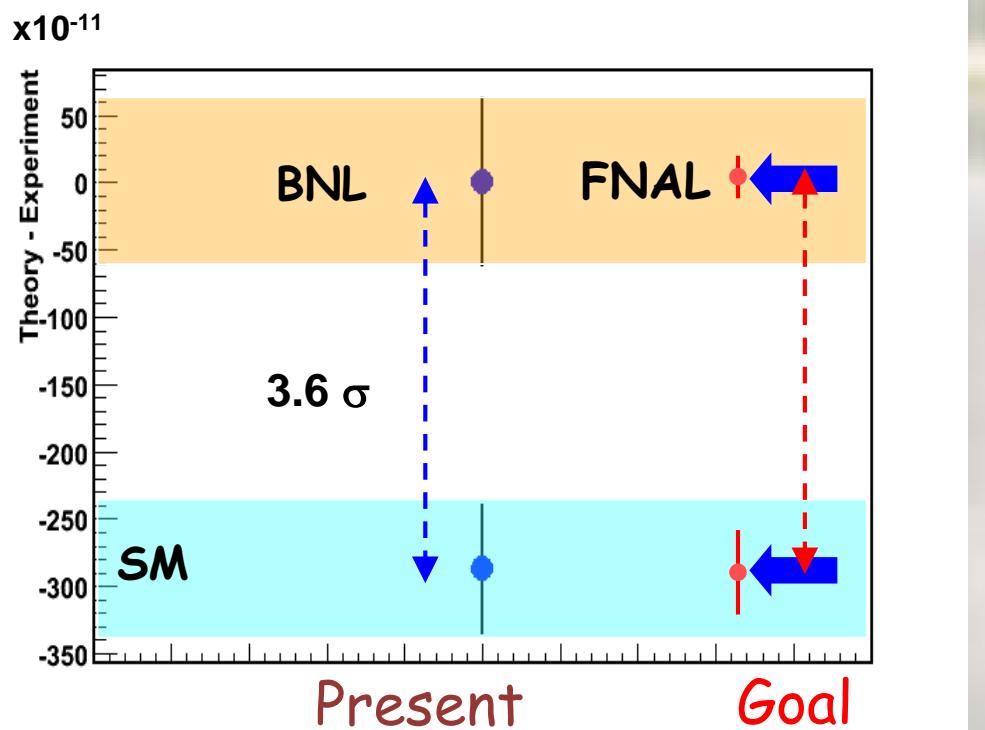


Experimental input and cross checks for HLbL contribution to muon $g-2$

A.Kupsc Uppsala U.



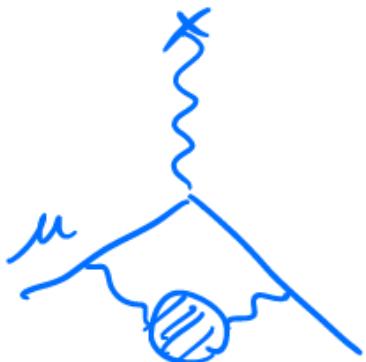
FCCP2019, Anacapri, 2019-08-29

Hadronic contribution

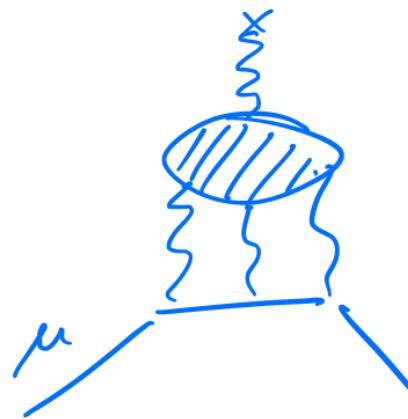
$$a_{\mu}^{\text{BNL}} = (116\ 592\ 091 \pm 63) \cdot 10^{-11}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (249 \pm 87) \cdot 10^{-11} \quad (3\sigma)$$

hadronic vacuum polarization
(HVP)



hadronic light-by-light scattering
(HLbL)



$$a_{\mu}^{\text{HVP}} = (6\ 923 \pm 42) \cdot 10^{-11}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} : 4\% \text{ HVP}$$

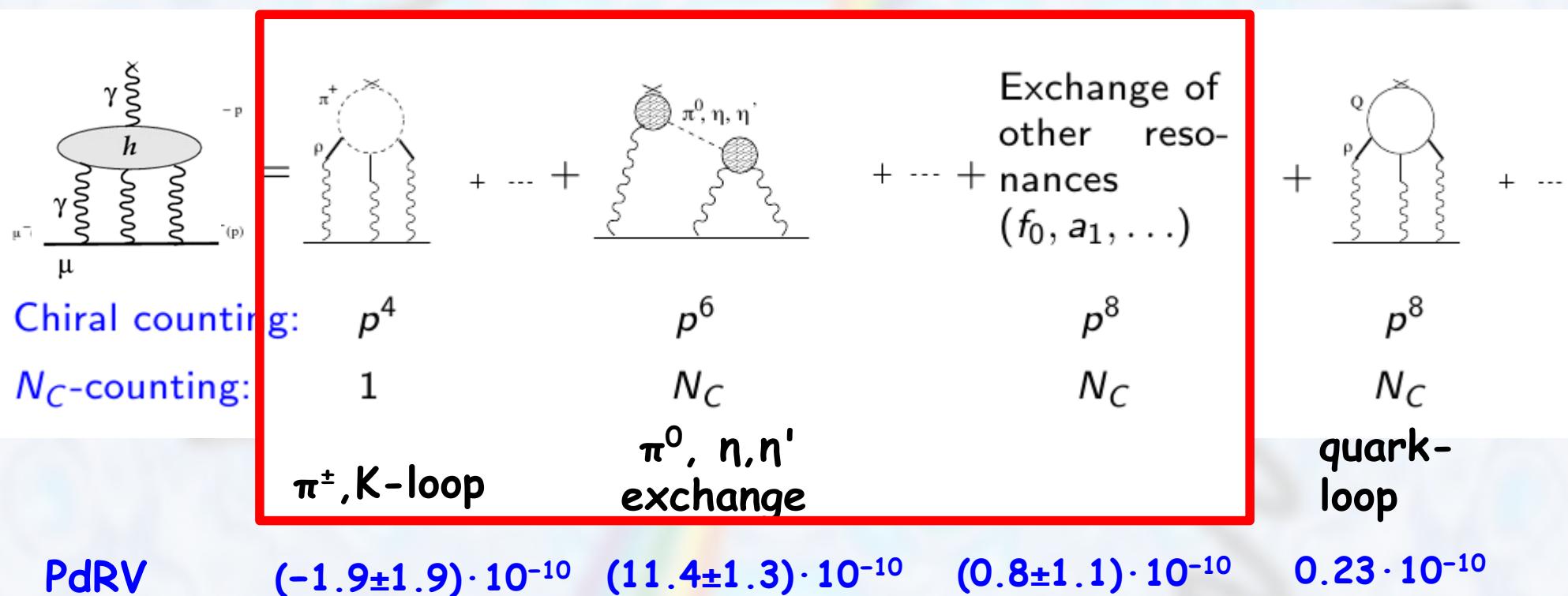
$$a_{\mu}^{\text{HLbL}} = (116 \pm 40) \cdot 10^{-11}$$

215% HLbL
(1% of leptonic LbL)

Hadronic Light by Light

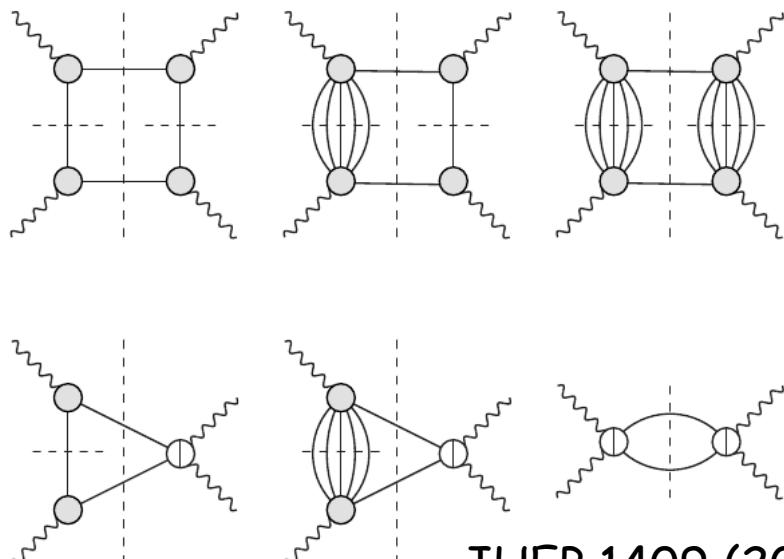
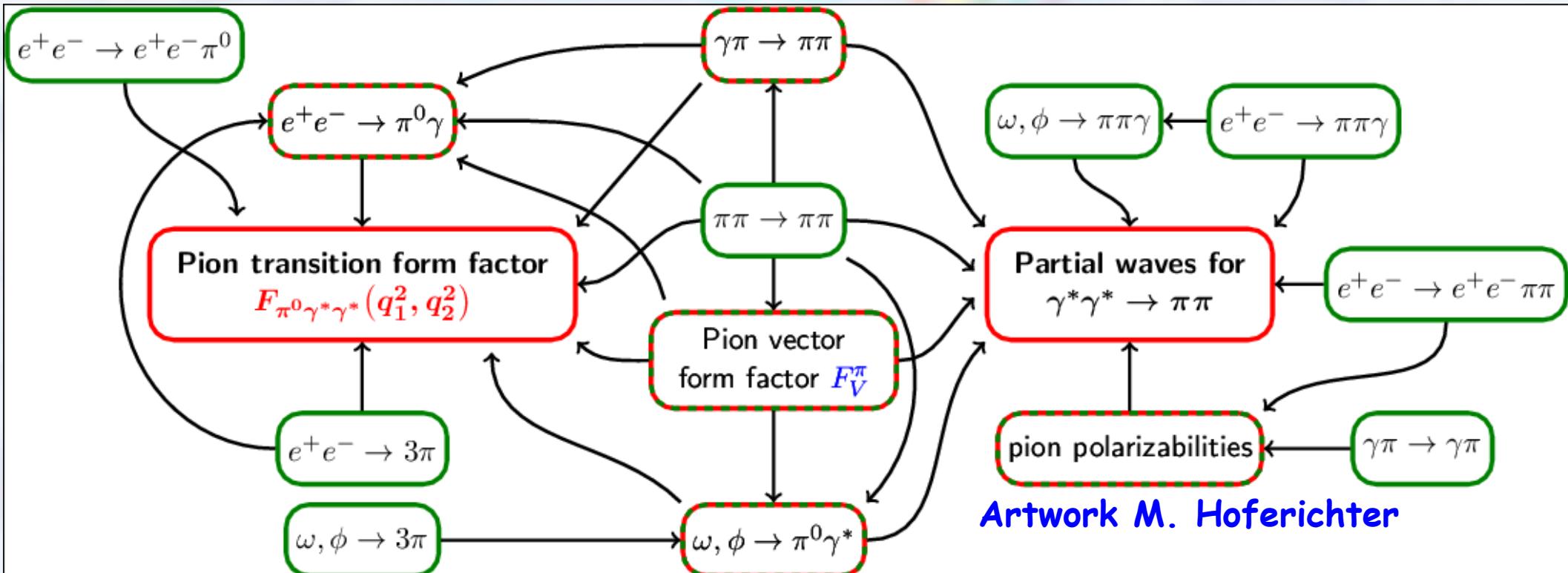
"must be calculated using hadronic models that correctly reproduce properties of QCD"

$$a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$$



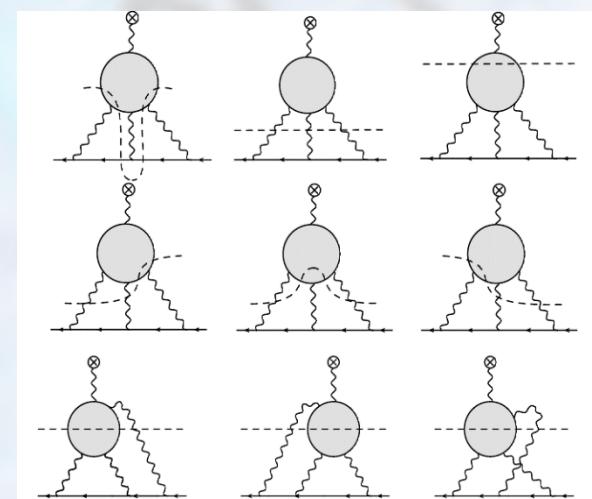
E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD,"
Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

Data driven dispersive HLbL



JHEP 1409 (2014) 091

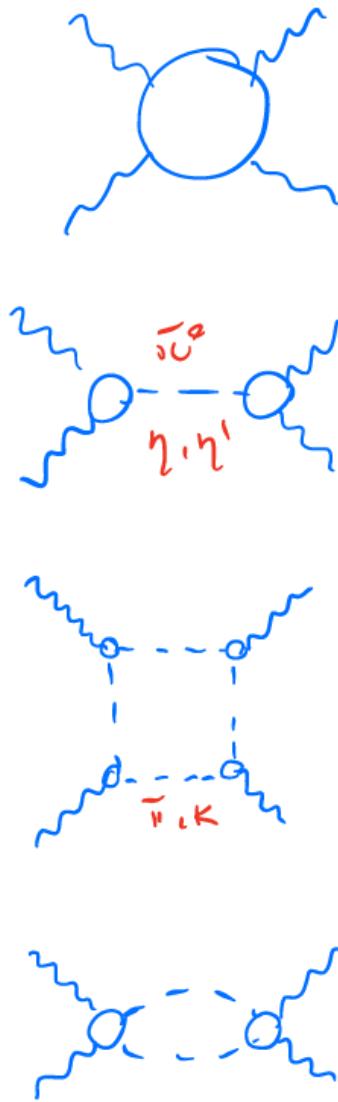
G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer



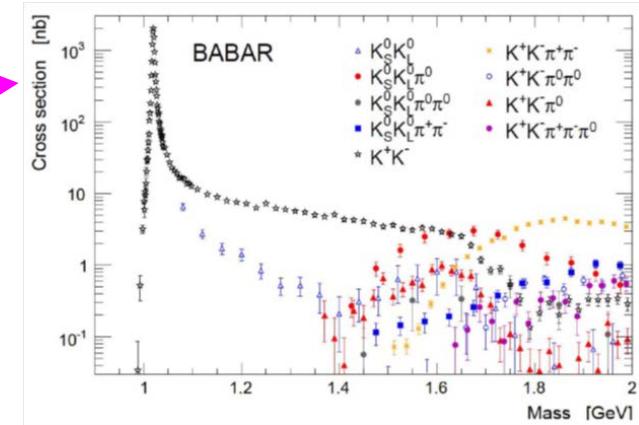
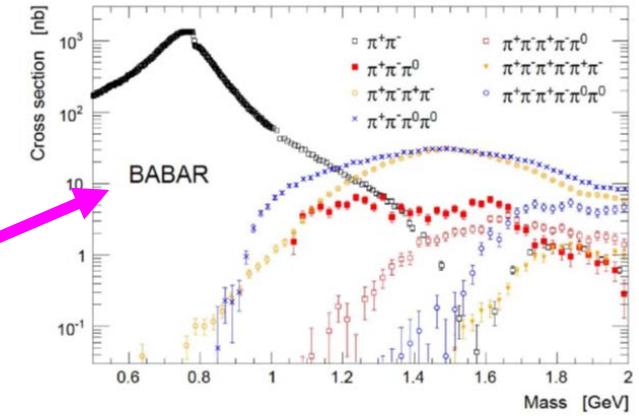
PRD90 (2014)113012

M. Vanderhaeghen, V. Pauk

Data for dispersive HLBL



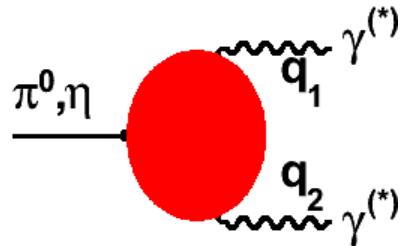
F_{π^0}
 $F_\eta, F_{\eta'}$
 F_ν
 F_K



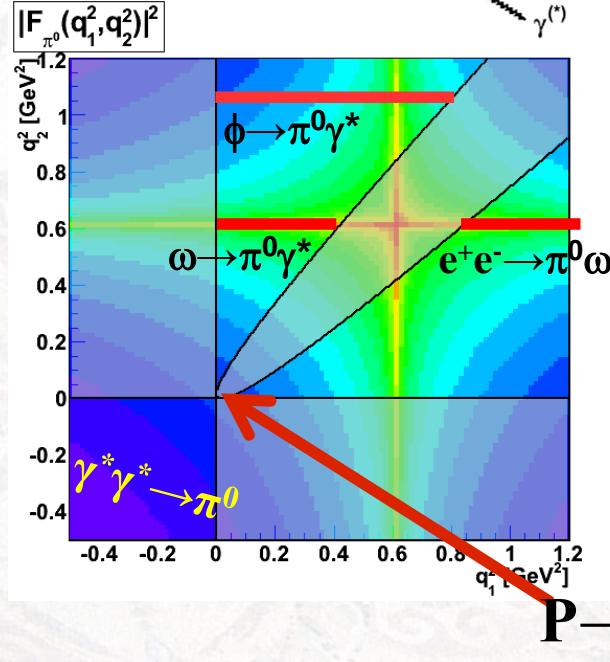
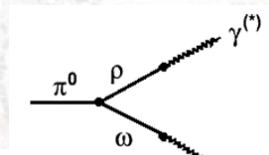
$\pi\pi, \pi K, \eta\pi, KK$ phase shifts

π^0, η, η' Transition Form Factors (TFF)

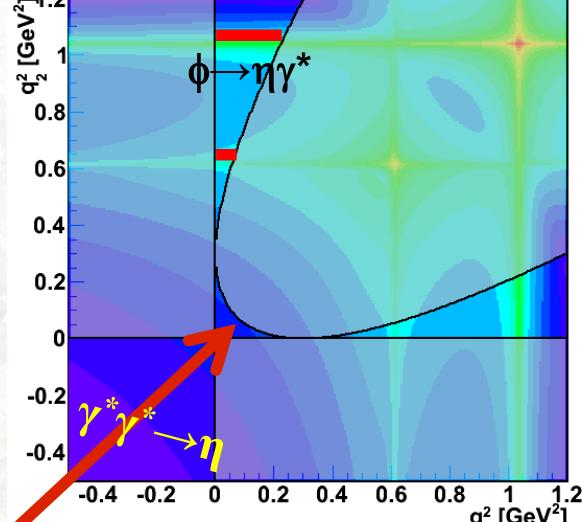
$$\Gamma(P \rightarrow \gamma\gamma)$$



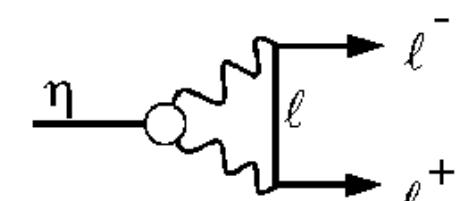
$$F_P(q_1^2, q_2^2)$$



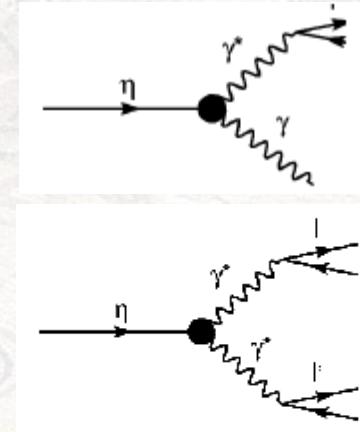
$$|F_\eta(q_1^2, q_2^2)|^2$$



$$\sum_{V=\rho, \omega, \phi} V \rightarrow \eta \gamma^{(*)}$$



Access to $q^2 < 0$



Radiative widths of η, π^0

$\eta: 5 \times 10^{-19}$ s; $\Gamma = 1.3$ keV	$\eta \rightarrow \gamma\gamma$
$\pi^0: 8 \times 10^{-17}$ s; $c\tau = 25$ nm	$\pi^0 \rightarrow \gamma\gamma$

Two exp. techniques:

$\gamma Z \rightarrow \eta, \pi^0$ Primakoff

PrimEx PRL 106, 162303(2011)

PrimEx-II:

$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.80 \text{ eV} \pm 0.7\% \text{ stat.} \pm 1.5\% \text{ syst.}$

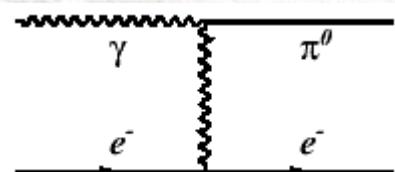
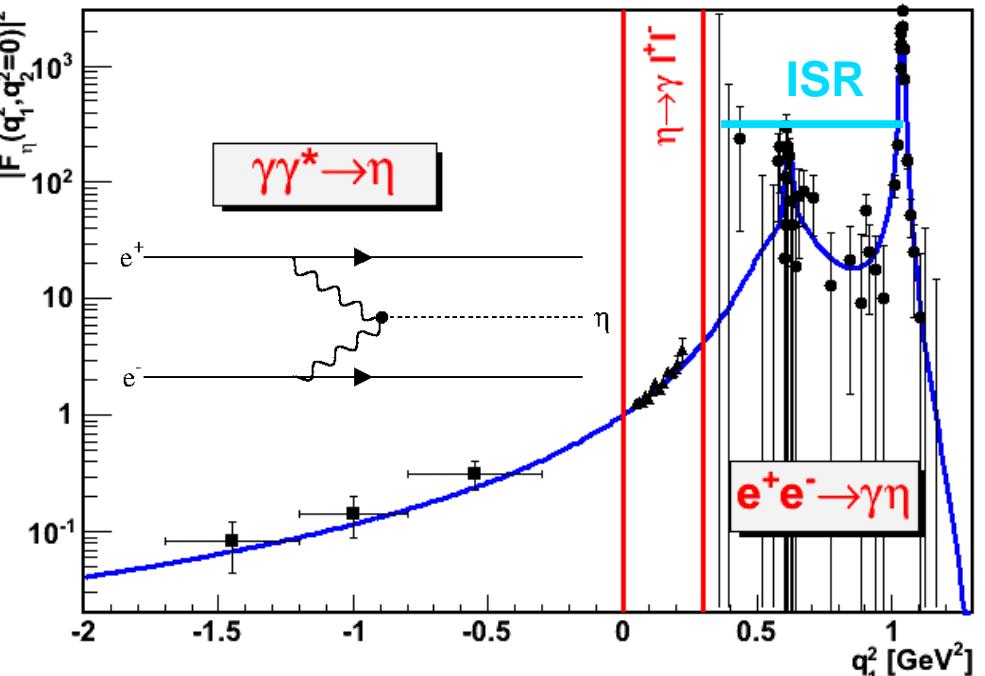
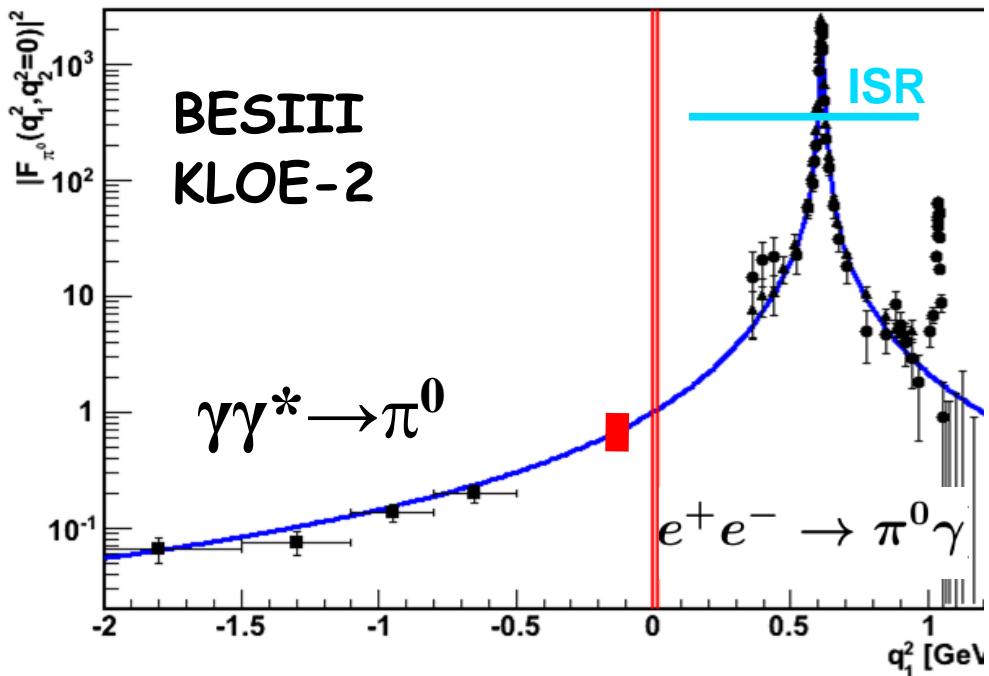
$e^+e^-: \gamma\gamma \rightarrow \pi^0$

KLOE-2 Taggers
 $5 \text{ fb}^{-1} \Rightarrow \delta\Gamma(\pi^0 \rightarrow \gamma\gamma)$

Details: [EPJC 72, 1917 (2012)]

VALUE (keV)	EVTS	DOCUMENT ID	TECN	COMMENT
0.515 ± 0.018	OUR FIT			
0.516 ± 0.018	OUR AVERAGE			
			$\delta\Gamma(\eta \rightarrow \gamma\gamma) \sim 3.5\%$	
$0.520 \pm 0.020 \pm 0.013$		BABUSCI	2013A	$e^+e^- \rightarrow e^+e^-\eta$
$0.51 \pm 0.12 \pm 0.05$	36	BARU	1990	$e^+e^- \rightarrow e^+e^-\eta$
$0.490 \pm 0.010 \pm 0.048$	2287	ROE	1990	$e^+e^- \rightarrow e^+e^-\eta$
$0.514 \pm 0.017 \pm 0.035$	1295	WILLIAMS	1988	$e^+e^- \rightarrow e^+e^-\eta$
$0.53 \pm 0.04 \pm 0.04$		BARTEL	1985E	$e^+e^- \rightarrow e^+e^-\eta$
••• We do not use the following data for averages, fits, limits, etc. •••				
0.476 ± 0.062		1 RODRIGUES	2008	CNTR Reanalysis
$0.64 \pm 0.14 \pm 0.13$		AIHARA	1986	$e^+e^- \rightarrow e^+e^-\eta$
0.56 ± 0.16	56	WEINSTEIN	1983	$e^+e^- \rightarrow e^+e^-\eta$
0.324 ± 0.046		BROWMAN	1974B	CNTR Primakoff effect
1.00 ± 0.22		2 BEMPORAD	1967	CNTR Primakoff effect

η, π^0 single off shell TFF

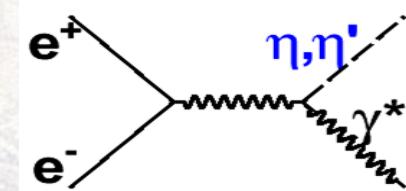


$$\frac{d\sigma}{dt}(e^-\gamma \rightarrow e^-P) = \frac{16}{3} \frac{\pi\alpha}{sm_P^3} \Gamma_{\gamma\gamma} |\mathcal{F}_P(t, 0)|^2 \frac{s - m_P^2 + t}{t}$$

$P \rightarrow \gamma^*\gamma$
Dalitz decays:
KLOE, WASA, CBall, BESIII
CLAS, NA48

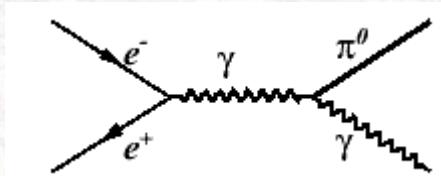
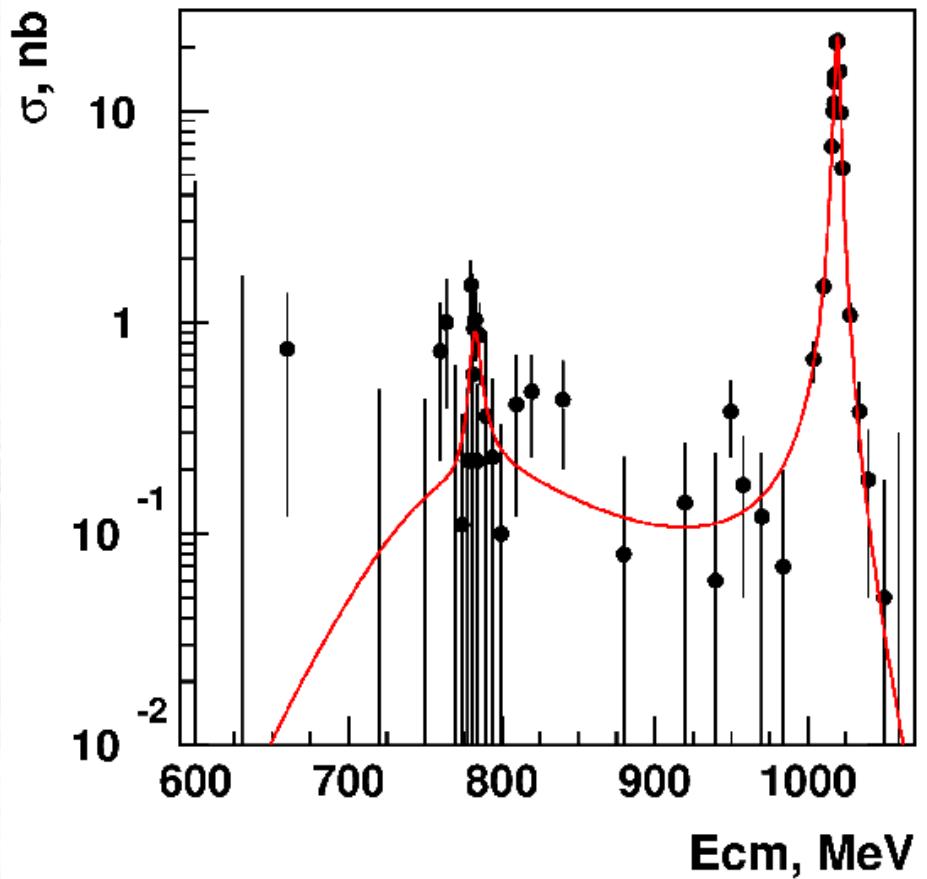
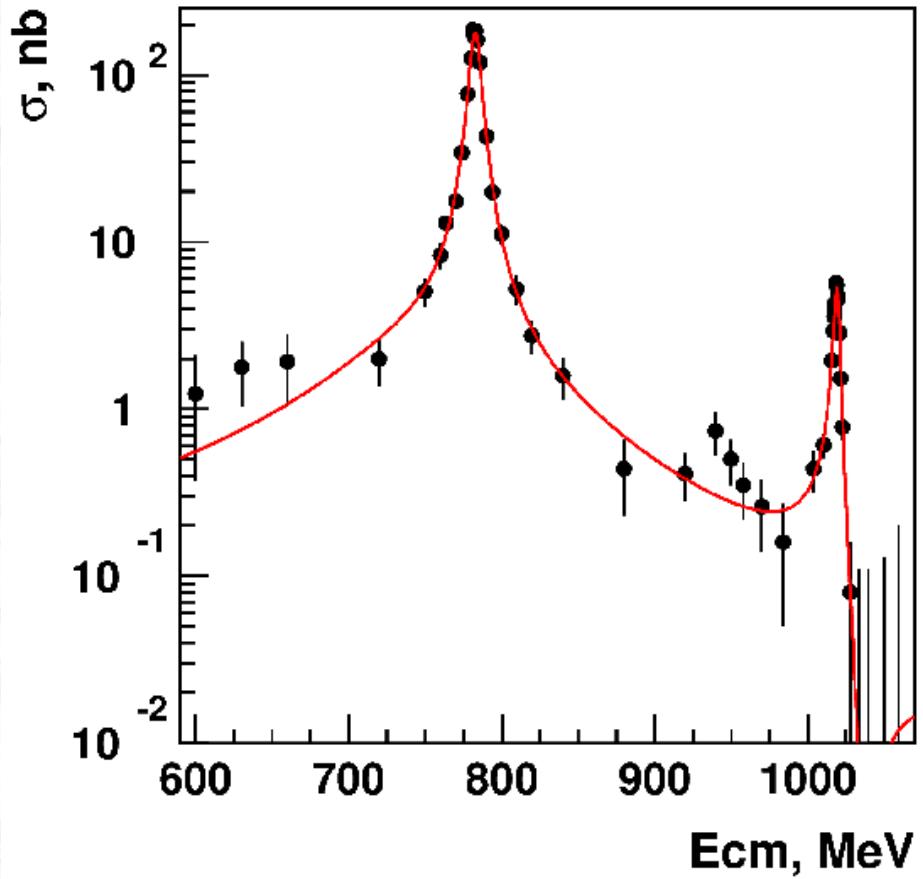
$\gamma^* \rightarrow P\gamma$
VEPP 2000 0.3-2 GeV
KLOE-2 ISR, BESIII

$$\sigma(e^+e^- \rightarrow P\gamma) = \frac{8}{3} \pi \alpha \Gamma_{\gamma\gamma} |\mathcal{F}_P(s, 0)|^2 \left(\frac{s - m_P^2}{sm_P} \right)^3$$



Data: CELLO, NA60, CB-MAMI, CMD-2, SND

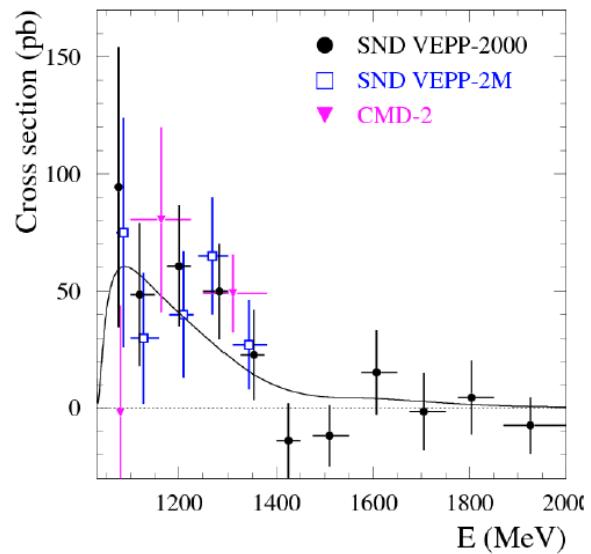
$\sigma(e^+e^- \rightarrow \pi^0\gamma, \eta\gamma)$



$$\sigma(e^+e^- \rightarrow P\gamma\gamma) = \frac{8}{3}\pi\alpha \Gamma_{\gamma\gamma} |F_P(s, 0)|^2 \left(\frac{s - m_P^2}{sm_P}\right)^3$$

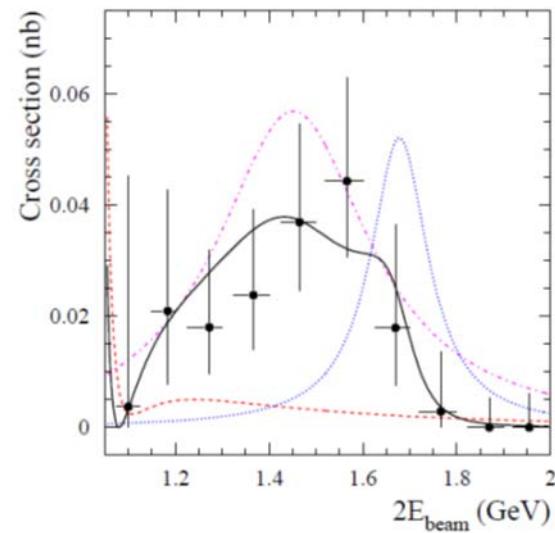
Data: CMD-2, SND

$e^+e^- \rightarrow \pi^0\gamma$ at SND



The first search above 1.4 GeV, preliminary
No signal above the background

$e^+e^- \rightarrow \eta\gamma$ at SND



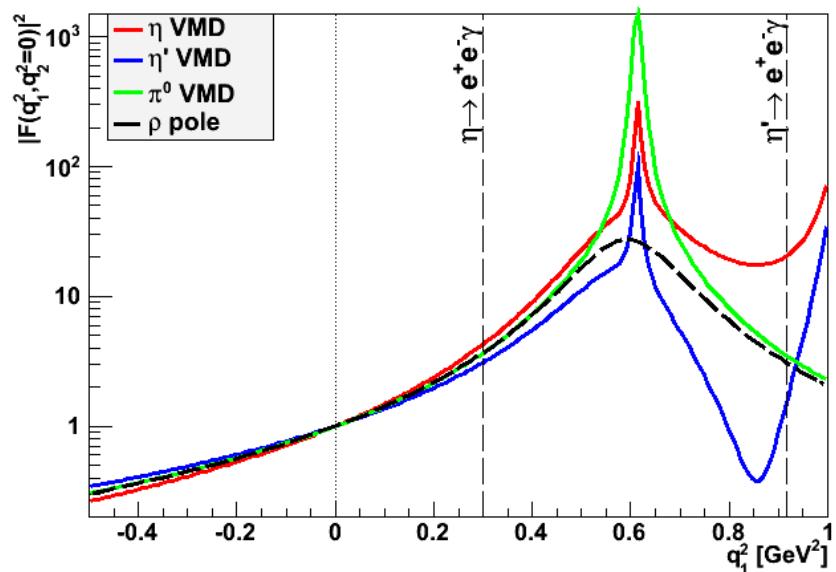
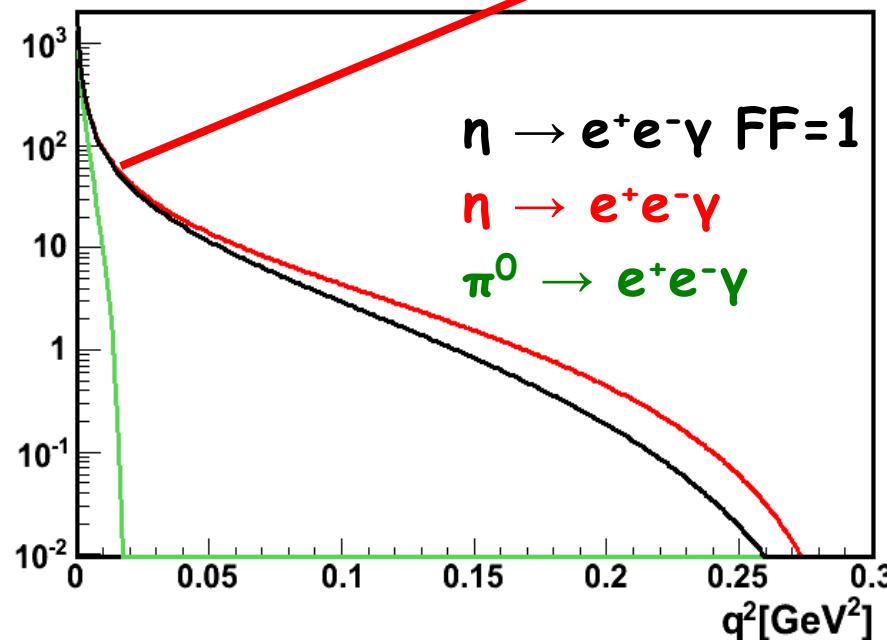
The first measurement above 1.4 GeV, Phys. Rev. D90 (2014) 032002
Dominated by the $\rho(1450)$ and $\phi(1680)$ mesons

Data: **CMD-2, SND**

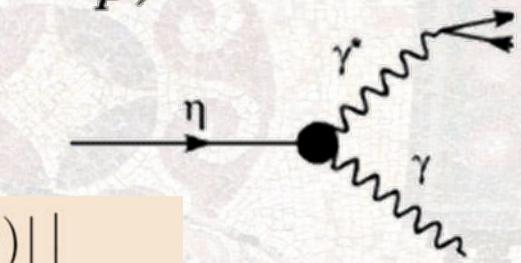
Dalitz decays

Single Dalitz decays

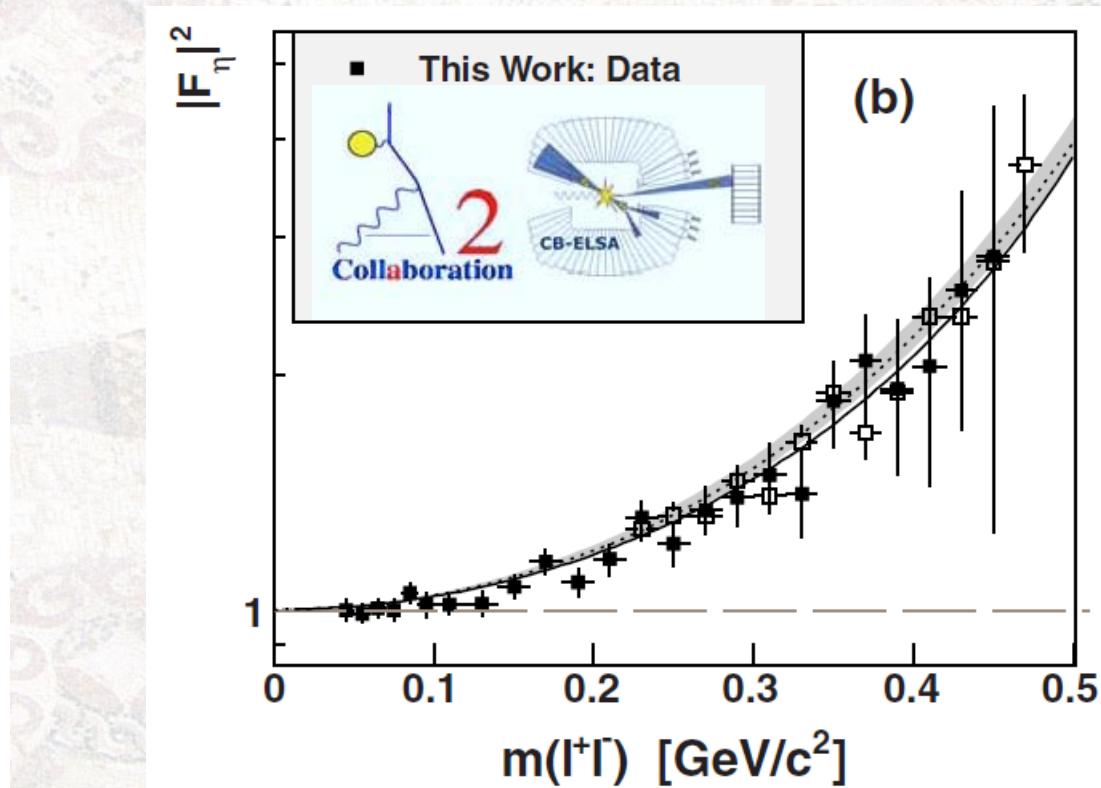
$$\frac{d\Gamma(P \rightarrow \ell^+ \ell^- \gamma)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi} \frac{1}{q^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \left(1 + \frac{2m_\ell^2}{q^2}\right) \left(1 - \frac{q^2}{M_P^2}\right)^3 |\mathbf{F}_P(q^2, 0)|^2$$



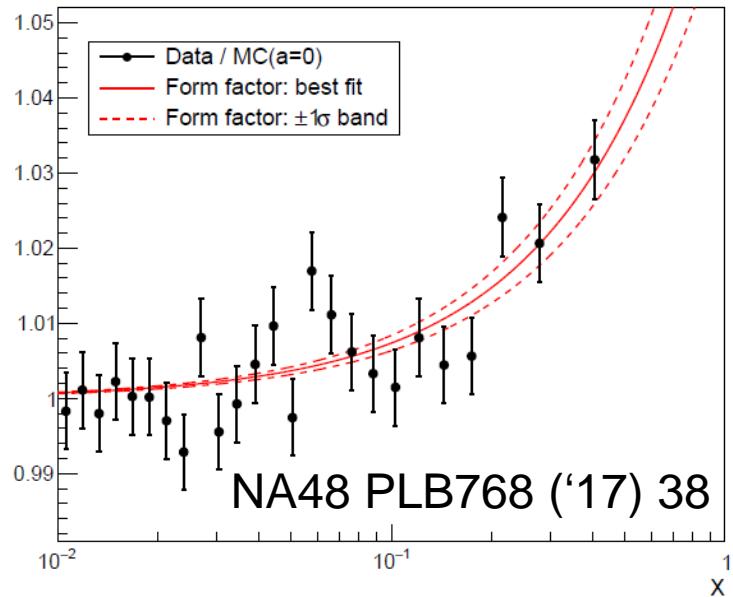
$$b_P = \left. \frac{d \ln |F_P(q^2)|}{dq^2} \right|_{q^2=0}$$



CB/TAPS: PRC89, 044608 (2014)



$$\pi^0 \rightarrow e^+ e^- \gamma$$



$$a_\pi = (3.68 \pm 0.57) \times 10^{-2}$$

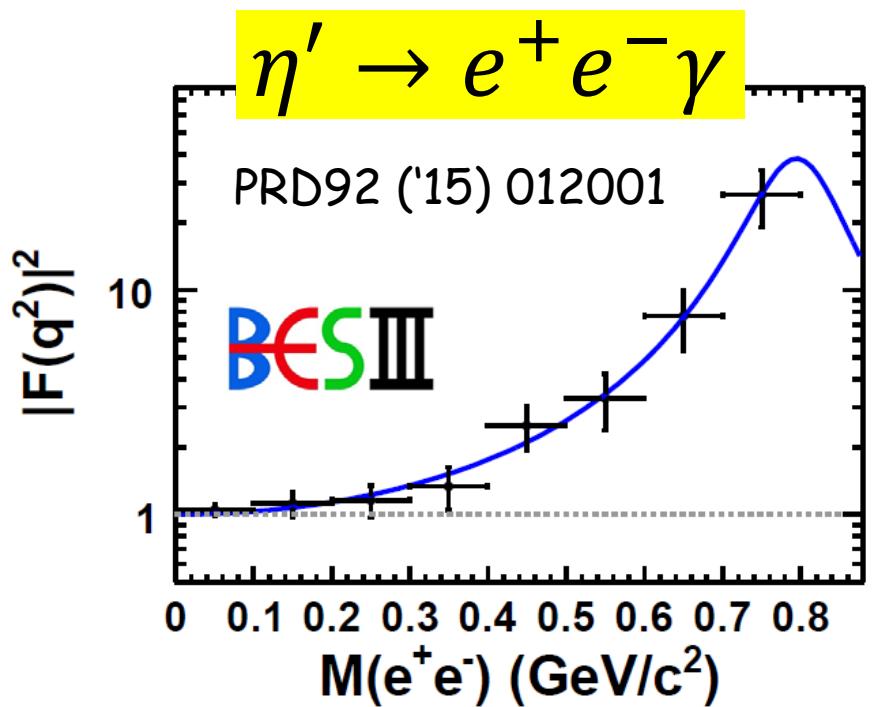
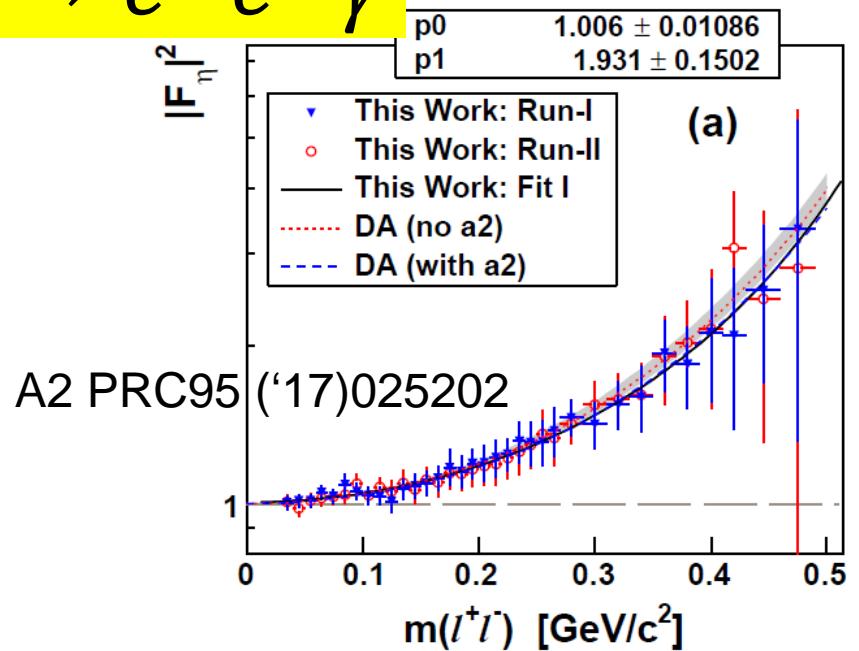
$$a_\pi = (3.0 \pm 1.0) \times 10^{-2} \quad \text{A2 PRC}$$

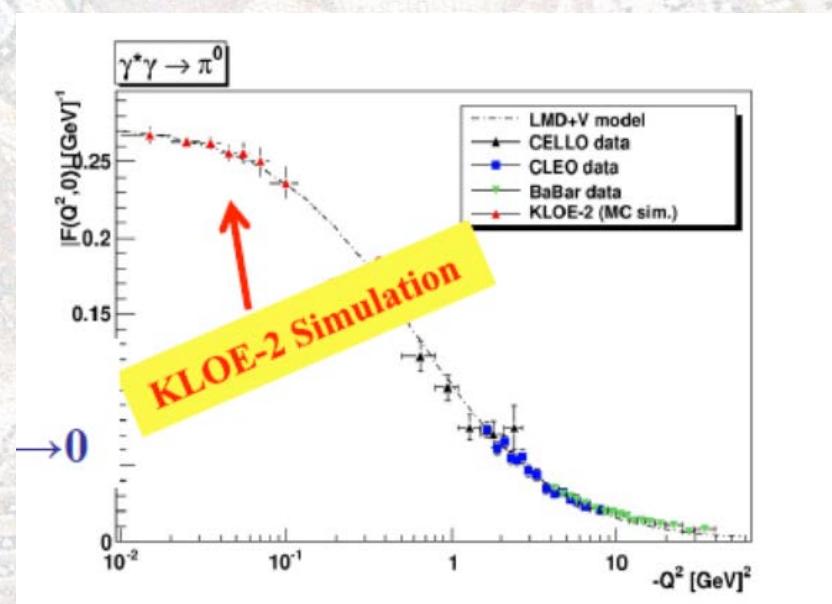
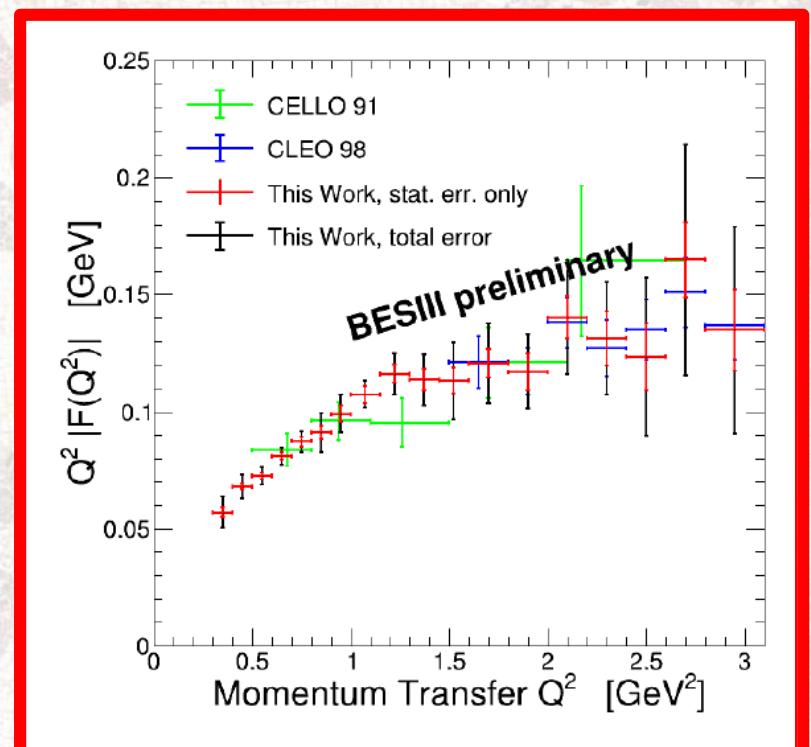
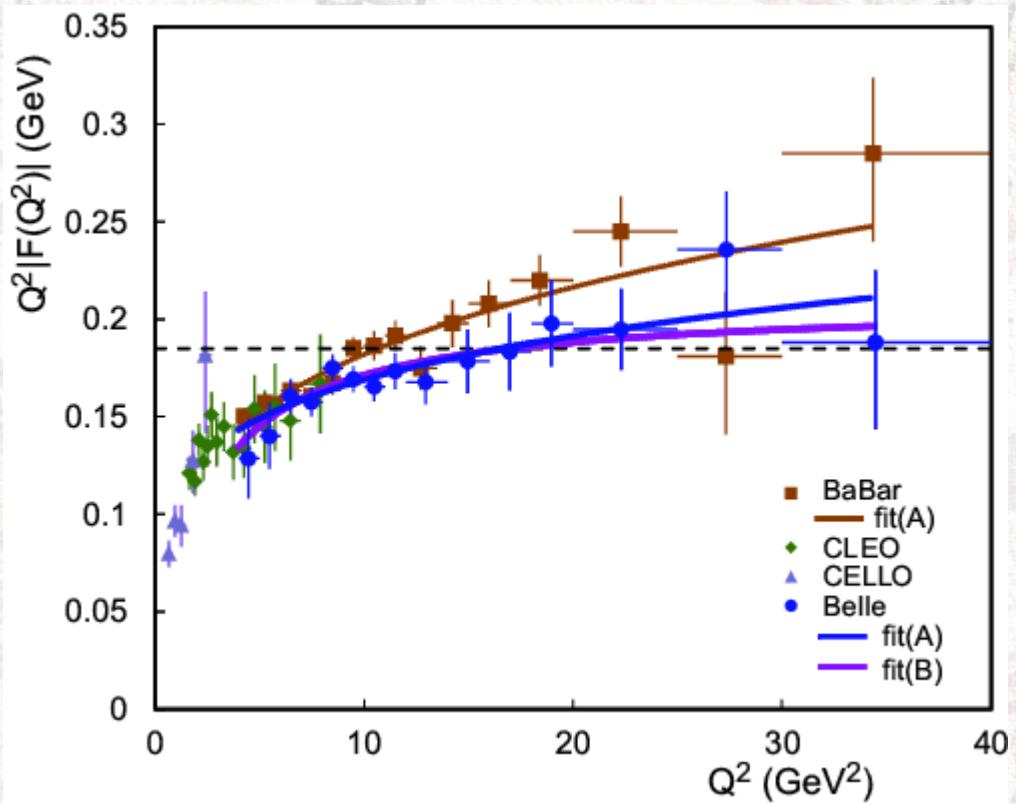
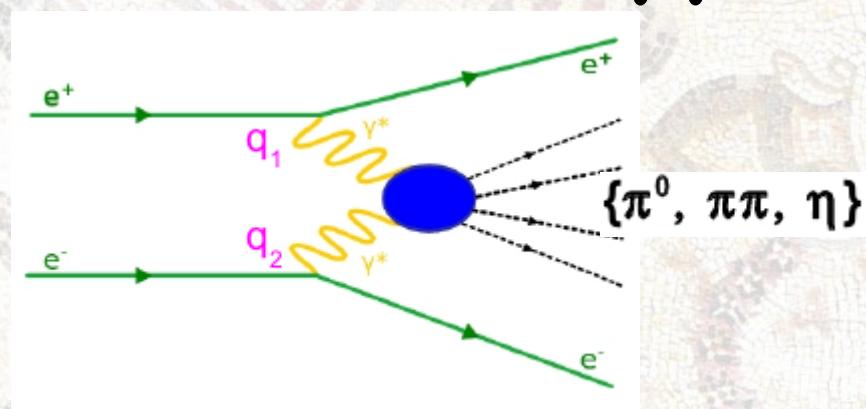
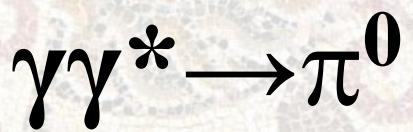
$$\text{PDG}_{\text{AVG}} \quad a_\pi = (3.35 \pm 0.31) \times 10^{-2}$$

$$\text{DR: } a_\pi = (3.15 \pm 0.09) \times 10^{-2} \quad \text{arXiv:1808.04823}$$

$$F_{\pi^0\gamma}(m_{ee}) = 1 + a_\pi \frac{m_{ee}^2}{m_{\pi^0}^2}$$

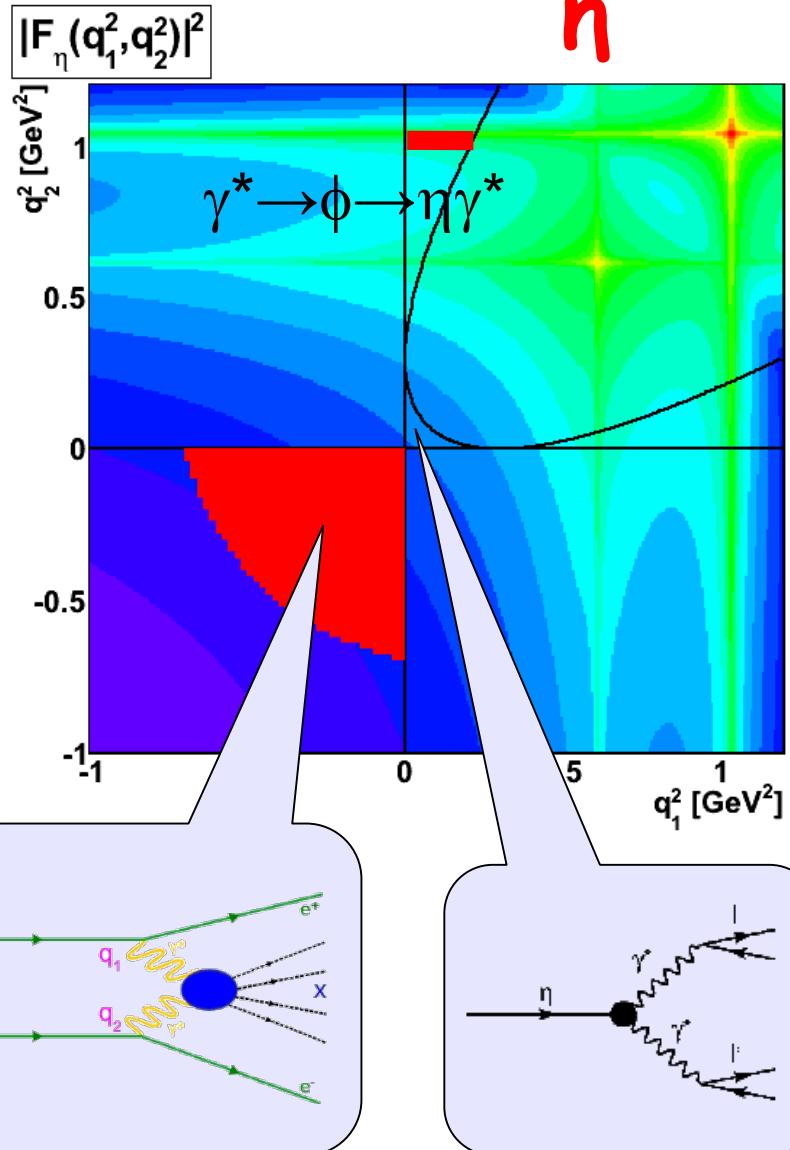
$$\eta \rightarrow e^+ e^- \gamma$$



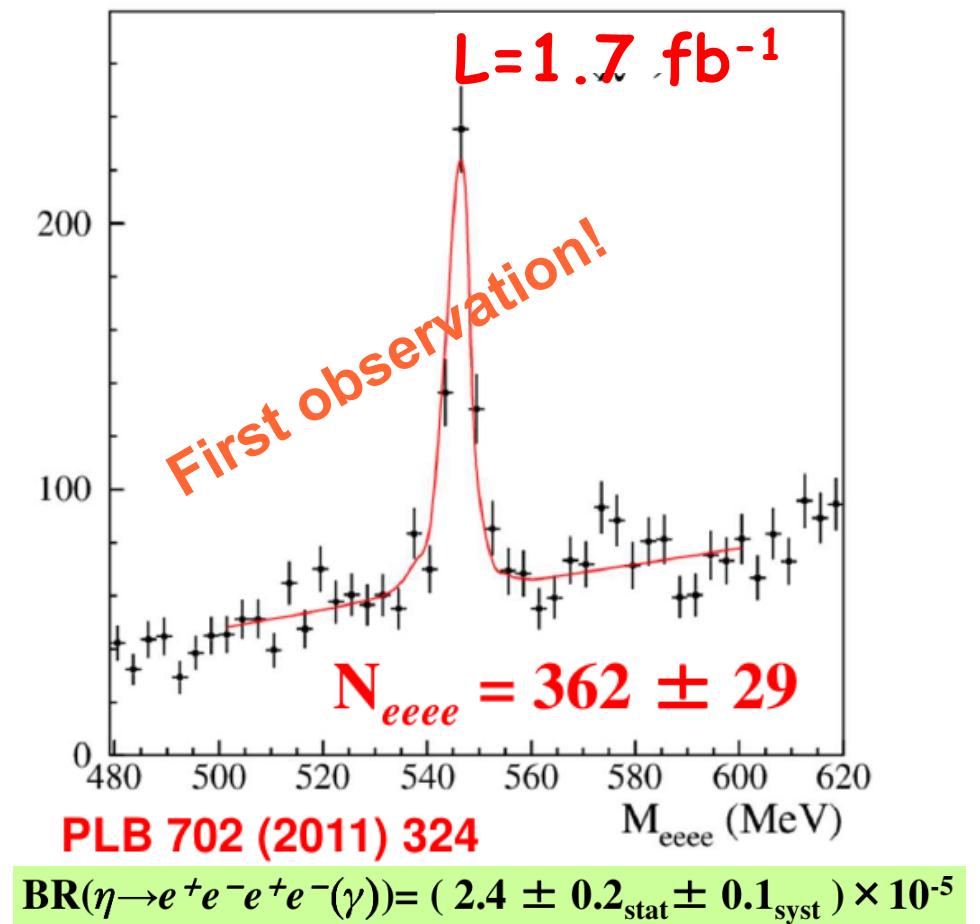


Double off shell TFF

η



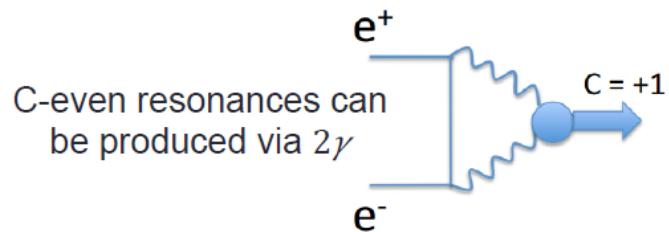
$\eta \rightarrow e^+ e^- e^+ e^-$



⇒ BESIII

Search for $e^+e^- \rightarrow \eta'(958)$

Phys.Lett. B740 (2015) 273-277



C-even resonances can
be produced via 2γ

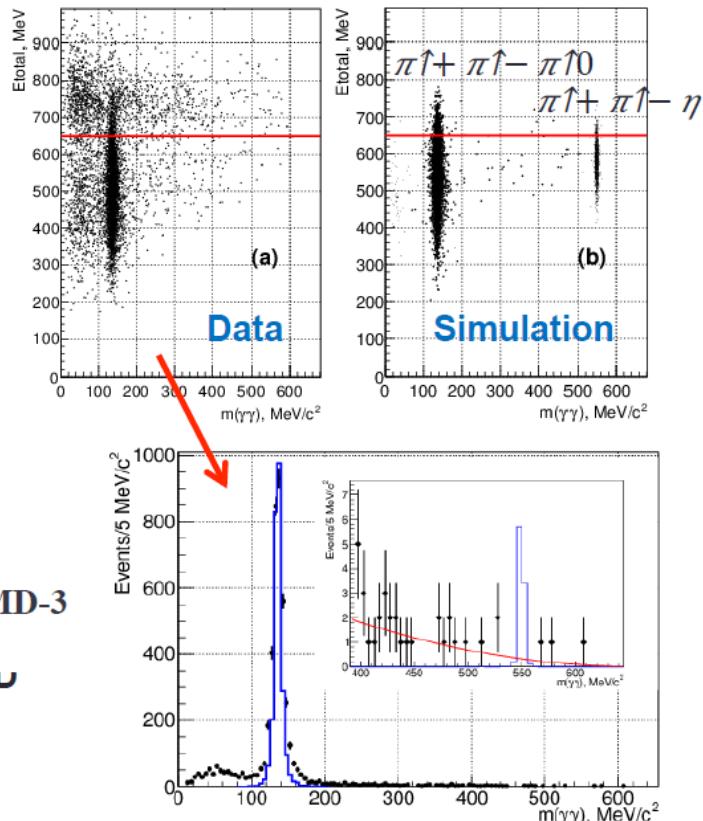
Theory: assuming real γ
 $B(\eta' \rightarrow e^+e^-) = 3.7 \cdot 10^{-11}$

γ virtuality and transition form factor can
enhance it

New limit:

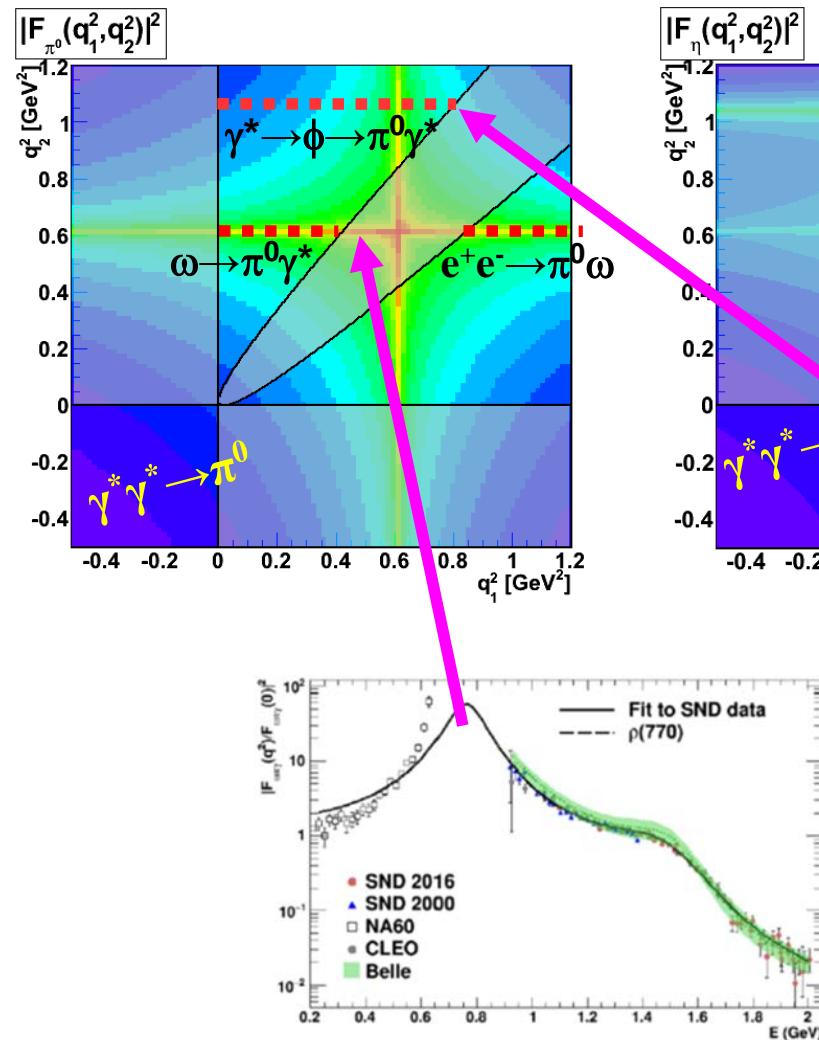
$B(\eta' \rightarrow e^+e^-) < 5.6 \times 10^{-9}$ (90% CL) - SND+CMD-3

Dedicated data taking at $\sqrt{s} = 2.76 \text{ TeV}$ INU
 Continuous beam energy monitoring
is crucial



SND+CMD-3

$V \rightarrow P\gamma^*$ and $e^+e^- \rightarrow PV$ processes



KLOE

result $b\eta(m_\phi^2)$
 $\phi \rightarrow \eta \gamma^*$ BR 10^{-4}

Phys.Lett. B742 (2015) 1-6

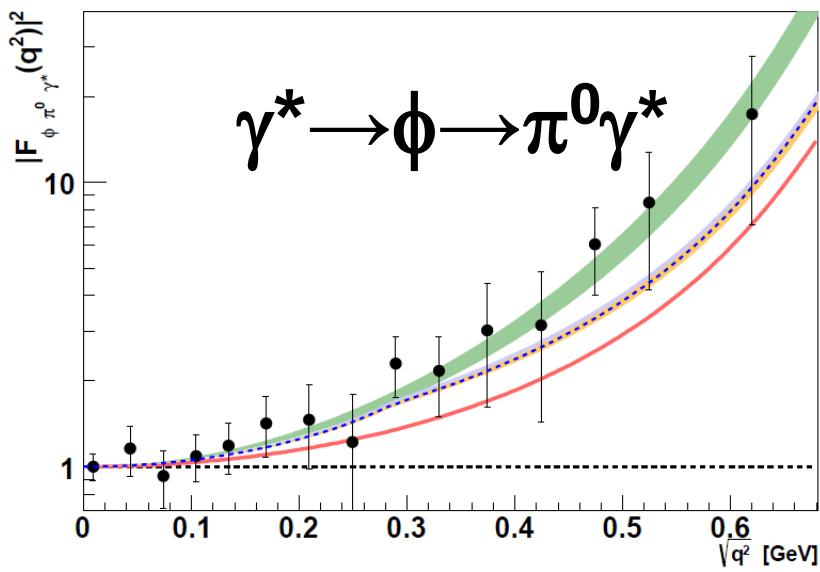
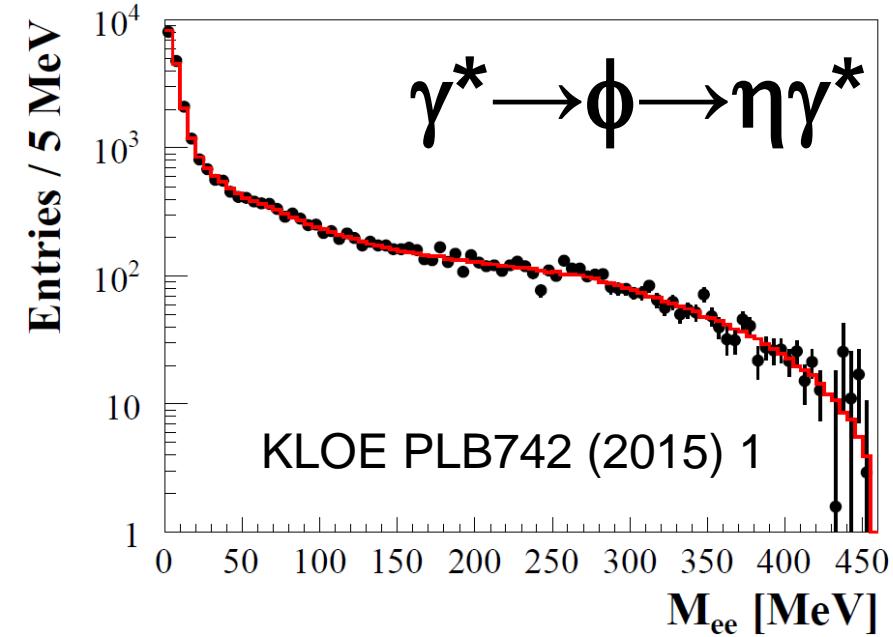
$$b_{\phi\eta} = (1.17 \pm 0.10^{+0.07}_{-0.11}) \text{ GeV}^{-2}$$

$$b\pi^0(m_\phi^2) \phi \rightarrow \pi^0 \gamma^* \text{ BR } 10^{-5}$$

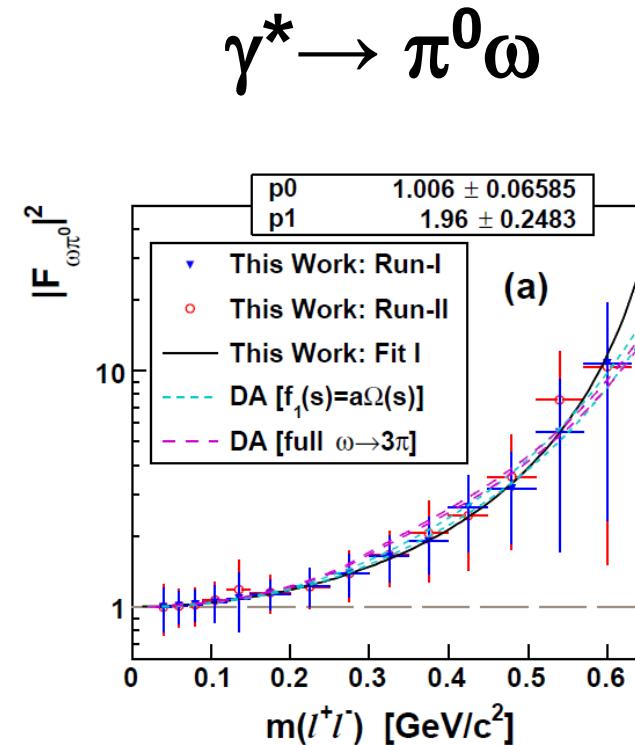
Phys.Lett. B757 (2016) 362-367

In addition to SND and NA60, other data on $\mathcal{F}(\gamma\omega\pi)$ exist:
 CLEO ($\tau^- \rightarrow \omega\pi^-\nu_\tau$), K.W. Edwards et al., Phys. Rev. D61 (2000) 072003
 Belle ($\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$), D. Matvienko et al., Phys. Rev. 92 (2015) 012013
 NA60 studied inclusively $\omega \rightarrow \pi^0 \mu^+ \mu^-$

$$b\varphi\pi^0 = (2.02 \pm 0.11) \text{ GeV}^{-2}$$

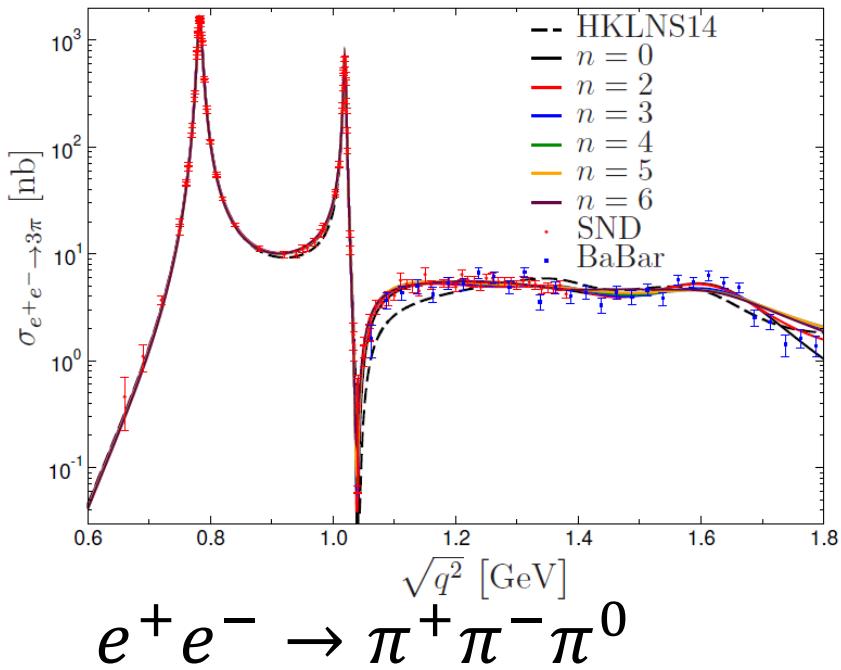


KLOE PLB757 (2016) 362

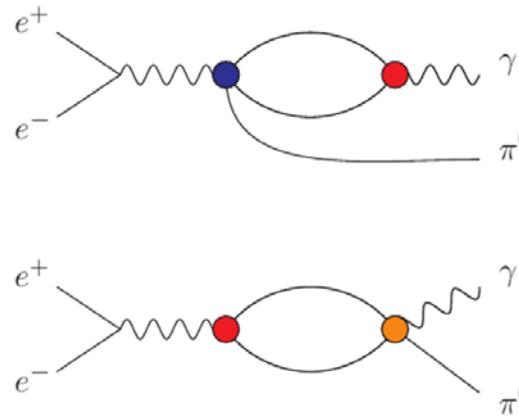


A2 Phys.Rev. C95 (2017) 025202

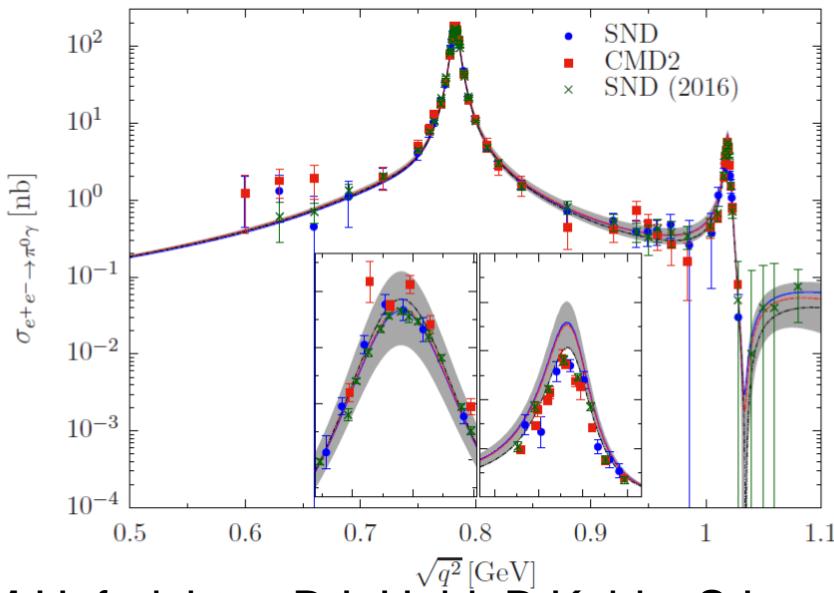
DP: $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ to π^0 TFF



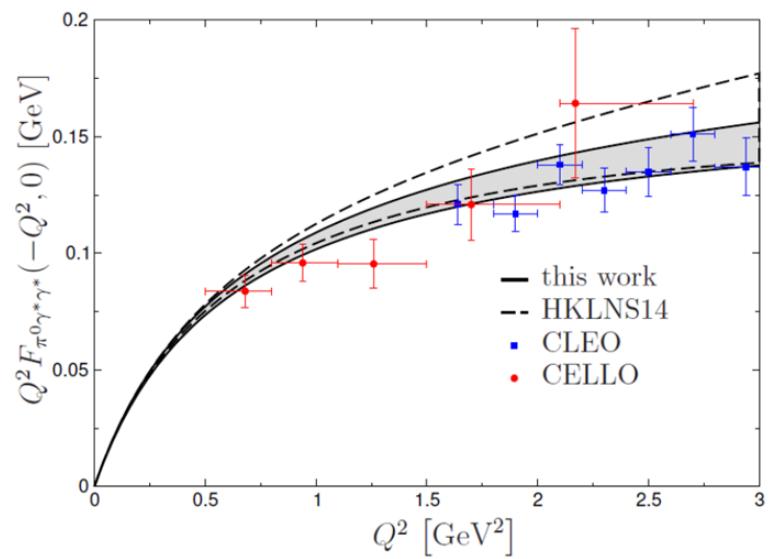
$$e^+e^- \rightarrow \pi^+\pi^-\pi^0$$



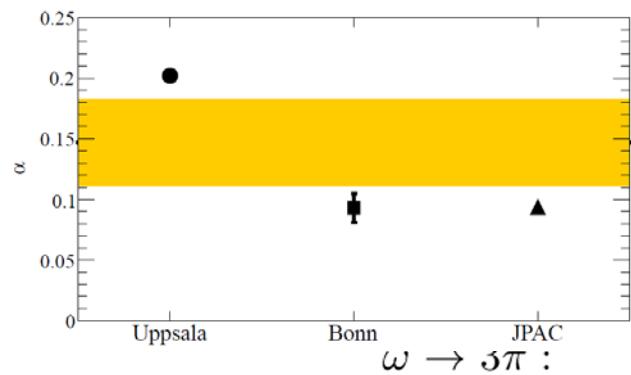
$\pi\pi$ phase shifts + $e^+e^- \rightarrow 3\pi$ data
Eur.Phys.J. C74 (2014) 3180



M Hoferichter, B-L Hoid, B Kubis, S Leupold, S P. Schneider
arXiv:1808.04823

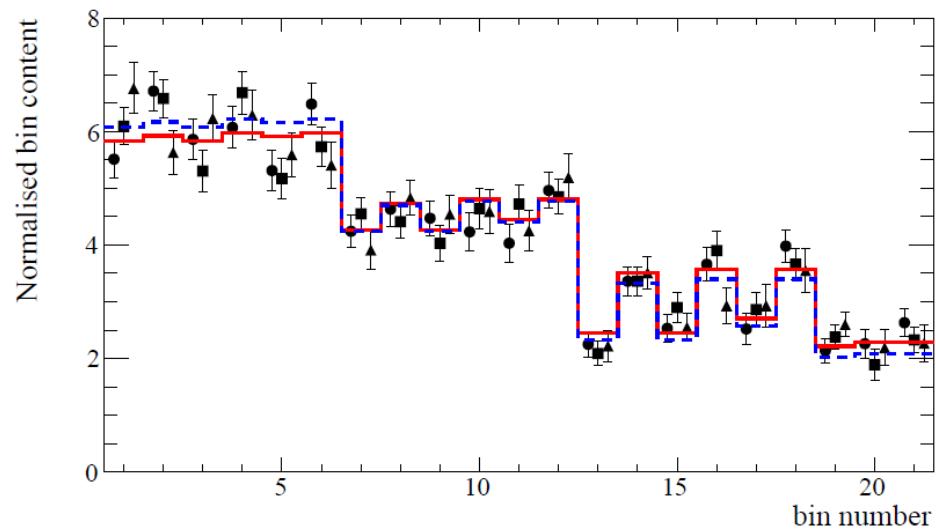


$$\omega/\phi \rightarrow \pi^+ \pi^- \pi^0$$

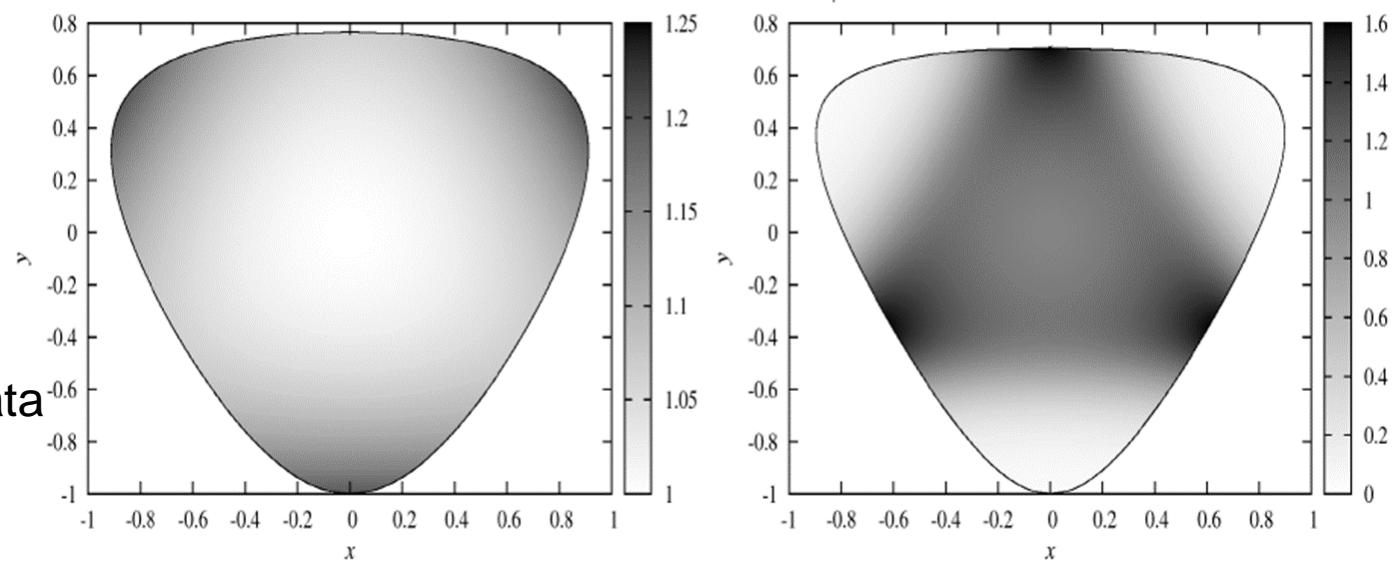


BESIII

for ϕ precise data
from CMD2
and KLOE

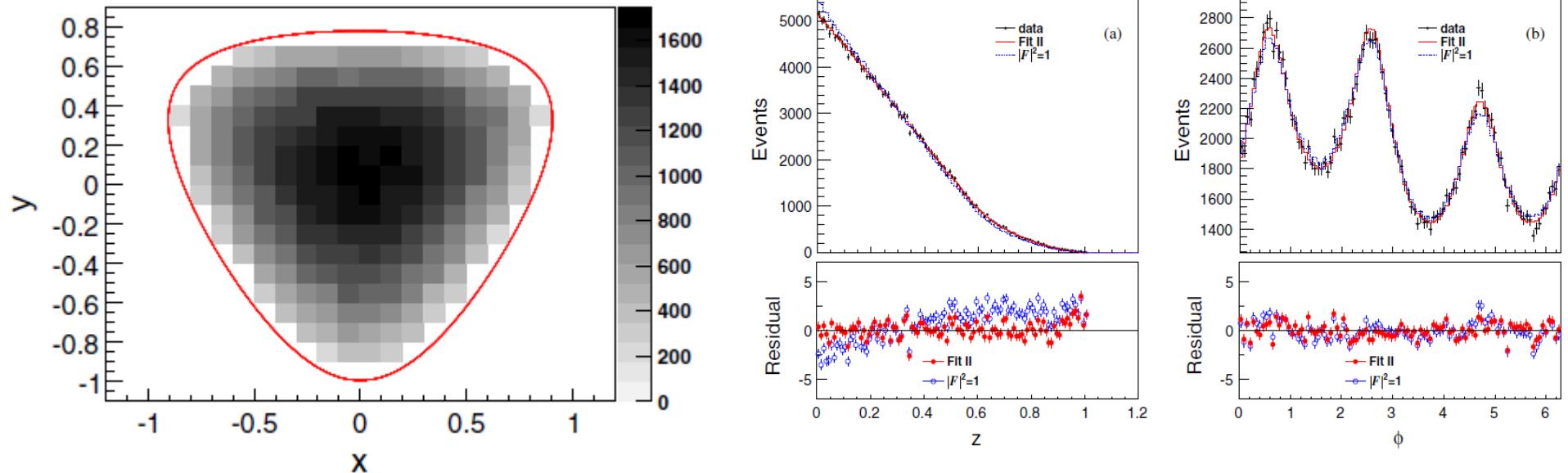


WASA PLB770 (2017) 418



Eur.Phys.J. C72 (2012) 2014

Slide: Bastian Kubis



BESIII: PRD98, 112007(2018)

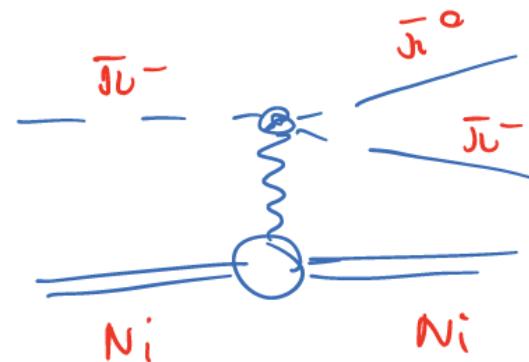
TABLE III. Predictions and fit results for the \mathcal{F} parametrizations. The predictions are from Danilkin *et al.* [4], Niecknig *et al.* [5], and Terschlüsen *et al.* [19]. Theoretical predictions without incorporating crossed-channel effects are indicated by w/o and those with crossed-channel effects by w.

Para. $\times 10^3$	Theoretical predictions				Experiment		
	Ref. [4]		Ref. [5]		Ref. [19]	BESIII	
	w/o	w	w/o	w			
Fit I	α	136	94	(137,148)	(84,96)	202	$132.1 \pm 6.7 \pm 4.6$
Fit II	α	125	84	(125,135)	(74,84)	190	$120.2 \pm 7.1 \pm 3.8$
	β	30	28	(29,33)	(24,28)	54	$29.5 \pm 8.0 \pm 5.3$

Goal of COMPASS experiment

- ▷ Test ChPT predictions for pion-photon reactions $\pi^- \gamma \rightarrow X^-$ with various final states X^-

$$\pi^- + \gamma^{(*)} \rightarrow \begin{cases} \pi^- + \gamma & \text{Compton reaction, pion polarisabilities} \\ \pi^- + \pi^0 & \text{single-pion production, chiral anomaly} \\ \pi^- + \pi^0 + \pi^0 & \text{double-pion prod., chiral tree \& loop} \\ \dots \end{cases}$$



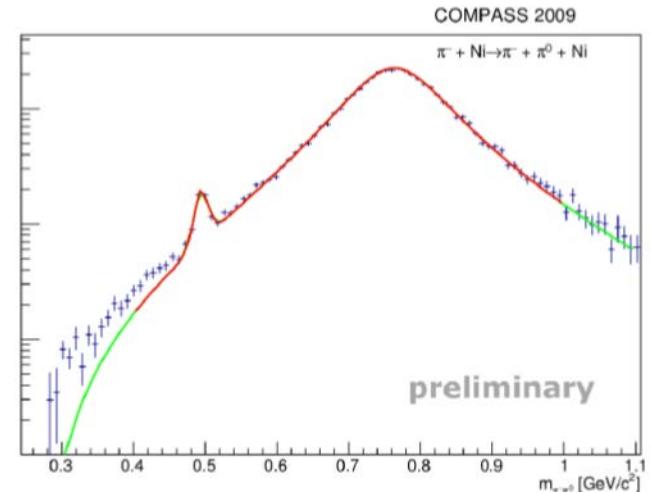
Dominik Steffen | g-2 workshop Mainz | 19.06.2018

COMPASS has acquired large Primakoff data set

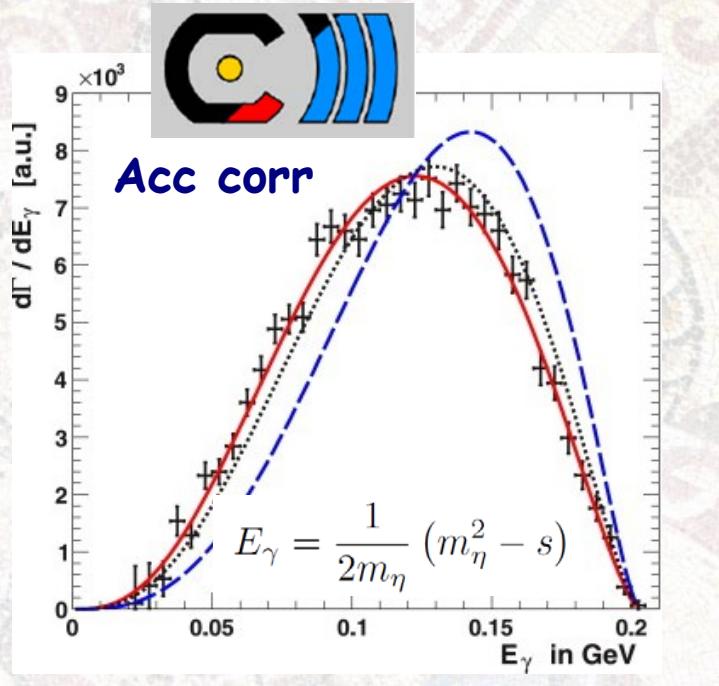
- ▷ Measurements from channel $\pi^- + \gamma^{(*)} \rightarrow \pi^- \pi^0$:
 - Chiral anomaly $\gamma \rightarrow 3\pi$
 - Radiative width of light-quark isovector mesons ($\rho(770)^- \rightarrow \pi^- \gamma$)
- ▷ Data from run 2009 give consistent picture:
 - Fit in good agreement with data
 - Normalizing to radiative width of $\rho(770)$ yields value for $F_{3\pi}$ in agreement with theoretical result
- ▷ 4x larger data set to come from 2012 data
- ▷ Possibility to extend analysis to measure radiative couplings of excited ρ states

$F_{3\pi}$ at COMPASS

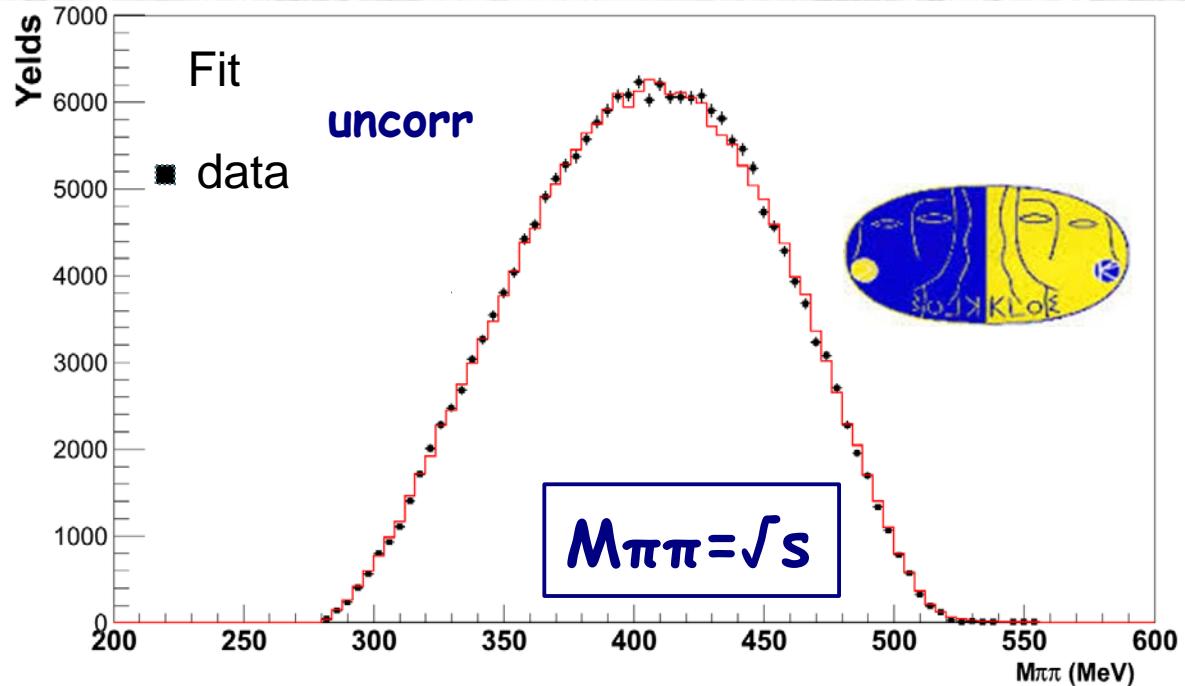
Conclusion



TFF from radiative processes (ex $\eta \rightarrow \pi^+\pi^-\gamma$)



WASA PLB707 (2012) 243



KLOE PLB718 (2013) 910

Model independent parametrization:

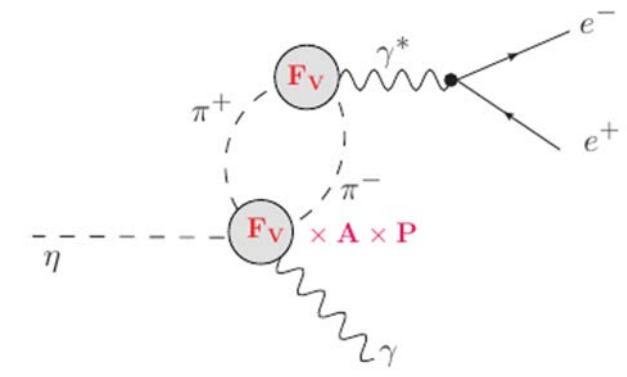
$$\frac{d\Gamma}{ds} = |A(1 + \alpha s + \dots) F_V(s)|^2 K_P(s)$$

PLB707 (2012) 184

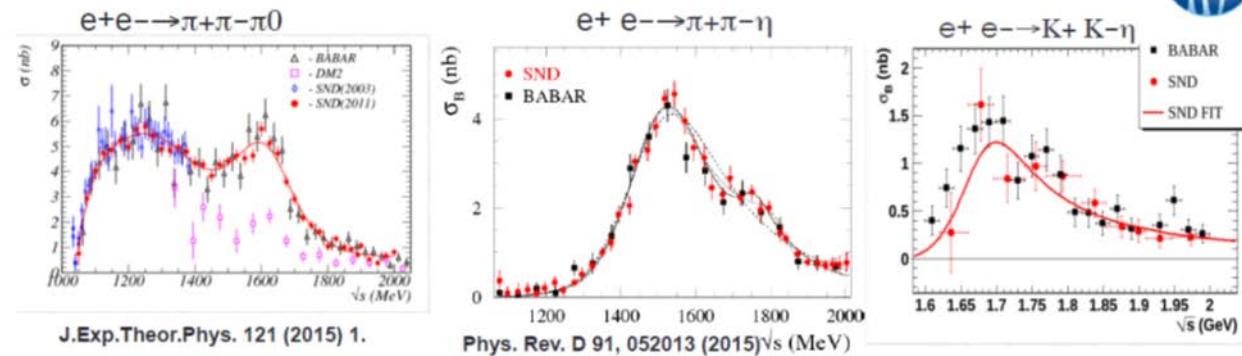
$e^+e^- \rightarrow \pi^+\pi^-$

$$\alpha = 1.89 \pm 0.25_{\text{stat}} \pm 0.59_{\text{syst}} \text{ GeV}^{-2}$$

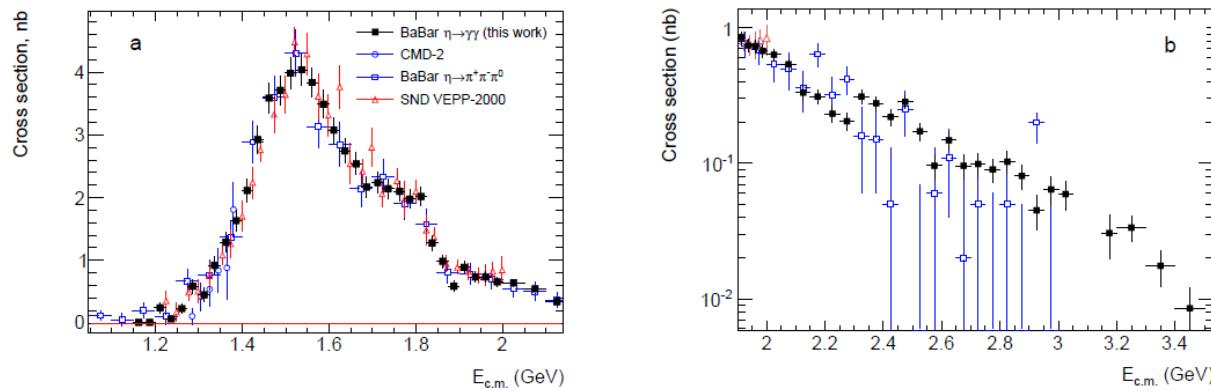
$$\alpha = 1.31 \pm 0.08_{\text{stat}} \pm 0.40_{\text{syst}} \text{ GeV}^{-2}$$



Some SND results overview



$e^+e^- \rightarrow \eta\pi^+\pi^-$ at BaBar



More precise result \Rightarrow first observation of the $\rho(1700)$ in $\eta\pi^+\pi^-$

J.P. Lees et al., Phys. Rev. D97 (2018) 052007

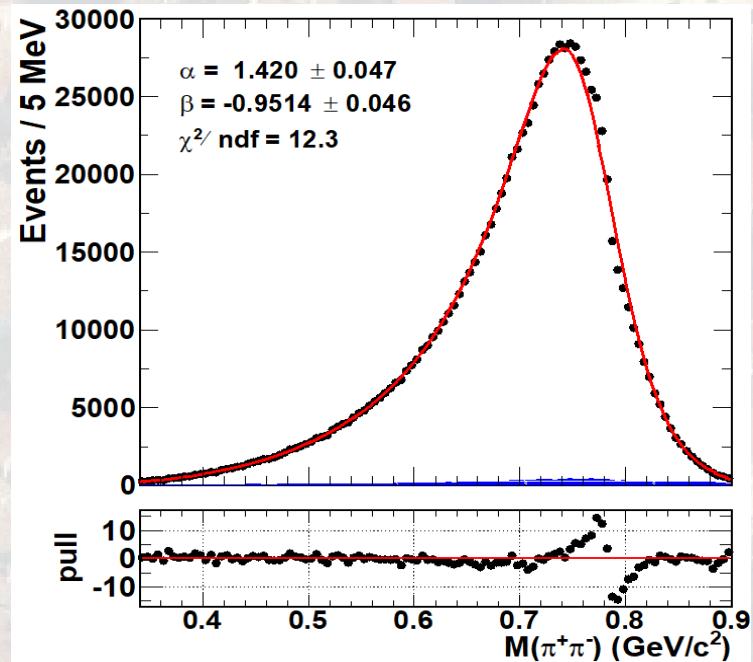
DP Similar strategy for η as for π^0
 From $e^+e^- \rightarrow \pi^+\pi^-\eta$ to η TFF

arXiv:1509.02194

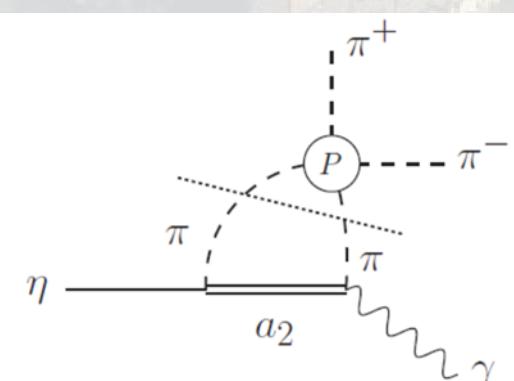
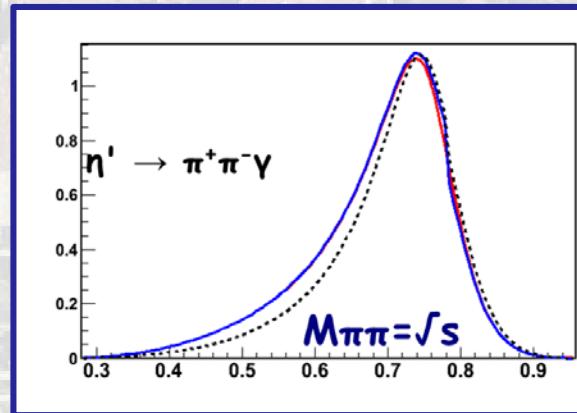
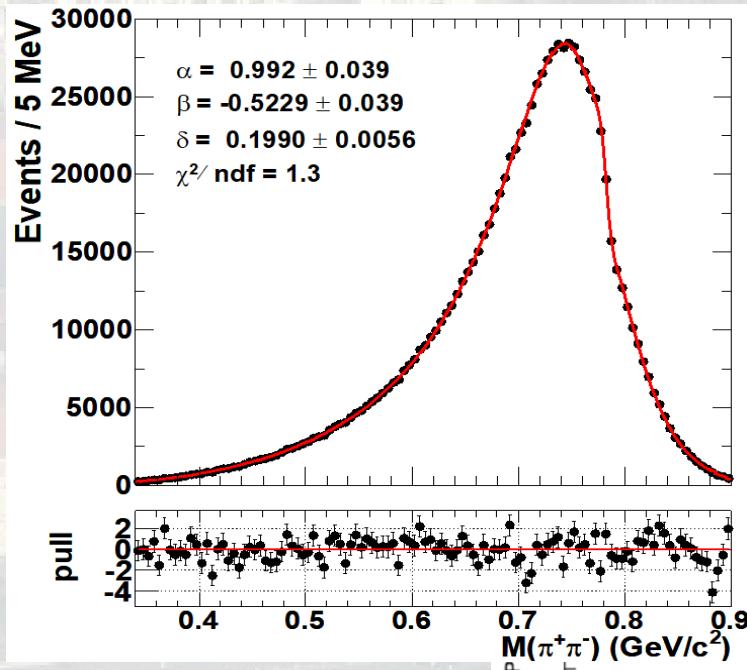
Analysis based on $0.9 \times 10^6 \eta' \rightarrow \pi^+\pi^-\gamma$

$$\frac{d\Gamma}{ds_{\pi\pi}} = |AP(s_{\pi\pi})F_V(s_{\pi\pi})|^2 \Gamma_0(s_{\pi\pi})$$

$$P(s_{\pi\pi}) = 1 + \alpha s_{\pi\pi} + \beta s_{\pi\pi}^2$$



$$P(s_{\pi\pi}) = 1 + \alpha s_{\pi\pi} + \beta s_{\pi\pi}^2 + \delta \text{BW}_\omega$$



ω contribution necessary

Linear polynomial is insufficient...

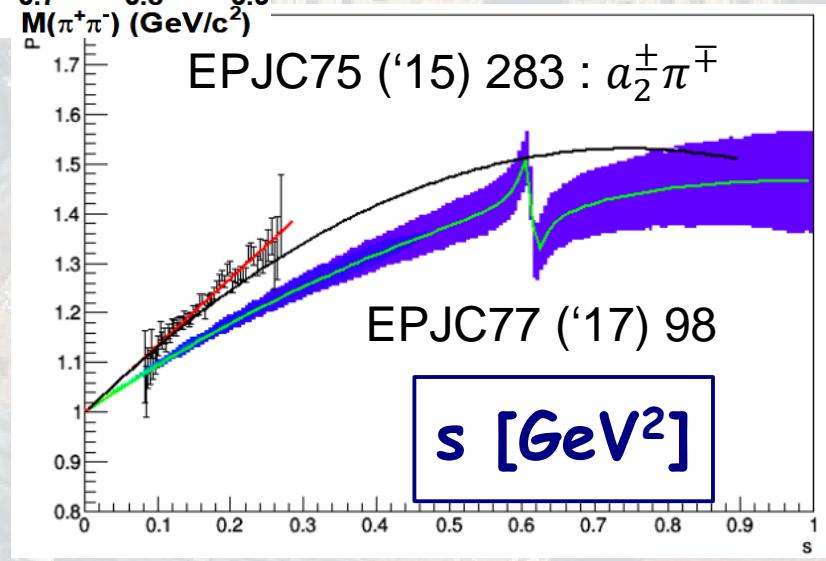
Phys.Rev.Lett. 120 (2018) 242003

BES III

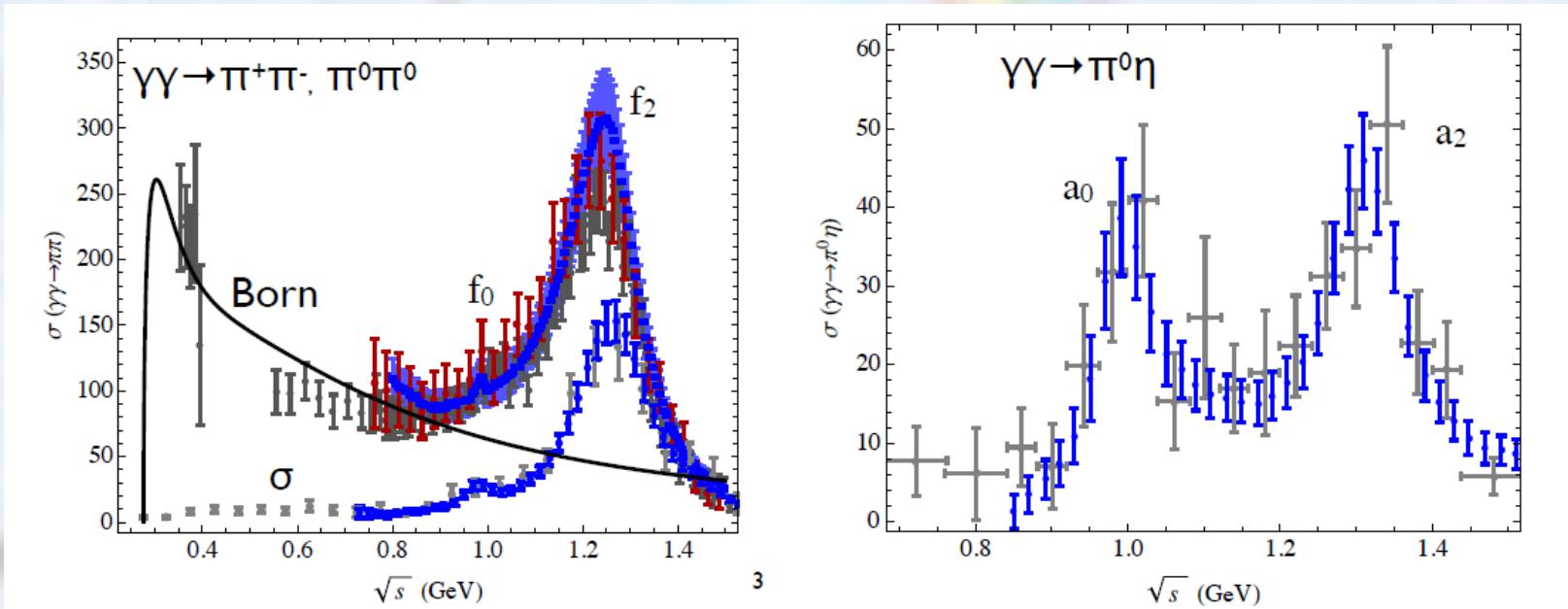
EPJC75 ('15) 283 : $a_2^\pm \pi^\mp$

EPJC77 ('17) 98

$s [\text{GeV}^2]$

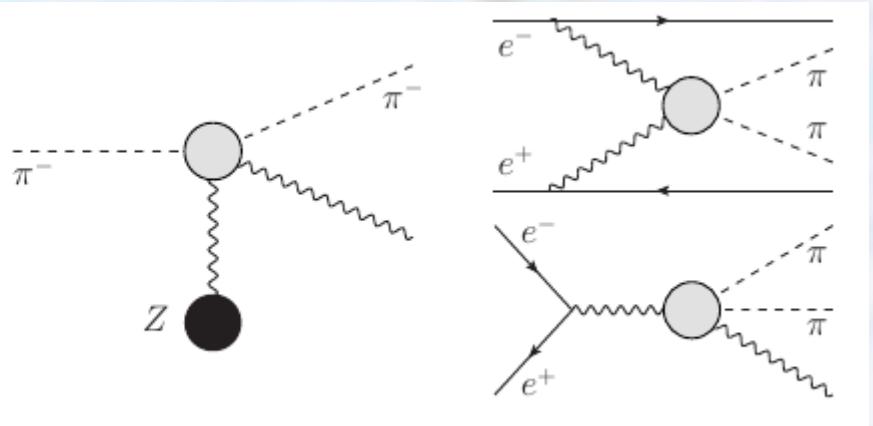


$\gamma^{(*)} \gamma^{(*)} \rightarrow \pi\pi; \eta\pi; K\bar{K}$



$\gamma\gamma \rightarrow \pi\pi, K\bar{K}, \eta\eta, \pi\eta$ (Belle: 07,08, 09,10, ..)
 $\gamma\gamma^* \rightarrow \pi\pi, \pi\eta$ (BESIII in progress)

- ▶ Low energy: pion polar., ChPT
- ▶ Primakoff: $\gamma\pi \rightarrow \gamma\pi$ at COMPASS, JLAB
- ▶ Scattering: $e^+e^- \rightarrow e^+e^-\pi\pi$, $e^+e^- \rightarrow \pi\pi\gamma$
- ▶ Decays: $\omega, \phi \rightarrow \pi\pi\gamma$



Data driven DR HLbL determination:

Transition Form Factors of π^0 , η , η'

- Direct measurements
- Input and checks for DR approach to TFF

Pion and Kaon Form Factors...

Two photon production of PS pairs

HLbL error:
- input experiments
- high energy behavior
=> error budget