Holographic approach to the Light-By-Light contribution to $(g-2)_\mu$

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Plan of the talk

- Holographic models of QCD
- 2-point functions: VV and AA Current-Current Crrelators
- 3-point functions: The Pion Transition Form Factor
- 4-point functions: The HLbL Tensor
- Conclusions and Outlook

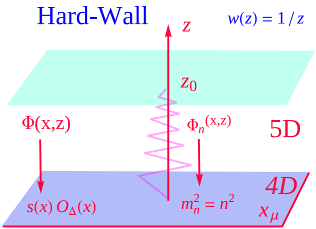
Holographic models of QCD: SS, HW1, HW2, SW

SS: [Sakai,Sugimoto(05)]
HW1: [Erlich, Katz, Son, Stephanov(05)],[Da Rold, Pomarol(05)]
HW2: [Hirn,Sanz(05)]
SW: [Karch, Katz, Son, Stephanov(06)]

Holographic models of QCD I: recipes & ingredients

HQCD models inspired by AdS/CFT duality between a 4D (conformal) (Large-N_c) gauge theory at strong coupling and a (classical) 5D field theory in a curved Anti-de Sitter space

$$\exp(iW[\mathbf{s}(\mathbf{x})]) \equiv \left\langle \exp\left(i \int d^4x\, \mathbf{s}(\mathbf{x}) O_\Delta(x) \right) \right\rangle_{QCD} = \exp\left(i\, S_5(\Phi_0(z,x))\right)$$



4D	5D
operator $O_\Delta(x)$	dual field $\Phi(x,z)$
source $\mathbf{s}(\mathbf{x})$ coupled to $O_\Delta(x)$	on-shell $\Phi_0(x,z) \to \mathbf{s}(\mathbf{x})$
conformal dimension Δ	mass m_Φ: $m_\Phi^2 = (\Delta - p)(\Delta + p - 4)$
$U(N_f)_L \times U(N_f)_R$ global symmetry	$U(N_f)_L \times U(N_f)_R$ gauge symmetry
vector current $\bar{q}\gamma^\mu t^a q$ $\quad \mathbf{v}_\mu^\mathbf{a}(\mathbf{x}) \leftarrow$	gauge field $V_\mu^a(x,z)$
axial current $\bar{q}\gamma^\mu\gamma_5 t^a q$ $\quad \mathbf{a}_\mu^\mathbf{a}(\mathbf{x}) \leftarrow$	gauge field $A_\mu^a(x,z)$
quark bilinear $\bar{q} t^a q$ $\quad \mathbf{s}(\mathbf{x}) \leftarrow$	scalar field $X^a(x,z)$

confinement	Hard-Wall: sharp cut-off $0 \le z \le z_0$ Soft-Wall: dilaton potential
Chiral Symmetry Breaking	5D profile $X(z)$ 5D parity/ ChSB boundary conditions

Holographic models of QCD II: 5D Lagrangians

$$S_5 = S_{\rm YM} + S_{\rm X} + S_{\rm CS}$$

with

$$S_{\rm YM} = -\int d^4x \int_0^{z_0} dz\; e^{-\Phi(z)} \frac{1}{8g_5^2} w(z)\, {\rm tr}\left[\mathcal{F}_L^2 + \mathcal{F}_R^2\right]$$

$$S_X = \int d^4x \int_0^{z_0} dz\; e^{-\Phi(z)} w(z)^3\, {\rm tr}\left[D^M X D_M X^\dagger + V(X^\dagger X)\right]$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int {\rm tr}\left(\mathcal{A}_L \mathcal{F}_L^2 - \frac{i}{2}\mathcal{A}_L^3 \mathcal{F}_L - \frac{1}{10}\mathcal{A}_L^5\right) - (L \to R)$$

In these equations:

- 5D metric $ds_5^2 = w(z)^2 \left(dx_\mu^2 - dz^2\right)$. For AdS, $w(z) = 1/z$.
- $\mathcal{F}_{MN} = \partial_M \mathcal{A}_N - \partial_N \mathcal{A}_M - i[\mathcal{A}_M, \mathcal{A}_N]$ and $\mathcal{A}_{L,R} = V \mp A$,
- In HW models z has a sharp cut-off at a finite value of z_0.
- In SW models, there is a background dilaton field $\Phi(z) = -\kappa^2 z^2$.
- In the HW1 models the 5D scalar field $X(x, z)$, dual to $\bar{q}q$, induces ChSB, by acquiring a non trivial 5D profile $X = v(z)$.
- In HW2 ChSB broken by different boundary conditions for V_μ and A_μ on the IR wall z_0 and the 4D chiral field $U(x)$ appears as the remnant of non trivial 5D Wilson line of A_z.

2-point Functions: Vector and Axial-vector Current-Current Correlators

$$\left\langle T\left\{J_V^\mu(x)J_V^\nu(y)\right\}\right\rangle \Longleftrightarrow \frac{\delta^2 S_5}{\delta v^\mu(x)\delta v^\nu(y)}$$

$$\left\langle T\left\{J_A^\mu(x)J_A^\nu(y)\right\}\right\rangle \Longleftrightarrow \frac{\delta^2 S_5}{\delta a^\mu(x)\delta a^\nu(y)}$$

2-point Function: Fixing the parameters of HW2

$$2i\int d^4x\, e^{iq\cdot x}\left\langle T\left\{J^{a,\mu}_{V,A}(x)J^{b,\nu}_{V,A}(0)\right\}\right\rangle = \delta^{ab}\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi_{V,A}(q^2)$$

$$\Pi_V(q^2) = \overset{L_{10},\,H_1}{\otimes\!\otimes} + \frac{q^2}{g_5^2}\int_0^{z0} dz\int_0^{z0} dz'\ \underset{\rho,\,\rho',\,\ldots}{\overset{V}{\otimes\!=\!=\!=\!\otimes}} = \frac{1}{q^2 g_5^2}\,\partial_z\partial_{z'}\,\overset{V}{\otimes\!=\!=\!=\!\otimes}\Big|_{z=z'=0}$$

$$\Pi_A(q^2) = \overset{f_\pi^2}{\otimes\!\bullet\!\otimes} + \overset{f_\pi}{\otimes\!\bullet}\underset{\pi}{- - -}\overset{f_\pi}{\bullet\!\otimes} + \overset{L_{10},\,H_1}{\otimes\!\otimes} + \frac{q^2}{g_5^2}\int_0^{z0} dz\int_0^{z0} dz'\ \underset{a_1,\,a_1',\,\ldots}{\overset{A}{\otimes\!=\!=\!=\!\otimes}}$$

$$= \frac{1}{q^2 g_5^2}\,\partial_z\partial_{z'}\,\overset{A}{\otimes\!=\!=\!=\!\otimes}\Big|_{z=z'=0}$$

QCD OPE for Large Euclidean momentum $Q^2 = -q^2$

$$\Pi_{V,A}(-Q^2) = -\frac{1}{g_5^2}\left(\log\frac{Q^2}{\mu^2}\right) + \cdots = -\frac{N_c}{12\pi^2}\left(\log\frac{Q^2}{\mu^2}\right) + \mathcal{O}\left(\frac{1}{Q^4}\right) \Longrightarrow g_5^2 = \frac{12\pi^2}{N_c}$$

Low momenta: pion field canonical normalization: $f_\pi^2 = \frac{2}{g_5^2 z_0^2} = \frac{N_c}{6\pi^2 z_0^2}$

$m_\rho = \frac{\gamma_{0,1}}{z_0} = \frac{2.405}{z_0} \Longrightarrow m_\rho = 776\,MeV$ fixes the *size* of the extra-dim. $z_0 = 3.103\,GeV^{-1}$

3-point Functions: The Pion Transition Form Factor

$$\langle \pi(x) | \, T \left\{ J^{\mu}_{e.m.}(y) J^{\nu}_{e.m.}(z) \right\} | \rangle \Longleftrightarrow \frac{\delta^3 S_5}{\delta\pi(x)\delta v_0^{\mu}(y)\delta v_0^{\nu}(z)}$$

3-point Function I: The Pion TFF from HW2

$$\int d^4x\, e^{-iq_1\cdot x}\,\langle P(q_1+q_2)|\,T\left\{J^{\mu}_{e.m.}(x)J^{\nu}_{e.m.}(0)\right\}|\rangle = \epsilon^{\mu\nu\rho\sigma} q_{1\,\rho} q_{2\,\sigma} \mathcal{F}_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$$

where $Q^2_{1,2} = -q^2_{1,2}$

For $P = \pi^0$, real photons normalization

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0) = \frac{N_c}{12\pi^2 f_\pi} \quad \text{(pointlike WZW vertex)}$$

Normalized TFF $K(Q_1^2, Q_2^2) \equiv \mathcal{F}_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)/\mathcal{F}_{P\gamma^*\gamma^*}(0,0) \to K(0,0) = 1$

Where is the pion field in HW2?

$$V_\mu(x,z) = v_\mu(x) + V_\mu^{(reson)}(x,z)$$

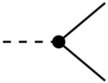
$$A_\mu(x,z) = \left(a_\mu(x) + \frac{\partial_\mu \pi(x)}{f_\pi}\right)\alpha(z) + A_\mu^{(reson)}(x,z)$$

Anomalous AVV amplitudes from trilinear terms in the CS action

$$S_{CS}^{(3)} = \frac{N_c}{24\pi^2}\int \mathrm{tr}\left(L(dL)^2 - R(dR)^2\right) \quad \text{with } L = V + A,\ R = V - A$$

3-point Functions II: The Pion Transition Form Factor cont'nd

$$K(Q_1^2, Q_2^2) = -\int_0^{z_0} v(Q_1, z) v(Q_2, z) \partial_z \alpha(z) dz \quad \Longrightarrow$$



Vector bulk-to-boundary propagator $v(q^2, z) = -w(z') \partial_{z'} G_V(z, z'; q^2)|_{z' \to 0}$

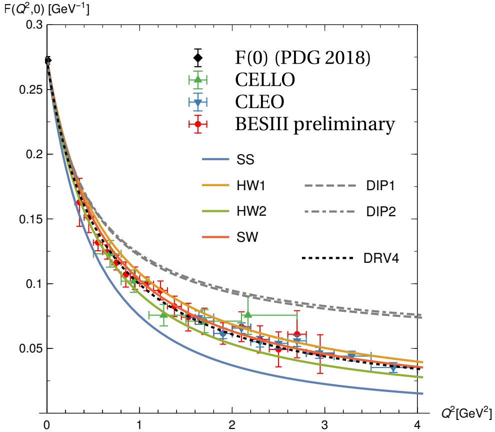
Low-Q^2

$$K(Q_1^2, Q_2^2) = 1 + \widehat{\alpha}\,(Q_1^2 + Q_2^2) + \widehat{\beta}\, Q_1^2 Q_2^2 + \widehat{\gamma}\,(Q_1^4 + Q_2^4) + \ldots$$

CELLO(91) : $\widehat{\alpha} = -1.76(22)\,GeV^{-2}$

NA62(17) : $\widehat{\alpha} = -1.76(22)\,GeV^{-2}$

$\longrightarrow$ W.A. : $\widehat{\alpha} = -1.84(17)\,GeV^{-2}$



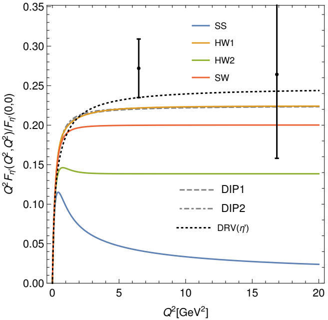
[Leutgeb,Mager,Rebhan(19)]

3-point Functions III: The Pion Transition Form Factor cont'nd

Large Euclidean momentum $Q^2 \gg \Lambda_{QCD}$

$$K^{pQCD}(Q^2, 0) = \frac{8\pi^2 f_\pi^2}{Q^2} \qquad K^{pQCD}(Q^2, Q^2) = \frac{8\pi^2 f_\pi^2}{3Q^2}$$

- The same expressions obtained in HW1 and HW2 and SW due to AdS metric
- However, with $z_0 = 3.103 GeV^{-1}$, in order to reproduce the value of the ρ meson mass, f_π is underestimated in HW2, and since $8\pi^2 f_\pi^2 = 4/z_0$, one gets 61.6% of the pQCD result, as shown in the figure.
- Possible solution: shrinking $z_0 = 3.103 GeV^{-1}$ one gets the physical value of $f_\pi = 92.4 MeV$, at the cost of overestimating $m_\rho = 987 MeV$



Double-virtual TFF with experimental data for η' from BaBar rescaled byf_π / f_η' [Leutgeb,Mager,Rebhan(19)]

3-point Func. VI: TFF and one-pion exchange HLbL diagrams

Ansätze for $\mathcal{F}_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$

$$WZW : -\frac{N_c}{12\pi^2 f_\pi}$$

$$VMD : -\frac{N_c}{12\pi^2 f_\pi} \frac{m_V^2}{(q_1^2 - m_V^2)} \frac{m_V^2}{(q_2^2 - m_V^2)}$$

$$LMD : \frac{f_\pi}{3} \frac{q_1^2 + q_2^2 - (N_c m_V^4/(4\pi^2 f_\pi^2))}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}$$

$$LMD + V : \frac{f_\pi}{3} \frac{P_6(q_1^2, q_2^2, M_{V_1}^2, M_{V_2}^2; h_1, h_2, h_5)}{(q_1^2 - m_{V_1}^2)(q_2^2 - m_{V_1}^2)(q_1^2 - m_{V_2}^2)(q_2^2 - m_{V_2}^2)}$$

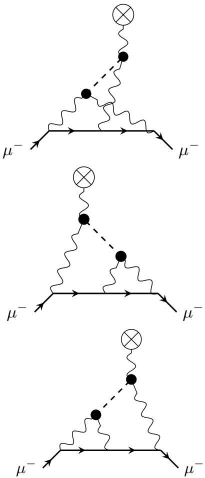
[Knecht, Nyffeler(01))]

$$DIP : -\frac{N_c}{12\pi^2 f_\pi}\left(1 + \lambda\left(\frac{q_1^2}{(q_1^2 - m_{V_1}^2)} + \frac{q_2^2}{(q_2^2 - m_{V_2}^2)}\right)\right.$$

$$\left. + \eta \sum_{i=1,2} \frac{q_1^2 q_2^2}{(q_1^2 - m_{V_i}^2)(q_2^2 - m_{V_i}^2)}\right)$$

[C, Cata,D'Ambrosio(10))]

HLbL One-pion exchange diagrams.



3-point Func. VI: $a_\mu^{\mathrm{HLbL},\pi^0}$ estimates

$$a_\mu^{\mathrm{HLbL},\pi^0} = -\frac{e^6}{48 m_\mu}\int \frac{d^4 q_1}{(2\pi)^4}\frac{d^4 q_2}{(2\pi)^4}\frac{1}{q_1^2 q_2^2 (q_1+q_2)^2}\frac{1}{(p+q_1)^2 - m_\mu^2}\frac{1}{(p-q_2)^2 - m_\mu^2}$$
$$\times\left[\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_3^2,0)}{q_3^2 - m_\pi^2} T_1(q_1,q_2;p) + \frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_3^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_2^2,0)}{q_2^2 - m_\pi^2} T_2(q_1,q_2;p)\right]$$

Using Gegenbauer polynomials techniques [Knecht Nyffeler 01] only a triple integral remains

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^2 \bar{T}_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau),$$

where $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. $\bar{\Pi}_i$ evaluated for the reduced kinematics

$$q_1^2 = -Q_1^2, \quad q_2^2 = -Q_2^2, \quad q_3^2 = -Q_3^2 = -Q_1^2 - 2Q_1 Q_2 \tau - Q_2^2, \quad q_4^2 = 0.$$

3-point Func. VI: $a_\mu^{\mathrm{HLbL},\pi^0}$ estimates contn'd

$a_\mu^{\mathrm{HLbL},\pi^0} \times 10^{-9}$		
VMD	5.7	KN(01)
LMD+V	6.3	KN(01)
DIP	6.58	CCD(11)
⟨HQCD's⟩	5.9(2)	LMR(19)
DVR interp.	5.64(25)	DVR(19)
Lattice	5.97± 0.23	GMN(19)

⟨HQCD's⟩LMR(19) $a_\mu^{\mathrm{HLbL},\pi^0} \times 10^{-9}$	
SS	4.83
HW1	6.13
HW2	5.66
SW	5.92

[Danilkin,Redmer,Vanderaeghen(19)], [Gérardin,Meyer, Nyffeler(19)]

However, there is a problem: The value for HW2 is obtained with the physical value $f_\pi = 92,4 MeV$ while taking $N_c = 3$ and $m_\rho = 776 MeV$, but as we already saw, **these three parameters are not independent in HW2 !** Different choices of fixing two of the parameters to their physical values (but not the third) all lead to a sensible increase of the value of $a_\mu^{\mathrm{HLbL},\pi^0} \sim 30\%$

4-point Function: The Hadronic Light-by-Light Tensor

$$\left\langle \left| T\left\{ J^{\mu}_{e.m.}(x) J^{\nu}_{e.m.}(y) J^{\lambda}_{e.m.}(z) J^{\sigma}_{e.m.}(w) \right\} \right| \right\rangle$$

$$\Longleftrightarrow \frac{\delta^4 S_5}{\delta v_0^{\mu}(x) \delta v_0^{\nu}(y) \delta v_0^{\lambda}(z) \delta v_0^{\sigma}(w)}$$

4-point Function I:The Hadronic Light-by-Light Tensor

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = -i\int d^4x d^4y d^4z\, e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)} < |T\{j^{\mu}_{\rm em}(x)j^{\nu}_{\rm em}(y)j^{\lambda}_{\rm em}(z)j^{\sigma}_{\rm em}(0)\}| >$$

$$q_4 = q_1+q_2+q_3$$

138 Lorentz structures

$$\Pi^{\mu\nu\lambda\sigma} = g^{\mu\nu}g^{\lambda\sigma}\,\Pi^1 + g^{\mu\lambda}g^{\nu\sigma}\,\Pi^2 + g^{\mu\sigma}g^{\nu\lambda}\,\Pi^3$$
$$+\sum_{i,j=1,2,3}\Big(g^{\mu\nu}q_i^{\lambda}q_j^{\sigma}\,\Pi^4_{ij} + g^{\mu\lambda}q_i^{\nu}q_j^{\sigma}\,\Pi^5_{ij} + g^{\mu\sigma}q_i^{\nu}q_j^{\lambda}\,\Pi^6_{ij}$$
$$+g^{\nu\lambda}q_i^{\mu}q_j^{\sigma}\,\Pi^7_{ij} + g^{\nu\sigma}q_i^{\mu}q_j^{\lambda}\,\Pi^8_{ij} + g^{\lambda\sigma}q_i^{\mu}q_j^{\nu}\,\Pi^9_{ij}\Big)$$
$$+\sum_{i,j,k,l=1,2,3} q_i^{\mu}q_j^{\nu}q_k^{\lambda}q_l^{\sigma}\,\Pi^{10}_{ijkl}$$

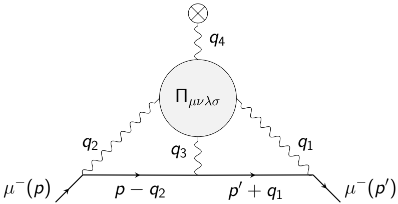
95 linearly independent relations from gauge invariance $\Longrightarrow$

$$\{q_{1\mu},q_{2\nu},q_{3\rho},q_{4\sigma}\}\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3)=0$$

Complete crossing symmetric, e.g. under

$$\mathcal{C}_{14}=\{q_1\leftrightarrow -q_4,\mu\leftrightarrow\sigma\},\quad \mathcal{C}_{13}=\{q_1\leftrightarrow q_3,\mu\leftrightarrow\lambda\}$$

The HLbL tensor in the HLbL diagram



43 linearly independent tensor structures

BTT basis: 54 (redundant) tensor structures, with scalar functions Π_i free of kinematic singularities [Colangelo et al.15]

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma}\Pi_i,$$

4-point Function II:The Master Formula for a_μ^{HLbL}

$$a_\mu^{\mathrm{HLbL}} = -\frac{e^6}{48 m_\mu}\int \frac{d^4 q_1}{(2\pi)^4}\frac{d^4 q_2}{(2\pi)^4}\frac{1}{q_1^2 q_2^2 (q_1+q_2)^2}\frac{1}{(p+q_1)^2-m_\mu^2}\frac{1}{(p-q_2)^2-m_\mu^2}$$
$$\times \mathrm{Tr}\left((\not{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\not{p}+m_\mu)\gamma^\mu(\not{p}+\not{q}_1+m_\mu)\gamma^\lambda(\not{p}-\not{q}_2+m_\mu)\gamma^\nu\right)$$
$$\times \sum_{i=1}^{54}\left(\frac{\partial}{\partial q_4^\rho}T^i_{\mu\nu\lambda\sigma}(q_1,q_2,q_4-q_1-q_2)\right)\Bigg|_{q_4=0}\Pi_i(q_1,q_2,-q_1-q_2).$$

Only 19 independent linear combinations of the 54 $T_i^{\mu\nu\rho\lambda}$ contribute to a_μ^{HLbL}. Using Gegenbauer polynomials techniques [Knecht Nyffeler 01], the symmetry of the loop integral and the propagators, there remain 12 different integrals containing 12 coefficients $\bar{\Pi}_i(q_1,q_2,-q_1-q_2)$.

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1\int_0^\infty dQ_2\int_{-1}^1 d\tau\sqrt{1-\tau^2}Q_1^3 Q_2^3\sum_{i=1}^{12}\bar{T}_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau),$$

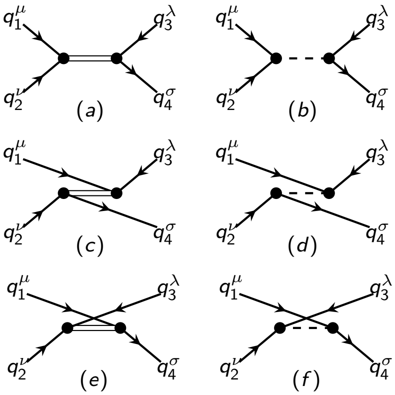
where $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. $\bar{\Pi}_i$ evaluated for the reduced kinematics

$$q_1^2=-Q_1^2,\quad q_2^2=-Q_2^2,\quad q_3^2=-Q_3^2=-Q_1^2-2Q_1Q_2\tau-Q_2^2,\quad q_4^2=0.$$

Integral kernels expressions $\bar{T}_i(Q_1,Q_2,\tau)$, in [Colangelo et al.15&17]

4-point Function III: HLbL tensor from HW2

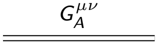
[Cata, C., D'Ambrosio, Greynat, Iyer, to appear]



Pion and Massive axial resonances anomalous AVV vertices from S_{CS}

Propagators (from S_{YM})

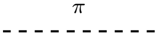
(Massive) axial resonances



5D axial Green function

$$G_A^{\mu\nu}(z,z';q^2) =$$
$$G_A^T(z,z';q^2)P_T^{\mu\nu}(q) + G_A^L(z,z')P_L^{\mu\nu}(q)$$
$$P_T^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right),$$
$$P_L^{\mu\nu}(q) = \frac{q^\mu q^\nu}{q^2}$$

Pion propagator



$$\frac{i}{q^2 - m_\pi^2}$$

4-point Function VI: HLbL tensor from HW2 contn'd

$$\Pi^{\mu\nu\lambda\sigma} = \underbrace{\Pi_L^{(\pi,\,A)\,\mu\nu\lambda\sigma}}_{\text{pion \& massive axial reson.}} + \underbrace{\Pi_T^{(A)\,\mu\nu\lambda\sigma}}_{\text{massive axial reson.}}$$

where, for the massive resonances contributions

$$\Pi_{L,T}^{(A)\,\mu\nu\lambda\sigma} = \underbrace{\left(g^{\mu\mu'} - \frac{q_1^\mu q_1^{\mu'}}{q_1^2}\right)\left(g^{\nu\nu'} - \frac{q_2^\nu q_2^{\nu'}}{q_2^2}\right)\left(g^{\lambda\lambda'} - \frac{q_3^\lambda q_3^{\lambda'}}{q_3^2}\right)\left(g^{\sigma\sigma'} - \frac{q_4^\sigma q_4^{\sigma'}}{q_4^2}\right)}_{\text{transverse projectors on external vector legs}}$$

$$\times \underbrace{\varepsilon_{\mu'\nu'\alpha\beta}\,\varepsilon_{\lambda'\sigma'\gamma\delta}}_{\text{anomalous couplings}} \times \underbrace{P_{L,T}^{\alpha\gamma}}_{\text{L,T proj. in } G_A} \times \underbrace{A_{L,T}^{\beta\delta}}_{z \text{ and } z' \text{ integrals}}$$

$A_{L,T}^{\beta\delta}$ contains combinations of the form $q_a^\beta q_c^\delta\, \mathcal{G}_A^{L,T}(q_a, q_b; q_c, q_d)$ with the convolution integrals

$$\mathcal{G}_A^L(q_a, q_b; q_c, q_d) = \int_0^{z_0} dz \int_0^{z_0} dz'\, v(z, q_a^2)\partial_z v(z, q_b^2) G_A^L(z, z') v(z', q_c^2)\partial_{z'} v(z', q_d^2)$$

$$\mathcal{G}_A^T(q_a, q_b; q_c, q_d) = \int_0^{z_0} dz \int_0^{z_0} dz'\, v(z, q_a^2)\partial_z v(z, q_b^2) G_A^T(z, z'; q_a+q_b) v(z', q_c^2)\partial_{z'} v(z', q_d^2)$$

4-point Function V: Asymptotic behaviour

Asymptotic behaviour of the HW2 4-point amplitude for large Euclidean momenta

- Quite interestingly Melnikov-Vainshtein [Melnikov,Vainshtein(04)] QCD OPE constraints are satisfied !

  While the pion contribution is dominating at low momenta, the massive axial resonance contribution gives the MV OPE behaviour for Large Euclidean momenta.

- In the literature the MV constraint
  - lead to an increase of the accepted estimate of the HLbL
  - was difficult to implement in models:
    For instance MV proposed a model with pointlike WZW at the vertex with physical photon, while [JegerlehnerNyffeler(09)] got the MV behaviour using LMD+V TFF's, with an elaborate choice of the parameters.

- the HW2 seems the first model to satisfy MV, without any of the above assumptions, that's why, dispite its simplicity, it is still worth studying

Preliminary results and Outlook

- HW2 gives a definite prediction for the contribution of the (full tower of) massive axial resonances to the one-particle exchange diagram of HLbL
- Preliminary numerical analysis: numerical results depend on the choice of the values of N_c, f_π and m_ρ, which are not independent in HW2.

  For instance, for $N_c = 3$, $f_\pi = 92.4 MeV$ and $m_\rho = 987 MeV$ one gets

  $$a_\mu^{\pi^0,\,\mathrm{HLbL}} = 7.5 \times 10^{-10} \quad a_\mu^{\mathrm{A},\,\mathrm{HLbL}} = 0.9 \times 10^{-10}$$

  Other choices also indicate similar increase of the pion contribution.
- The massive axial resonances contribution is compatible with results in the literature, e.g. [Pauk, Vanderaeghen(14)]