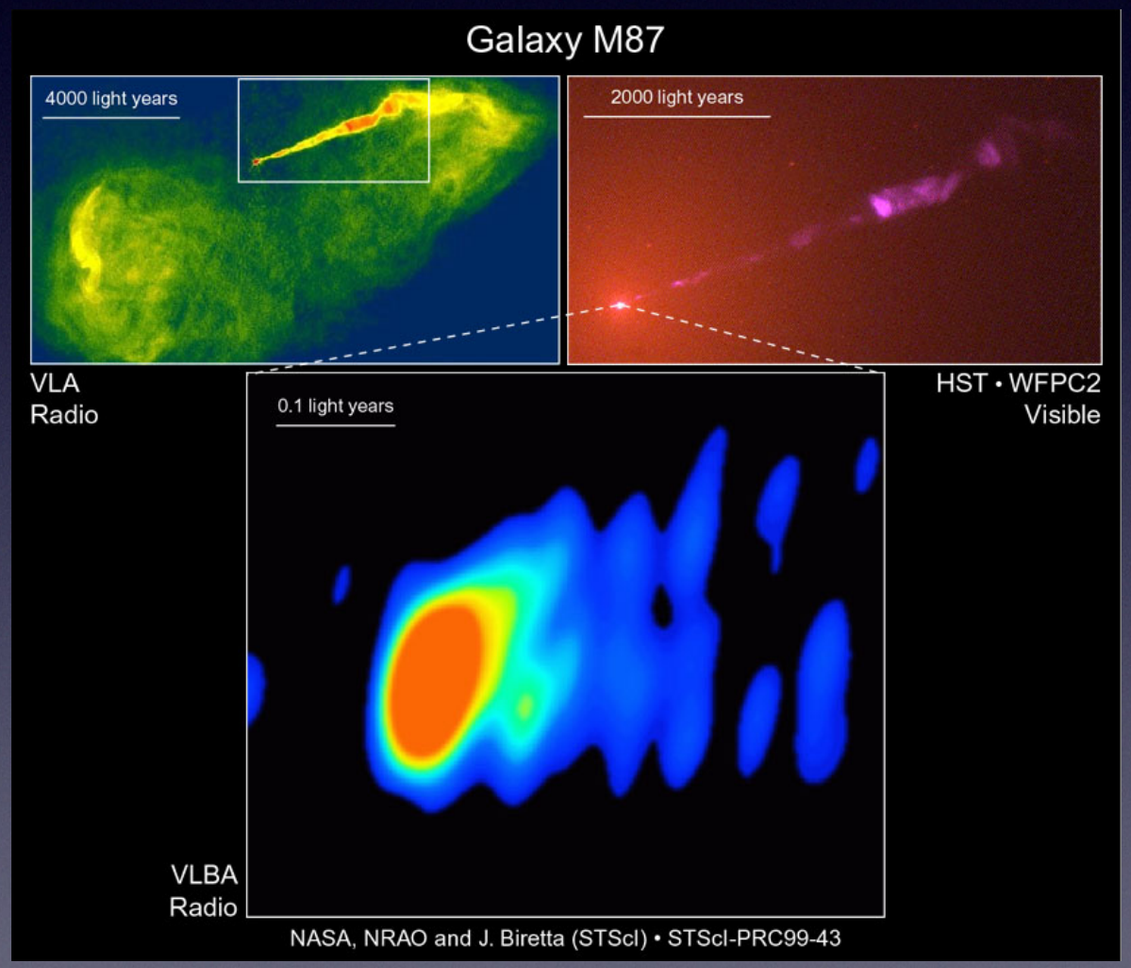
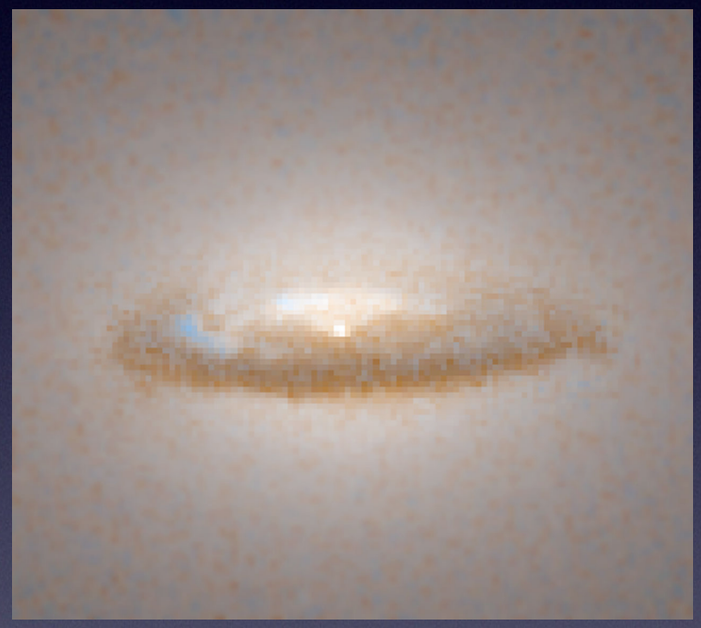


EB 2012
Figure credits: Lucy Ward

What links large and small scale?

- Small to large: BH jets or disk winds transfer kinetic energy to the galaxy and keep it “hot”, quenching star formation (”AGN feedback”). Needed to reconcile ΛCDM bottom-up structure formation with observed “downsizing” of cosmic galaxies



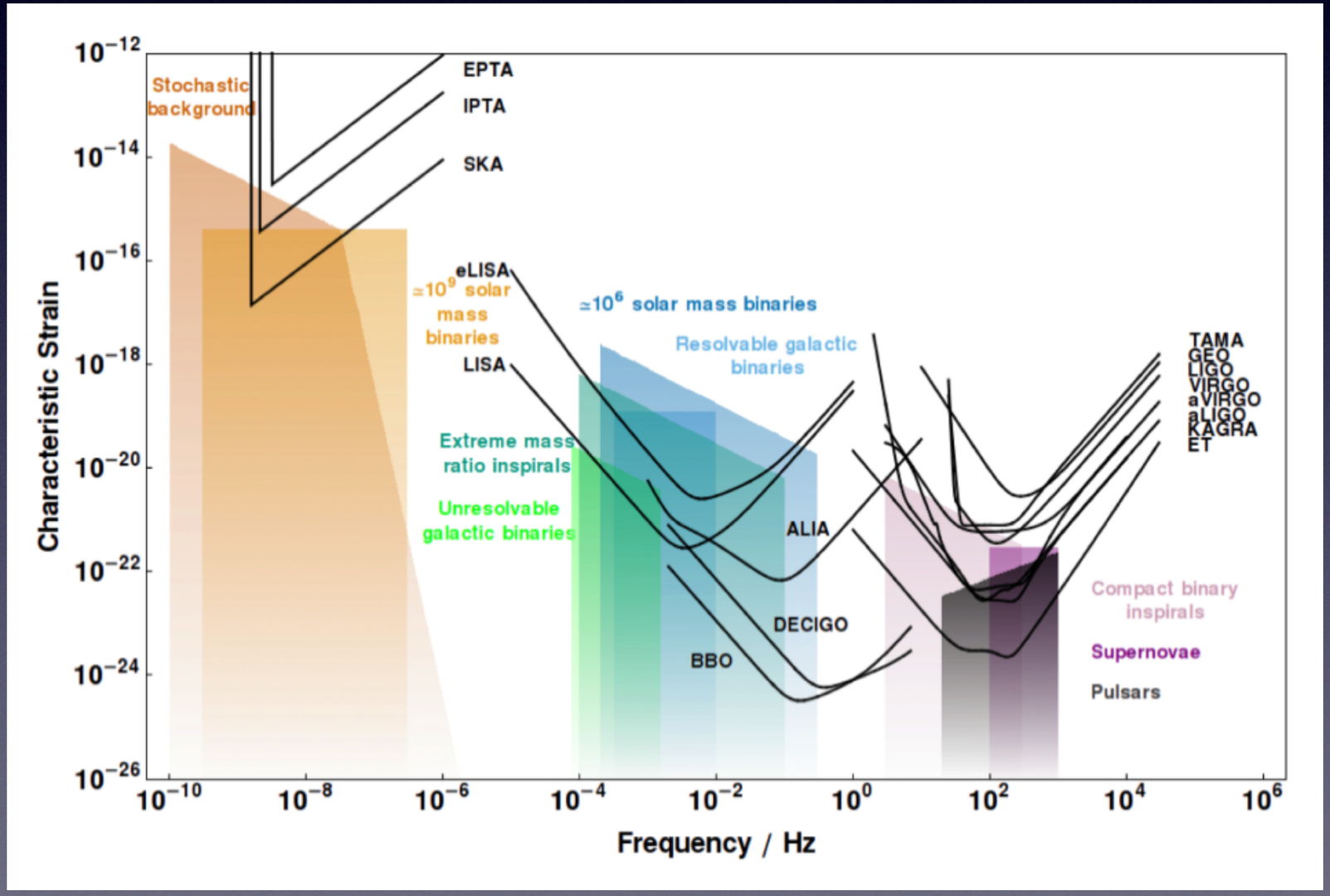


Disk of dust and gas around the massive BH in NGC 7052

- Large to small: galaxies provide fuel to BHs to grow (”accretion”)

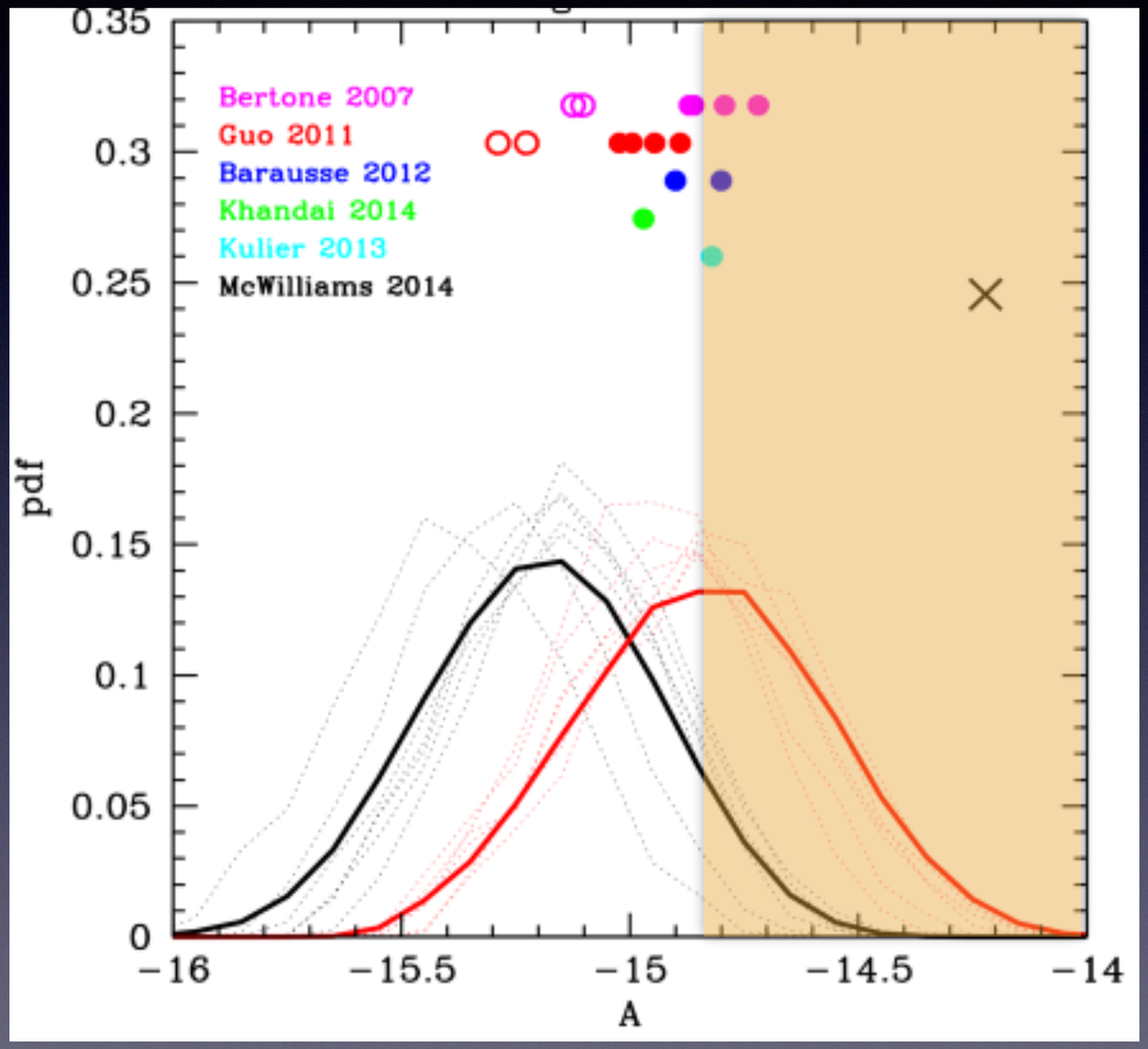
GWs from massive BHs

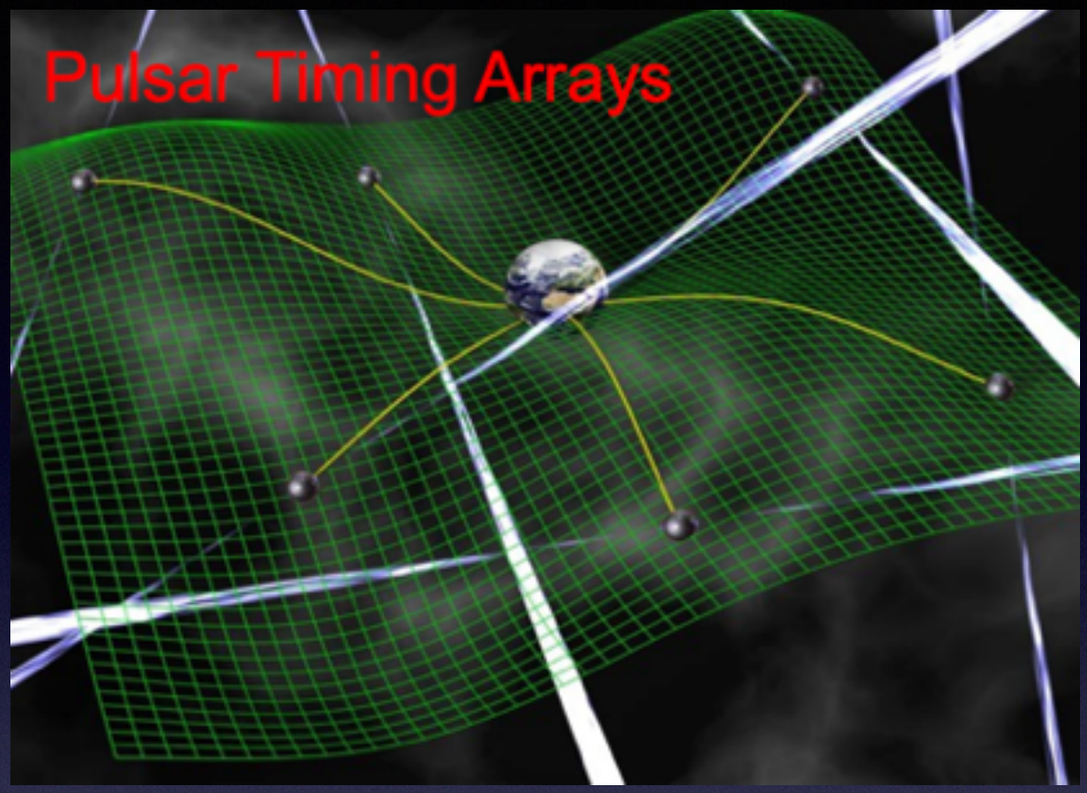
$$f_{\rm GW} = \frac{6\times 10^4}{\tilde{m}\tilde{R}^{3/2}}{\rm Hz} \qquad \tilde{R} = R/(Gm/c^2) \qquad \tilde{m} = m/M_\odot$$

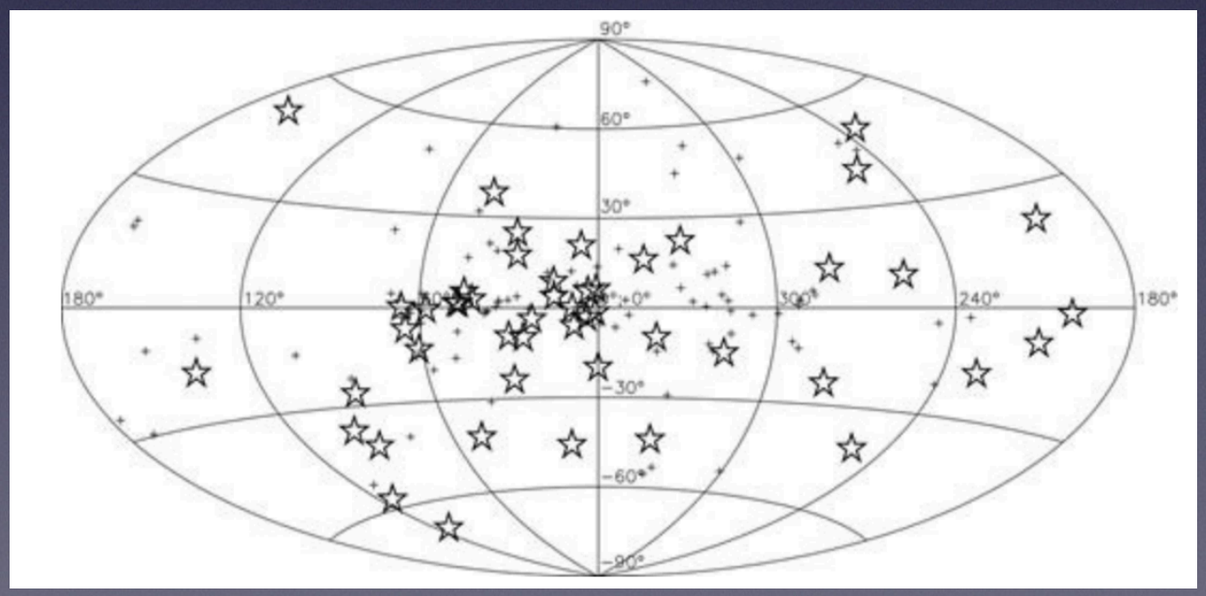


Problem: terrestrial detectors blind at f ≲ 1-10 Hz (seismic noise)

The space race!

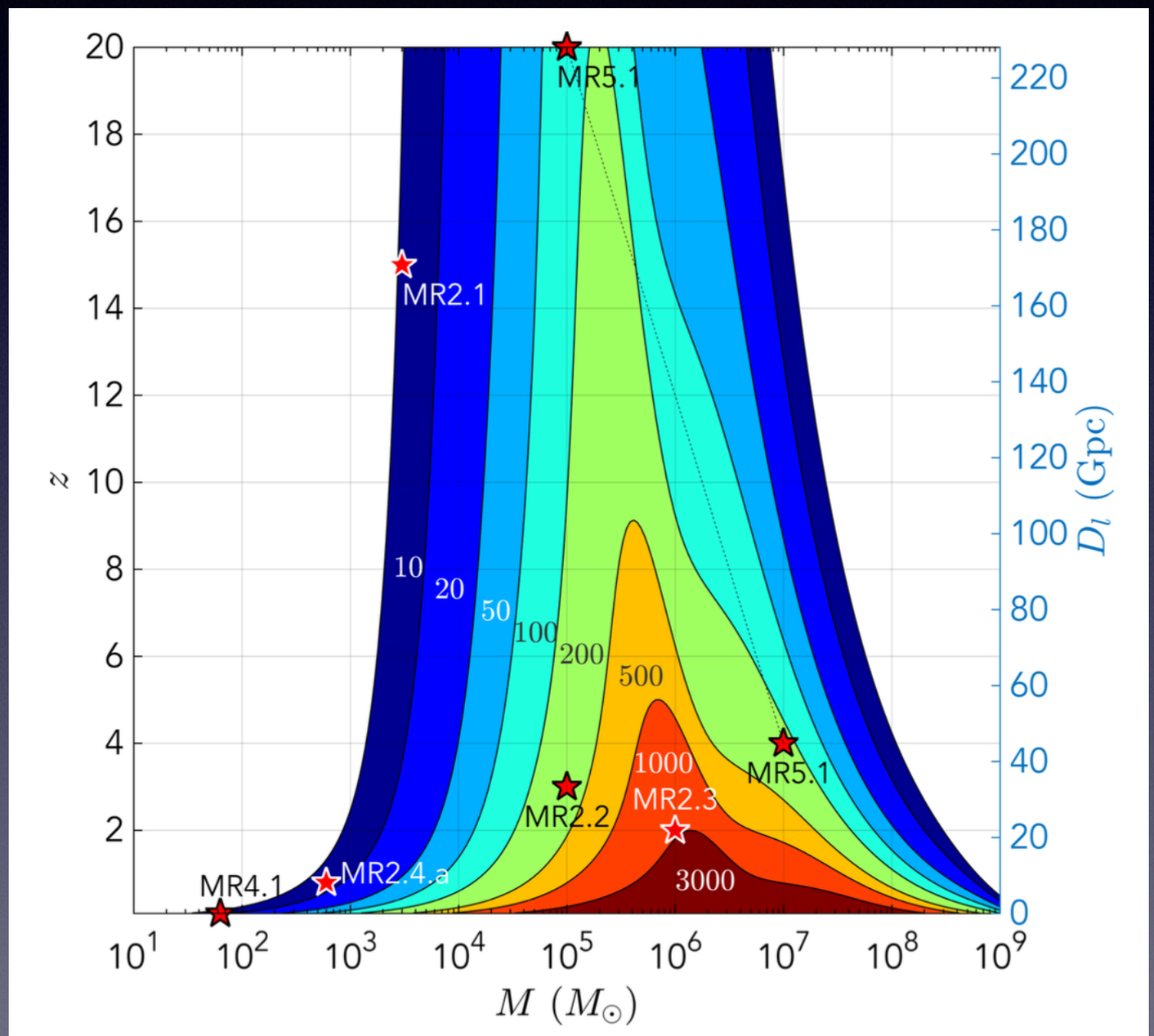


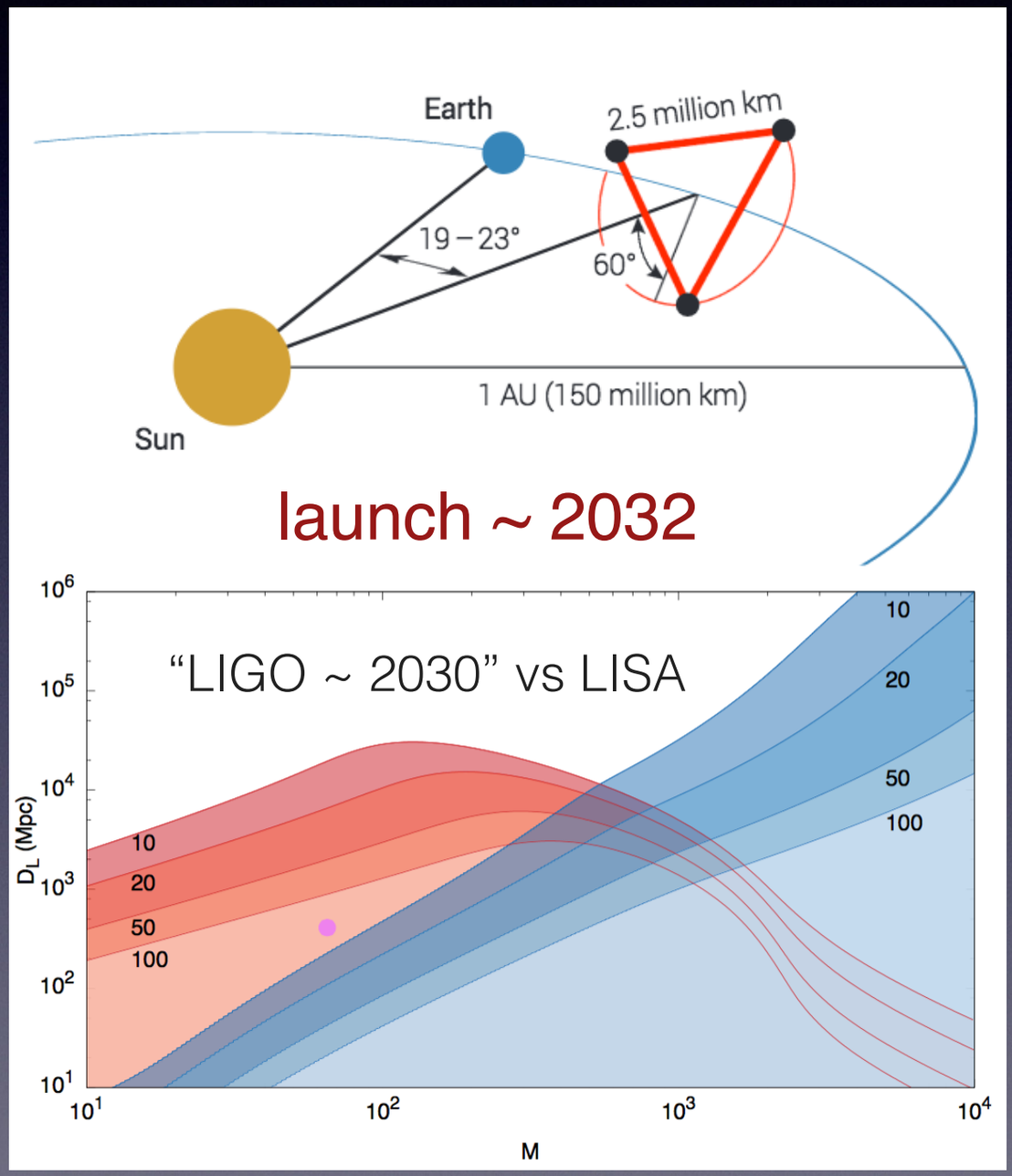




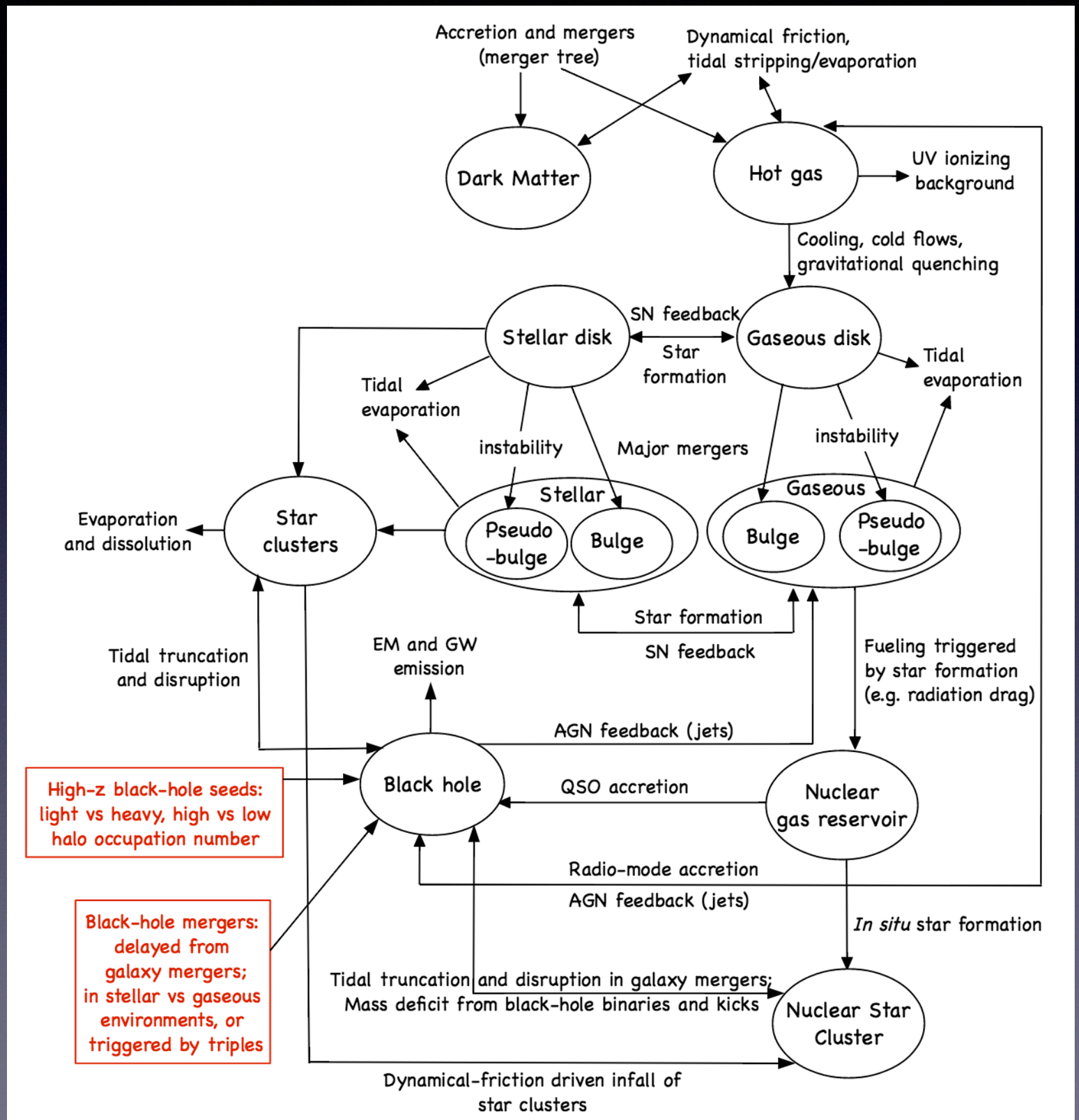
Background characteristic strain at f=1/yr is $A<1.45 \times 10^{-15}$ (Nanograv 2018)

Laser Interferometer Space Antenna (LISA)





Galaxy/BH co-evolution



EB 2012

How big are baby black holes?

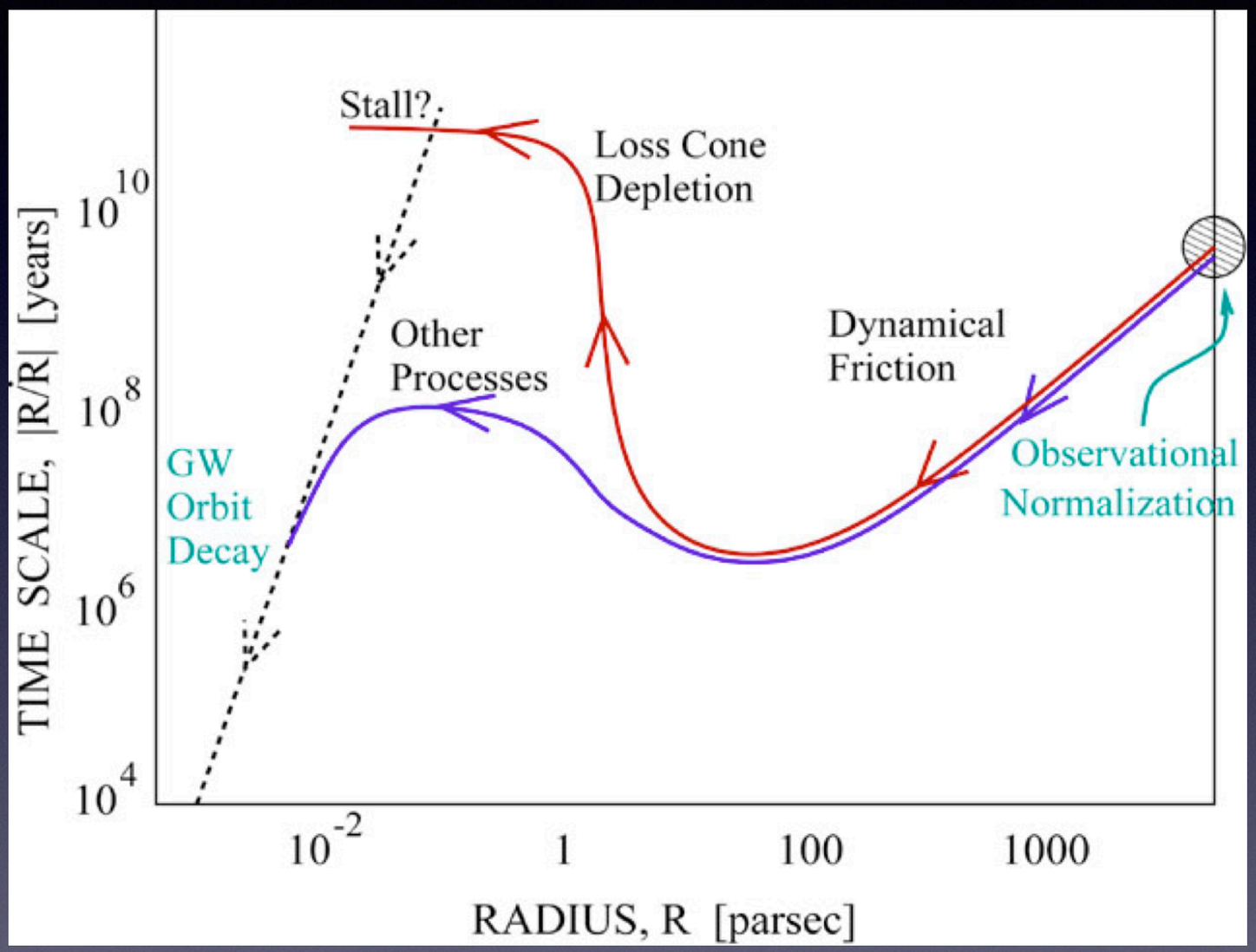
vs

Light seeds from PopIII stars (~100 M_{sun})

vs

Heavy seeds from instabilities of protogalactic disks (~10^5 M_{sun})

The “final pc problem”

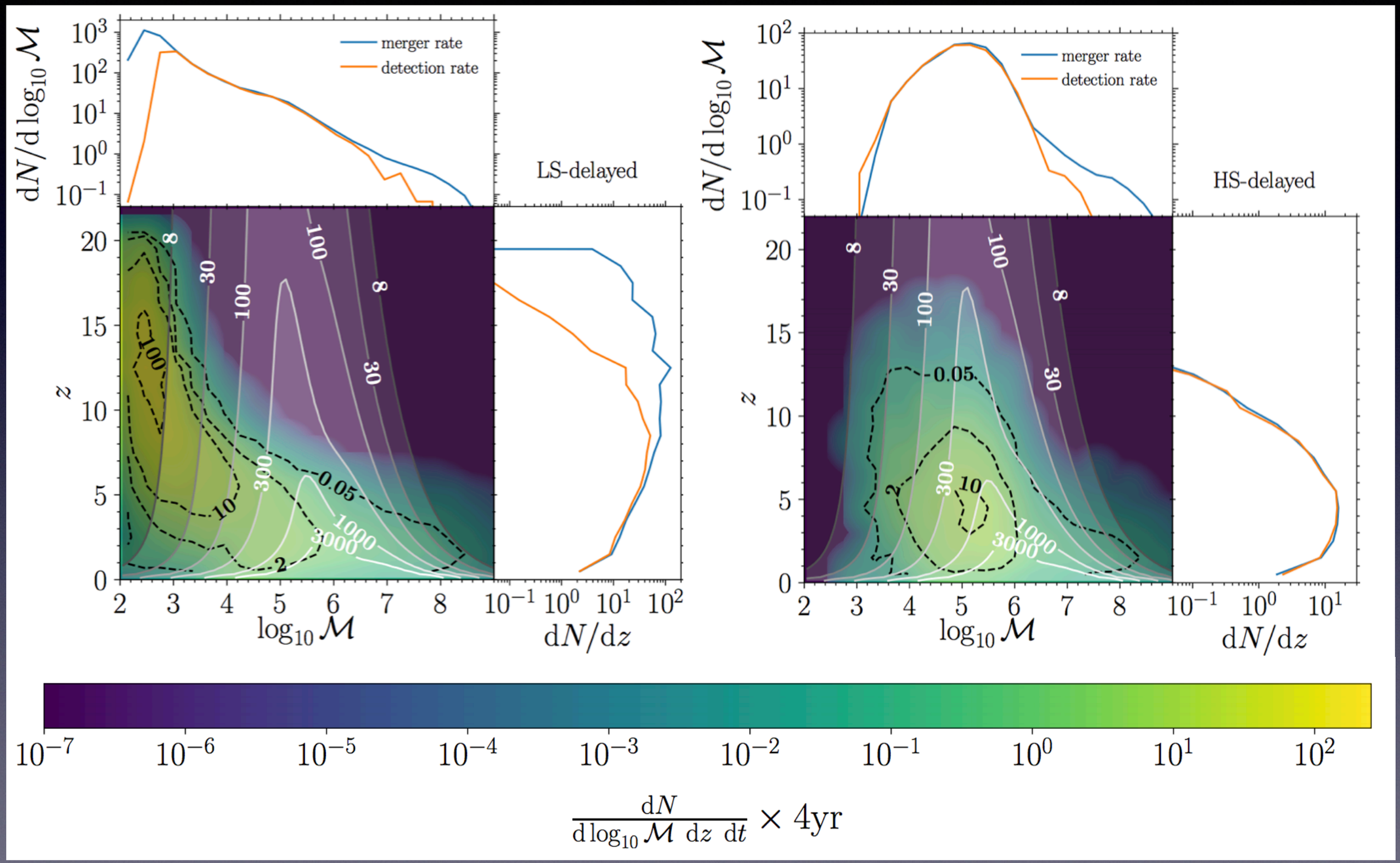


Begelman, Blandford & Rees 1980

Delays between galaxy and BH mergers depend on environment/presence of gas:

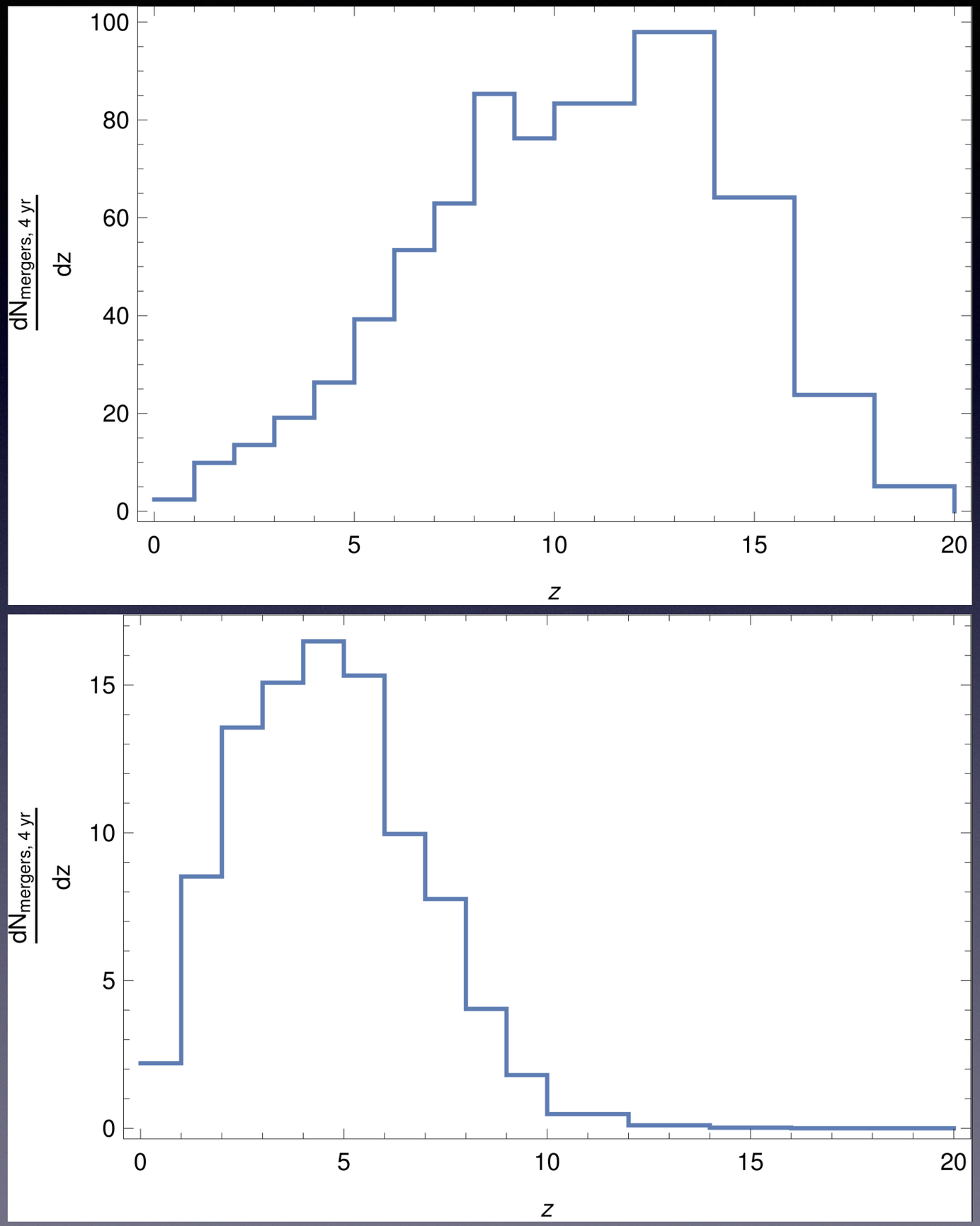
- 3-body interactions with stars on timescales of 1-10 Gyr
- Gas-driven planetary-like migration on timescales ≳ 10 Myr
- Triple massive BH systems on timescales of 0.1-1 Gyr

Detection rates



Bonetti, Sesana, Haardt, EB & Colpi 2019

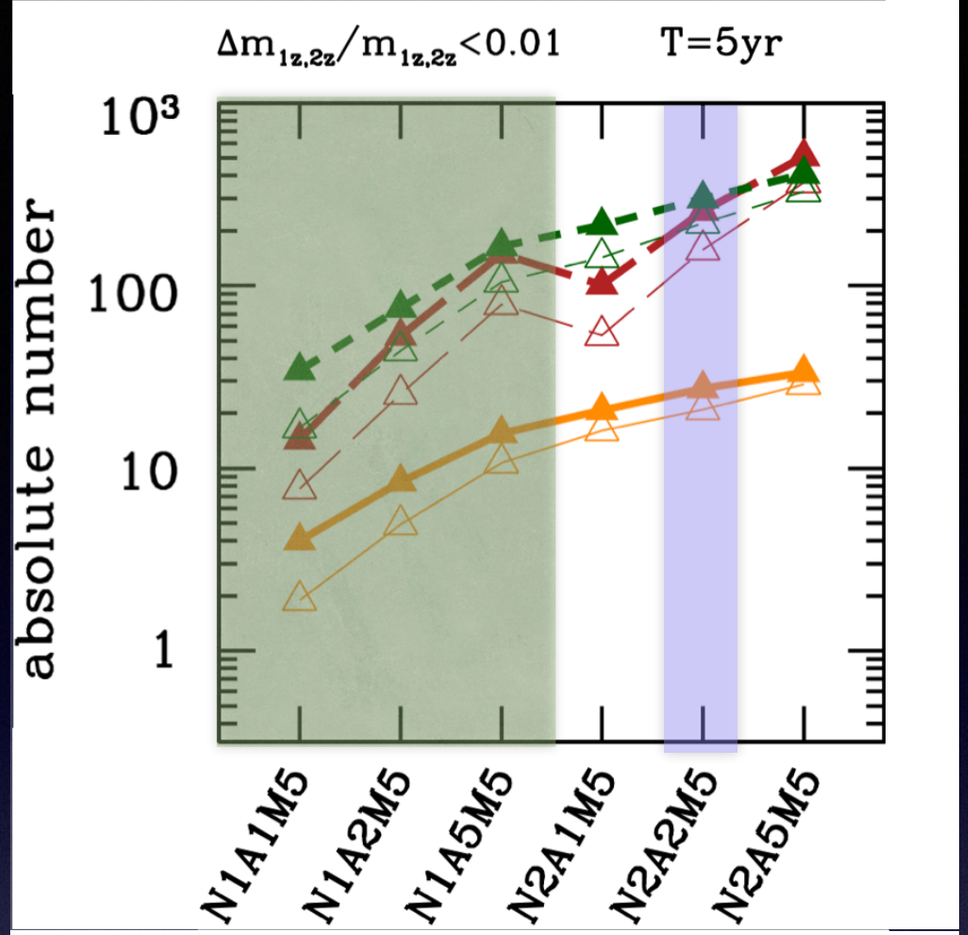
Detection rates

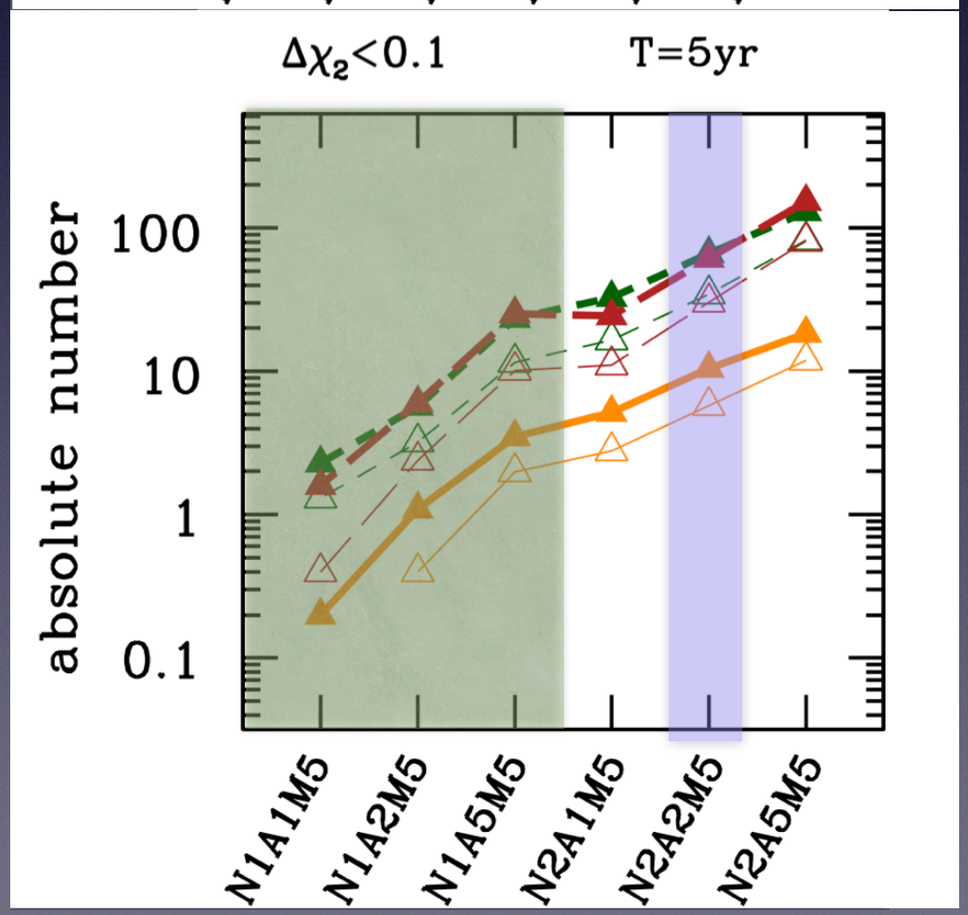


Light seeds

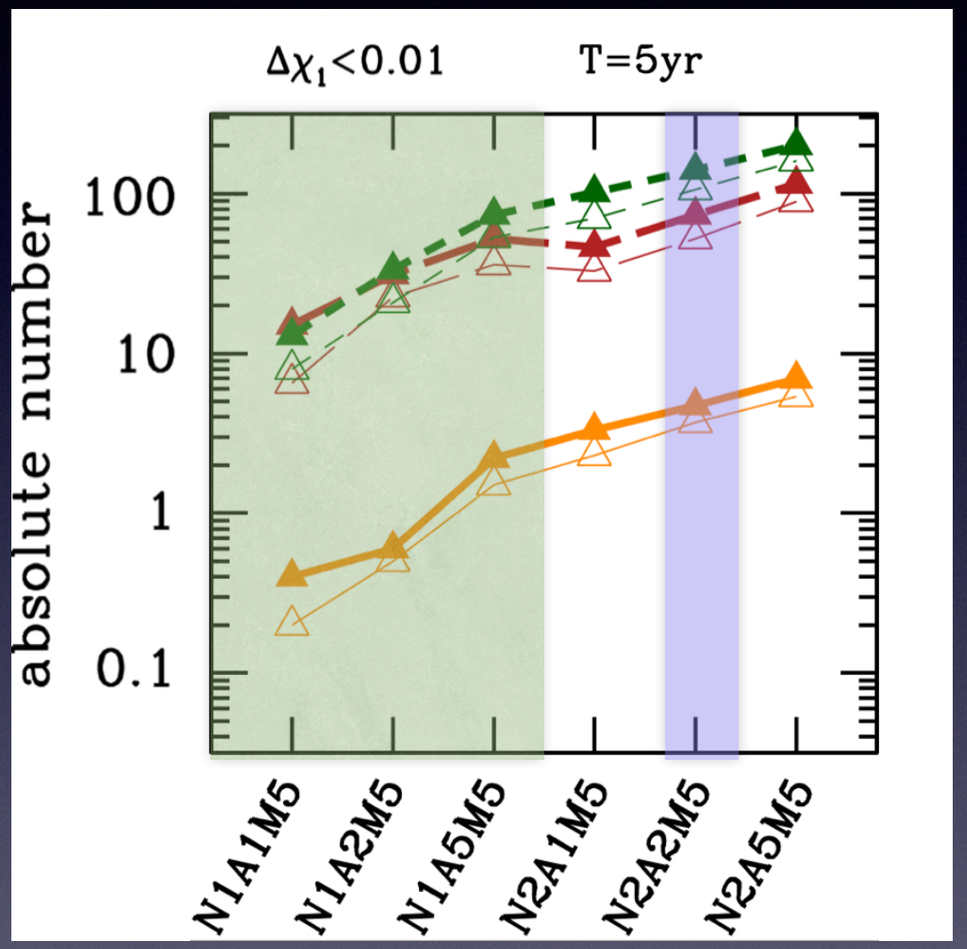
Heavy seeds

Errors on individual masses/spins





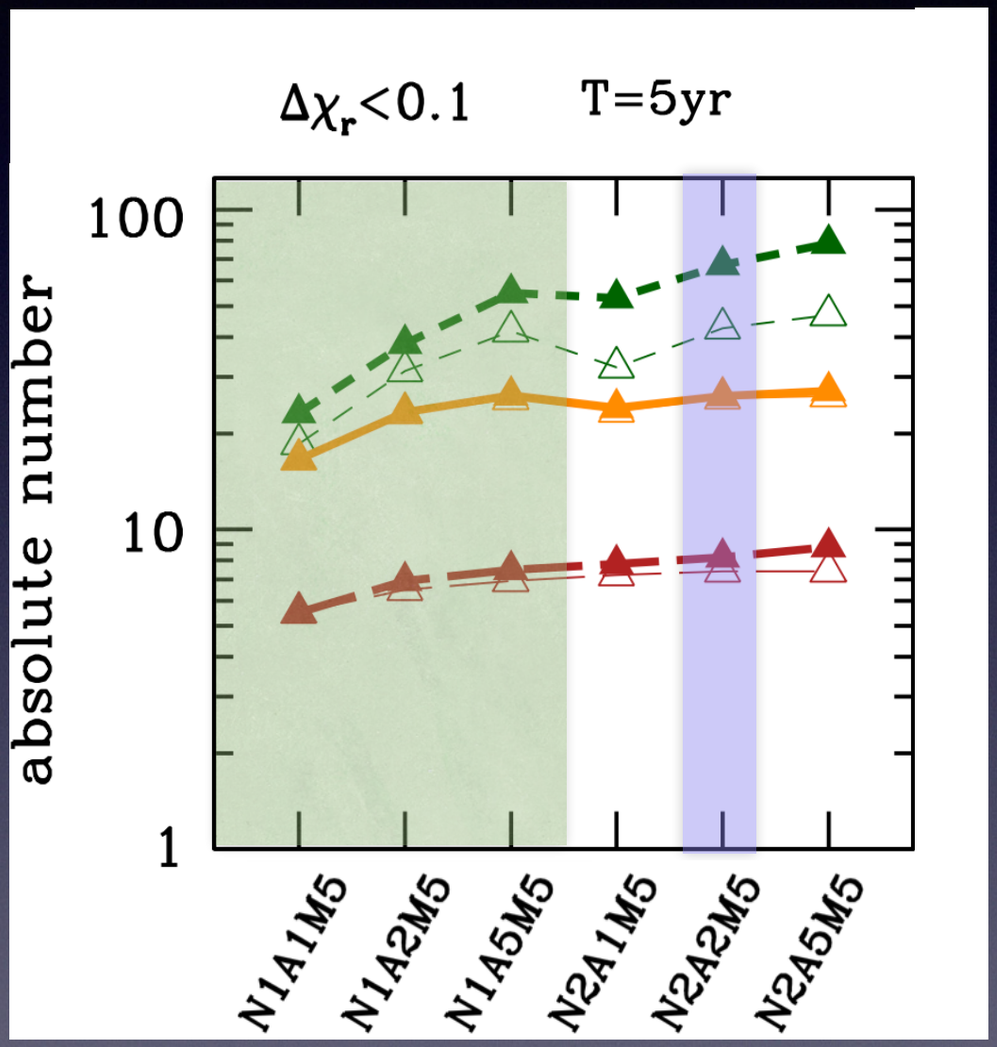
Klein EB et al 2015



Red = popIII, orange = Q3-d, green = Q3-nod
thick = six links (L6), thin = four links (L4)

Provides information about properties of BH accretion and BH mass history

Errors on spin inclinations and final spin



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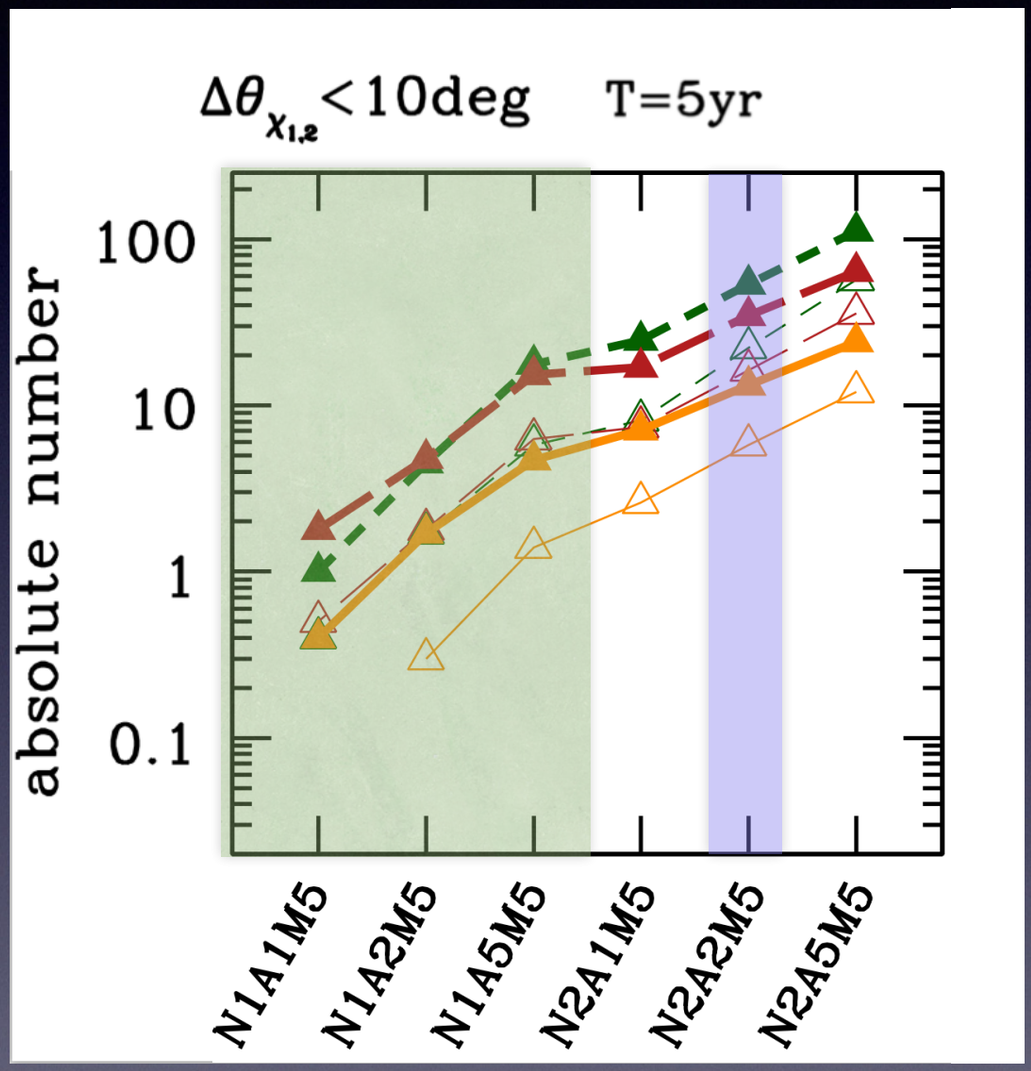
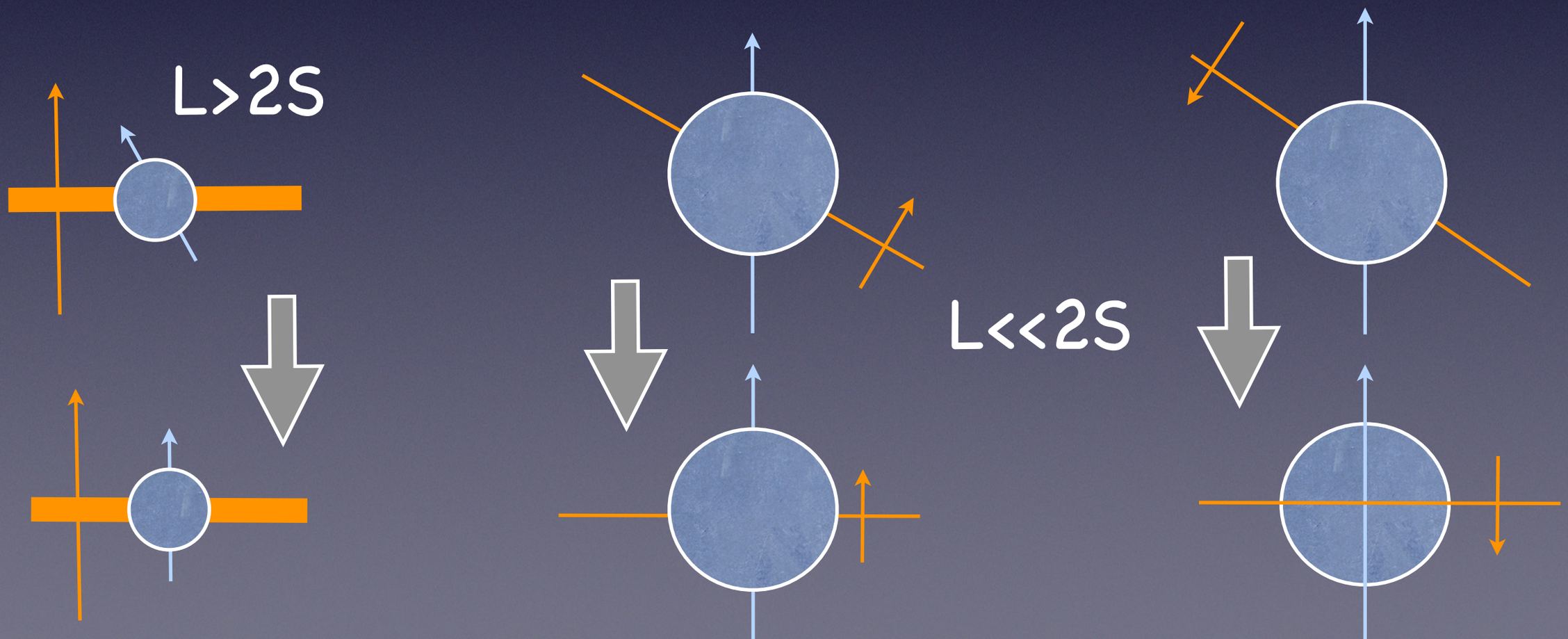


Provides information about interactions with gas (Bardeen-Petterson effect) and ringdown tests of GR

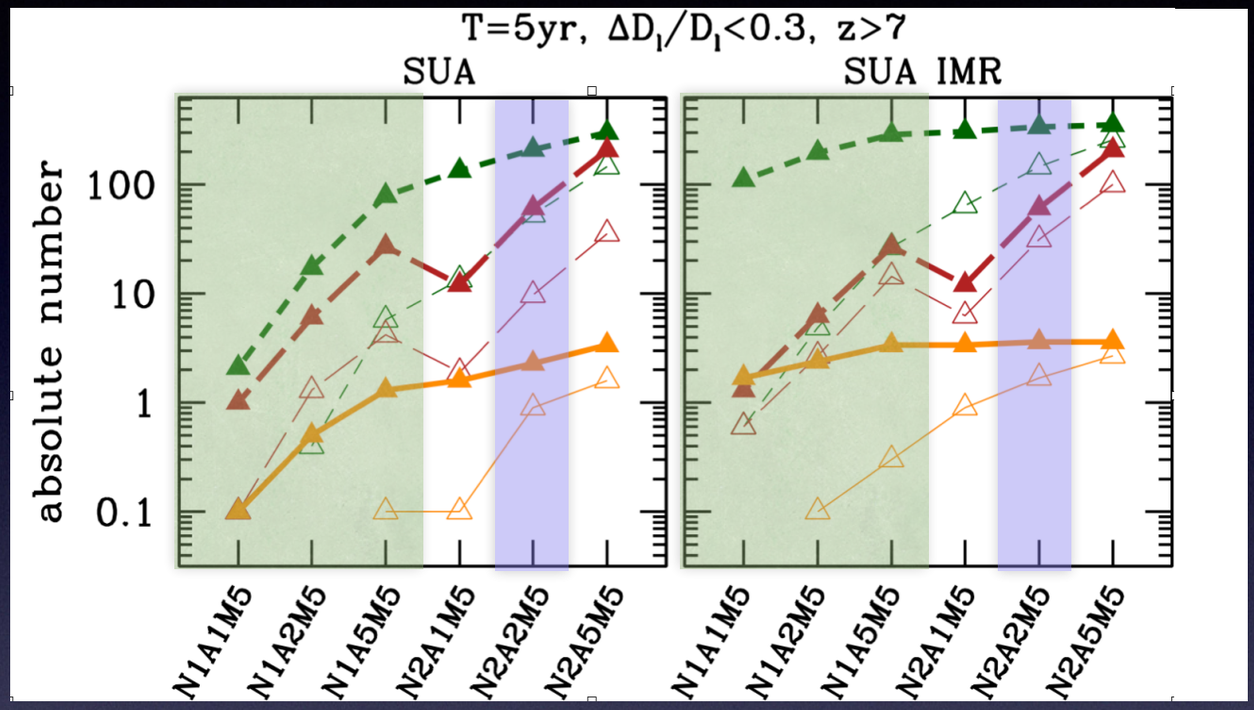
Klein EB et al 2015

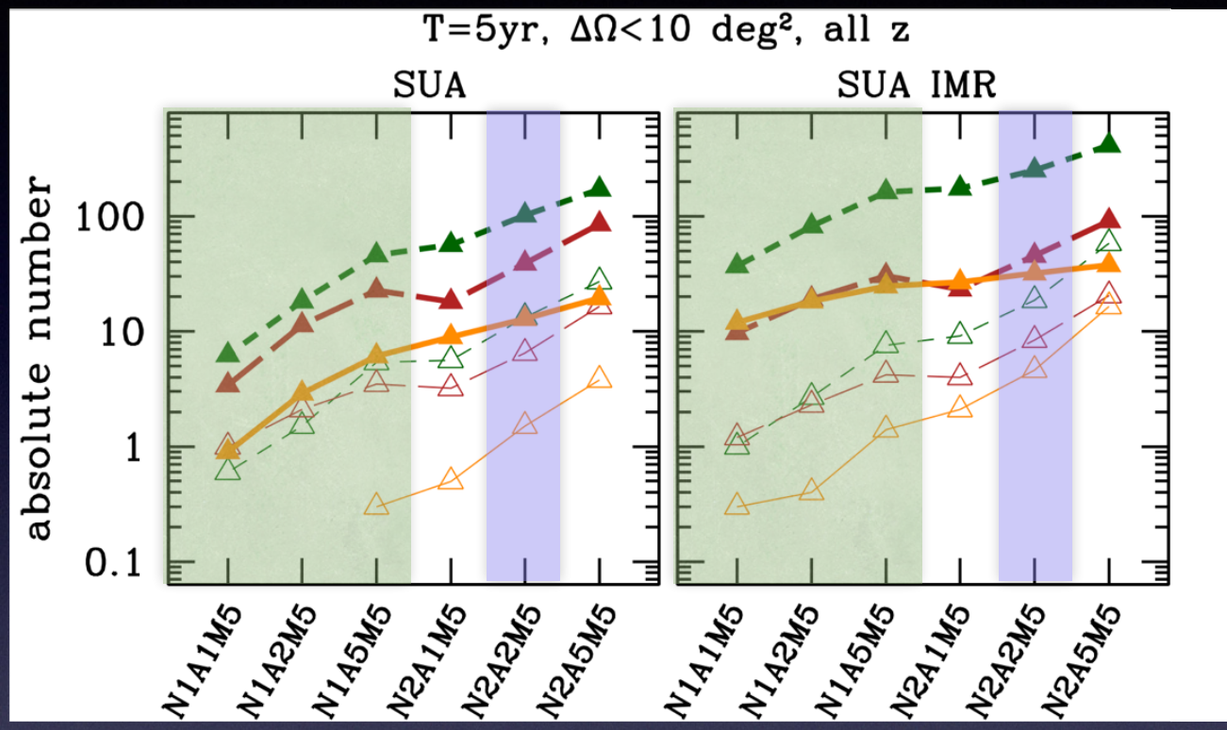
The Bardeen Petterson effect

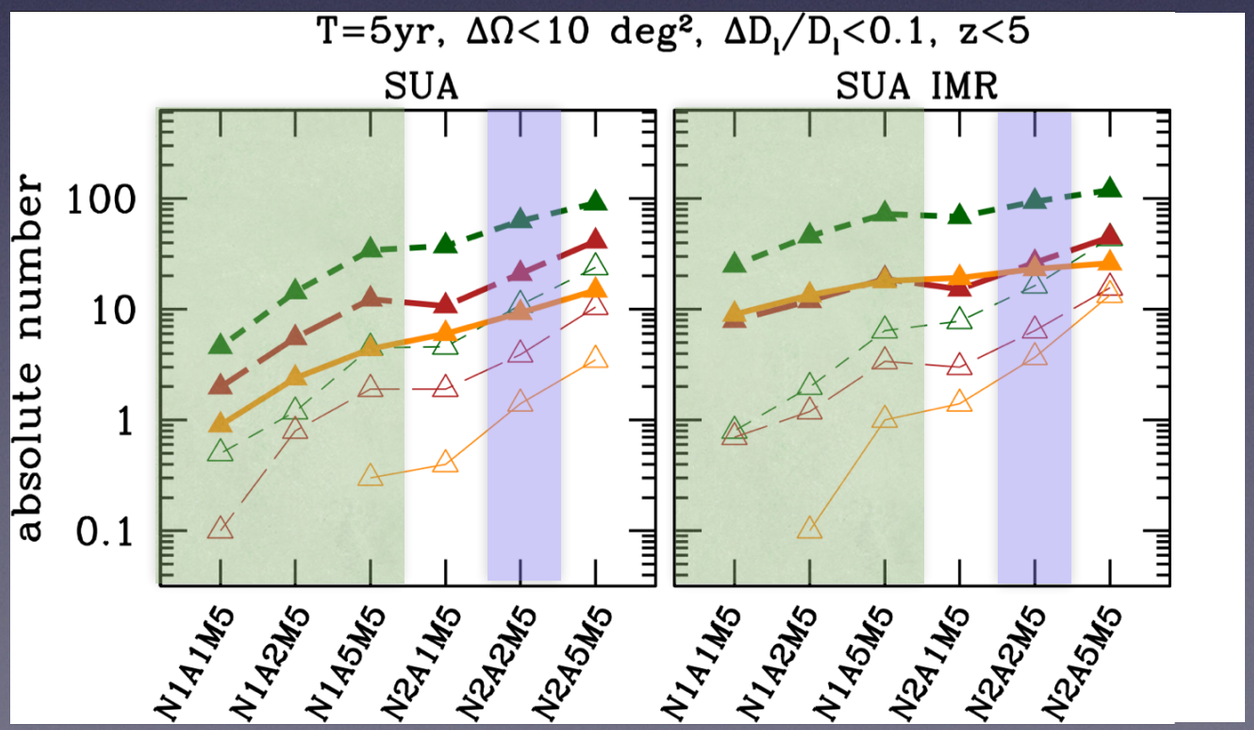
- Coupling between BH spin S and angular momentum L of misaligned accretion disk + dissipation
- Either aligns or anti-aligns S and L in ~10^5 yrs (for MBHs) << accretion timescale
- Anti-alignment only if disk carries little angular momentum (L < 2S) and is initially counterrotating



Cosmography (“standard sirens”) and probes of massive BH formation







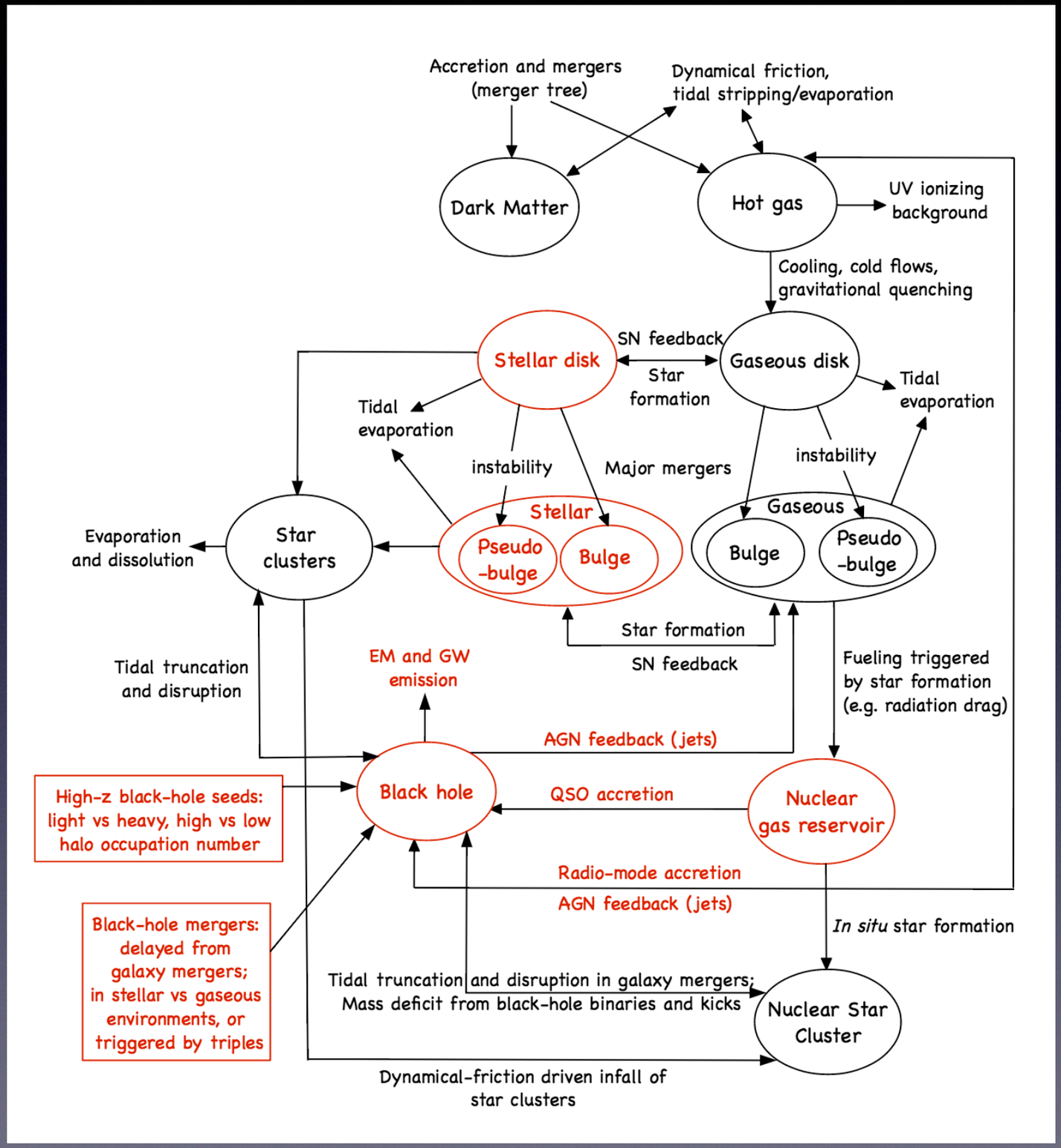
brown = popIII, orange = Q3-d, green = Q3-nod
thick = six links (L6), thin = four links (L4)

SUA = inspiral
SUA IMR = inspiral-merger-ringdown

From Klein EB et al 2015; see also Tamanini EB et al 2016

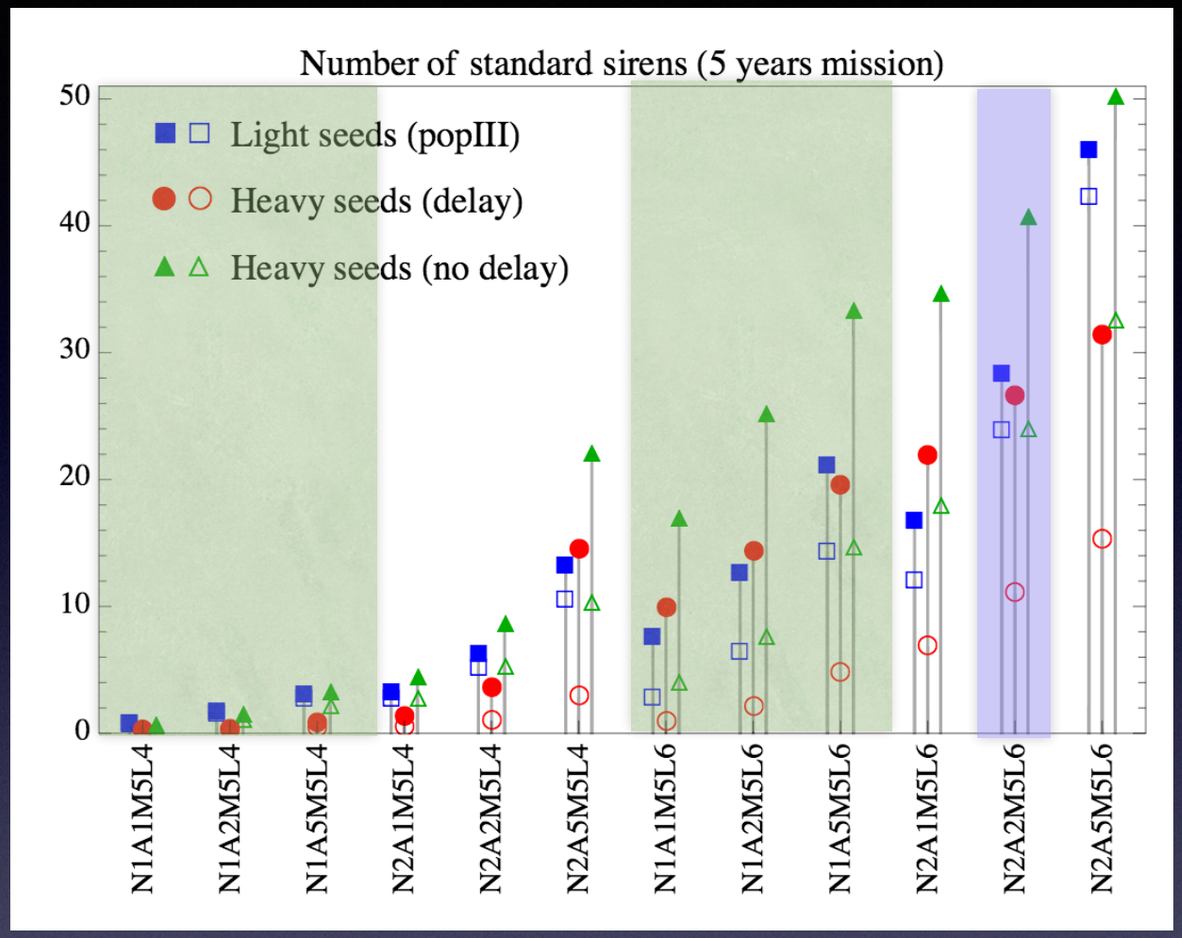
Electromagnetic counterparts

- GWs provide measurement of luminosity distance (though degraded by weak lensing) but not redshift
- In order to do cosmography in a non-statistical way, we need redshift
- Electromagnetic (spectroscopic or photometric) redshift measurement needs presence of gas, e.g. radio jet+ follow-up optical emission



EB 2012

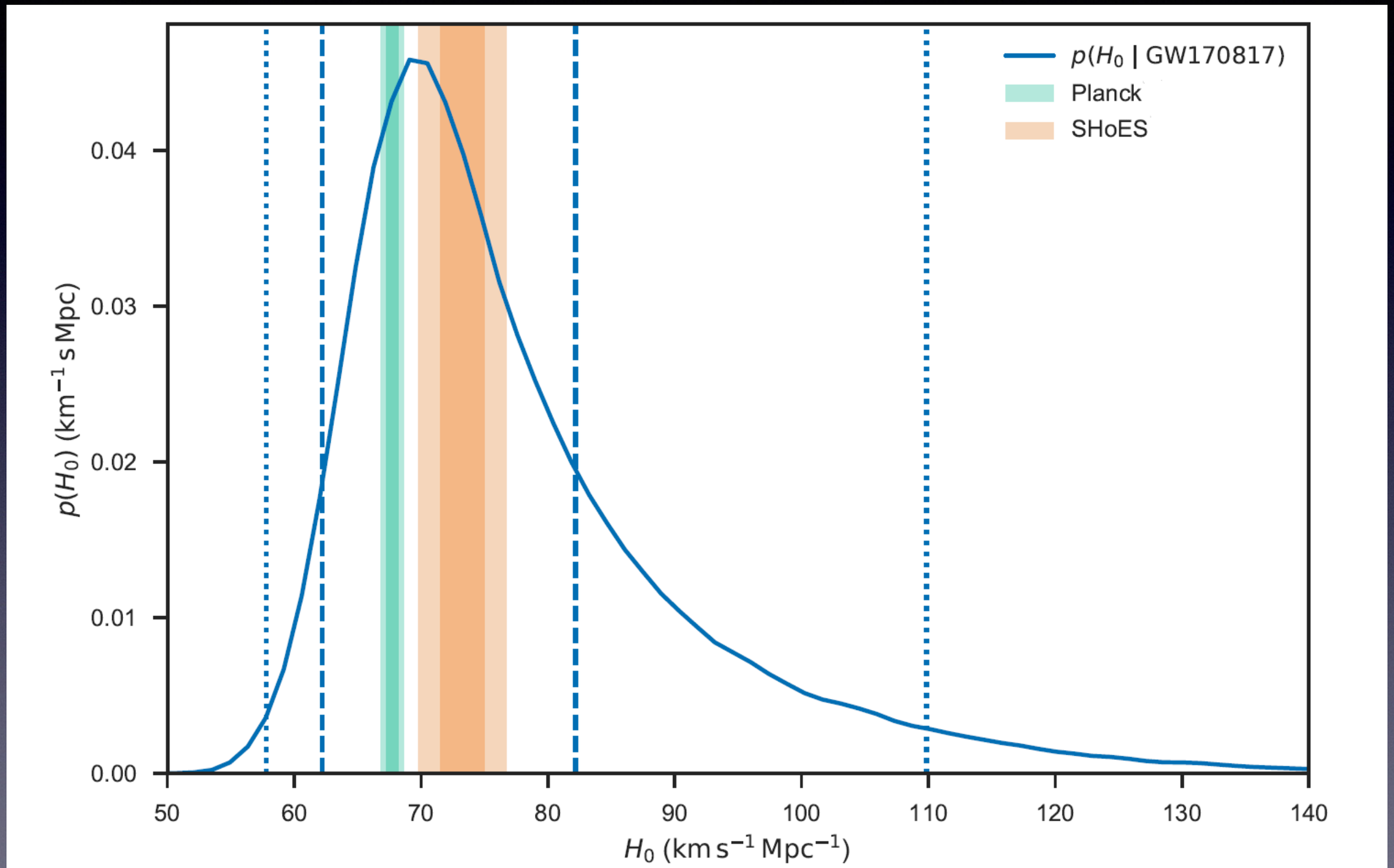
Cosmography with LISA



Tamanini, EB et al 2016

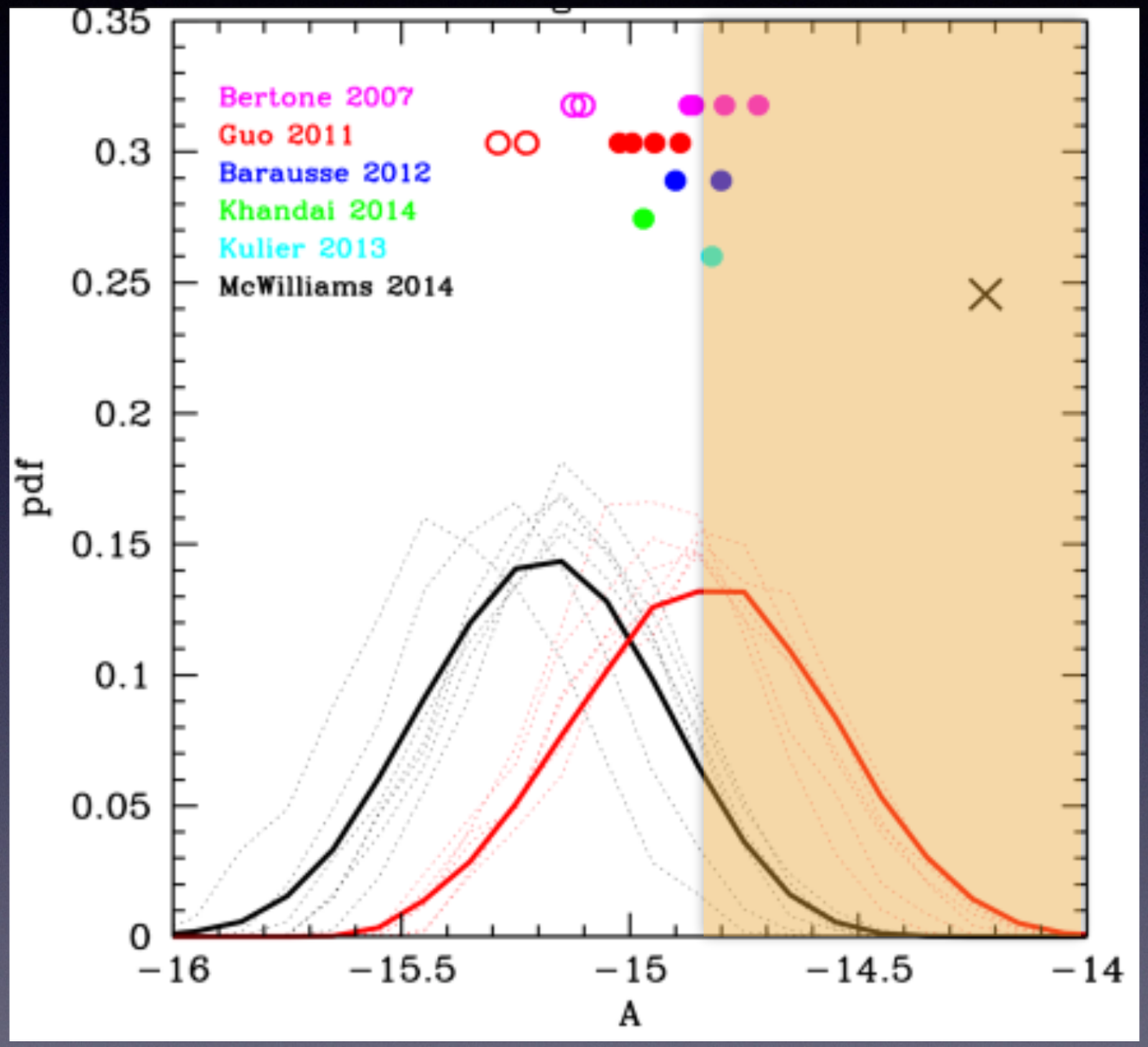
- LISA may measure h under different systematics than present probes
- Measurement of Ω_m slightly better than SNIa with best designs
- Measurement of combination of Ω_m and Ω_Λ different from SNIa/CMB (i.e. potential to break degeneracy)
- Discovery space: LISA sensitive to cosmological evolution at $z \sim 1 - 8$
- Understanding of EM emission crucial (inspiral vs merger sky localisation)

Cosmography from 2G and 3G



LSC 2018

What can we learn from PTA limits?



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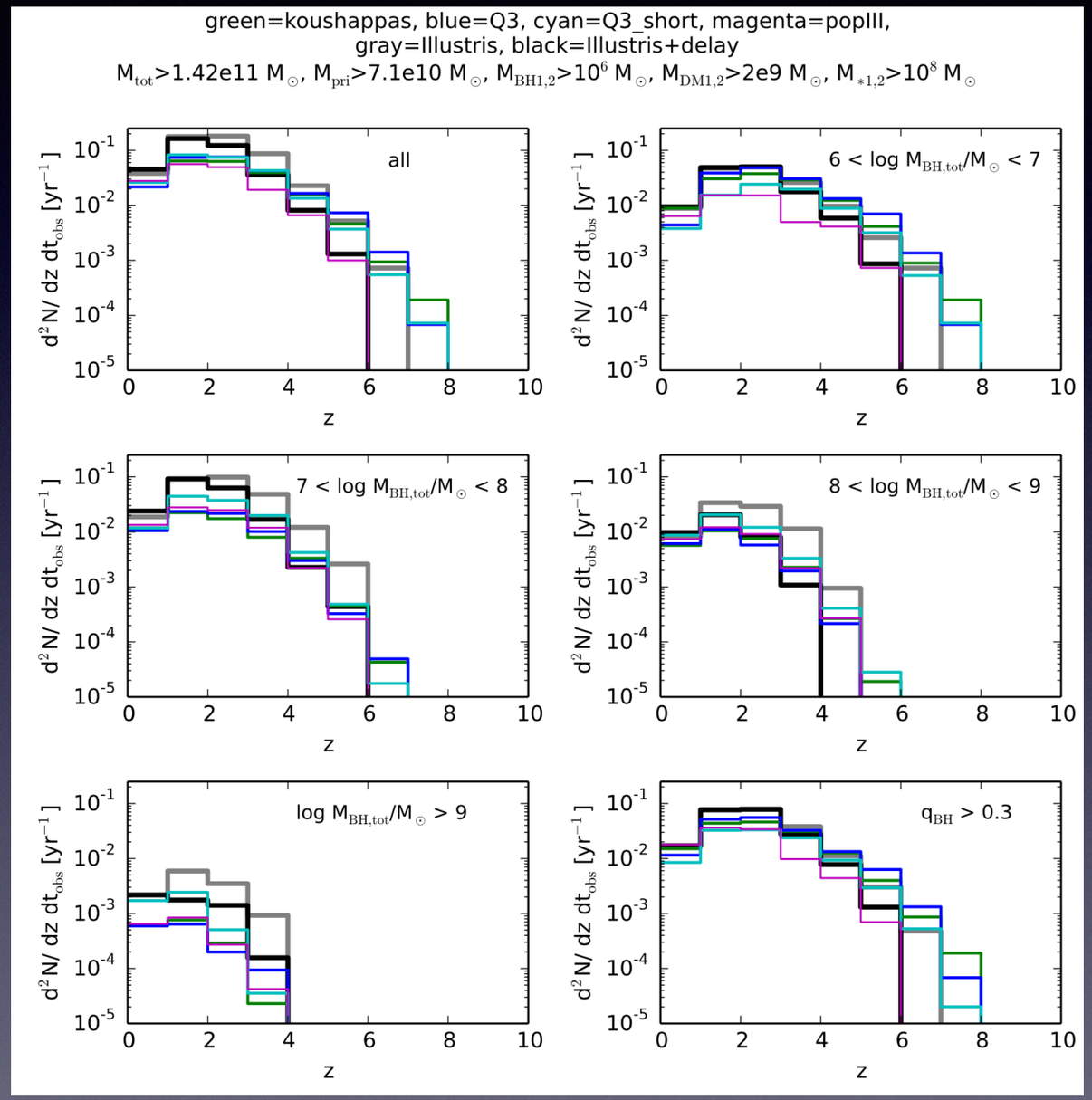


SAM vs simulations
(EB 2012 vs Illustris)

Why are we seeing nothing?

Predictions assume:

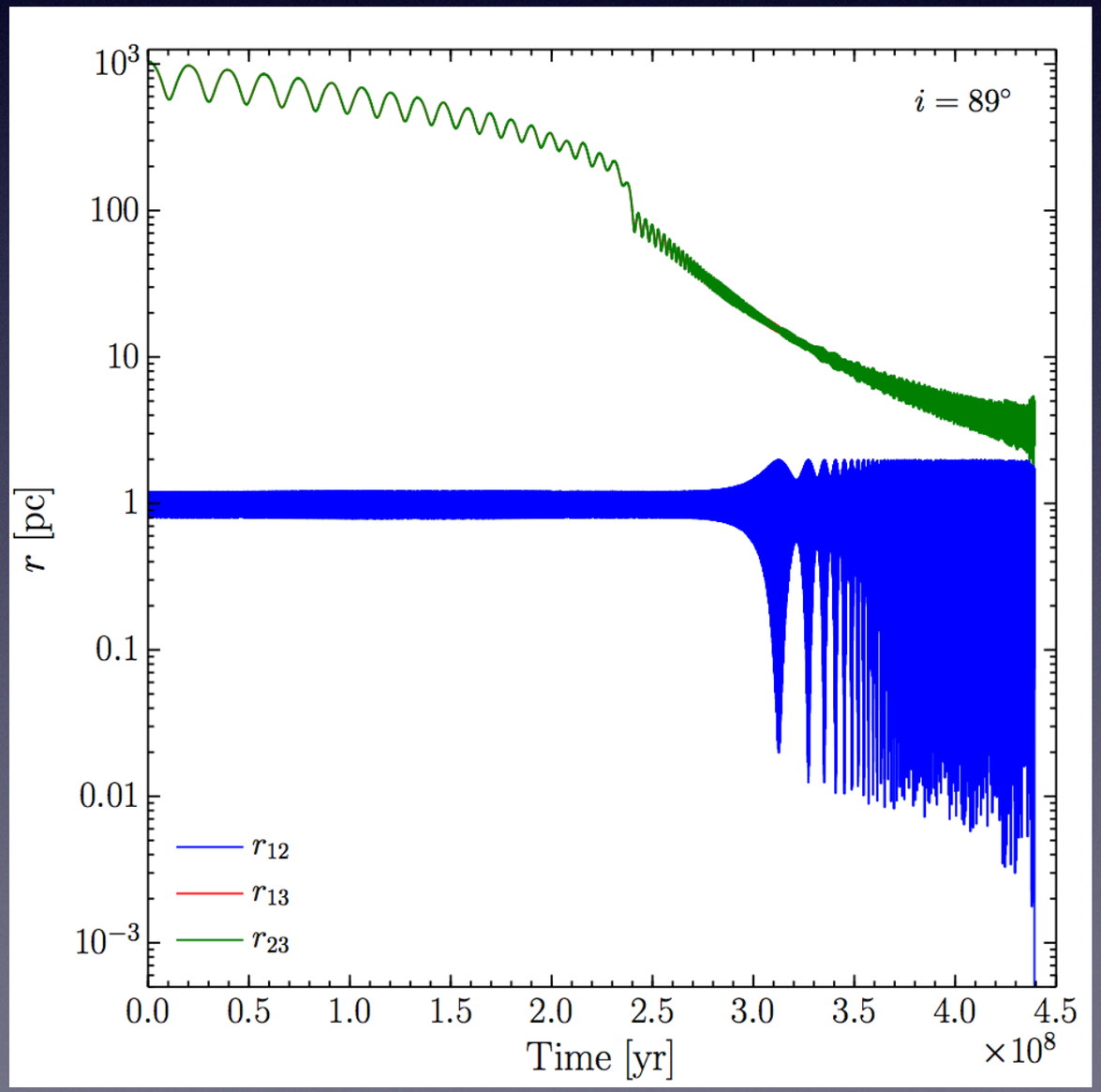
- GW driven binaries
- Circular orbits
- Efficient formation of bound massive BH binaries after galaxy mergers
- M-σ relation

Loopholes:

- Binaries may merge faster than expected based on GW emission alone (hence less time in band)
- Eccentric binaries (more power at high frequencies) due e.g. to strong environmental effects/triple systems
- Last pc problem (binaries stall)
- M-σ relation may be biased

The final parsec “problem”

- If BH binaries stall and do not merge, triple systems naturally form as a result of later galaxy mergers
- Merger induced by Kozai-Lidov resonances (secular exchange between eccentricity and orbital inclination)

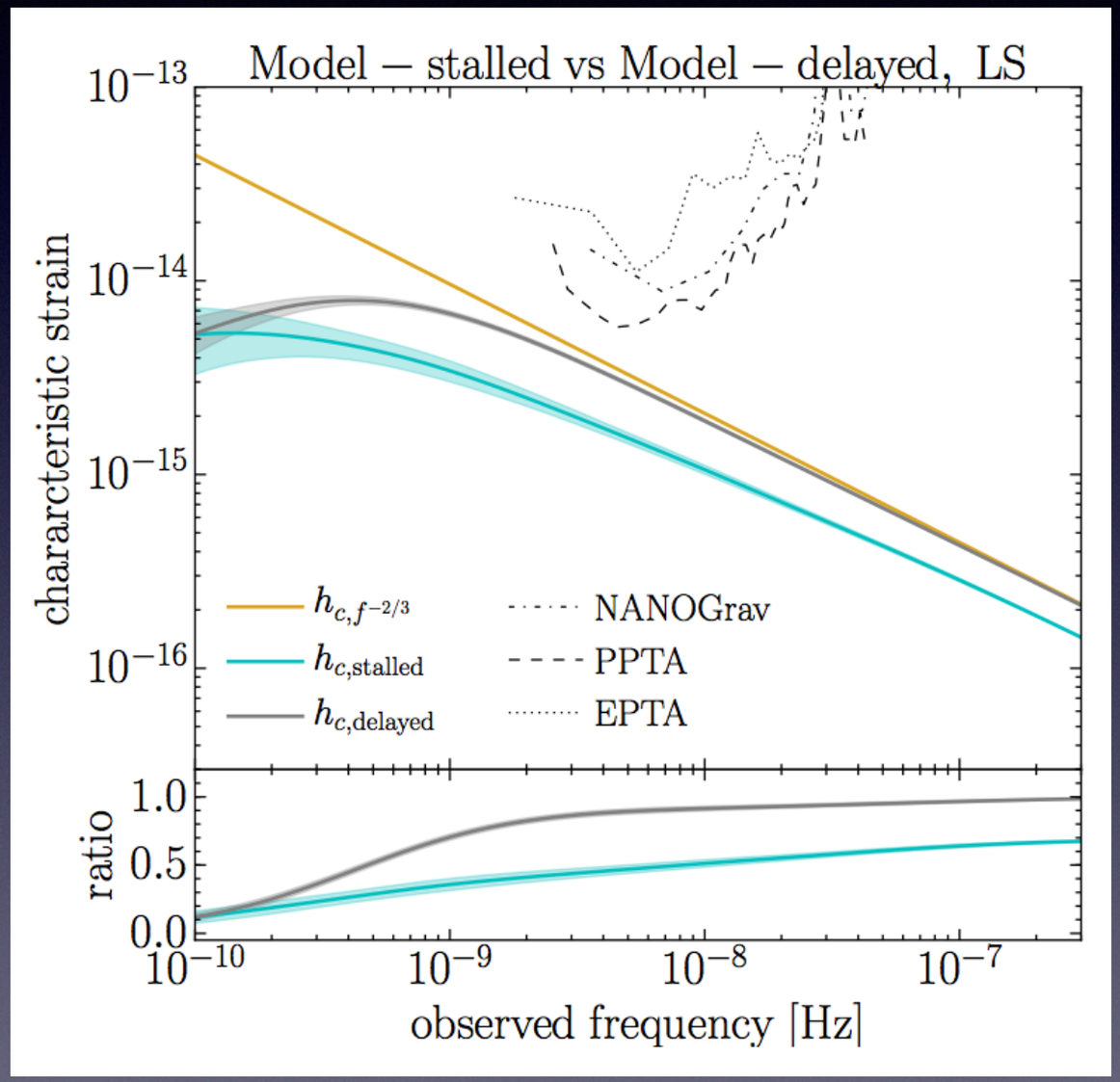


$$t_{\rm KL} \sim \frac{a_{\rm out}^3(1-e_{\rm out}^2)^{3/2}\sqrt{m_1+m_2}}{G^{1/2}a_{\rm in}^{3/2}m_3} \simeq 2\times 10^6 \ \ {\rm yrs},$$

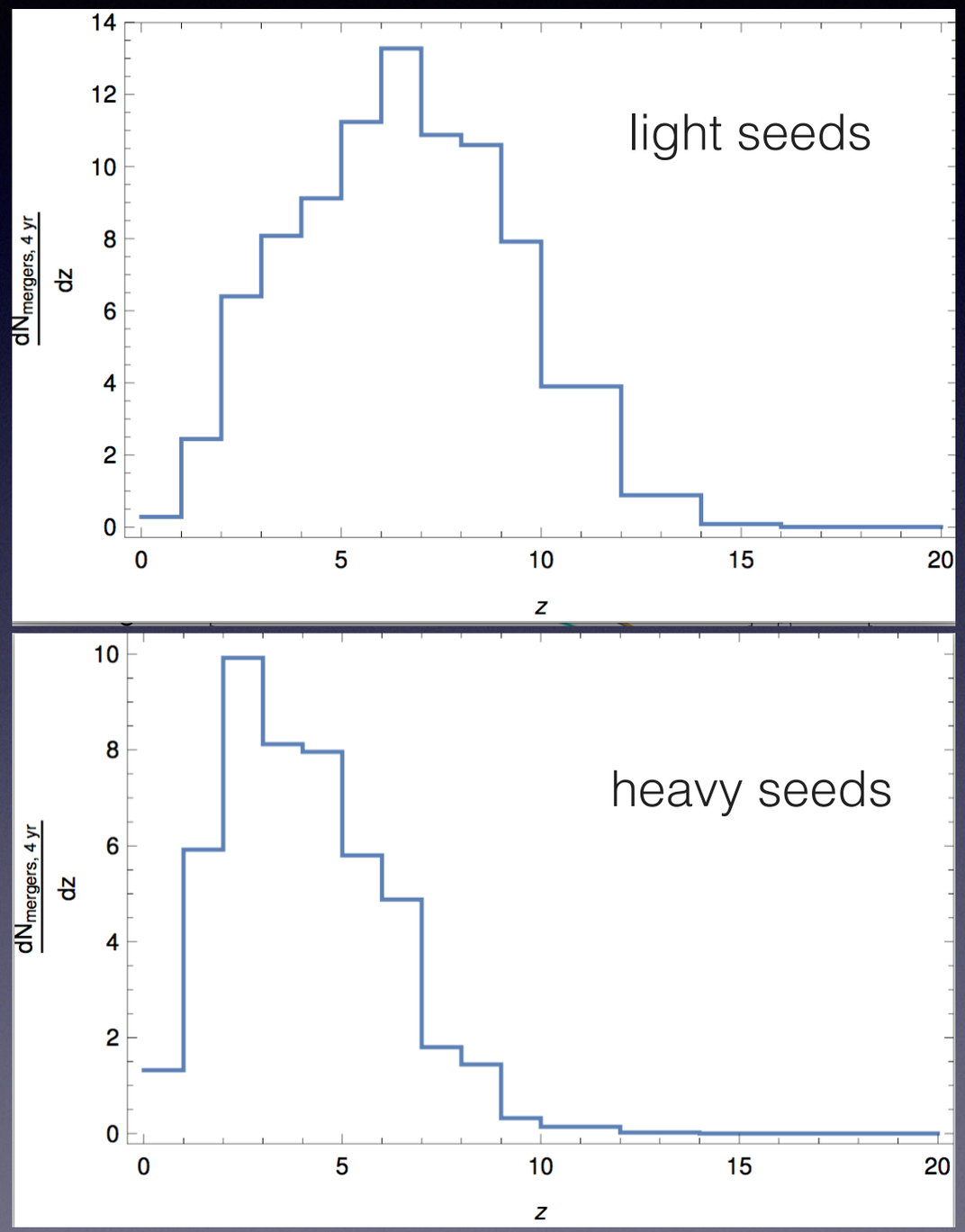
$m_1 = m_2 = m_3 = 10^8$ M$_\odot$, $a_{\rm in} = 1$ pc, $a_{\rm out} = 10$ pc, and $e_{\rm out} = 0$.

PN 3-body simulation in a stellar environment, with m_1=10^8 M_{sun}, m_2=3 x 10^7 M_{sun}, m_3=5 x 10^7 M_{sun} (Bonetti, Haardt, Sesana & EB 2016)

Triple-induced BH mergers: PTA and LISA

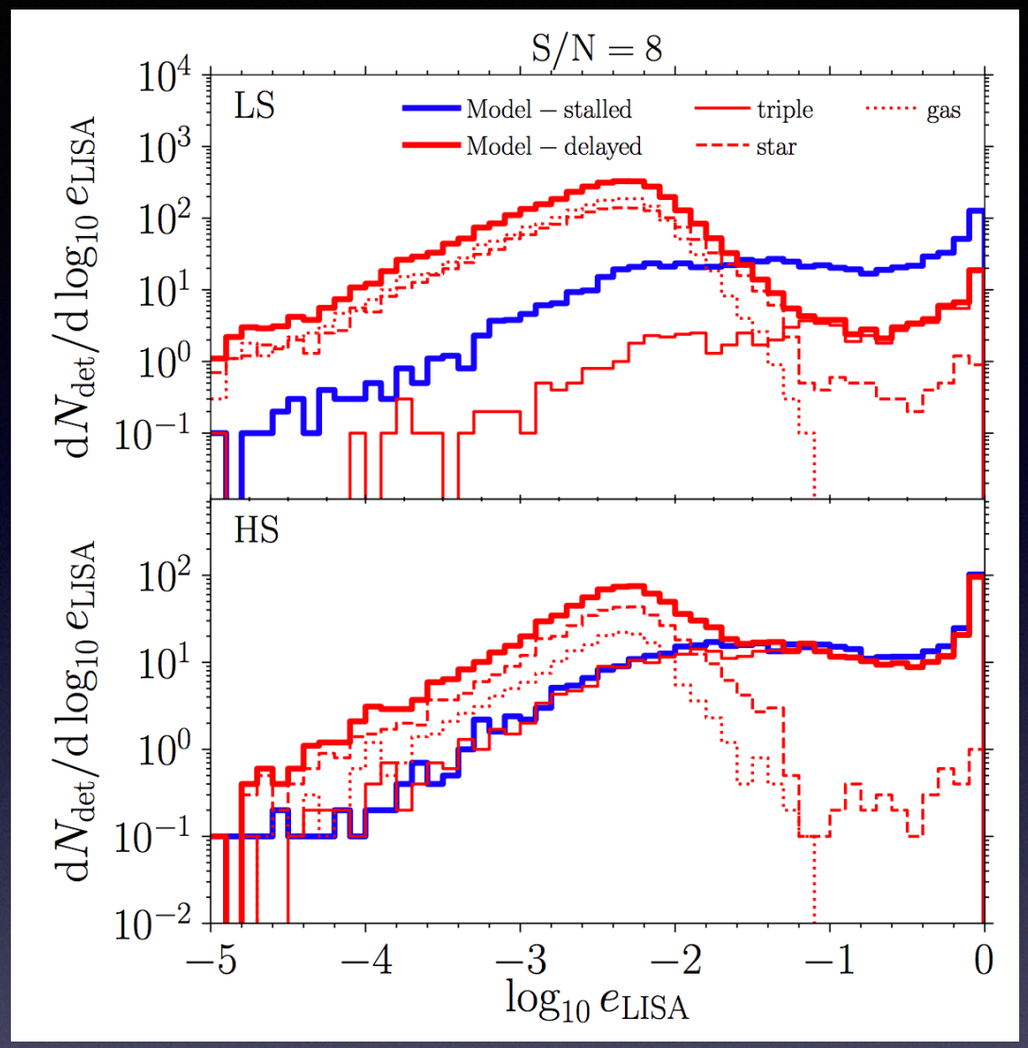


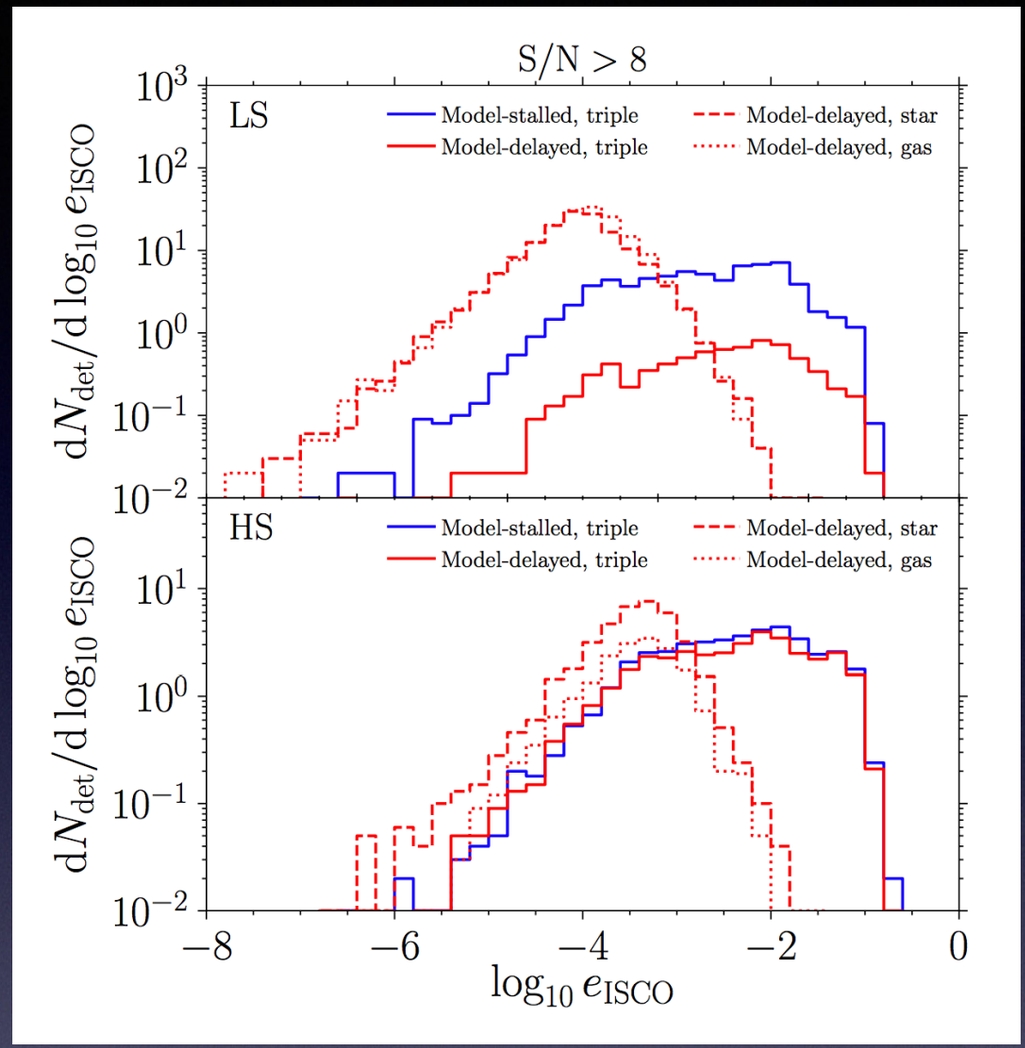
Bonetti, Sesana, EB, Haardt 2018



Bonetti, Sesana, Haardt, EB & Colpi 2019

Eccentric inspirals and bursts?



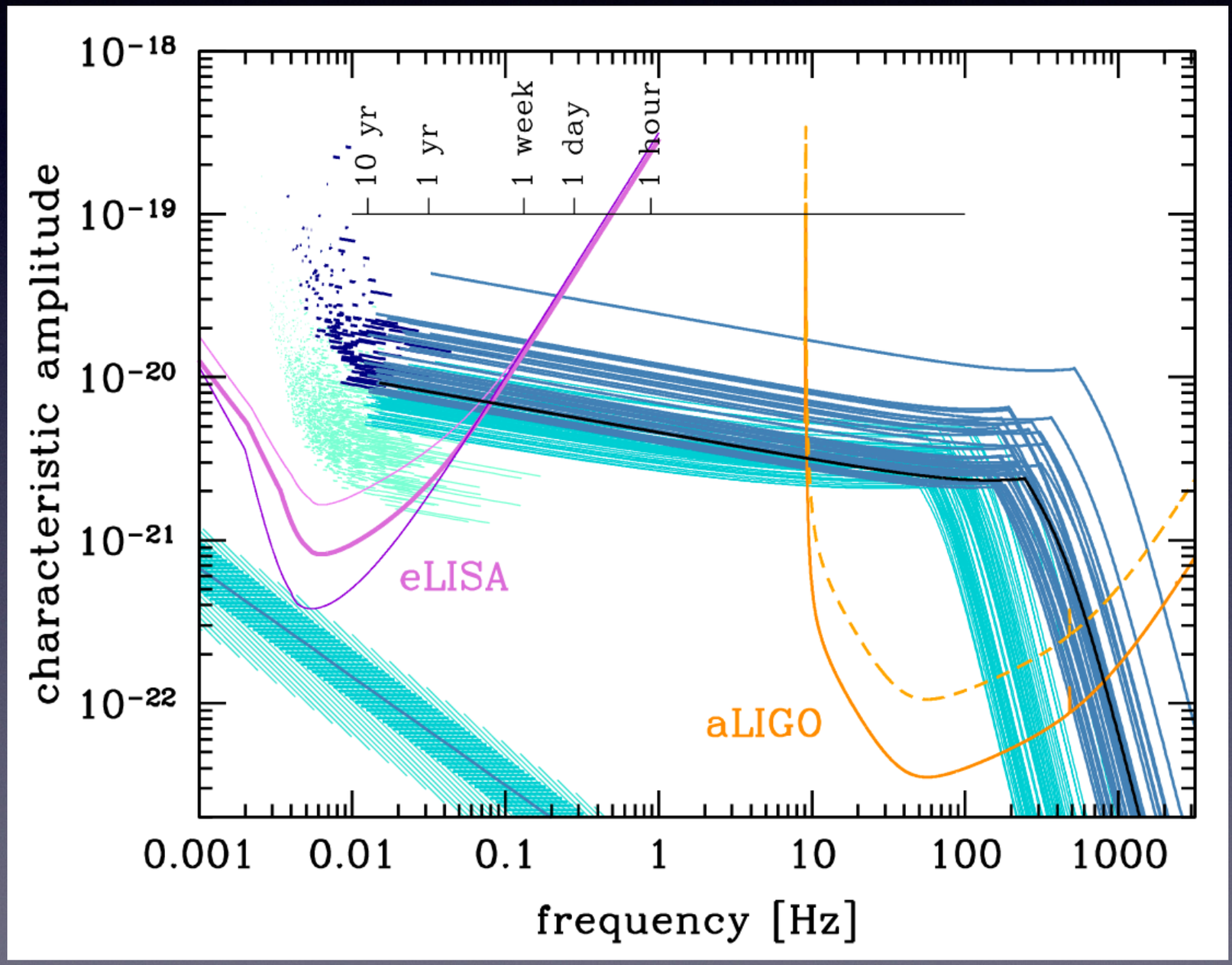


- Standard hardening channels (gas, stars) produce small eccentricities by the time MBHBs get into band
- Triple driven systems can display eccentricities >0.99 (at band entrance)!
- Possibility to see eccentric non-merging sources (bursts)
- Possibility of an unresolved signal?

Model	$N_{\mathrm{det}}(e_{\mathrm{LISA}} > 0.1)$	$N_{\mathrm{det,triple}}$	$N_{\mathrm{det,total}}$
LS-*stalled*	35.4	77.8	77.8
LS-*delayed*	4.6	7.9	310.9
HS-*stalled*	22.5	49.4	49.4
HS-*delayed*	19.8	41.7	96.7

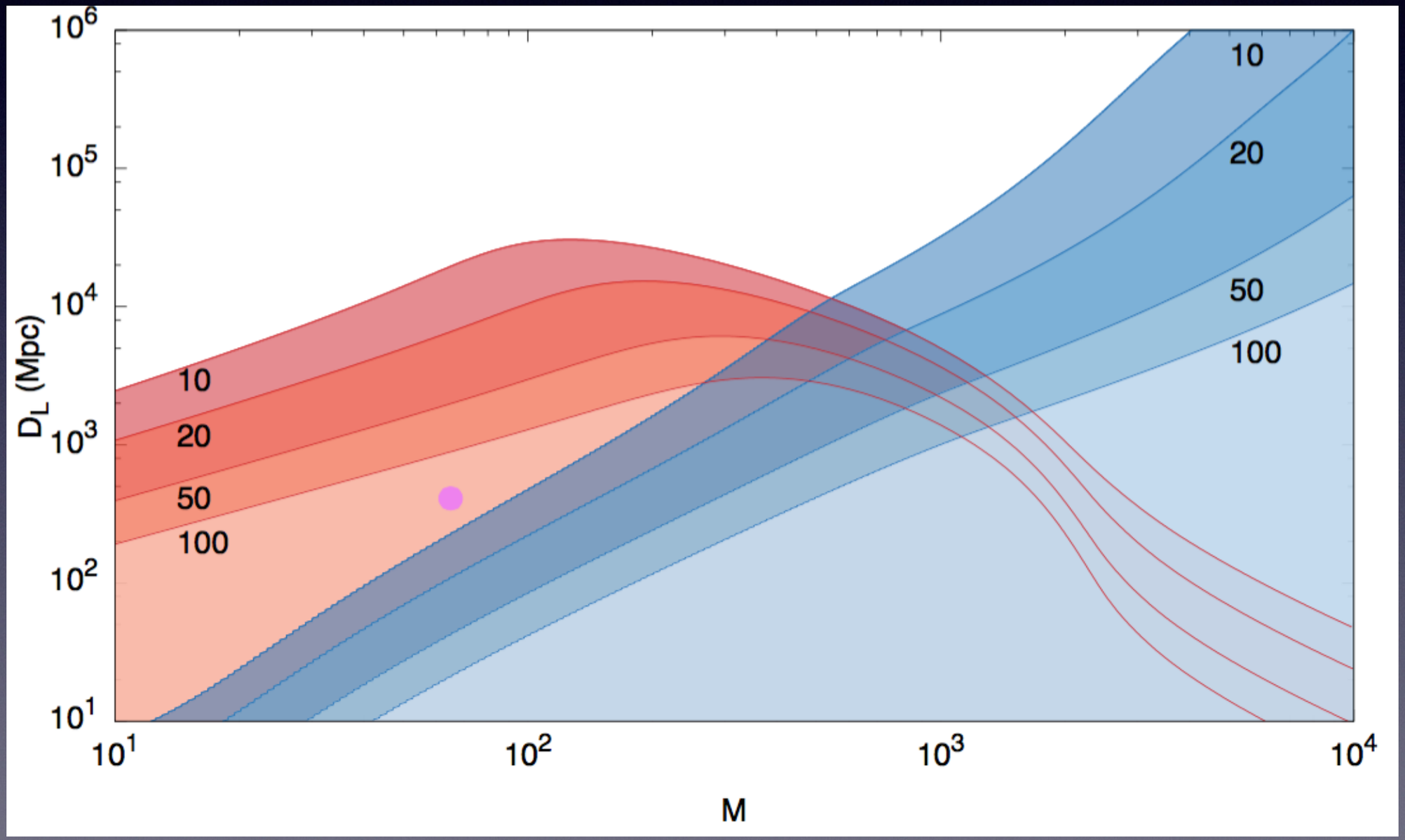
Bonetti, Sesana, Haardt, EB & Colpi 2019

Multi-band gravitational-wave astronomy



Sesana 2016

Multi-band gravitational-wave astronomy

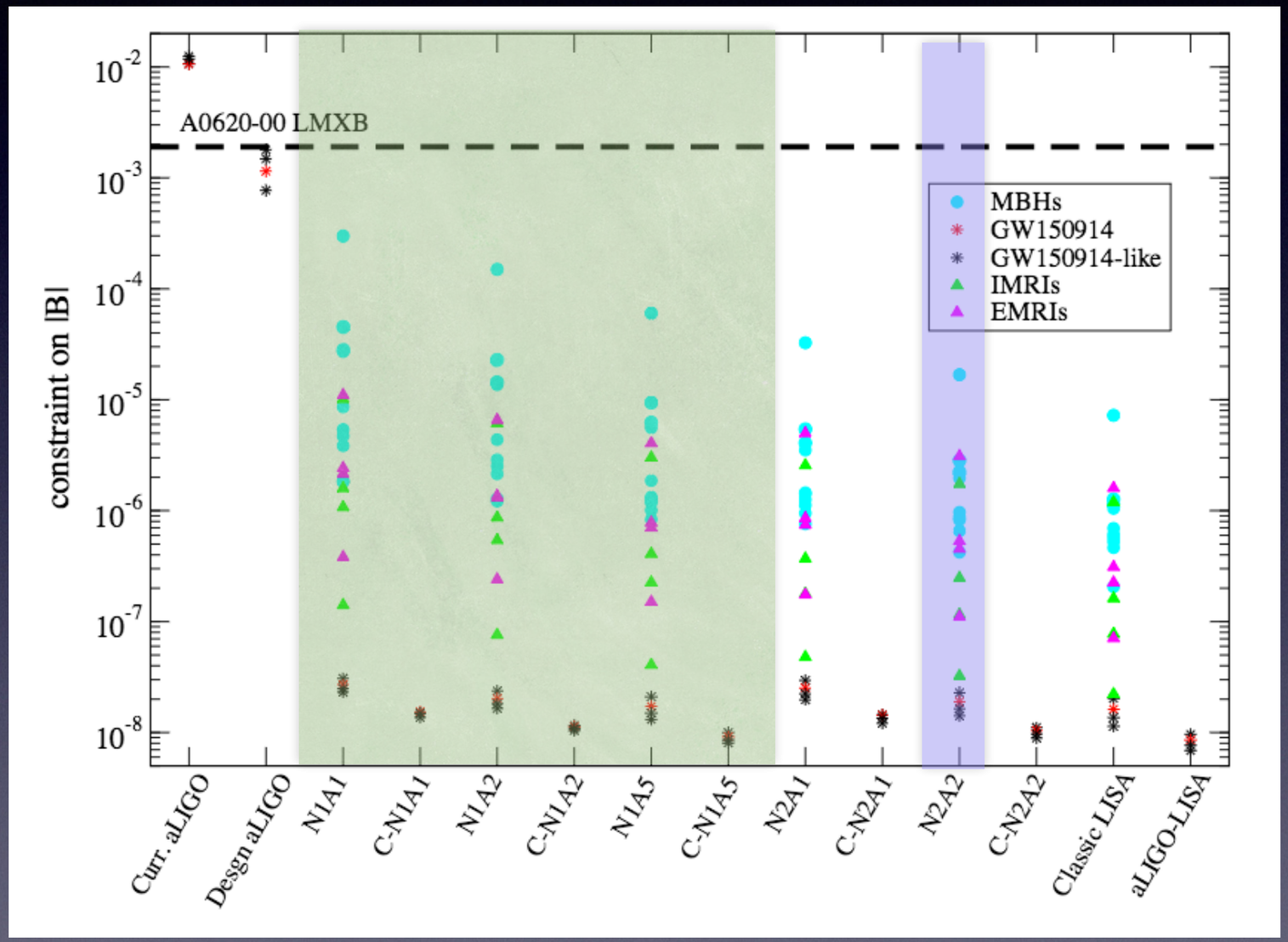


Tests of the equivalence principle with multi-band observations

- Smoking-gun sign of deviation from GR/BH "hairs" would be BH-BH dipole emission (-1PN term in phase/flux)

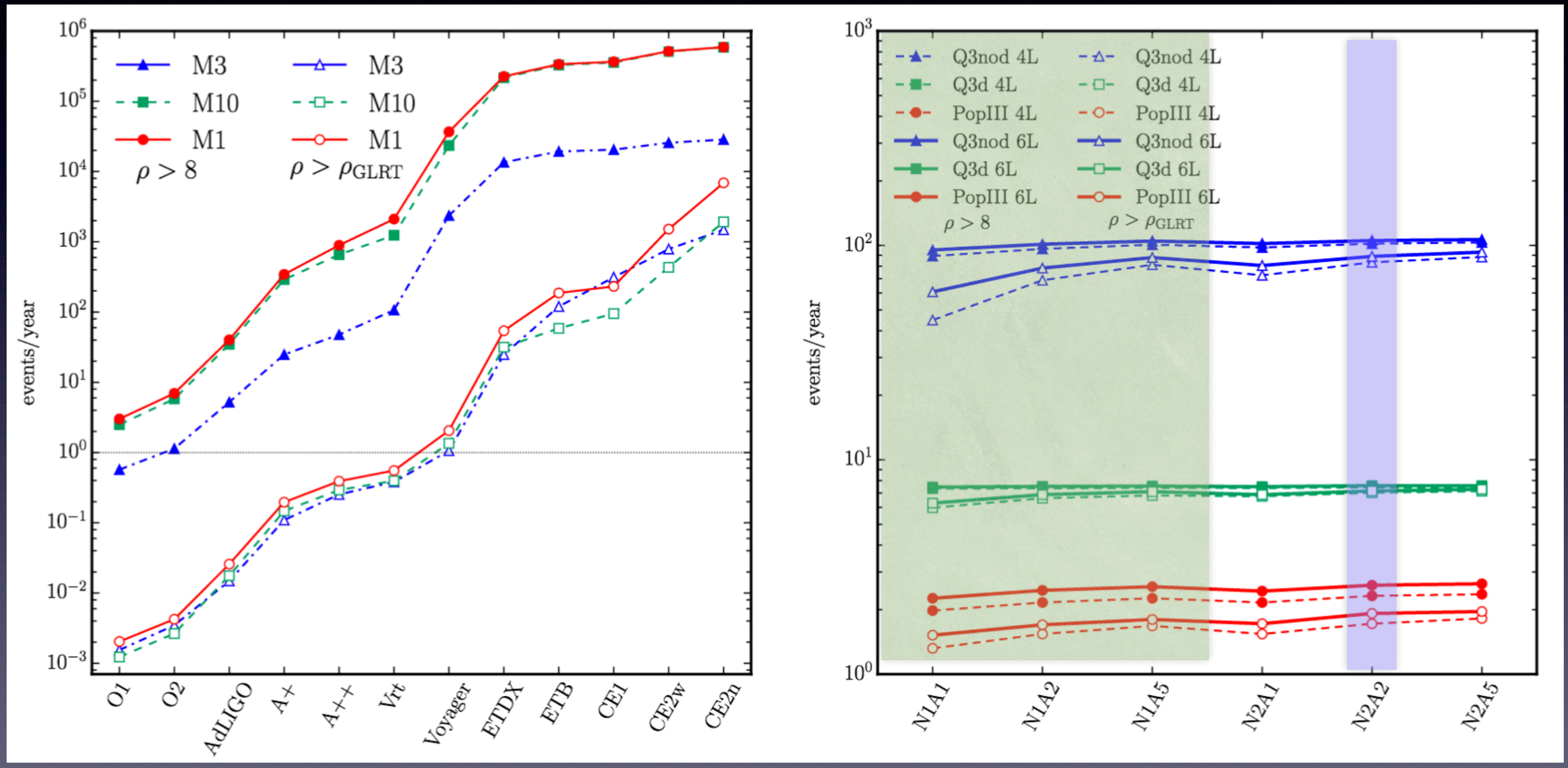
$$\dot{E}_{\rm GW} = \dot{E}_{\rm GR}\left[1 + B\left(\frac{Gm}{r_{12}c^2}\right)^{-1}\right]$$

- Pulsar constrain |B| $\lesssim 10^{-7}$, GW150914-like systems + LISA will constrain same dipole term in BH-BH systems to comparable accuracy



From EB, Yunes & Chamberlain 2016

Ringdown tests



$$\rho_{\rm GLRT} \equiv \min(\rho^{2,3}_{\rm GLRT}, \rho^{2,4}_{\rm GLRT})$$

But overtones of 22 mode may help (Giesler+2019)

Berti, Sesana, EB, Cardoso, Belczynski, 2016

More science with LISA…

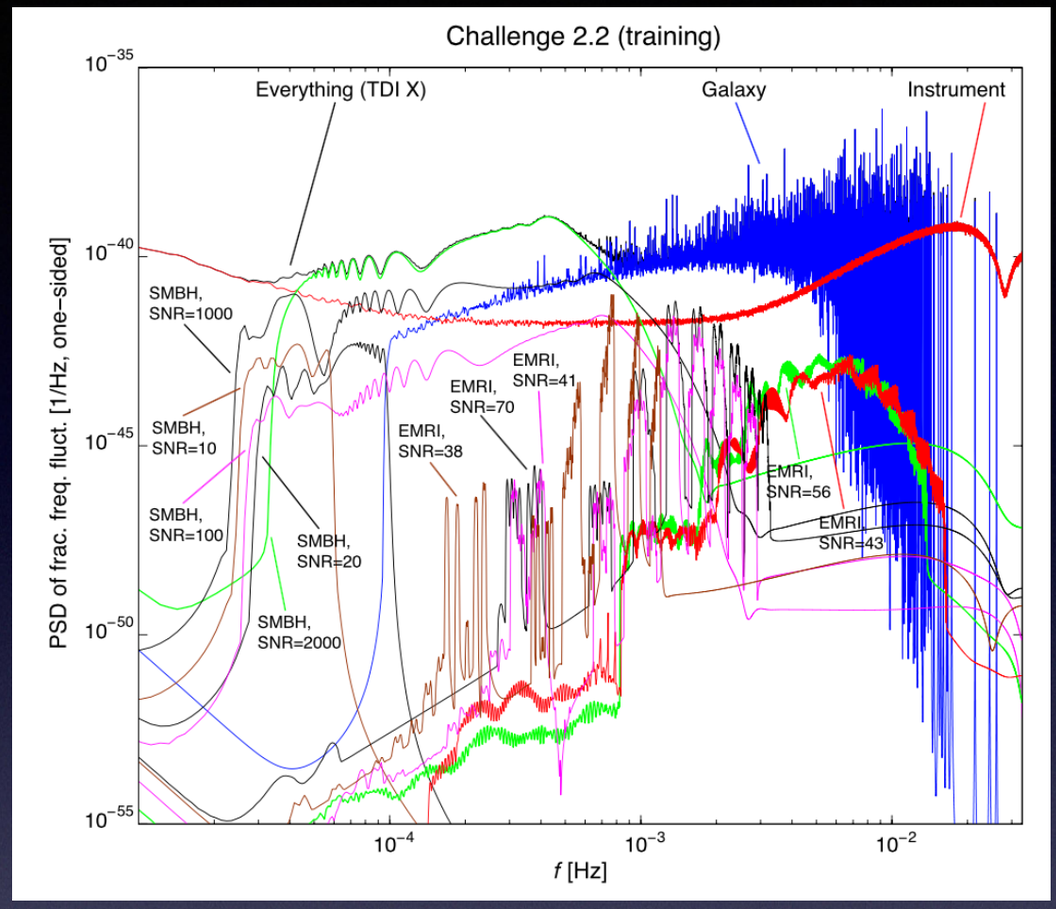
- Galactic white-dwarf binaries

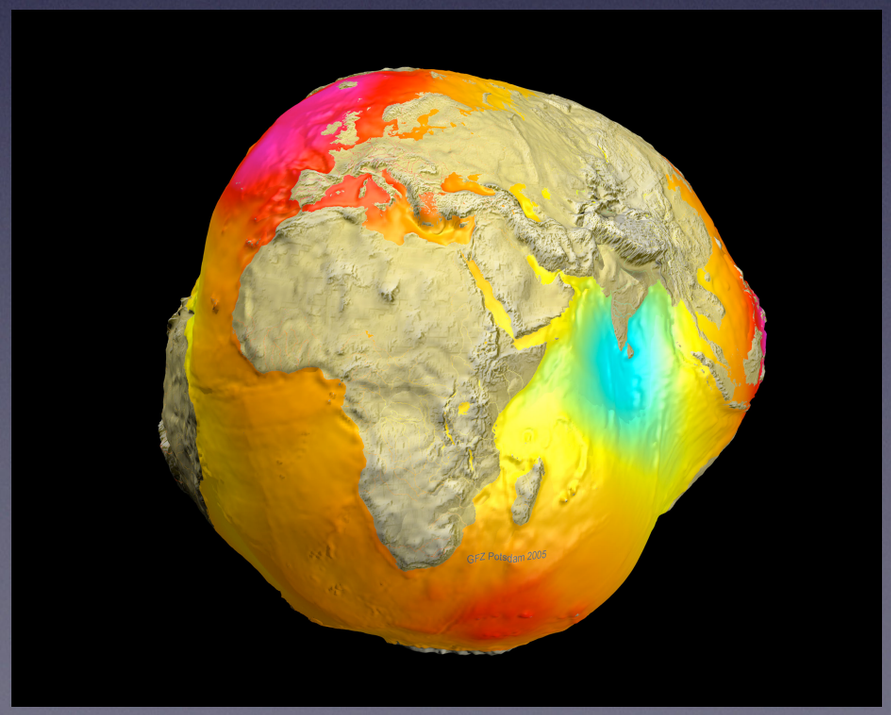
- Extreme mass ratio inspirals: star or “LIGO” BH + massive BH:

- Will test the “no hair” theorem

- Akin to mapping Earth’s gravitational field with artificial satellites

- Stochastic backgrounds from inflation/phase transitions in the early universe





Light bosons and GWs

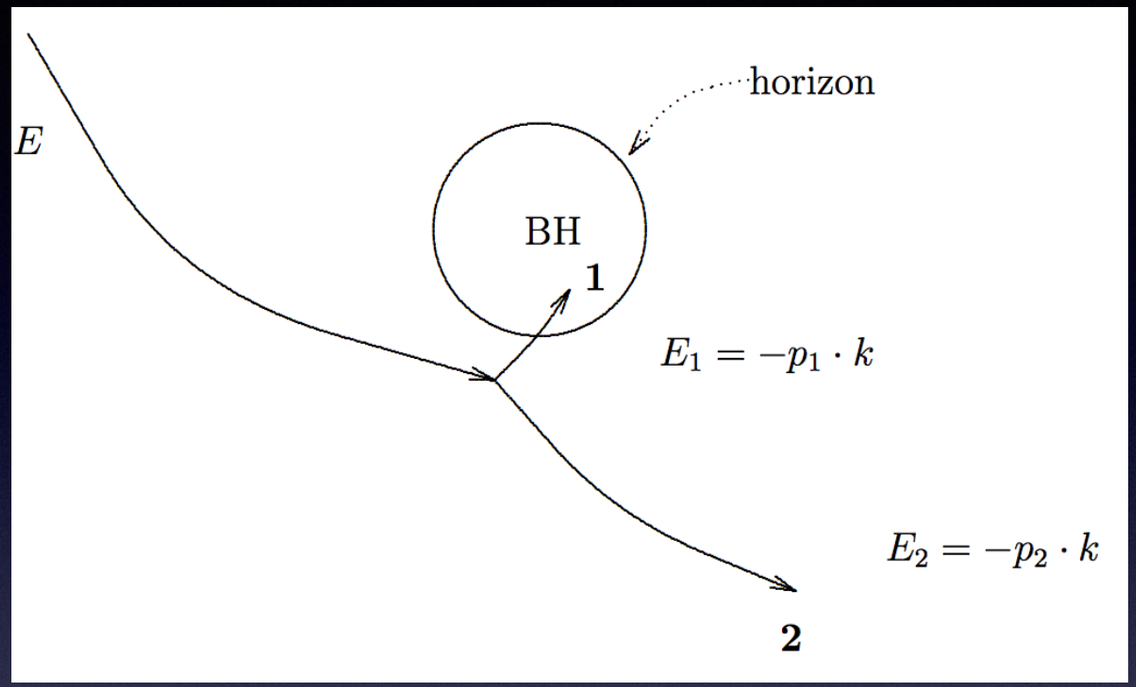
- Scalars ubiquitous in string theory, inflation, dark matter models (e.g. fuzzy/axionic dark matter)
- Useful as toy models for unknown phenomena/ interactions (e.g. modifications of GR)

- “Light” means <~ 1.e-10 eV
- Effect of mass term expected to be qualitatively similar for all boson degrees of freedom
- Can form condensates around rotating BHs if Compton wavelength ~ BH size

BH-boson condensates

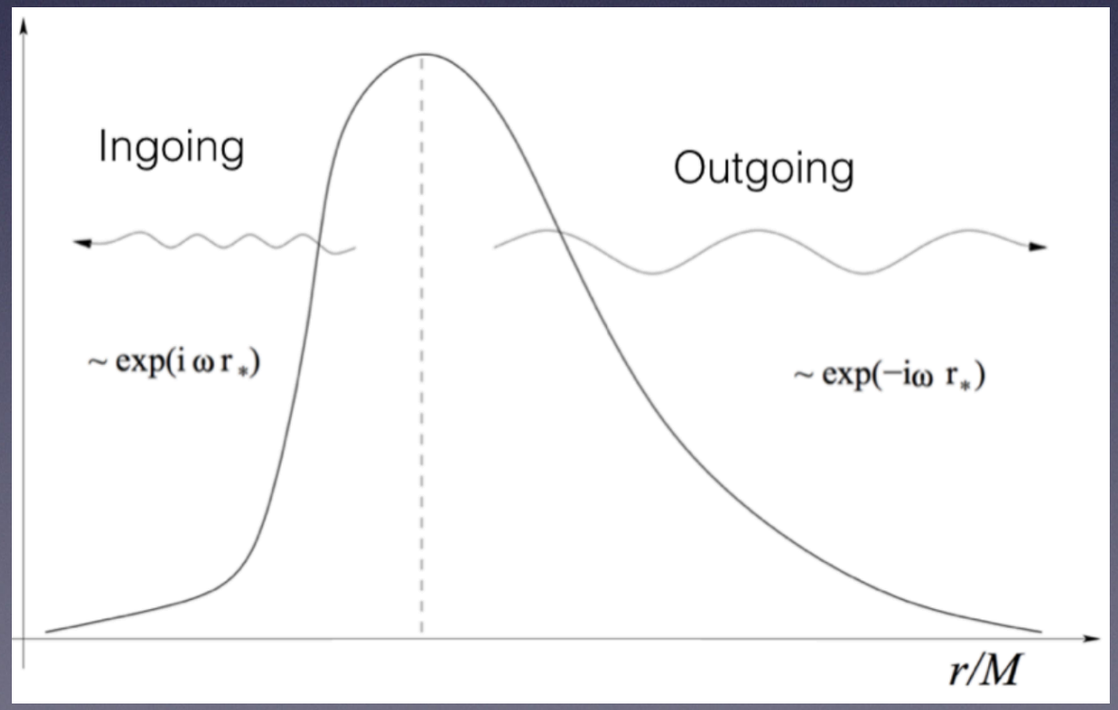
- Formation linked to superradiant instabilities/Penrose process (amplification of scattered waves with $\omega < m\Omega_H$)

- BH with high enough spin and “mirror” are superradiance unstable (BH bomb; Zeldovich 71, Press & Teukolsky 72, Cardoso et al 04)



- In ergoregion, negative energy modes can be produced but are confined (only positive energy modes can travel to infinity)

- By energy conservation, more and more negative energy modes can be produced, which may cause instability according to boundary conditions (at horizon and spatial infinity)



BH-boson condensates

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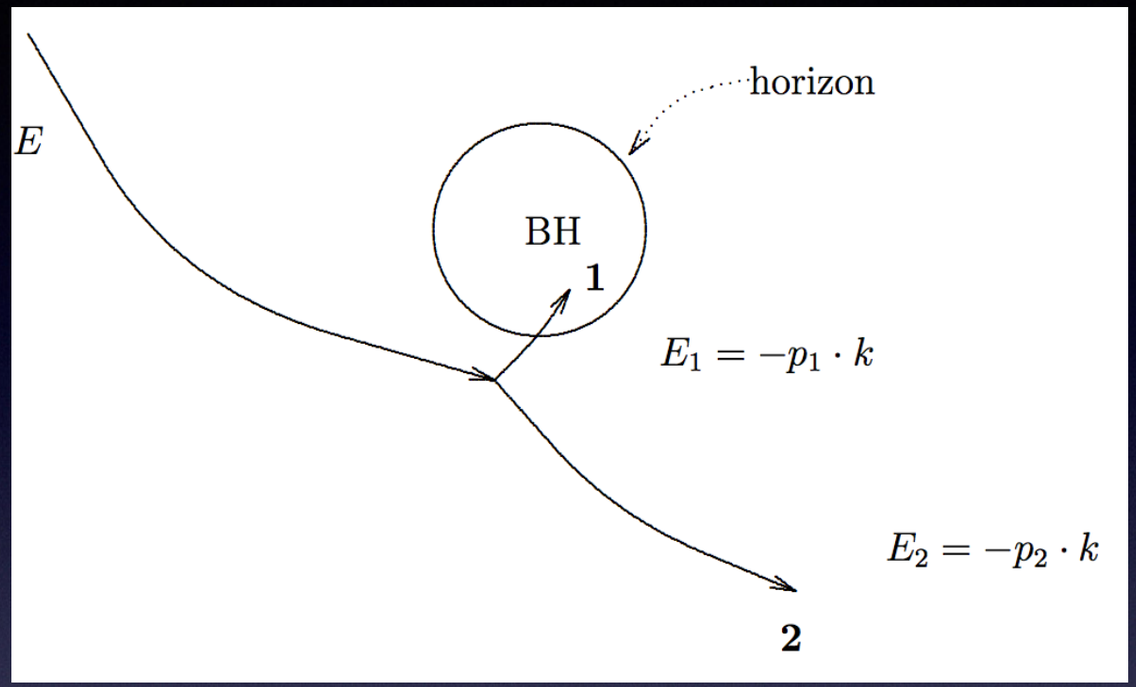
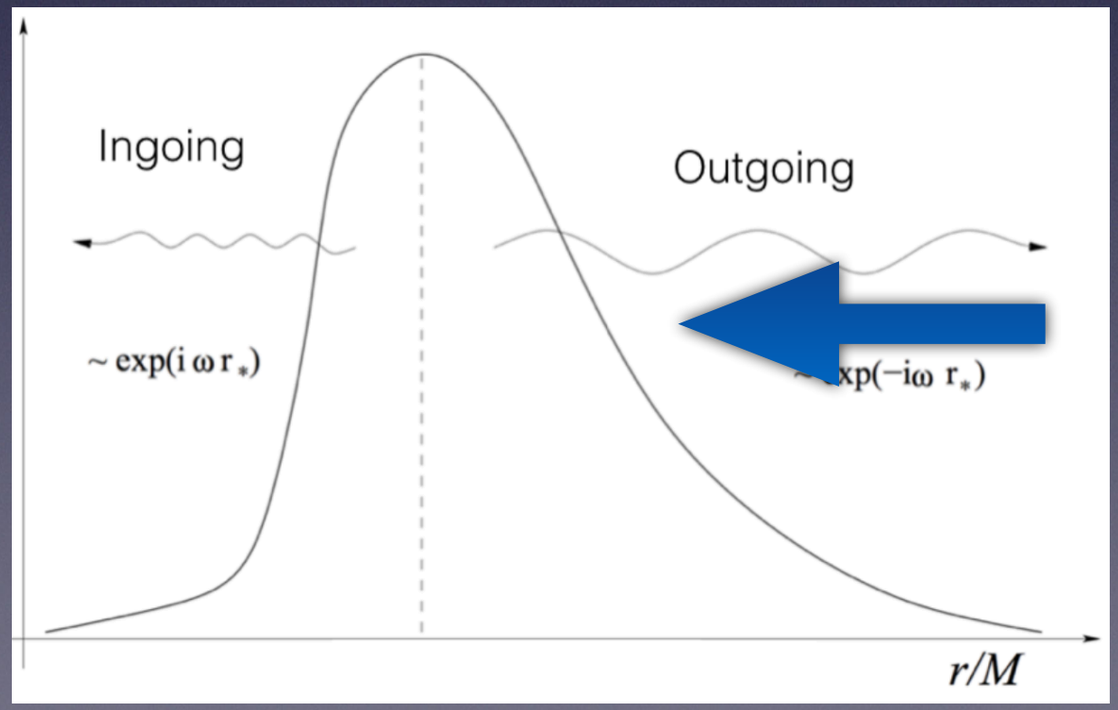
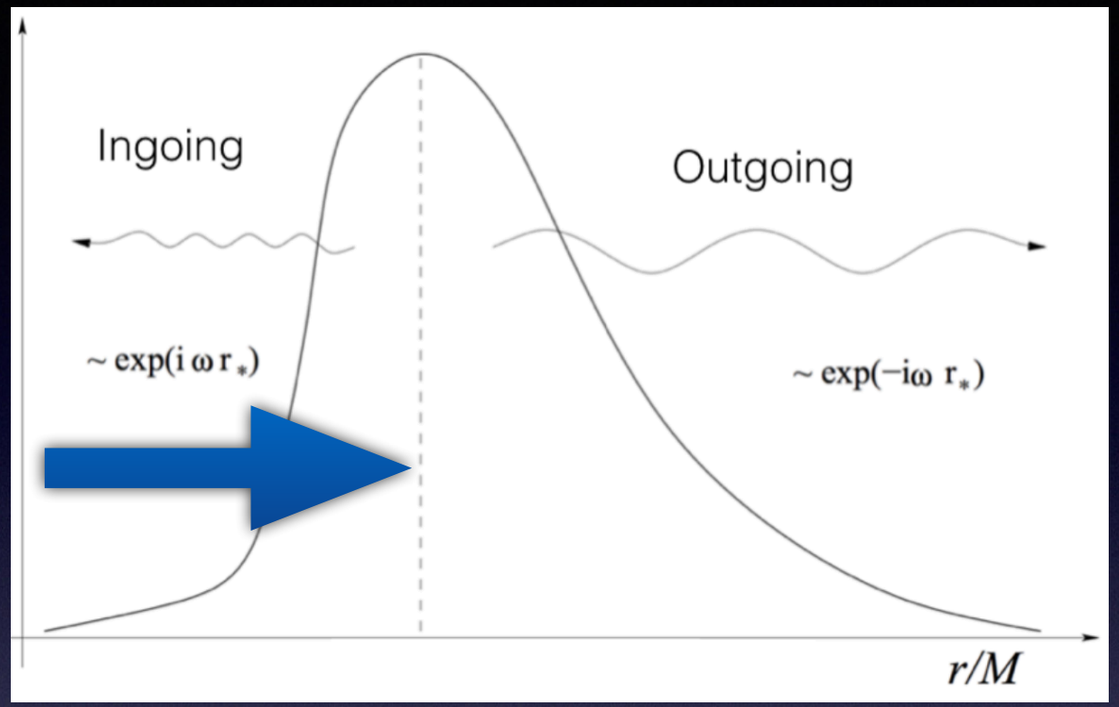


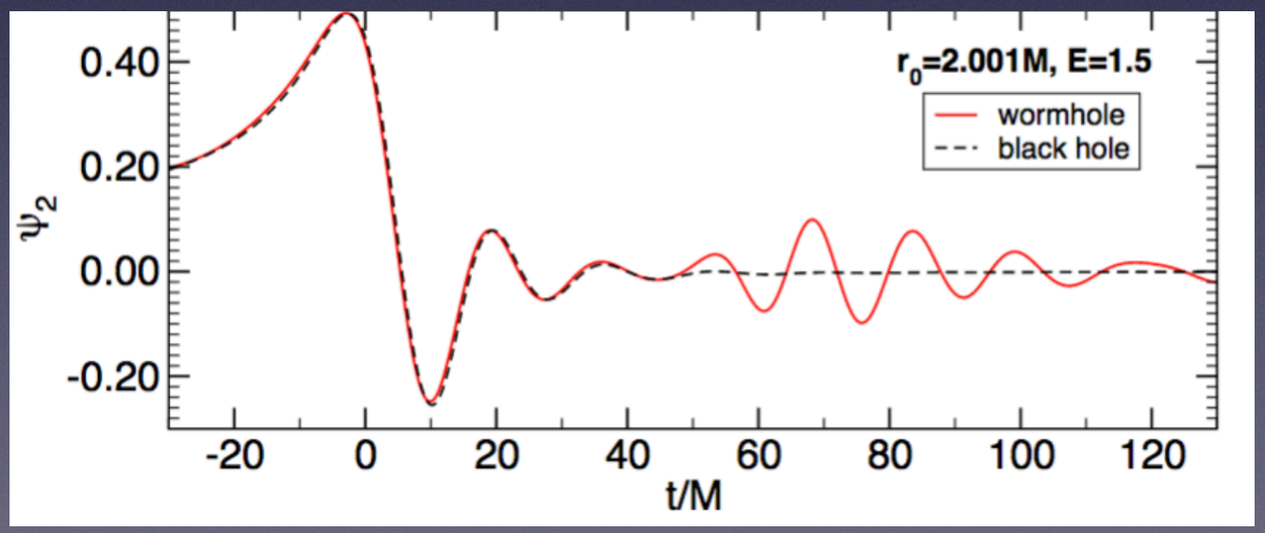
- In ergoregion, negative energy modes can be produced but are confined (only positive energy modes can travel to infinity)
- By energy conservation, more and more negative energy modes can be produced, which may cause instability according to boundary conditions (at horizon and spatial infinity)



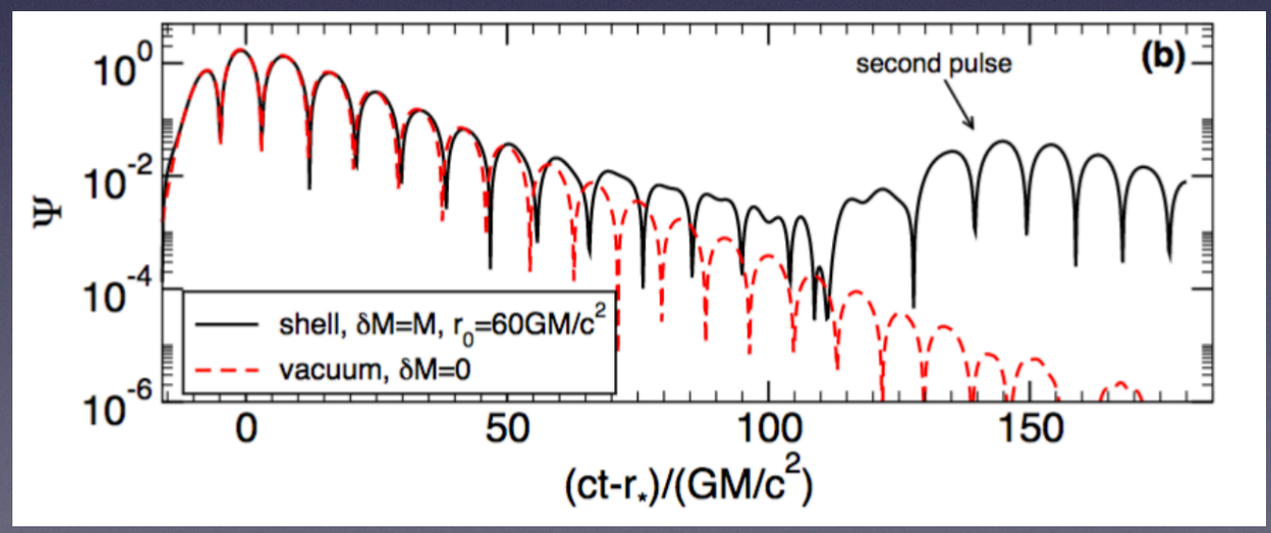
Superradiance from near horizon physics



- Deviations away from Kerr geometry near horizon (e.g. firewalls, gravastars, wormholes, Lorentz violations, etc) can produce significant changes in QNM spectrum
- Delays $\Delta t \sim \log[r_0/(2M) - 1]$



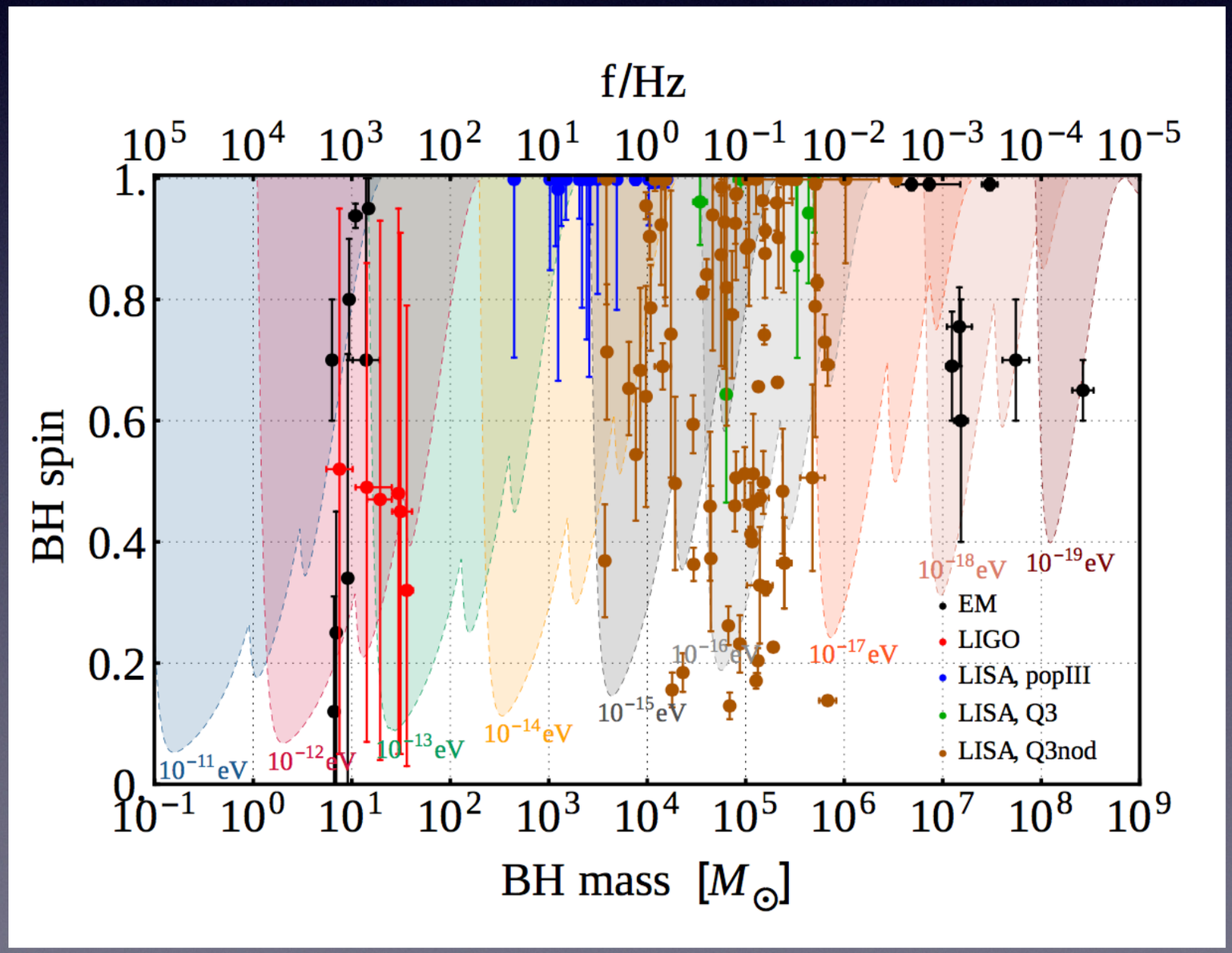
Cardoso, Franzin & Pani 2016



EB, Cardoso & Pani 2014

BH-boson condensates

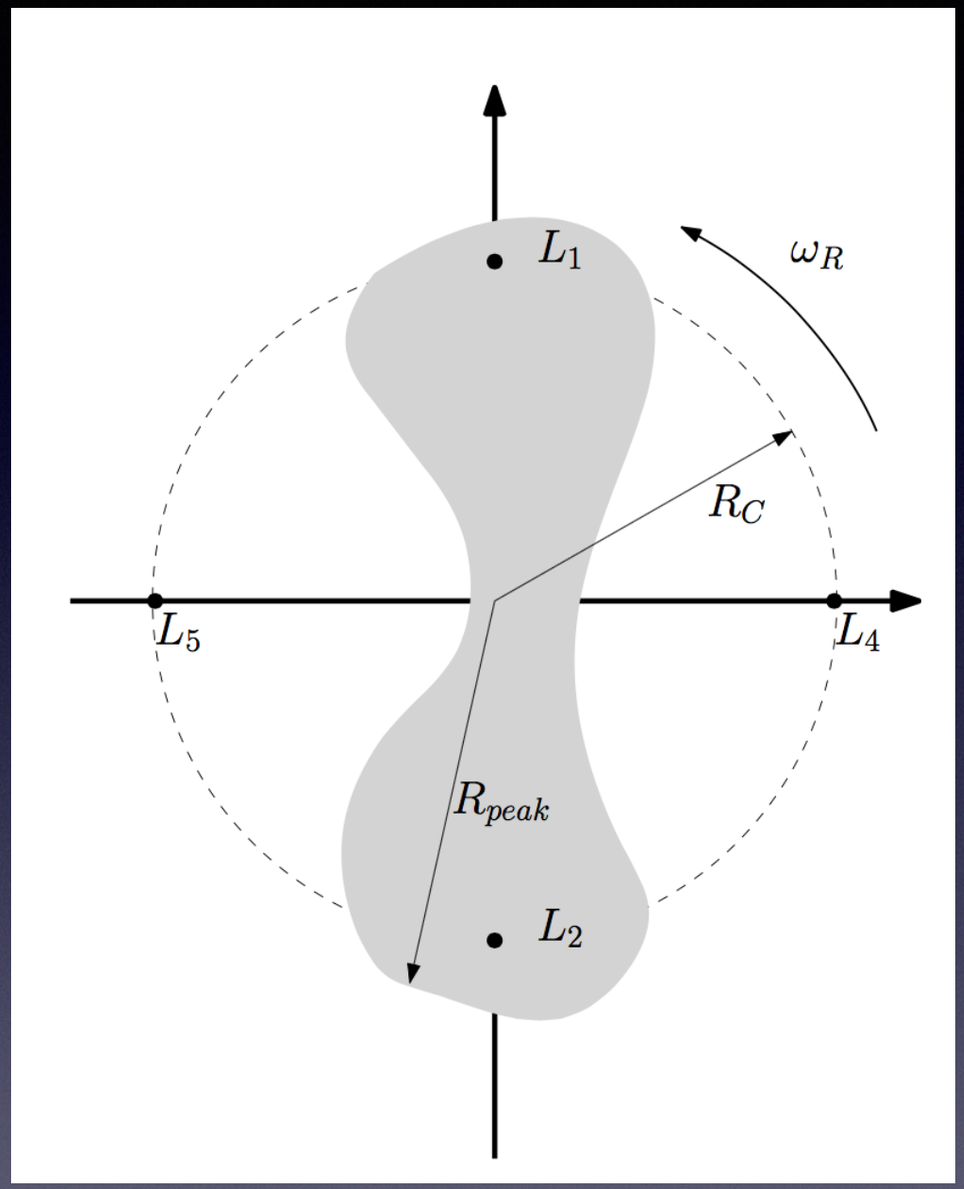
- Same instability of spinning BH + massive boson (mass acts as “mirror” and allows for bound states), but NOT for fermions. Cf Damour, Deruelle & Ruffini 76



Brito EB, et al 2017

Instability end point

- BH sheds excess spin (and to a lesser degree mass) into a mostly dipolar rotating boson cloud …



$$m_s \equiv \mu\hbar,$$

$$\omega_R \sim \mu - \frac{M^2\mu^3}{8}$$

$$\Phi = A_0 g(r)\cos(m_\phi\phi - \omega_R t)\sin\theta\,,$$

- … till instability saturates

$$\mu \sim m\Omega_{\rm H}$$

$$\tau_{\rm inst} \sim 0.07\,\chi^{-1}\left(\frac{M}{10\,M_\odot}\right)\left(\frac{0.1}{M\mu}\right)^9\ {\rm yr}\,,$$

(for Mμ<<1 and χ<<1; max instability for Mμ=0.42)

GW emission

- Long-lived rotating scalar dipole produces almost monochromatic GWs via quadrupole formula on timescale

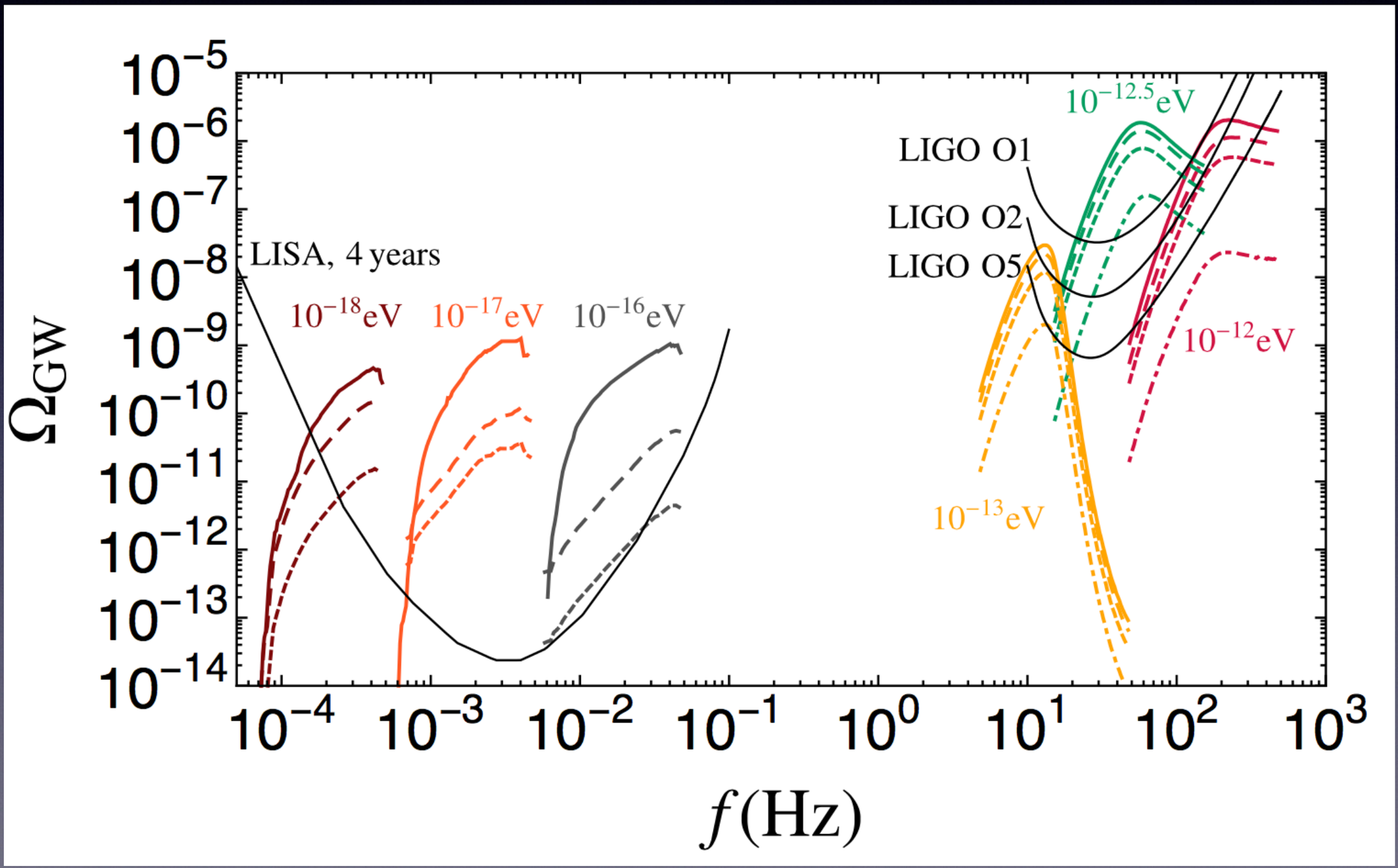
$$\tau_{\rm GW} \sim 6 \times 10^4 \, \chi^{-1} \left(\frac{M}{10\, M_\odot} \right) \left(\frac{0.1}{M\mu} \right)^{15} \, {\rm yr}$$

$$h = \sqrt{\frac{2}{5\pi}} \frac{GM}{c^2 r} \left(\frac{M_S}{M} \right) A(\chi, f_s M) \, ,$$

rms strain amplitude

$\omega \sim \mu$ frequency

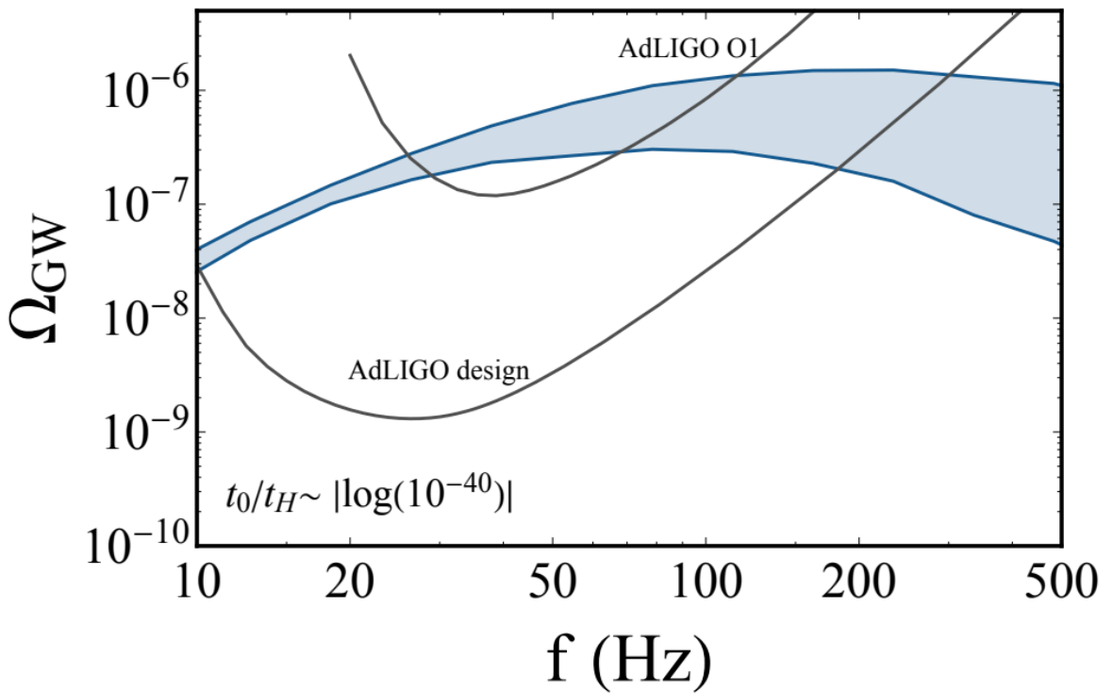
Background from isolated spinning BHs

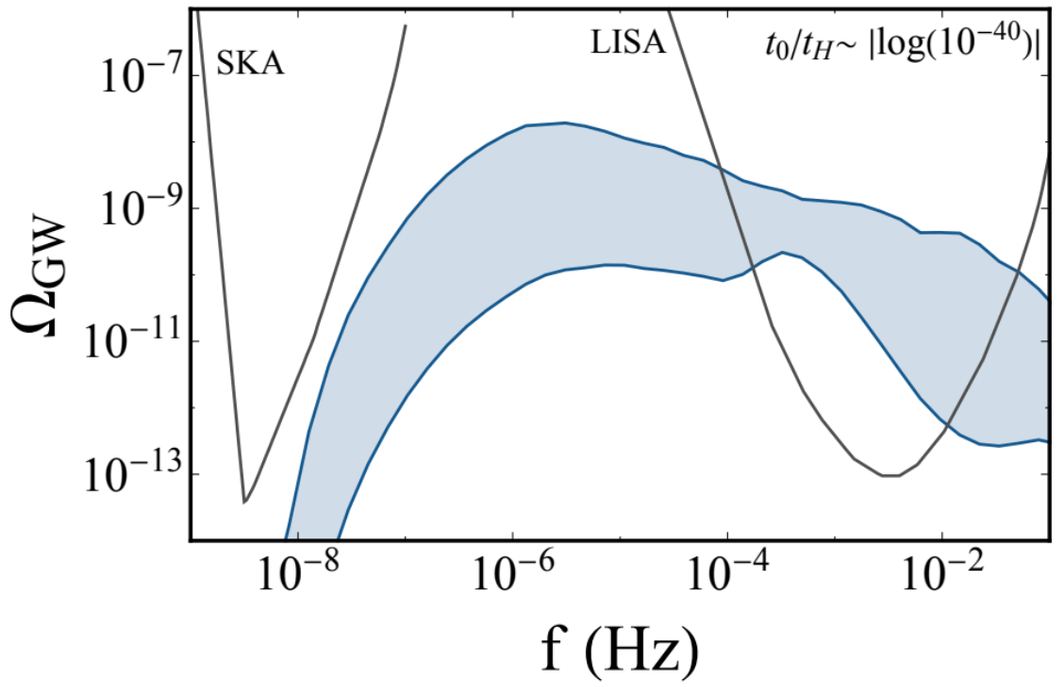


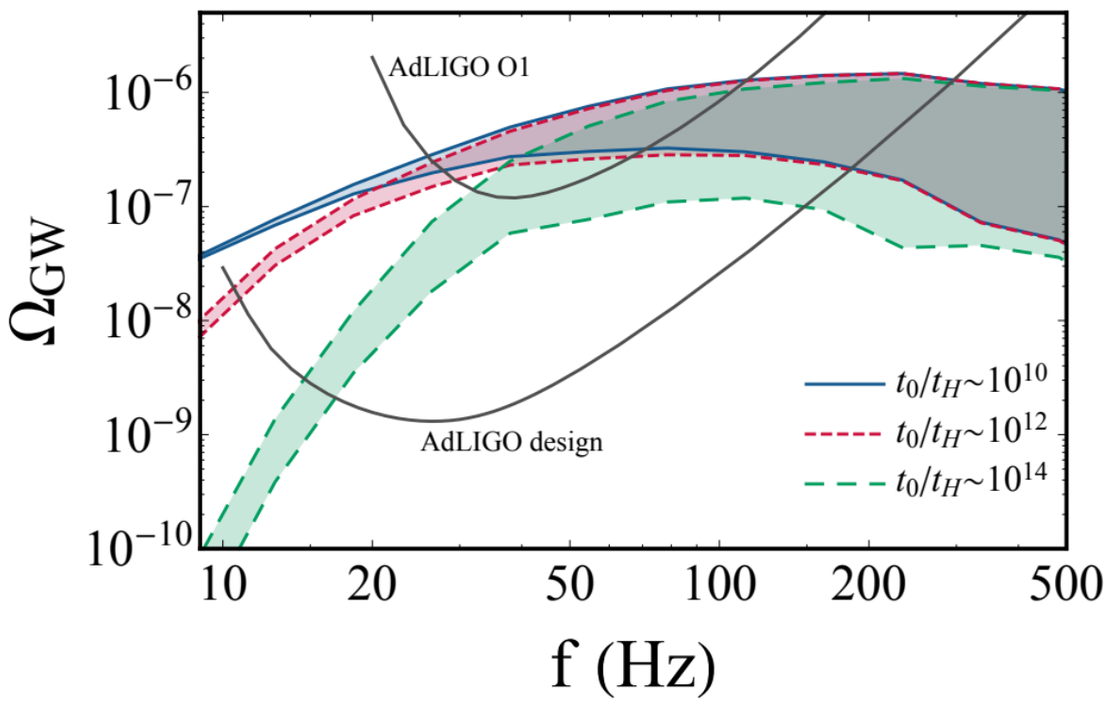
Brito EB, et al 2017

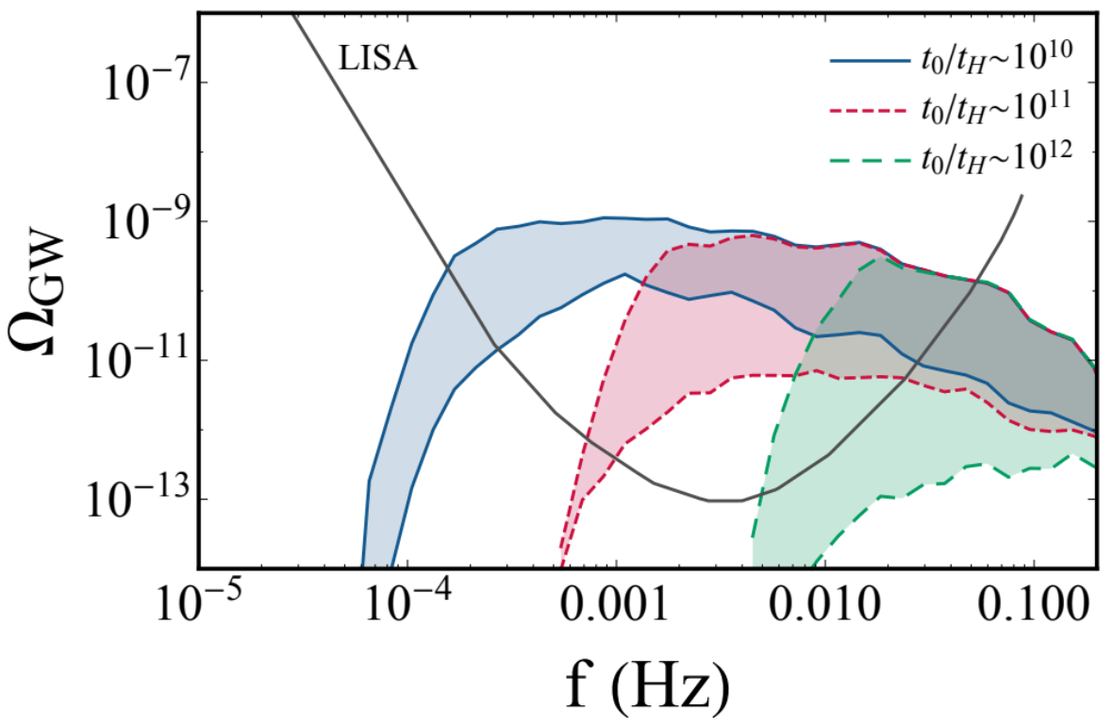
Bounds on BH mimickers

BH mimickers with no horizon are unstable to superradiance



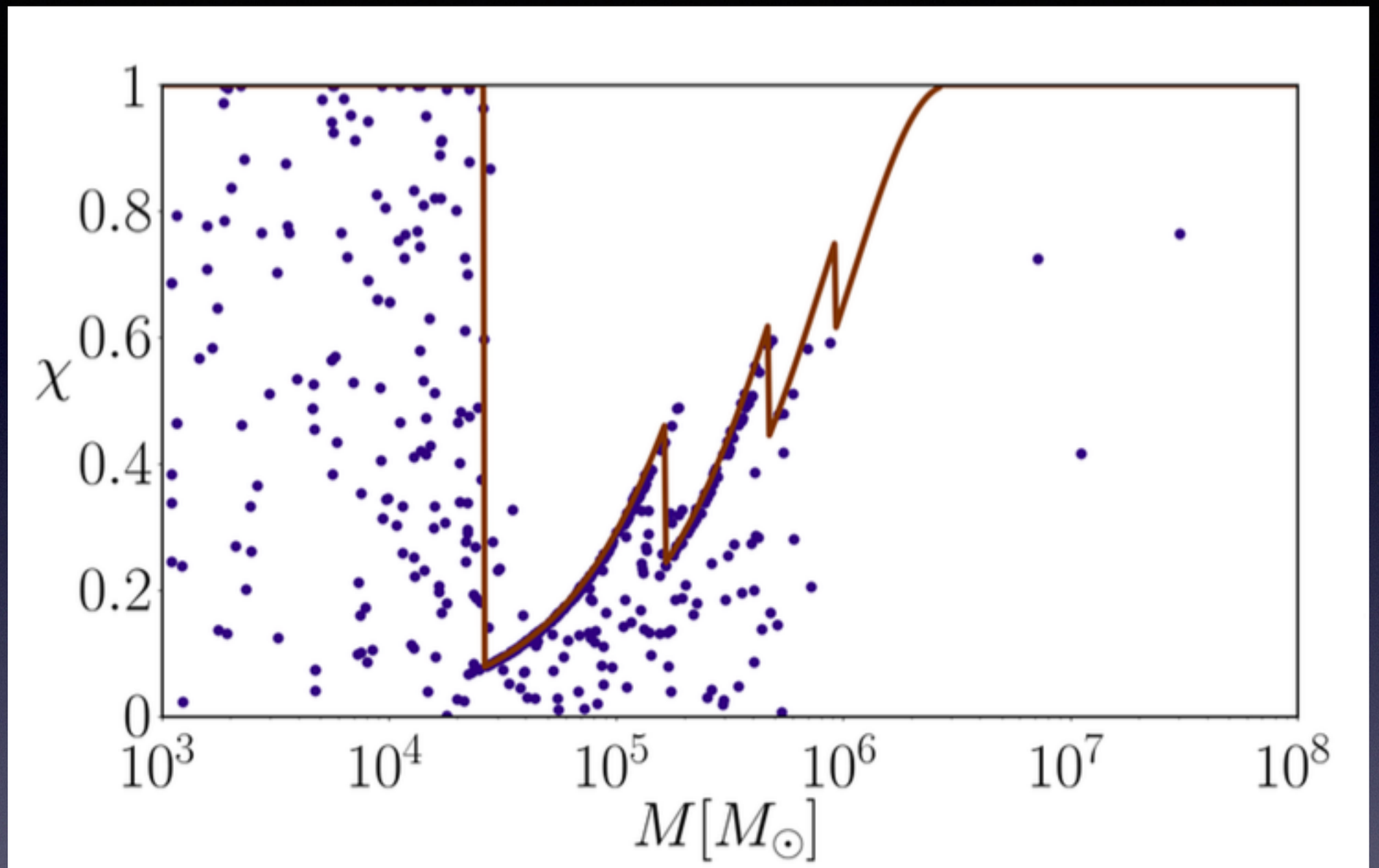






EB, et al 2018

Regge plane “holes”



Brito EB, et al 2017

Look for “accumulation” near instability threshold
to avoid having to make assumptions
on astrophysical model

Conclusion

Gravitational waves have opened a new window on the Universe, and the LIGO/Virgo detections are just the beginning…