Monte Carlo for the MUonE experiment

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Flavour Changing and Conserving Processes 2019

Villa Orlandi, Anacapri, August 29-31, 2019

with M. Chiesa, G. Montagna, O. Nicrosini and F. Piccinini

Outline

- Introduction
- QED & EWK NLO corrections to $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}$ (and their Monte Carlo implementation)

  M. Alacevich *et al.*, JHEP 02 (2019) 155

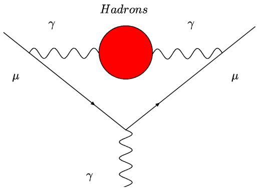
  - Details of the calculation
  - Phenomenology of NLO corrections
    - ✓ QED corrections (and splitting into gauge-invariant subsets)
    - ✓ EWK corrections
    - ✓ finite electron-mass effects
- Towards NNLO: "easy" deliverables at NNLO
- Conclusion and Outlook

Master formula

- Standard approach

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{e^+e^-\to\text{had}}(s)$$

$$K(s) = \int_0^1 \frac{x^2(1-x)}{x^2+(1-x)\frac{s}{m_\mu^2}}$$

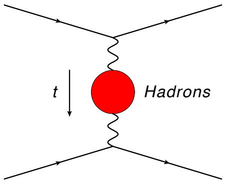


- Alternatively (exchanging s and x integrations in a_μ^{HLO})

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



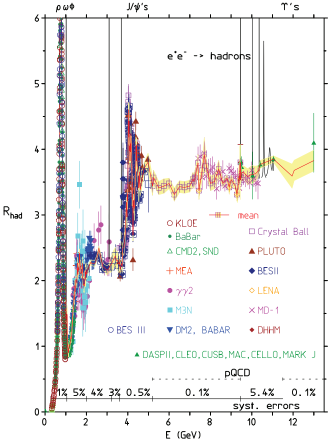
⋆ $\Delta\alpha_{\text{had}}(t)$ can be directly measured in a (single) experiment involving t-channel (space-like) scattering

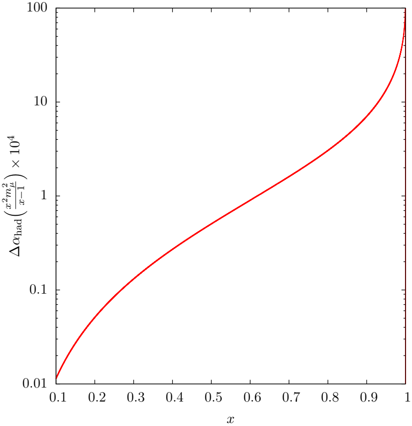
Arbuzov *et al.* EPJC 34 (2004) 267

Abbiendi *et al.* (OPAL) EPJC 45 (2006) 1

Time-like $\mapsto$ Space-like

F. Jegerlehner, EPJ Web Conf. 118





Smooth function

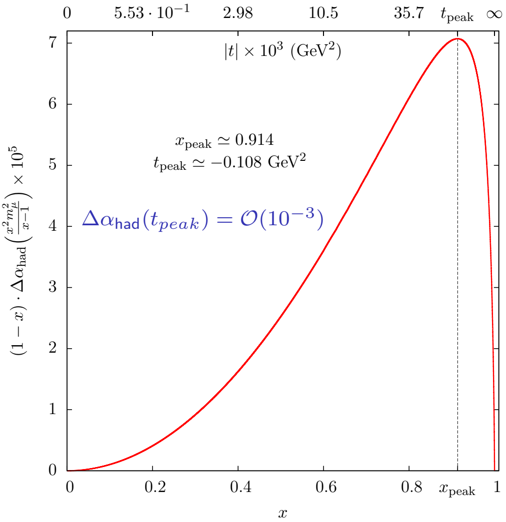
CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

$\mapsto$ **Time-like:** combination of many experimental data sets,
control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required.

$\mapsto$ **Space-like:** in principle, one single experiment.
Need to measure a one-loop effect, very high accuracy needed.

General considerations

- integrand function $(1-x)\Delta\alpha_{\rm had}[t(x)]$



$$x_{\rm peak} \simeq 0.914 \qquad t_{\rm peak} \simeq -0.108 \text{ GeV}^2 \simeq -(329 \text{ MeV})^2$$

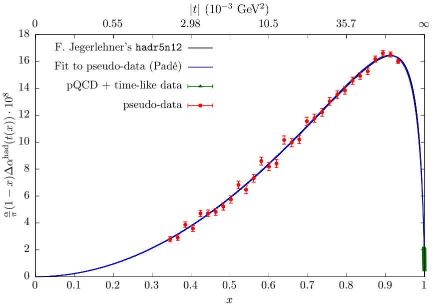
General considerations

- To get $\Delta\alpha_{\rm had}(t)$, the goal is to measure the running of $\alpha_{\rm QED}(t)$

  $\rightarrow$ **The idea:** Bhabha events at e^+e^- (low-energy) colliders **[original proposal]**
  CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

  $\rightarrow$ or μe scattering events in a fixed target experiment **[MUonE proposal]**
  Abbiendi *et al.* EPJC 77 (2017) no.3, 139

$$\alpha(t) = \frac{\alpha}{1 - \Delta\alpha_{\text{all the rest}}(t) - \Delta\alpha_{\text{had}}(t)} \qquad \Delta\alpha_{\text{had}}(t) = 1 - \Delta\alpha_{\text{all the rest}}(t) - \frac{\alpha}{\alpha(t)}$$



Strategy:

- measure $\Delta\alpha_{\rm had}(t)$ within the exp. range
- get large $|t|$ values from elsewhere (time-like data, lattice)
- fit $\Delta\alpha_{\text{had}}(t)$
- integrate to get a_μ^{HLO}

Roughly, to be competitive with the current evaluations, $\Delta\alpha_{\rm had}(t)$ needs to be known at the sub-% level

$\mu e \to \mu e$ scattering in fixed target experiment

Abbiendi *et al.* EPJC 77 (2017) no.3, 139

Part of the CERN *Physics Beyond Colliders* program, **LoI under review by the SPSC**

↦ A 150 GeV high-intensity ($\sim 10^7$ μ's/s) muon beam is available at CERN NA

↦ Muon scattering on a low-Z target ($\mu e \to \mu e$) looks an ideal process

⋆ it is a "pure" t-channel process →

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha}\right|^2 \qquad \frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\rm had}, \qquad \Delta\alpha_{\rm had}(t_{peak}) = \mathcal{O}(10^{-3})$$

⋆ Assuming a 150 GeV incident muon beam we have

$s \simeq 0.164$ GeV^2 $\quad -0.143 \lesssim t < 0$ GeV^2 $\quad 0 < x \lesssim 0.93$ **it spans the peak!**

Pros: **it can cover 87% of the $a_\mu^{\rm HLO}$ integral!**

⋆ existing μ-beam at CERN with all requirements (M2 beam line)

⋆ highly boosted kinematics

⋆ the same detector and process can be exploited for signal and normalization: for $x \lesssim 0.3$, $\Delta\alpha_{\rm had}(t) < 10^{-5}$ → normalization region

Cons:

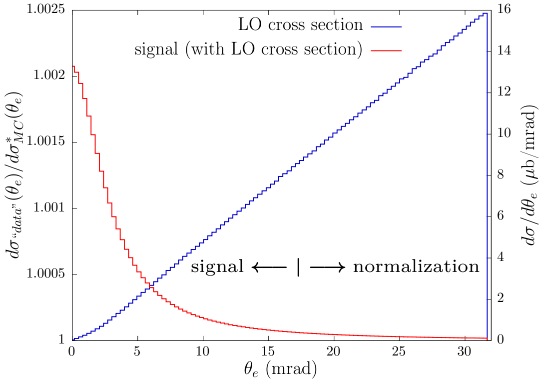
⋆ high accuracy needed: control of systematics at the 10^{-5} level

Our "signal"

$$\text{Our signal} \equiv \frac{dN_{data}(O_i)}{dN_{MC}(O_i)|_{\Delta\alpha_{\rm had}(t)=0}} \equiv \frac{dN_{data}(O_i)}{dN^*_{MC}(O_i)} =$$

$$= \frac{d\sigma_{data}(O_i)}{d\sigma^*_{MC}(O_i)} = \frac{dN_{data}(O_i)}{N^{norm}_{data}} \times \frac{\sigma^{norm}_{MC}}{d\sigma^*_{MC}(O_i)} \simeq$$

$$\simeq 1 + 2\left[\Delta\alpha_{\rm lep}(O_i) + \Delta\alpha_{\rm had}(O_i)\right] \qquad \text{(at LO)}$$



A first step, radiative corrections at NLO in QED

- The μe cross section and distributions must be known as precisely as possible
  $\mapsto$ radiative corrections (RCs) are mandatory

⋆ First step are QED $\mathcal{O}(\alpha)$ (i.e. QED NLO, next-to-leading order) RCs

The NLO cross section is split into two contributions,

$$\sigma_{NLO} = \sigma_{2\to 2} + \sigma_{2\to 3} = \sigma_{\mu e\to \mu e} + \sigma_{\mu e\to \mu e\gamma}$$

$\mapsto$ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$

$\mapsto$ $[2\to 2]/[2\to 3]$ phase space splitting at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$

- $\mu e \to \mu e$

$$\sigma_{2\to 2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2 (|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^{*} \times \mathcal{A}_{NLO}^{virtual}(\boldsymbol{\lambda})])$$

- $\mu e \to \mu e\gamma$

$$\sigma_{2\to 3} = \frac{1}{F}\int_{\omega > \boldsymbol{\lambda}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2\right)$$

$$= \Delta_s(\boldsymbol{\lambda}, \boldsymbol{\omega_s}) \int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2$$

- the integration over the 2/3-particles phase space is performed with MC techniques and **fully-exclusive events are generated**

NLO: method and cross-checks

- Calculation performed in the on-shell renormalization scheme
- **Full mass dependency kept everywhere**, fermions' helicities kept explicit
- Diagrams manipulated with the help of `FORM`, independently by at least two of us
  [perfect agreement]
  J. Vermaseren, `https://www.nikhef.nl/~form`
- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with `LoopTools` and `Collier` libraries
  [perfect agreement]
  T. Hahn, `http://www.feynarts.de/looptools`
  A. Denner, S. Dittmaier, L. Hofer, `https://collier.hepforge.org`
- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations
  [perfect agreement]
- Comparisons with past/present independent results
  [all good]
  T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725
  D. Y. Bardin and L. Kalinovskaya, DESY-97-230, `hep-ph/9712310`
  N. Kaiser, J. Phys. G **37** (2010) 115005
  Fael & Passera
- Also NLO weak RCs calculated [tiny, see later]

Simulation setups & RCs

- 4 setups have been considered for $E_\mu^{\text{beam}} = 150$ GeV.

  Notice: $\sqrt{s} \simeq 0.4055$ GeV $\qquad t_{ee,\mu\mu}^{min} = -\lambda(s, m_\mu^2, m_e^2)/s \simeq -0.143$ GeV^2

Setup 1:

- E_e (in the lab) ≥ 0.2 GeV $\quad (\to t_{ee}^{max} \lesssim -2.04 \cdot 10^{-4}$ GeV^2) and $\theta_e, \theta_\mu \leq 100$ mrad

Setup 2:

- E_e (in the lab) ≥ 1 GeV $\quad (\to t_{ee}^{max} \lesssim -1.02 \cdot 10^{-3}$ GeV^2) and $\theta_e, \theta_\mu \leq 100$ mrad

Setup 3:

- **Setup 1 + acoplanarity cut**, *i.e.* acoplanarity $\equiv |\pi - (\phi_e - \phi_\mu)| \leq 3.5$ mrad

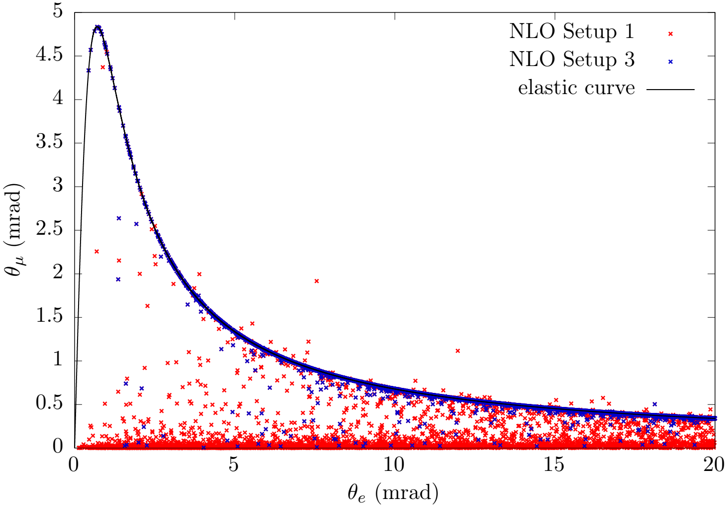
Setup 4:

- **Setup 2 + acoplanarity cut**

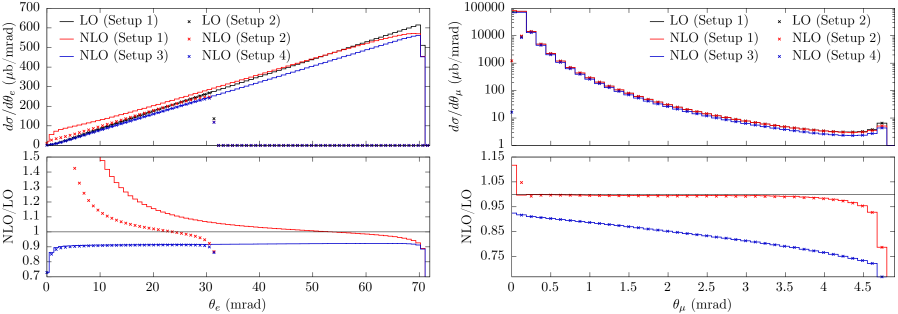
- both processes $\mu^\pm e^- \to \mu^\pm e^-$ considered
- full QED NLO, gauge-invariant subsets (e-, μ-line corrections, interference), $m_e \to 0$ limit, weak LO & NLO RCs, **any VP switched off**

[More realistic elasticity cuts are being explored together with experimental colleagues]

θ_e-θ_μ correlation (in the lab. frame)

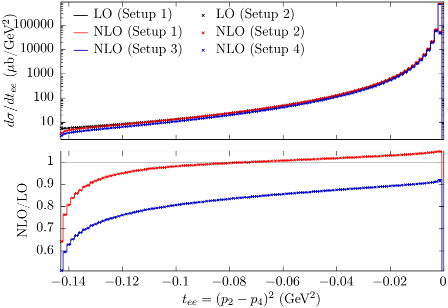


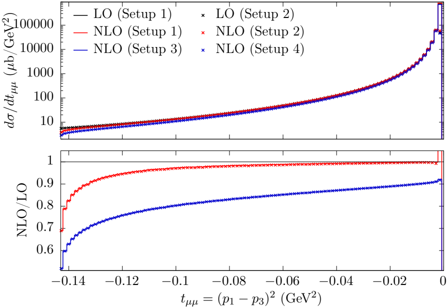
QED RCs on θ_e & θ_μ (incoming μ^+)



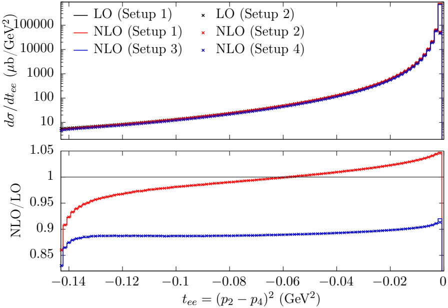
- Large RCs on θ_e in Setup 1 & 3 induced by hard bremsstrahlung

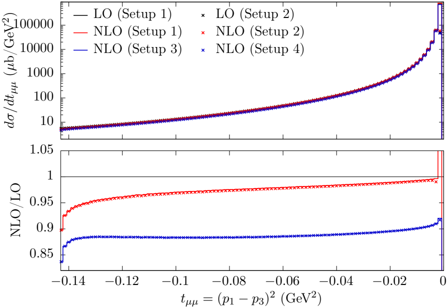
QED RCs on t_{ee} & $t_{\mu\mu}$ (incoming μ^+)





QED RCs on t_{ee} & $t_{\mu\mu}$ (incoming μ^-)



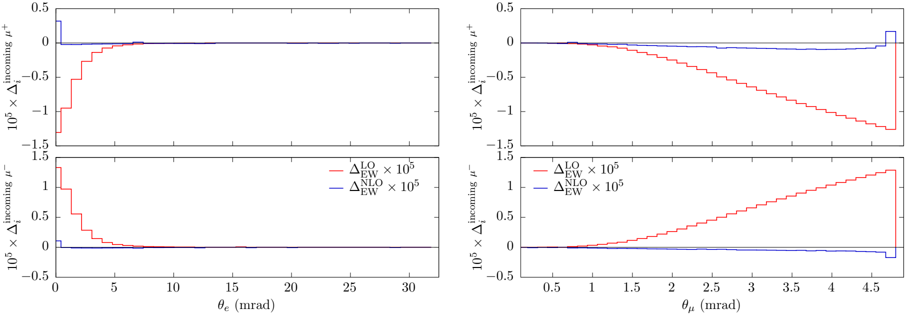


EWKology on θ_e & θ_μ

$\mapsto$ Full EWK RCs calculated in the on-shell (complex mass) scheme with **RECOLA**

S. Actis *et al.*, JHEP 04:037, 2013

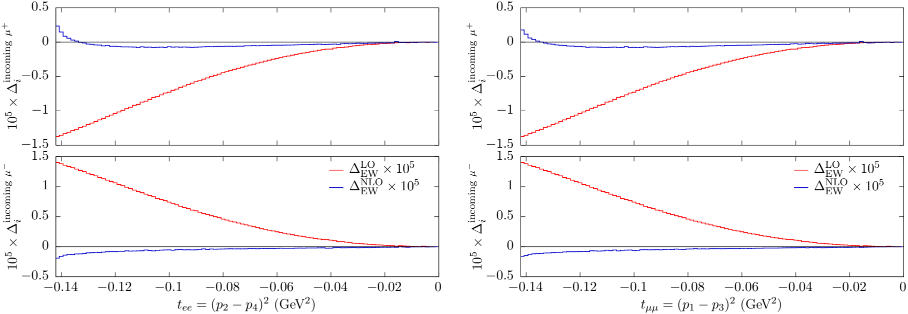
S. Actis *et al.*, CPC 214:140–173, 2017



$$\Delta_{\text{EW}}^{\text{LO}} = \frac{d\sigma_{\text{EW}}^{\text{LO}} - d\sigma_{\text{QED}}^{\text{LO}}}{d\sigma_{\text{QED}}^{\text{LO}}} \qquad \Delta_{\text{EW}}^{\text{NLO}} = \frac{\left(d\sigma_{\text{EW}}^{\text{NLO}} - d\sigma_{\text{EW}}^{\text{LO}}\right) - \left(d\sigma_{\text{QED}}^{\text{NLO}} - d\sigma_{\text{QED}}^{\text{LO}}\right)}{d\sigma_{\text{QED}}^{\text{NLO}}}$$

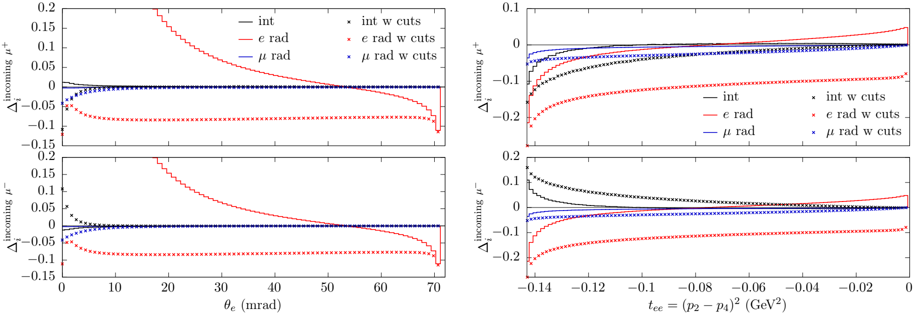
- $\Delta_{\text{EW}}^{\text{NLO}}$ measures the (gauge-invariant) purely weak RC, in QED NLO units

EWKology on t_{ee} & $t_{\mu\mu}$



- tree-level Z-exchange important at the 10^{-5} level
- purely weak RCs (in QED NLO units) at a few 10^{-6} level

Gauge-invariant subsets on θ_e and t_{ee} (Setup 1 & 3)



$$\Delta_i^{\text{incoming } \mu^\pm} = \frac{d\sigma_i^{\text{NLO}} - d\sigma^{\text{LO}}}{d\sigma^{\text{LO}}}$$

- "$e(\mu)$ rad": QED RCs only on electron (muon) current
- "int": full − [e rad] − [μ rad]
- ⋆ in general: $|e \text{ rad}| > |\text{int}| > |\mu \text{ rad}|$

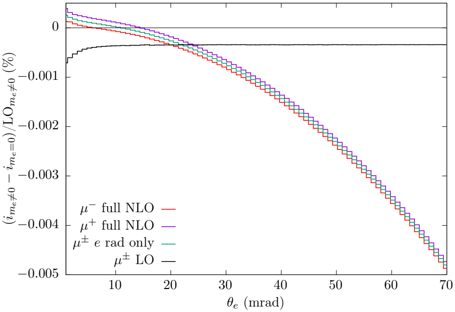
Electron mass effects ($m_e \to 0$ limit)

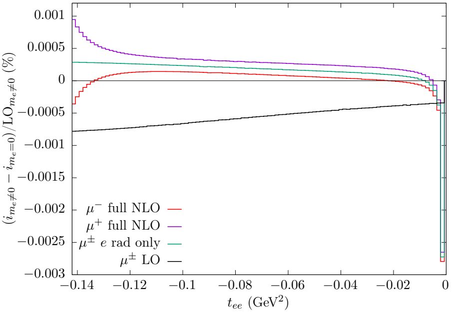
✓ Studied at NLO.
Can it give a grasp of finite m_e effect at NNLO?

1. Fully massive 4-momenta, phase space and flux kept.
   [Otherwise the frame where e^- is at rest can't be defined...]
2. $2 \to 2$ amplitudes expressed as functions of s and t.
3. Virtual amplitudes: fully reduced to scalar functions.
   Everything $\propto \log \lambda$ is kept massive [**IR part**].
   In the non-IR part, $u = 2m_\mu - s - t$ and everything $\propto m_e$ is neglected, *except* $\log m_e^2$.
4. Soft real: similarly, full m_e dependency in IR terms $\propto \log(\omega_s/\lambda)$, m_e neglected in the remainder, *except* $\log m_e^2$.
5. Real $(\omega \geq \omega_s)$: m_e kept everywhere.
   Finite m_e corrections come from the interplay [phase-space integration]/[matrix elements], difficult to disentangle unambiguously.

- Following **1.-5.**, no spurious IR dependence is left.

Electron mass effects (limit $m_e \to 0$) at NLO



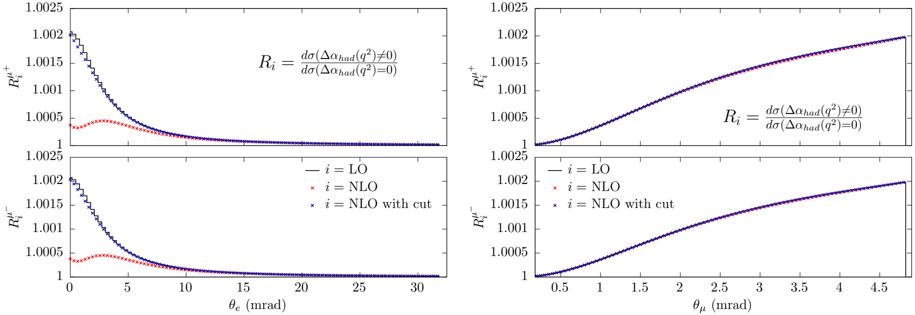


- with our definitions, finite m_e effects at NLO lie in the range of some 10^{-5}, dominated by e current corrections
- ***Educated guess***: at NNLO, it's likely finite e mass can be neglected

Signal sensitivity to RCs

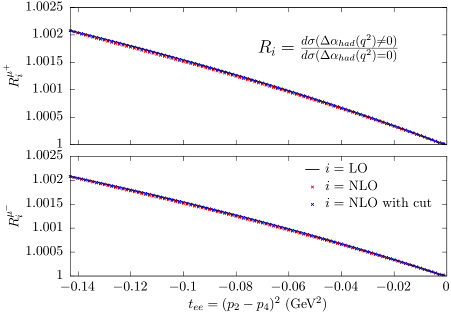
$$\text{Our “signal” on observable } O \equiv \frac{d\sigma^{\text{best}}(O, \Delta\alpha_{\text{had}}(q^2) \neq 0)}{d\sigma^{\text{best}}(O, \Delta\alpha_{\text{had}}(q^2) = 0)}$$

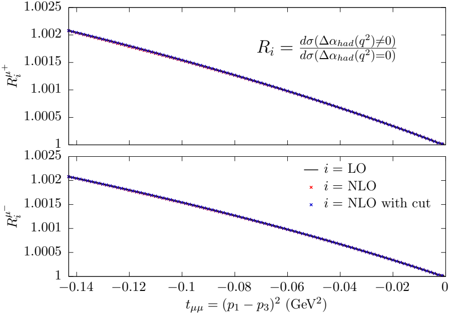
- Does it survive radiative corrections?



- Elasticity cuts mandatory to keep signal sensitivity on θ_e
- θ_μ is more “robust” under RCs (in particular “hard” photon radiation)

Signal sensitivity to RCs





"Quick" deliverables at NNLO (from Monte Carlo point of view)

- An impressive amount of work is currently put in NNLO/resummation calculations

M. Fael and M. Passera, PRL 122 (2019) 19, 192001

M. Fael, JHEP 1902 (2019) 027

S. Di Vita *et al.*, JHEP 1809 (2018) 016

P. Mastrolia *et al.*, JHEP 1711 (2017) 198

2nd ThinkStart/WorkStop: Theory of μ-e scattering @ 10ppm, Zurich, February 4-7 '19

see next talk by Jonathan Ronca

- QED NLO to $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}\gamma$
  - ✓ (almost) straightforward

- $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}e^{+}e^{-}, \to \mu^{\pm}e^{-}\mu^{+}\mu^{-}$ (NNLO leptonic real pair corrections)
  - ✓ partially cancelled by NNLO virtual leptonic RCs
  - ✗ how the experiment will deal with these final states?

- $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}\pi^{0}(\to \gamma\gamma), \to \mu^{\pm}e^{-}\pi^{+}\pi^{-}$
  - ✓ collaboration with Henryk Czyż

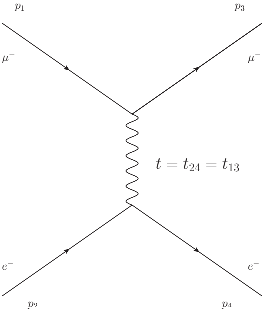
- Consider first full NNLO QED RCs to e and μ currents separately (already known)?
  - ✓ it can be the first step towards full fixed-order NNLO MC
  - ✓ it can be the testing playground to implement matching with exponentiation at NNLO
    (*e.g.* along the lines of NLO matching with QED Parton Shower in `BabaYaga@NLO`)

Conclusion & Outlook

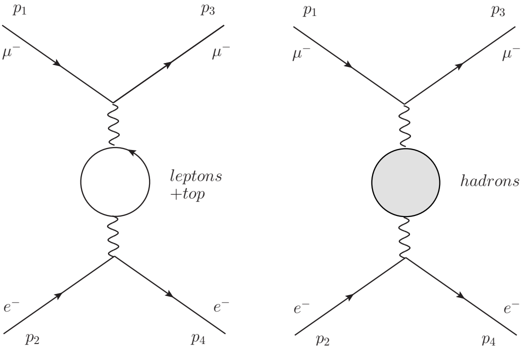
- $\mu^\pm e^- \to \mu^\pm e^-$ under control at NLO in the SM and available into a MC generator
- MC easy to be extended to fixed order NNLO
  (once amplitudes are available, also partially or in sound approximation)
- Need to define an elasticity region, preserving sensitivity to $\Delta\alpha_{\text{had}}(t)$ on "golden" observables
- Full QED NNLO mandatory
- Leptonic and hadronic pairs need to be studied with realistic exp. cuts
- QED resummation/exponentiation needed
- Consistent matching with fixed order NNLO needs to be developed

SPARES

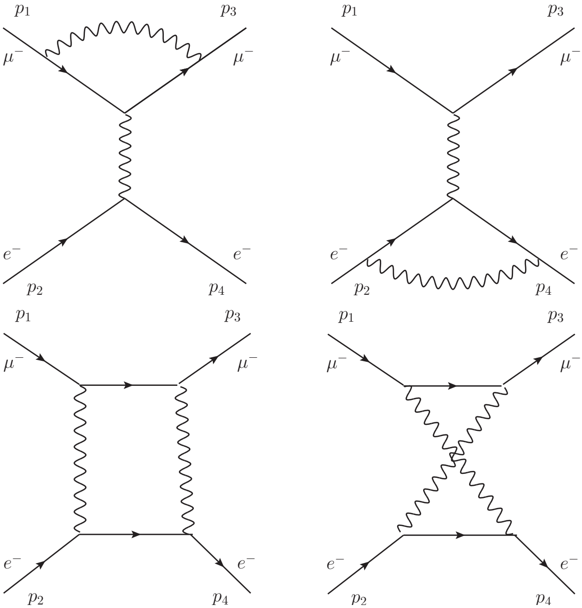
- $\mathcal{A}_{LO}$



- $\mathcal{A}_{NLO}^{virtual}$

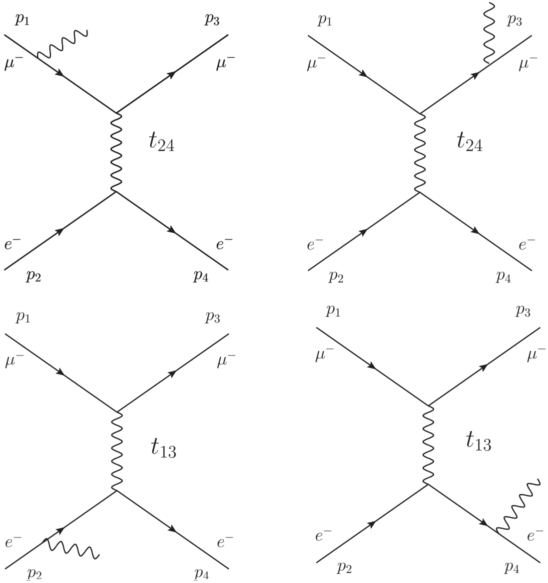


NLO virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)

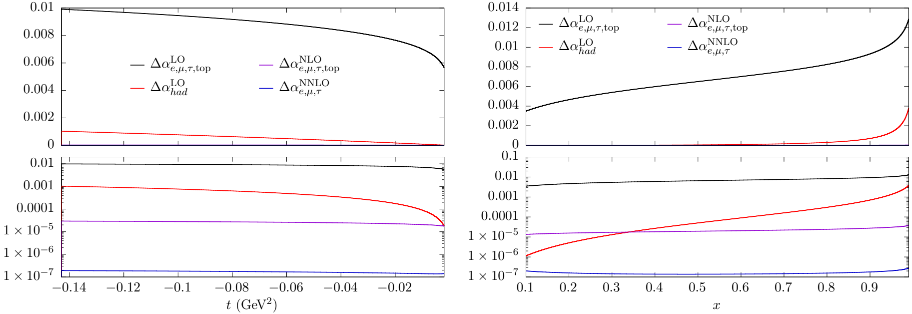


+ counterterms

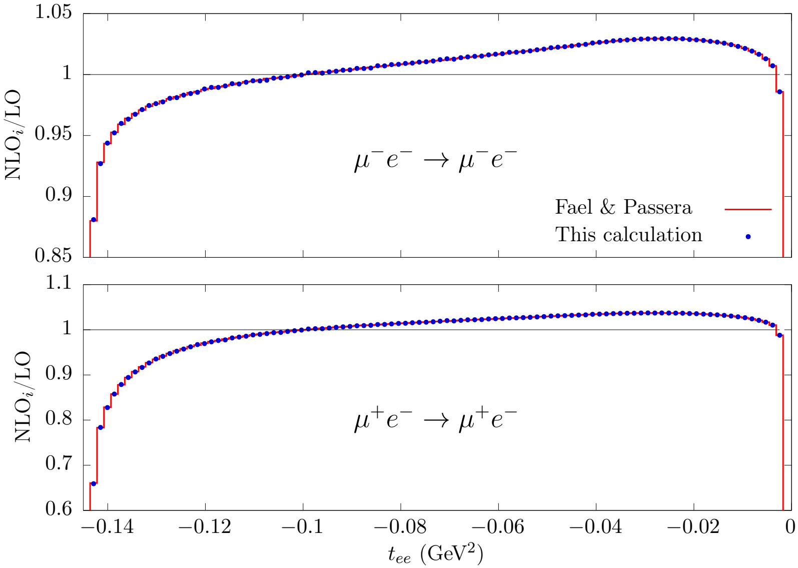
NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$



$\Delta\alpha_{\text{lep}}(t)$ at higher orders



Tuned comparison with Fael & Passera



Matching NLO and PS (in BabaYaga@NLO)

As originally developed for $e^+e^- \to e^+e^-, \to \mu^+\mu^-, \to \gamma\gamma$ at flavour factories

Balossini *et al.*, Nucl. Phys. **B758** (2006) 227, CMCC *et al.*, Nucl. Phys. Proc. Suppl. **131** (2004) 48

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma^{\infty}_{PS} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{PS} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma^{SV}_{PS}(\varepsilon) + d\sigma^{H}_{PS}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm NLO} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma^{SV}_{\rm NLO}(\varepsilon) + d\sigma^{H}_{\rm NLO}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles

(2 fermions + an arbitrary number of photons)

Any approximation is confined into matrix elements

⇝ The same QED PS & NLO matching framework successfully applied also to Drell-Yan processes (HORACE) and $H \to 4\ell$ (Hto4l)

CMCC *et al.*, JHEP 0710 (2007) 109; CMCC *et al.*, JHEP 0612 (2006) 016; S. Boselli *et al.*, JHEP 1506 (2015)

Matching NLO and PS (in BabaYaga@NLO)

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- Exponentiation of higher orders LL (PS) contributions is preserved
- The cross section is still fully differential in the momenta of the final state particles (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV \mid H,i} \otimes$ [leading-logs]

  G. Montagna et al., **PLB** 385 (1996)
- The theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO) *not infrared*, singly collinear terms: naively and roughly (for photonic corrections)

$$\frac{1}{2!}\alpha^2 L \equiv \frac{1}{2!}\alpha^2 \log\frac{s}{m_e^2} \sim 3.5 \times 10^{-4}$$

- **Need to generalize to NNLO matching!**
  Then, the error will be at the level of

$$\frac{1}{3!}\alpha^3 L^2 \sim 1.1 \times 10^{-5}$$

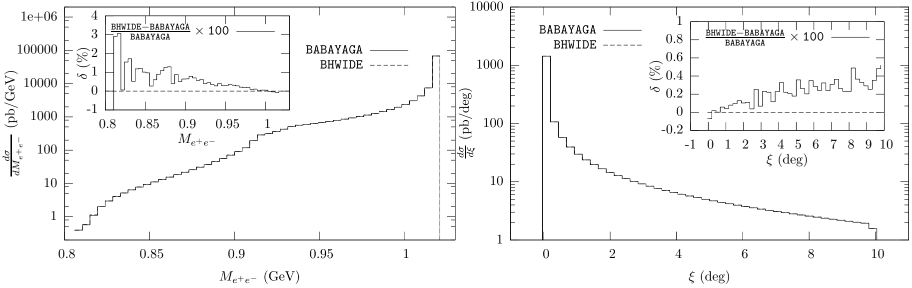
- Can we further improve with analytic resummation?

S. Actis et al. Eur. Phys. J. C **66** (2010) 585

"Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data"

- It is extremely important to compare independent calculations/implementations/codes, in order to
  - ↦ asses the technical precision, spot bugs (with the same th. ingredients)
  - ↦ estimate the theoretical error when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `Bhwide` at KLOE

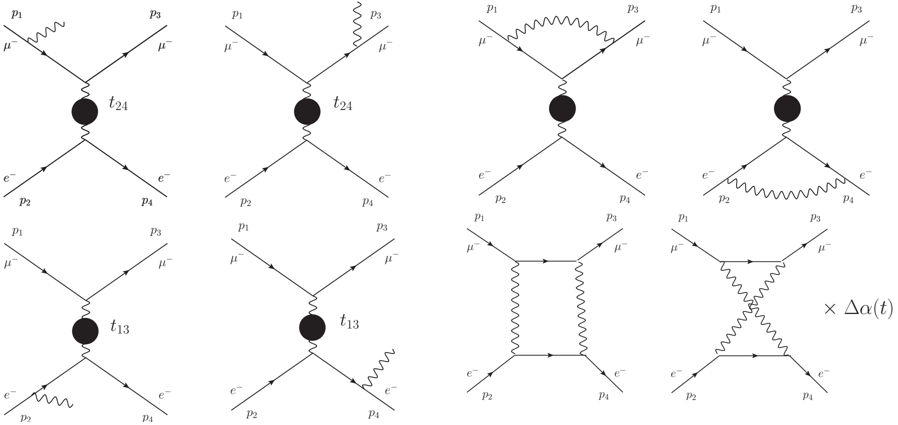
S. Jadach *et al.* PLB 390 (1997) 298



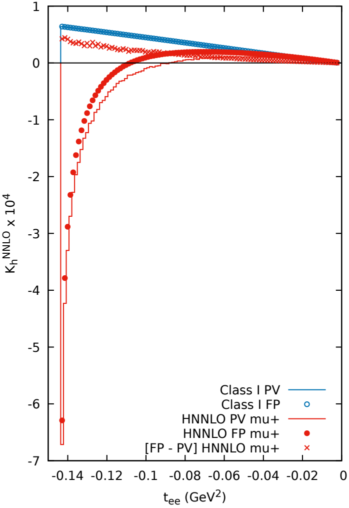
Approximating NNLO fermionic & hadronic corrections

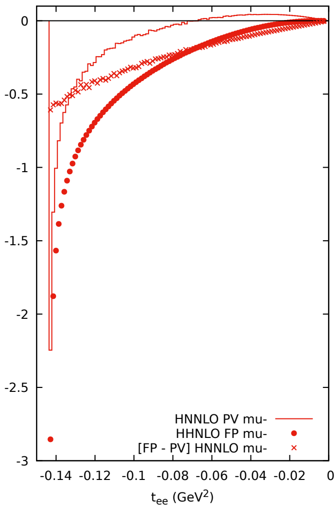
see also CMCC *et al.*, JHEP **1107** (2011) 126

- VP ($\Delta\alpha(q^2)$) can be inserted into QED NLO to approximate fermionic NNLO
  - $\rightsquigarrow$ exactly into $\mu^\pm e^- \to \mu^\pm e^- \gamma$ and into "non-loop γ" of vertex corrections
  - $\rightsquigarrow$ insertion into box diagrams can be approximated as $\mathcal{A}^{\text{box}} \cdot \mathcal{A}^0 \to \mathcal{A}^{\text{box}} \cdot \mathcal{A}^0 \times \Delta\alpha(t)$
  - $\rightsquigarrow$ VP insertion into "loop γ" at the vertex is missed



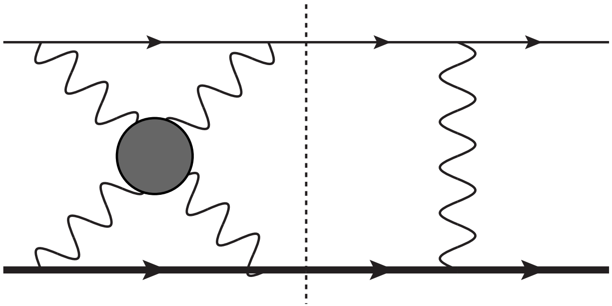
- E.g., isolating only corrections $\propto \Delta\alpha_{\text{had}}$ and comparing with Fig. 2 of M. Fael and M. Passera, PRL 122 (2019) 19, 192001





- The bulk of HNNLO corrections is catched (better for μ^+ than μ^-)

No LbL up to N^{3}LO in μe scattering

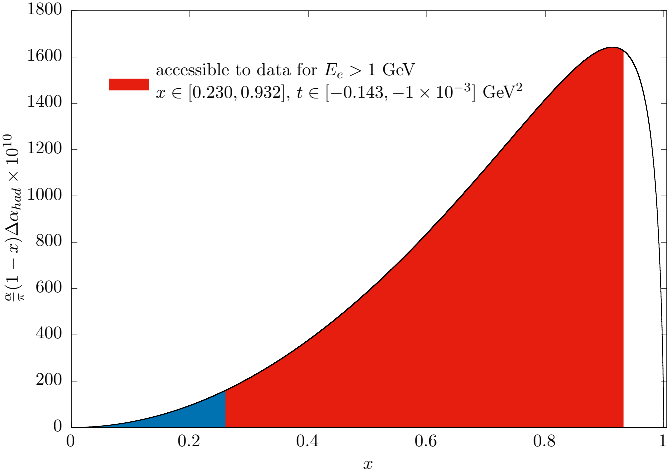


M. Fael, talk at MITP topical workshop, February 2018

is of $O(\alpha^5)$.

[i.e. $\mathcal{O}(\alpha^3)$ w.r.t. LO]

Choosing $\Delta\alpha_{\text{had}}^{\text{fit}}(t)$



$$10^{10} \times a_\mu^{\text{HLO}} = 10^{10} \times \frac{\alpha}{\pi} \int_0^1 dx\ (1-x)\Delta\alpha_{\text{had}}^{\text{Fred}}\,(t\,(x)) = 688.57^{\pm 5.03}$$

$$\int_0^{x_{max}} = \int_0^{x_{min}} + \int_{x_{min}}^{x_{max}} = 13.92^{\pm 0.10} + 586.84^{\pm 4.18} = 600.75^{\pm 4.28} \simeq 87\% \times \int_0^1$$

$$\text{to be fitted} = \frac{d\boldsymbol{\sigma}^{\text{best}}(O, \Delta\alpha_{\text{had}}^{\text{fit}}(q^2, \{P_1, \cdots, P_n\}))}{dO}$$

- Which functional form to choose for $\Delta\alpha_{\text{had}}^{\text{fit}}(q^2, \{P_1, \cdots, P_n\})$?

Choosing $\Delta\alpha_{\text{had}}^{\text{fit}}(t)$

explored in L. Pagani's master degree thesis, Bologna U., Sept. '17 (sup. Marconi, Passera, CC)

$$\boldsymbol{f^{\text{pol}}(t, P_{1\ldots3})} = P_1 t + P_2 t^2 + P_3 t^3 \qquad \boldsymbol{f^{\text{Padè}}(t, P_{1\ldots3})} = P_1 t \cdot \frac{1 + P_2 t}{1 + P_3 t}$$

$$\underbrace{\boldsymbol{f^{\text{f.l.}}(t, P_1, P_2)}}_{\text{“fermion-like”}} = \frac{P_1}{3}\left(-\frac{5}{3} - \frac{4P_2}{t} + \frac{\frac{8P_2^2}{t^2} + \frac{2P_2}{t} - 1}{\sqrt{1 - \frac{4P_2}{t}}} \log\left|\frac{1 - \sqrt{1 - \frac{4P_2}{t}}}{1 + \sqrt{1 - \frac{4P_2}{t}}}\right|\right)$$

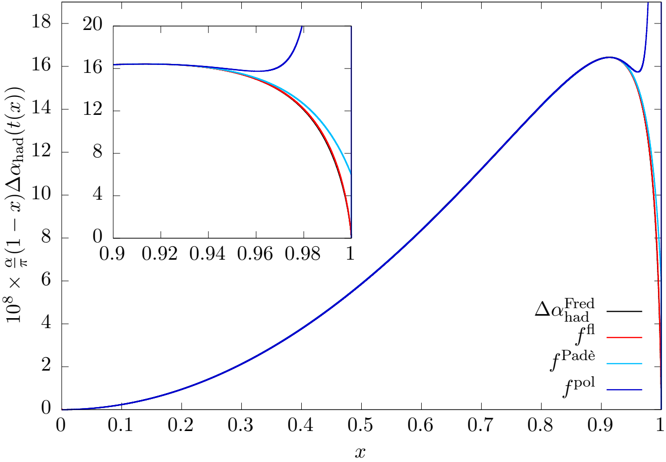
- $f^{\text{f.l.}}(t, \{P_i\})$ inspired (why not?) by subtracted LO fermionic contribution to VP, with “coupling” $\propto \sqrt{P_1}$ and “mass” $\sqrt{P_2}$
  - ✓ easy (?) to plug-in into HNNLO
  - ✓ it has polinomial limit as $t \to -0$ and a nice log behaviour as $t \to -\infty$

$$f^{\text{f.l.}}(t, P_1, P_2) = \begin{cases} \frac{P_1}{3}\left(\log\frac{-t}{P_2} - \frac{5}{3}\right) & t \to -\infty \\ -\frac{P_1}{15}\frac{t}{P_2} + \mathcal{O}(t^2) & t \to -0 \end{cases}$$

- by fitting $\Delta\alpha_{\text{had}}^{\text{Fred}}(t)$ from (old) **`hadr5n12.f` in the MUonE accessible range**

	P_1	P_2	P_3
$f^{\textbf{pol}}$	$-9.13776 \cdot 10^{-3}$	-0.0176392	-0.0300122
$f^{\textbf{Padè}}$	$-9.15347 \cdot 10^{-3}$	-0.693855	-2.74919
$f^{\textbf{f.l.}}$	$2.39479 \cdot 10^{-3}$	0.0523448	
		[→ “mass” 0.23 GeV]	

Extrapolating the fits



$$10^{10} \times \frac{\alpha}{\pi}\int_0^{x_{max}} dx\ (1-x)[f^{\rm pol}, f^{\rm Padè}, f^{\rm f.l.}, \Delta\alpha_{\rm had}^{\rm Fred}](t(x)) = [600.6, 601.0, 600.9, \underline{600.8}]$$

$$10^{10} \times \frac{\alpha}{\pi}\int_0^{1} dx\ (1-x)[f^{\rm pol}, f^{\rm Padè}, f^{\rm f.l.}, \Delta\alpha_{\rm had}^{\rm Fred}](t(x)) = [\infty, 692.8, \mathbf{688.9}, \underline{\mathbf{688.6}}]$$