FCCP2019 Villa Orlandi, Anacapri, 29-31 August 2019

The Leading Hadronic Contributions to the muon g-2

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LIVERPOOL THEORETICAL PHYSICS

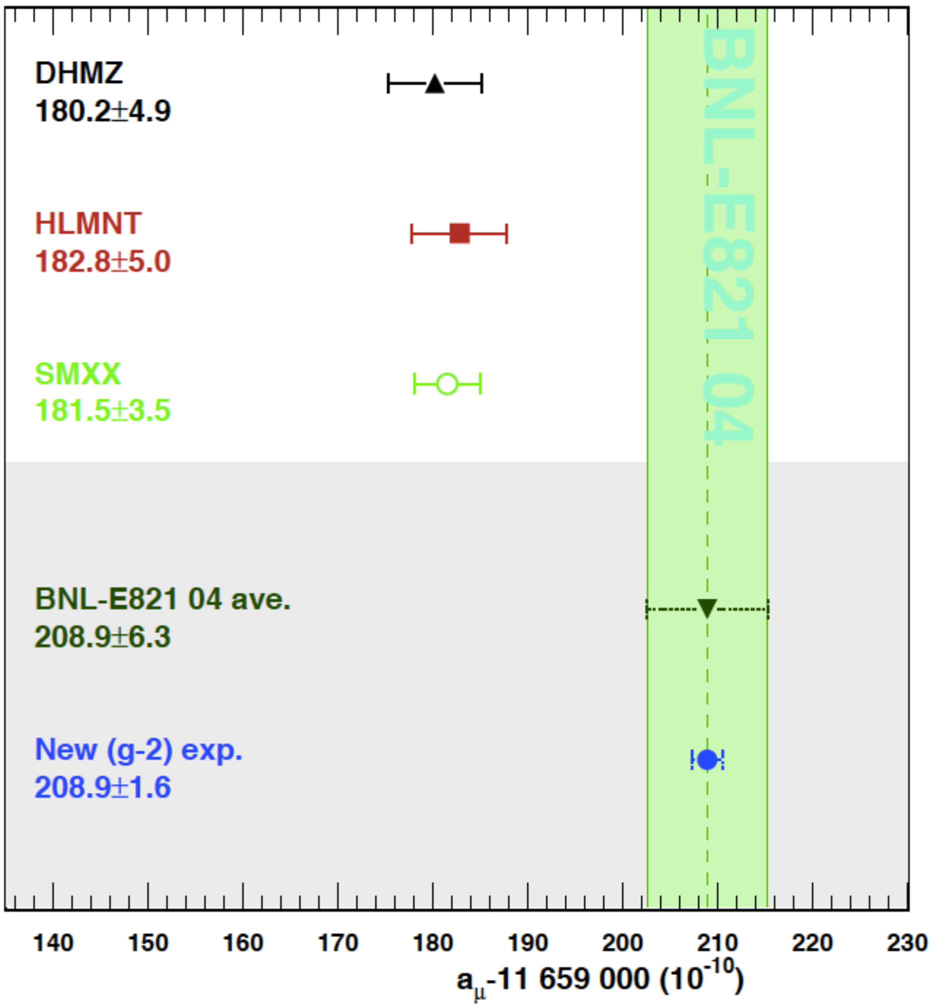
- Introduction
- Features and results of the KNT compilation
- Other approaches and recent developments
- Outlook

a_μ: Status and future projection → charge for SM TH

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

From: arXiv:1311.2198
`The Muon (g-2) Theory Value: Present and Future'

- if mean values stay and with no a_μ^{SM} improvement: 5σ discrepancy
- if also EXP+TH can improve a_μ^{SM} `as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8σ
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...



“Muon g-2 Theory Initiative” formed in June 2017

group photo from June 2018 workshop, https://indico.him.uni-mainz.de/event/11/overview

“map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results”

“Muon g-2 Theory Initiative”

- Next meeting: 9-13 September 2019, UoW, Seattle
- http://www.int.washington.edu/PROGRAMS/19-74W/
- The main emphasis of the upcoming workshop will be on producing a comprehensive white paper,
- presenting and discussing the current state-of-the-art for the SM prediction for g-2,
- tracing differences between different approaches,
- and, if possible, conclude with the best possible SM prediction

`KNT18' =

PHYSICAL REVIEW D **97,** 114025 (2018)

Editors' Suggestion | Featured in Physics

Muon $g-2$ and $\alpha(M_Z^2)$: A new data-based analysis

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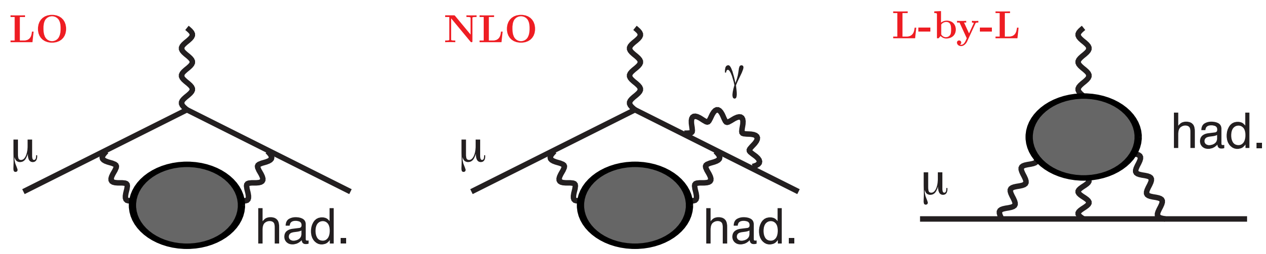
This work presents a complete reevaluation of the hadronic vacuum polarization contributions to the anomalous magnetic moment of the muon, $a_\mu^{\mathrm{had,VP}}$, and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\mathrm{had}}(M_Z^2)$, from the combination of $e^+e^- \to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \to$ hadrons cross section data have been analyzed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and perturbative QCD above that range results in estimates of the hadronic vacuum polarization contributions to $g-2$ of the muon of $a_\mu^{\mathrm{had,LO\,VP}} = (693.26 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\mathrm{had,NLO\,VP}} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\mathrm{SM}} = (11659182.04 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavor hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

DOI: 10.1103/PhysRevD.97.114025

a_μ hadronic

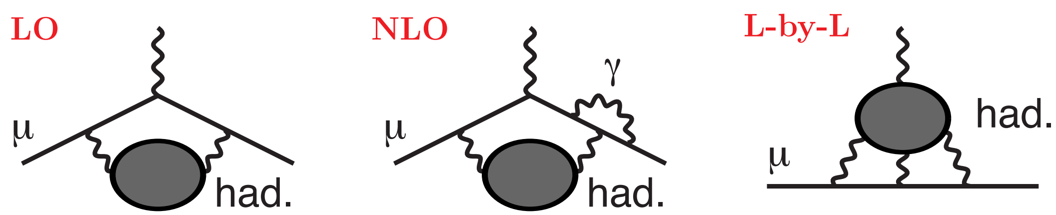
- Hadronic: non-perturbative, the limiting factor of the SM prediction? ✗ → ✓

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$

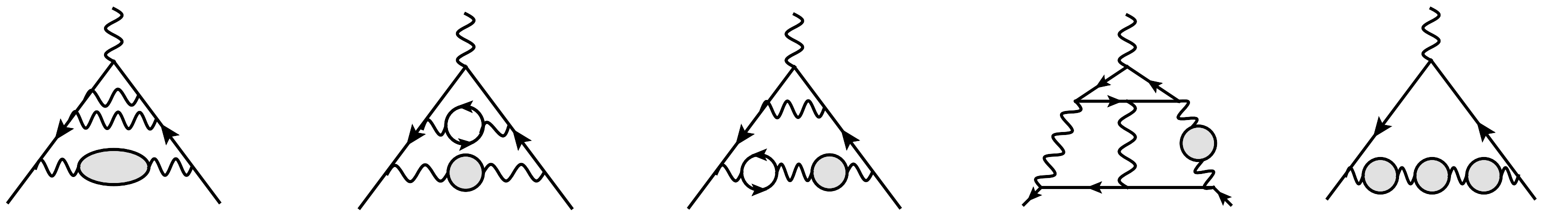


HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and well known dispersion integrals

- done at LO and NLO (see graphs)

- and recently at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]

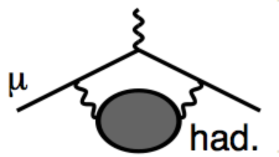
  $a_\mu^{\text{HVP, NNLO}} = +\ 1.24 \times 10^{-10}$ not so small, from e.g.:



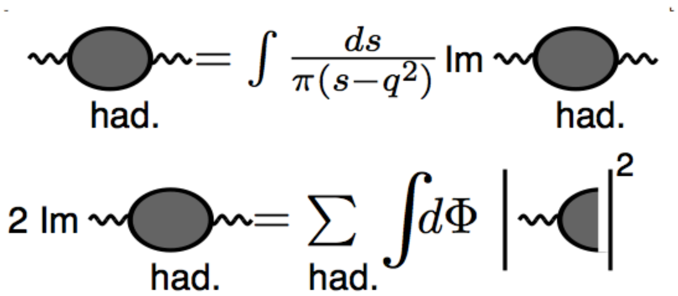
- Alternative: lattice QCD, but need QED and iso-spin breaking corrections.
  Lots of activity by several groups, errors coming down, QCD+QED started.

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$\boxed{a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

How to get the most precise $\sigma^0_{\rm had}$? e^+e^- data:

- Low energies: sum ~35 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, ..., [use iso-spin relations for missing channels]

- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of data combination (locally in √s): many experiments, different energy bins, stat+sys errors from different sources, correlations; must avoid inconsistencies/bias

- traditional \`direct scan' (tunable $e^+e^-$ beams) vs. \`Radiative Return' [+ τ spectral functions]

- $\sigma^0_{\rm had}$ means \`bare' σ, but WITH FSR: RadCorrs [HLMNT '11: $\delta a_\mu^{\rm had,\,RadCor\ VP+FSR} = 2 \times 10^{-10}$!]

Channel	Energy range [GeV]	$a_\mu^{\mathrm{had,LO\,VP}} \times 10^{10}$	$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) \times 10^4$	New data
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \leq \sqrt{s} \leq 0.600$	0.12 ± 0.01	0.00 ± 0.00	...
$\pi^+\pi^-$	$2m_\pi \leq \sqrt{s} \leq 0.305$	0.87 ± 0.02	0.01 ± 0.00	...
$\pi^+\pi^-\pi^0$	$3m_\pi \leq \sqrt{s} \leq 0.660$	0.01 ± 0.00	0.00 ± 0.00	...
$\eta\gamma$	$m_\eta \leq \sqrt{s} \leq 0.660$	0.00 ± 0.00	0.00 ± 0.00	...
Data based channels ($\sqrt{s} \leq 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \leq \sqrt{s} \leq 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \leq \sqrt{s} \leq 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \leq \sqrt{s} \leq 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \leq \sqrt{s} \leq 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \leq \sqrt{s} \leq 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\mathrm{no}\eta}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.99 ± 0.09	0.33 ± 0.03	...
$3\pi^+3\pi^-$	$1.313 \leq \sqrt{s} \leq 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\mathrm{no}\eta\omega}$	$1.322 \leq \sqrt{s} \leq 1.937$	1.35 ± 0.17	0.51 ± 0.06	...
K^+K^-	$0.988 \leq \sqrt{s} \leq 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \leq \sqrt{s} \leq 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \leq \sqrt{s} \leq 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \leq \sqrt{s} \leq 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \leq \sqrt{s} \leq 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \leq \sqrt{s} \leq 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\mathrm{no}\omega}$	$1.333 \leq \sqrt{s} \leq 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.01	0.03 ± 0.00	...
$\eta\omega$	$1.333 \leq \sqrt{s} \leq 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \leq \sqrt{s} \leq 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.42 ± 0.03	0.15 ± 0.01	...
$\phi \to$ unaccounted	$0.988 \leq \sqrt{s} \leq 1.029$	0.04 ± 0.04	0.01 ± 0.01	...
$\eta\omega\pi^0$	$1.550 \leq \sqrt{s} \leq 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to \mathrm{npp})K\bar{K}_{\mathrm{no}\phi\to K\bar{K}}$	$1.569 \leq \sqrt{s} \leq 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
Estimated contributions ($\sqrt{s} \leq 1.937$ GeV)				
$(\pi^+\pi^-3\pi^0)_{\mathrm{no}\eta}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.50 ± 0.04	0.16 ± 0.01	...
$(\pi^+\pi^-4\pi^0)_{\mathrm{no}\eta}$	$1.313 \leq \sqrt{s} \leq 1.937$	0.21 ± 0.21	0.08 ± 0.08	...
$KK3\pi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.02	0.02 ± 0.01	...
$\omega(\to \mathrm{npp})2\pi$	$1.285 \leq \sqrt{s} \leq 1.937$	0.10 ± 0.02	0.03 ± 0.01	...
$\omega(\to \mathrm{npp})3\pi$	$1.322 \leq \sqrt{s} \leq 1.937$	0.17 ± 0.03	0.06 ± 0.01	...
$\omega(\to \mathrm{npp})KK$	$1.569 \leq \sqrt{s} \leq 1.937$	0.00 ± 0.00	0.00 ± 0.00	...
$\eta\pi^+\pi^-2\pi^0$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.04	0.03 ± 0.02	...
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \leq \sqrt{s} \leq 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	...	6.26 ± 0.19	7.07 ± 0.22	...
ψ'	...	1.58 ± 0.04	2.51 ± 0.06	...
$\Upsilon(1S-4S)$	...	0.09 ± 0.00	1.06 ± 0.02	...
pQCD	$11.199 \leq \sqrt{s} \leq \infty$	2.07 ± 0.00	124.79 ± 0.10	...
Total	$m_\pi \leq \sqrt{s} \leq \infty$	693.26 ± 2.46	276.11 ± 1.11	...

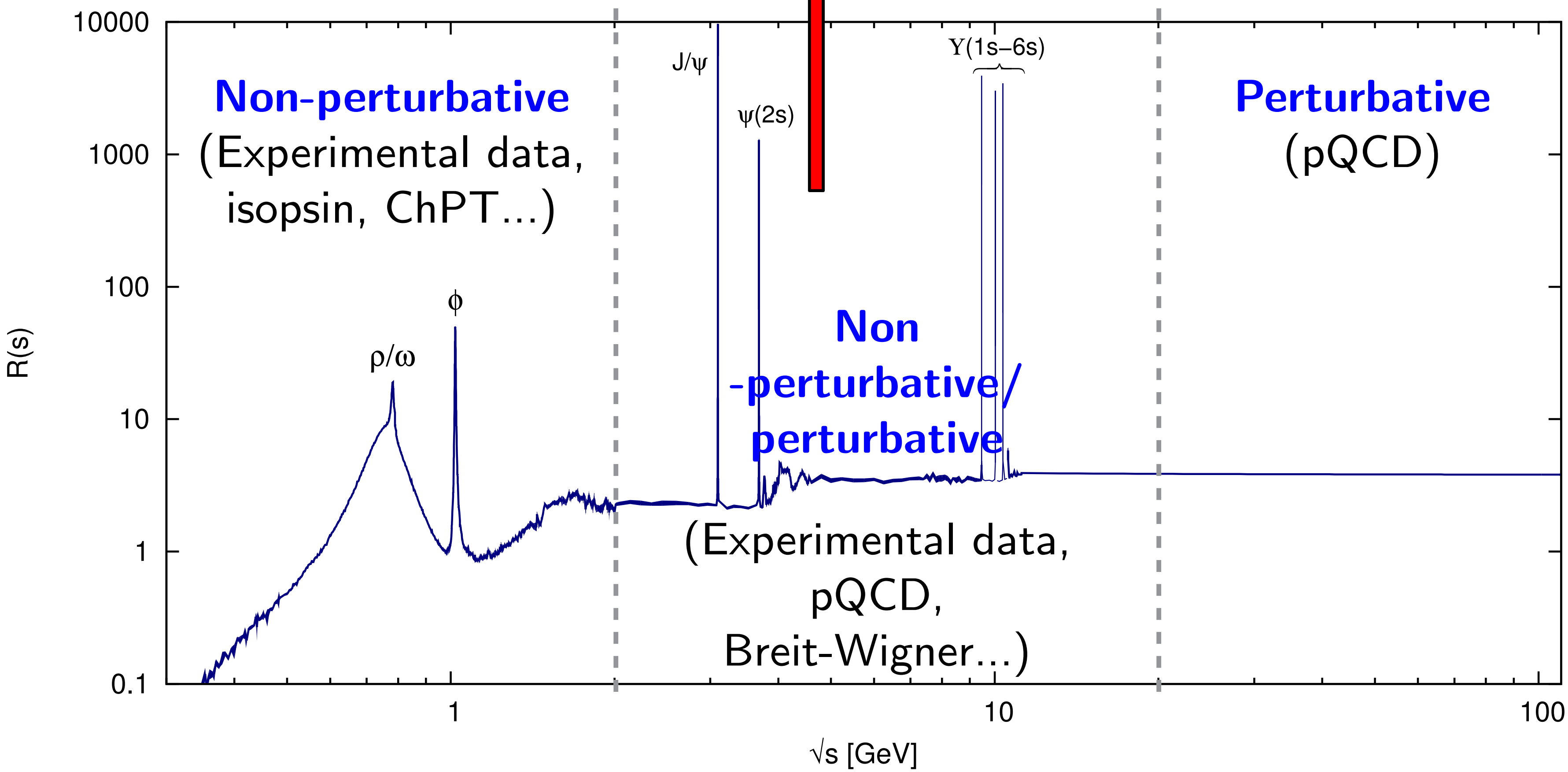
KNT18:

breakdown of HVP contributions in ~ 35 hadronic channels

From 2-11 GeV, use of inclusive data, pQCD only > 11 GeV

HVP cross section input

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty} \frac{\mathrm{d}s}{s} R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

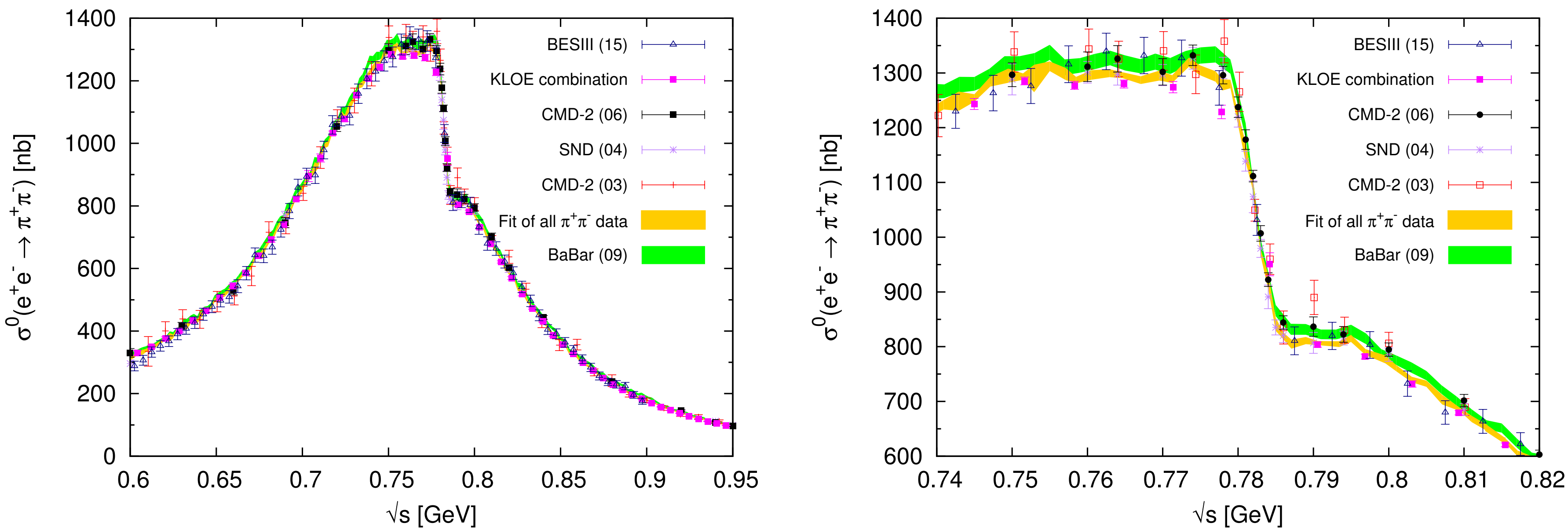


Must build full hadronic cross section/R-ratio...

HVP: $\pi^+\pi^-$ channel [KNT18, PRD97, 114025]

$\Rightarrow$ $\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\rm had,\, LO\, VP}$

$\rightarrow$ Combines 30 measurements totalling nearly 1000 data points



$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \le \sqrt{s} \le 1.937 \text{ GeV}] = 502.97 \pm 1.14_{\rm stat} \pm 1.59_{\rm sys} \pm 0.06_{\rm vp} \pm 0.14_{\rm fsr}$$

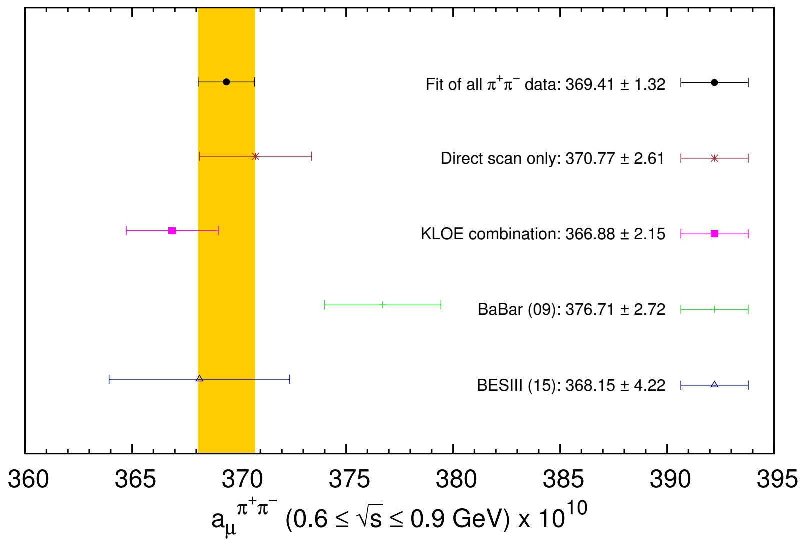
$$= 502.97 \pm 1.97_{\rm tot} \quad \text{HLMNT11: } 505.77 \pm 3.09$$

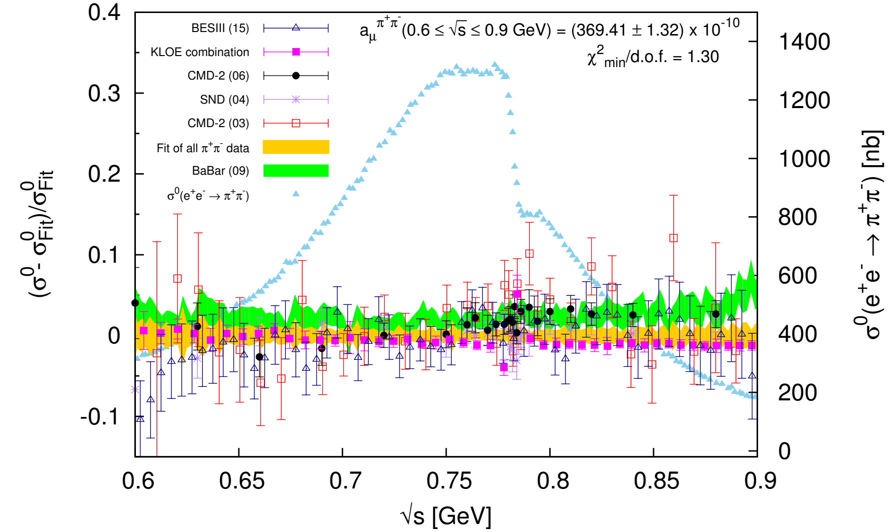
$\Rightarrow$ 15% local $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation due to tensions in clustered data

HVP: $\pi^+\pi^-$ channel [KNT18, PRD97, 114025]

$\Rightarrow$ Tension exists between BaBar data and all other data in the dominant ρ region.

$\rightarrow$ Agreement between other radiative return measurements and direct scan data largely compensates this.



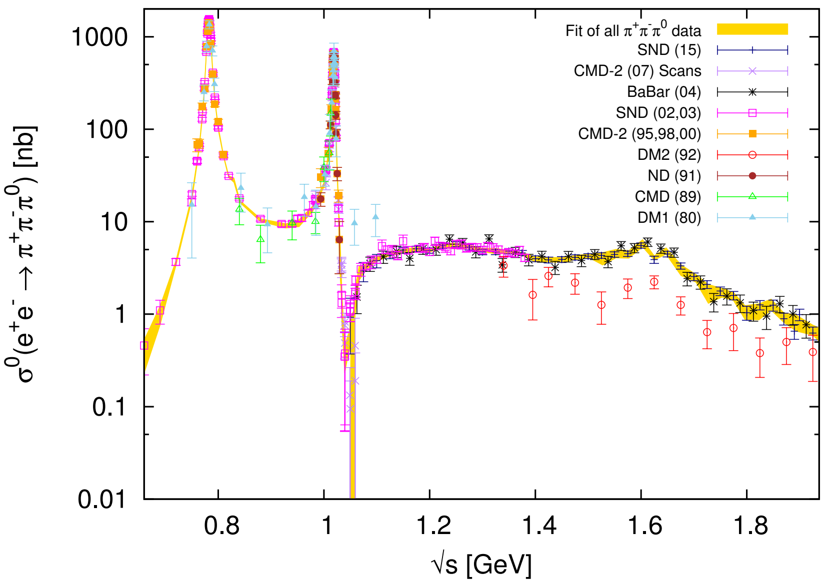


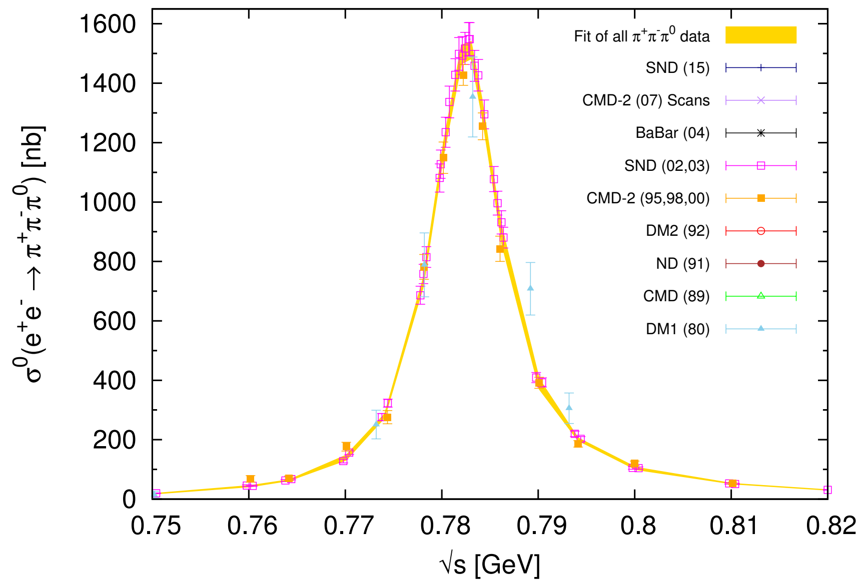
BaBar data alone $\Rightarrow a_\mu^{\pi^+\pi^-}(\text{BaBar data only}) = 513.2 \pm 3.8.$

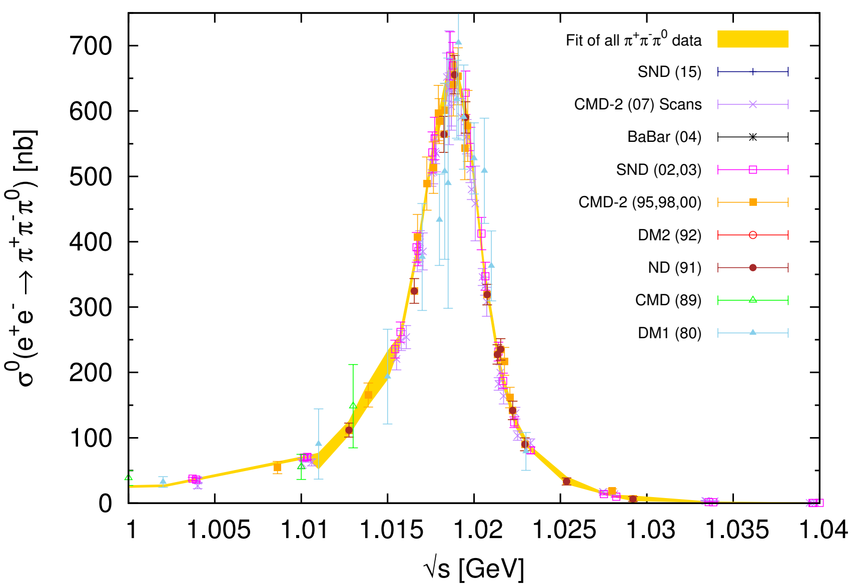
Simple weighted average of all data $\Rightarrow a_\mu^{\pi^+\pi^-}(\text{Weighted average}) = 509.1 \pm 2.9.$
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

HVP: $\pi^+\pi^-\pi^0$ channel [KNT18, PRD97, 114025]







Improvement for 3π also

New data:

SND: [J. Exp. Theor. Phys. 121 (2015), 27.]

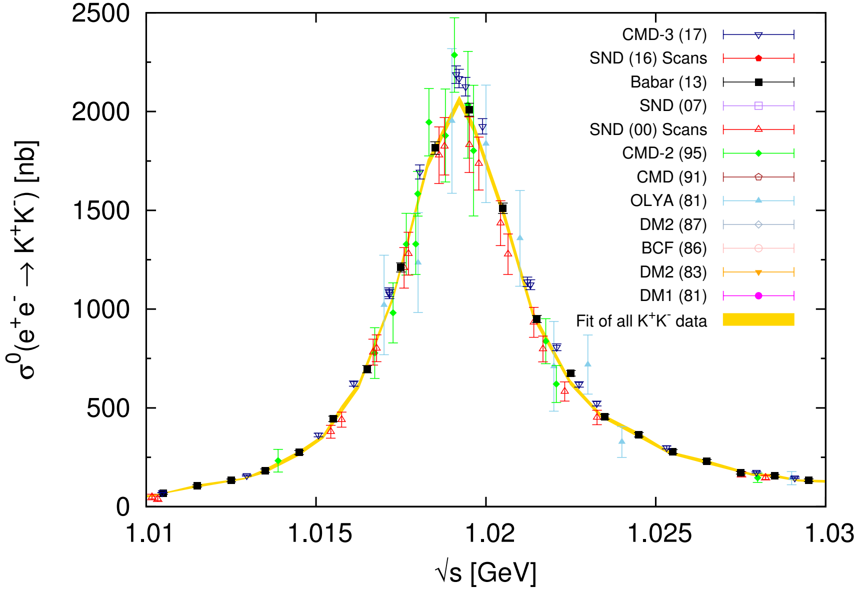
$$a_\mu^{\pi^+\pi^-\pi^0} = 47.79 \pm 0.22_{\rm stat} \pm 0.71_{\rm sys} \pm 0.13_{\rm vp} \pm 0.48_{\rm fsr}$$

$$= 47.79 \pm 0.89_{\rm tot}$$

HLMNT11: $47.51 \pm 0.99_{\rm tot}$

HVP: KK channels [KNT18, PRD97, 114025]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

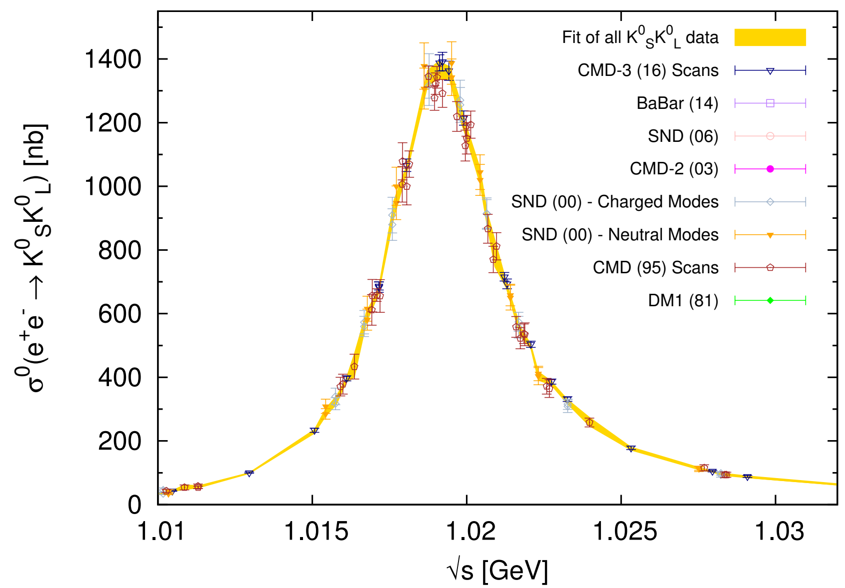
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\text{tot}}$$

HLMNT11: $22.15 \pm 0.46_{\text{tot}}$

Large increase in mean value

$K^0_S K^0_L$



New data:

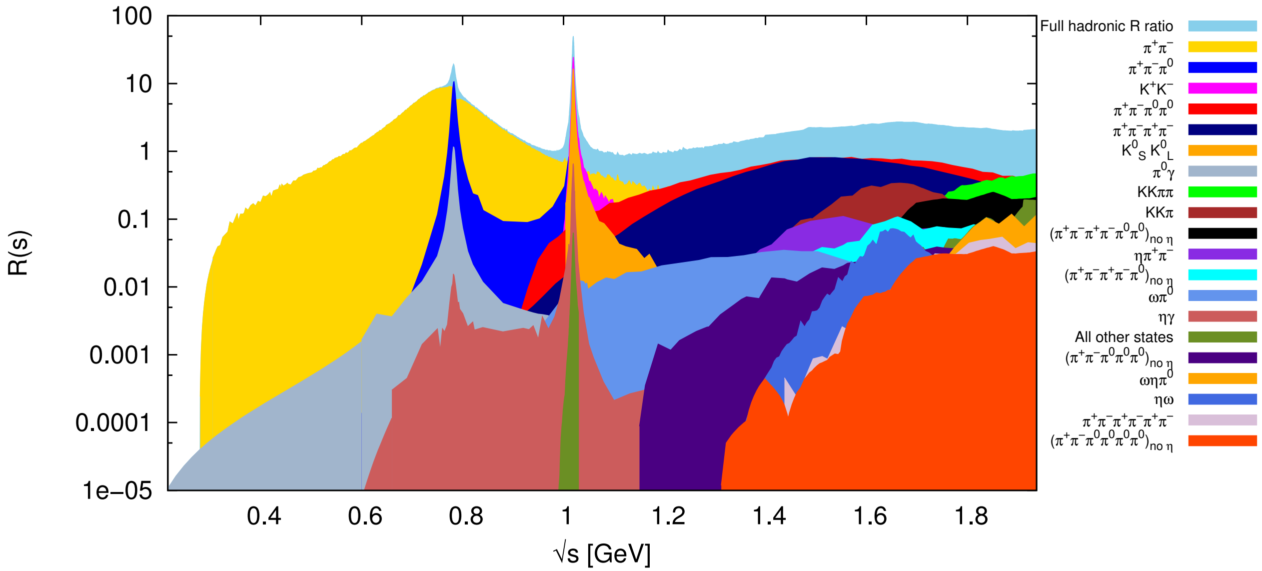
BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

$$a_\mu^{K^0_S K^0_L} = 13.04 \pm 0.19_{\text{tot}}$$

HLMNT11: $13.33 \pm 0.16_{\text{tot}}$

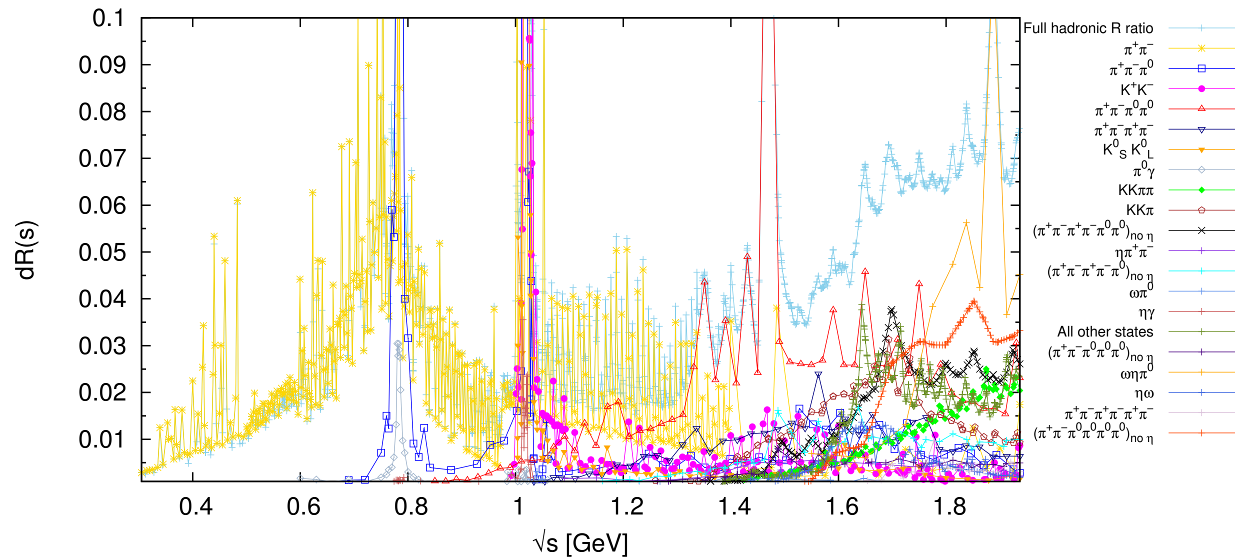
Large changes due to new precise measurements on ϕ

HVP: $\boldsymbol{\sigma}_{\mathrm{had}}$ channels below 2 GeV [KNT18, PRD97, 114025]



→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

→ Large improvement to cross section and uncertainty from new 4π data



HVP: $\sigma_{\rm had}$ excl → inclusive transition [KNT18]

⇒ New KEDR data allow reconsideration of exclusive/inclusive transition point

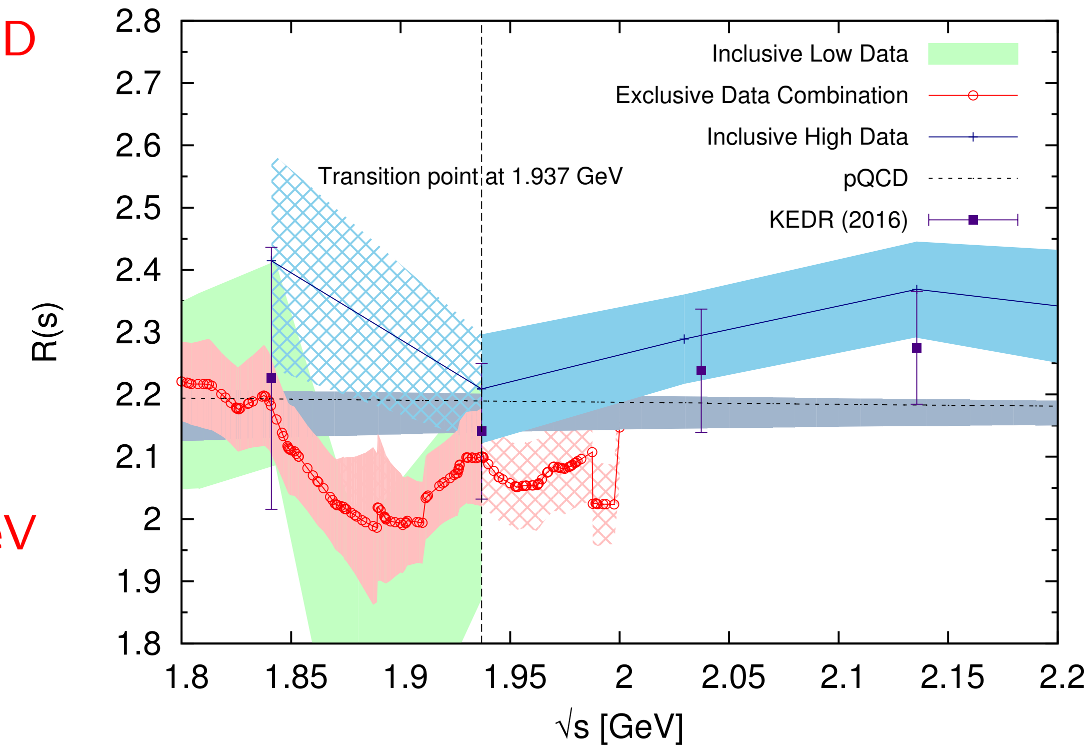
→ KNT18 aim to avoid use of pQCD and keep a data-driven analysis

→ Disagreement between sum of exclusive states and inclusive data/pQCD

→ New $\pi^+\pi^-\pi^0\pi^0$ data result in reduction of the cross section

→ Previous transition point at 2 GeV no longer the preferred choice

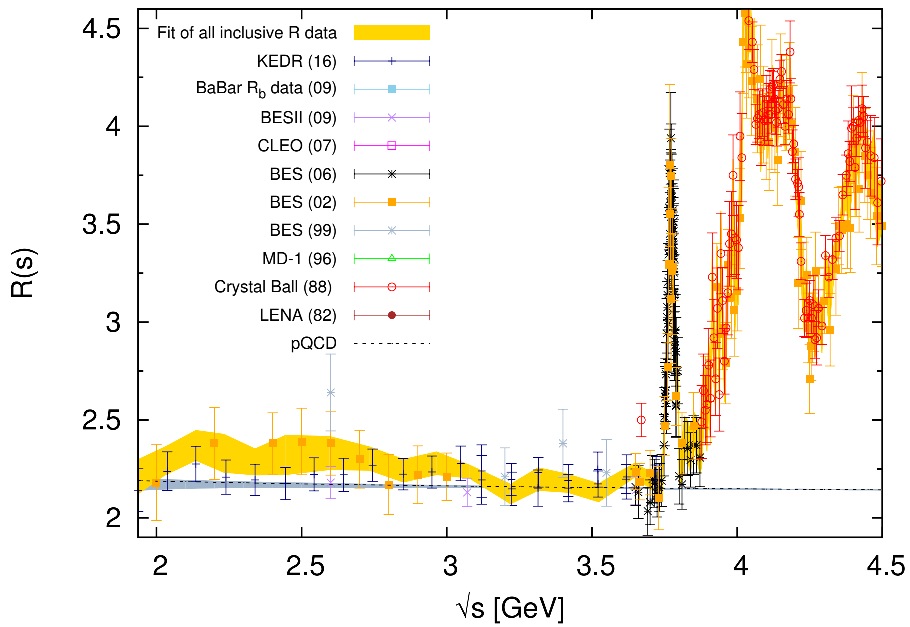
→ More natural choice for this transition point at 1.937 GeV



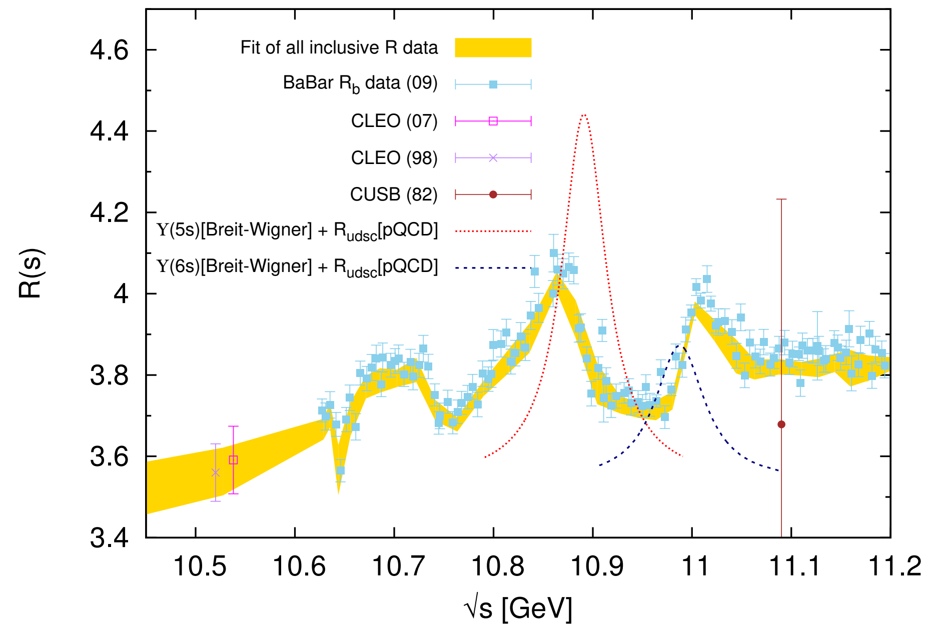
Input	$a_\mu^{\rm had,\,LO\,VP}[1.841 \le \sqrt{s} \le 2.00\ \text{GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive (< 1.937 GeV) + inclusive (> 1.937 GeV)	6.23 ± 0.13

HVP: σ_{had} inclusive region [KNT18]

$\Rightarrow$ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

HVP: KNT18 total and comparison w. other work

$$\text{HLMNT(11)}: \ 694.91 \pm 4.27$$
$$\downarrow$$

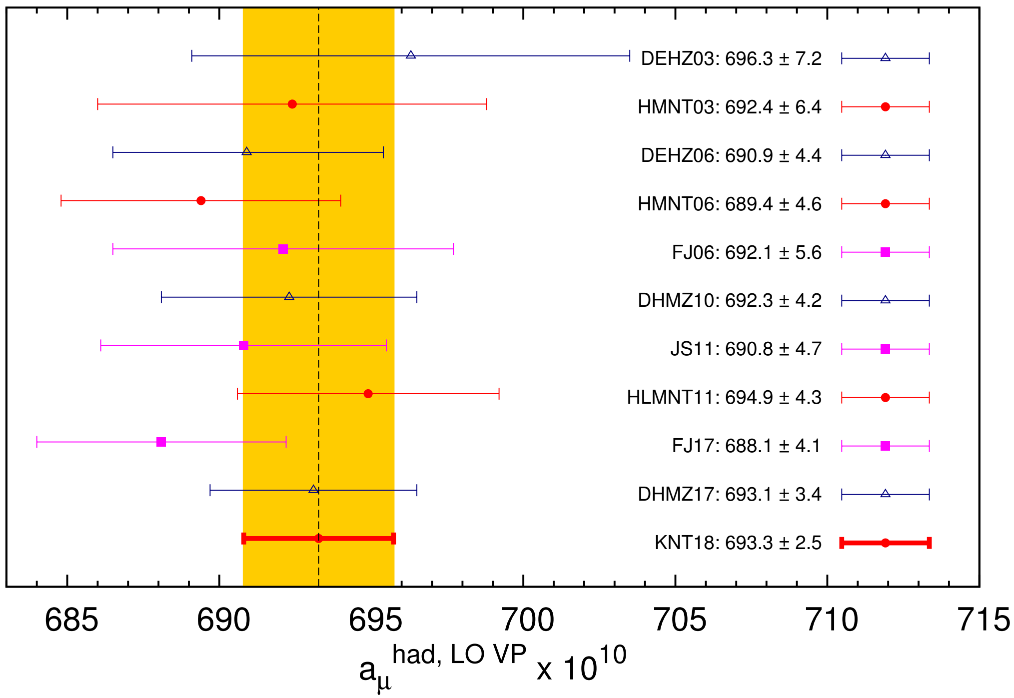
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

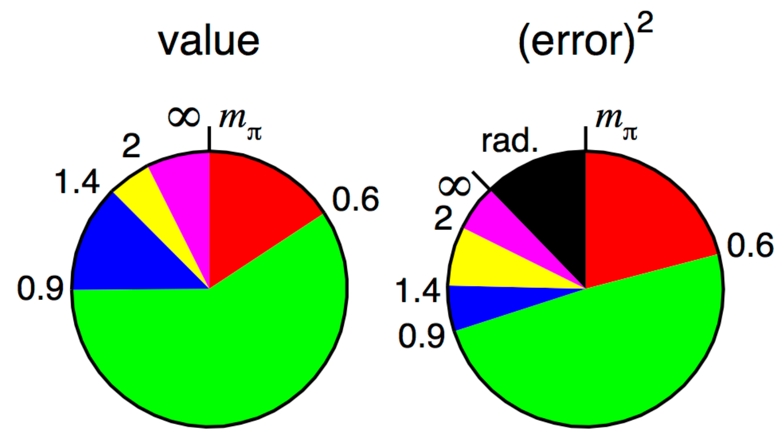
$$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$$

$$= 693.27 \pm 2.46_{\text{tot}}$$

$$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$$

$\Rightarrow$ Accuracy better then 0.4% (uncertainties include all available correlations)





$\Rightarrow$ 2π dominance

KNT18: comparison with data driven compilations

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K_S^0 K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
$1.8 \leq \sqrt{s} \leq 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

$\Rightarrow$ Total estimates from two analyses in very good agreement

$\Rightarrow$ Masks much larger differences in the estimates from individual channels

$\Rightarrow$ Unexpected tension for 2π considering the data input likely to be similar

$\rightarrow$ Points to marked differences in way data are combined

$\rightarrow$ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

$\Rightarrow$ Compensated by lower estimates in other channels

$\rightarrow$ For example, the choice to use pQCD instead of data above 1.8 GeV

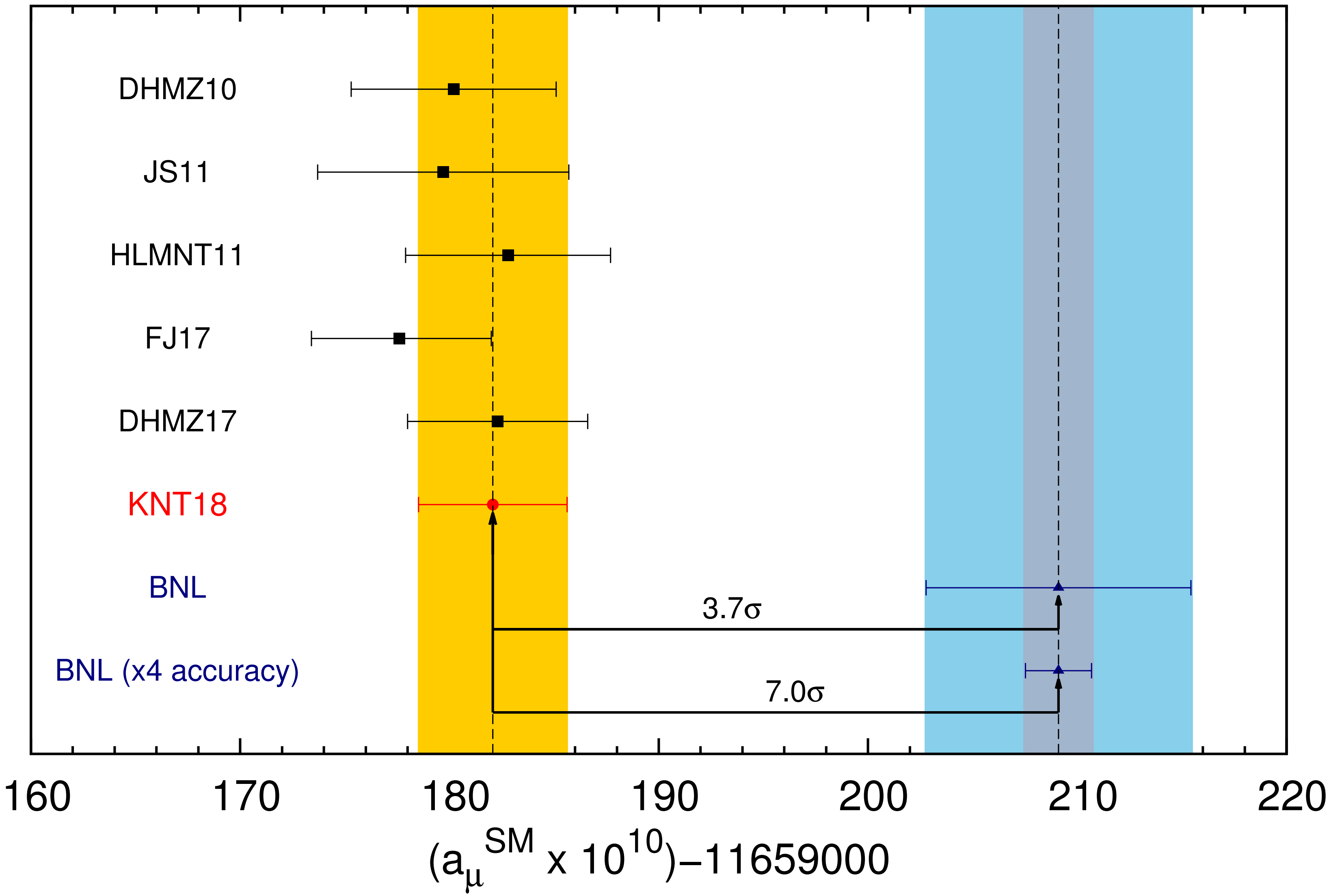
$\Rightarrow$ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

$\rightarrow$ Much lower mean value, but in agreement within errors

KNT18 a_μ^{SM} update

	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

a_μ^{SM} vs. a_μ^{EXP} discrepancy



7σ if E989 obtains same mean value with projected improvement in error

HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006

- Comprehensive dispersive study of the 2π vector form factor, including space-like data and phase shift analysis,
- leading to stronger constraints compared to pure direct data fit and integration.
- For good fit quality, energy calibration for narrow resonances crucial.

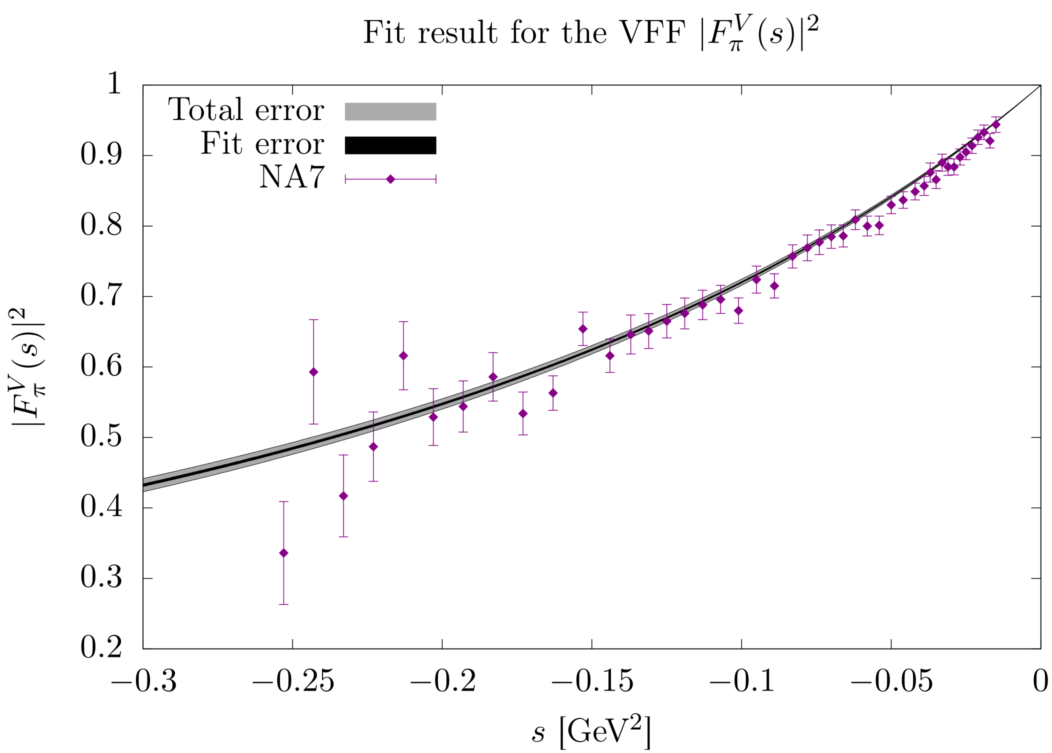


Figure 9: Fit result for the pion VFF in the space-like region, together with the NA7 data.

HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006

- Detailed analysis and comparison with other work on a region-by-region basis,
  -> allows improved understanding of differences between different groups
  -> important input for Theory Initiative white paper

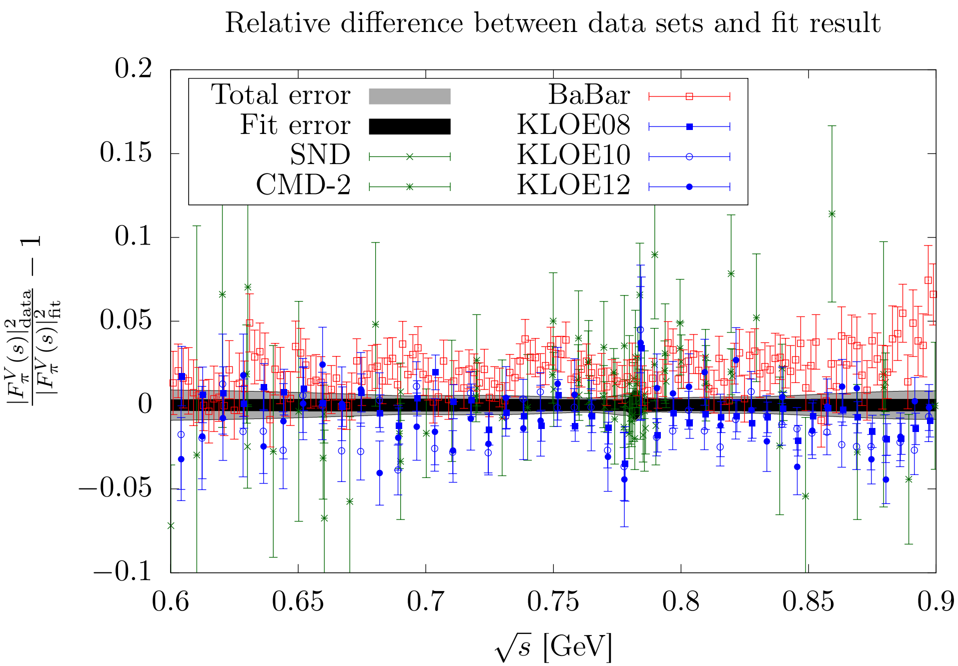


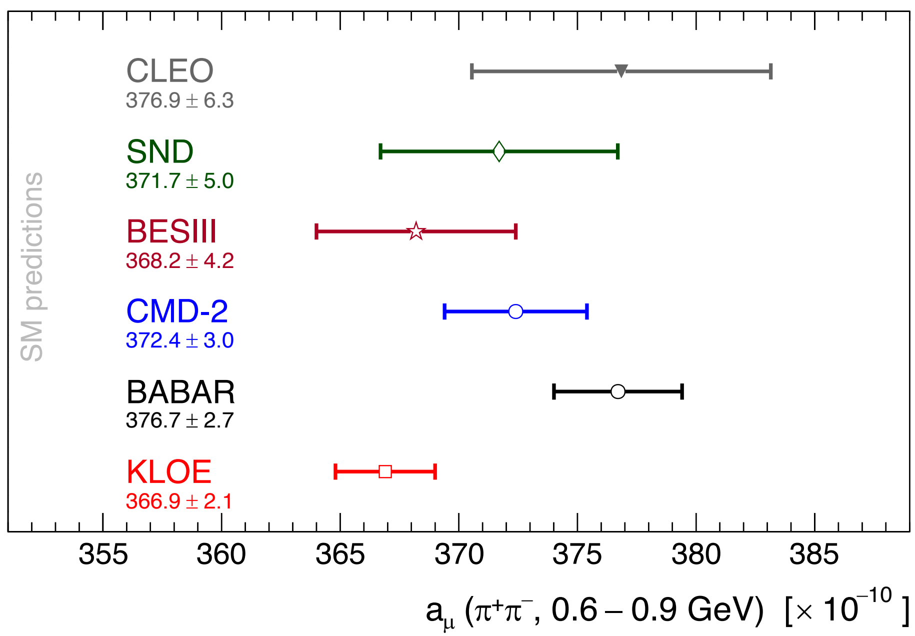
Figure 12: Relative difference between the data points (including the energy rescaling (4.10)) and the fit result for the VFF, normalized to the fit result for $|F_\pi^V(s)|^2$. As in all plots, we show fit errors and total uncertainties as two separate error bands. The total uncertainty is given by the fit error and the systematic uncertainty, added in quadrature.

HVP: new developments from other groups

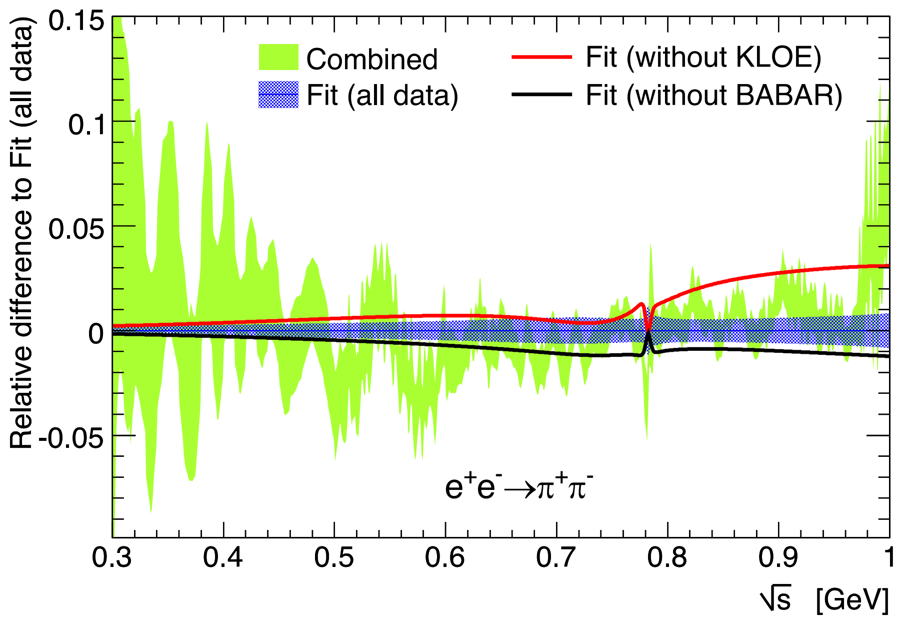
DHMZ: arXiv:1908.00921

- Add latest data. Use fit, based on analyticity&unitarity, similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraint/lower errors at low energies.
- For 2π, based on difference between result w/out KLOE and BaBar, sizeable additional sys. error is applied and mean value adjusted.

arXiv:1908.00921 Figure 5:



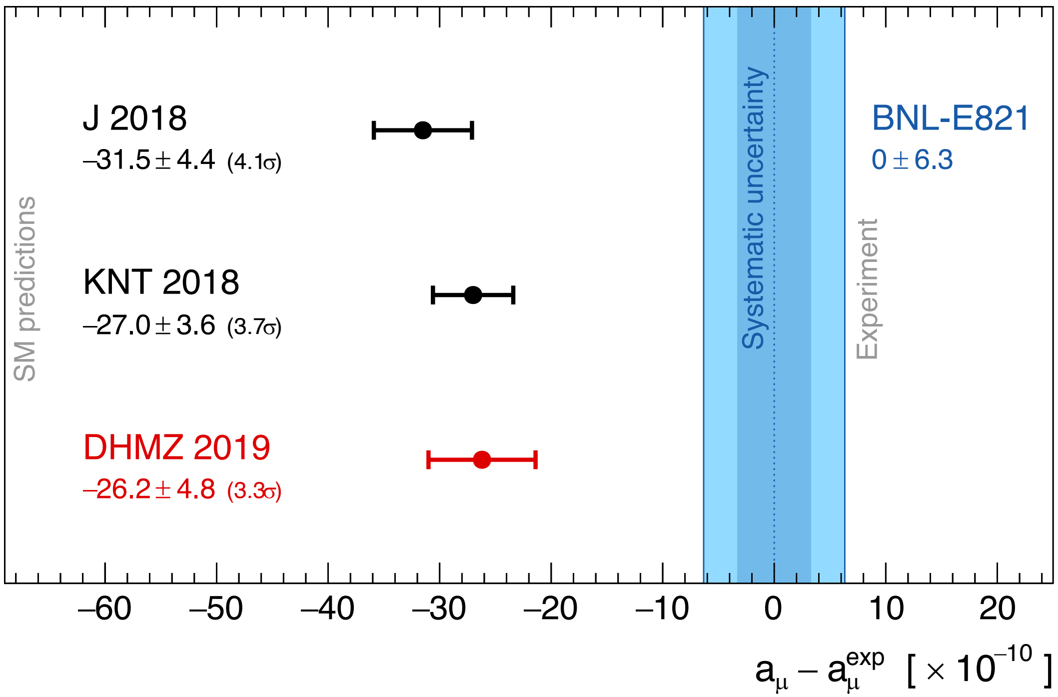
arXiv:1908.00921 Figure 6:



HVP: new developments from other groups

DHMZ: arXiv:1908.00921

- The resulting mean value is similar,
  for total LOHVP in (10^{-10}): 693.9±4.0 [vs. KNT's 693.3±2.5]
  but this description inflated the error beyond what a local error inflation of a combined fit does.
- Adding all contributions, they then quote 3.3**σ**.



arXiv:1908.00921
Figure 9

HVP: new developments from other groups

Benayoun+DelBuono+Jegerlehner: arXiv:1903.11034

- New analysis using effective theory based on (broken) Hidden Local Symmetry

Channel	BHLS	BLHS$_2$ (BS) (excl. τ)	BLHS$_2$ (RS) (incl. τ)	BLHS$_2$ (RS) (excl. τ)	Exp. Value.
$\pi^+\pi^-$	493.73 ± 0.70	494.52 ± 0.92	494.51 ± 0.83	494.50 ± 1.04	497.82 ± 2.80
$\pi^0\gamma$	4.42 ± 0.03	4.48 ± 0.03	4.42 ± 0.03	4.42 ± 0.03	3.47 ± 0.11
$\eta\gamma$	0.63 ± 0.01	0.63 ± 0.01	0.64 ± 0.01	0.64 ± 0.01	0.55 ± 0.02
$\pi^+\pi^-\pi^0$	42.56 ± 0.54	43.03 ± 0.55	42.97 ± 0.55	43.12 ± 0.50	41.38 ± 1.28
K^+K^-	18.10 ± 0.14	18.05 ± 0.13	18.14 ± 0.16	18.11 ± 0.14	17.37 ± 0.55
$K_L K_S$	11.53 ± 0.08	11.70 ± 0.08	11.65 ± 0.09	11.65 ± 0.10	11.98 ± 0.36
HLS Sum	570.97 ± 0.92	572.42 ± 1.08	572.32 ± 1.03	572.44 ± 1.20	572.57 ± 3.15
$\chi^2/N_{\rm pts}$	949.1/1056	1062.2/1152	1128.0/1237	1038.2/1152	$\times$
Probability	96.7%	91.6 %	94.6 %	96.7 %	$\times$

Table 7: HLS contributions to $10^{10} \times a^{\rm HVP-LO}$ integrated up to 1.05 GeV, including FSR. The first data column displays the results using the former BHLS [25, 27] and, the second one, those derived from the Basic Solution for BHLS$_2$, the τ decay data being discarded. The next two data columns refer to the results obtained using the Reference Solution for BHLS$_2$ using the largest set of data samples, keeping or discarding the τ data. The last data column refers to the numerical integration for each channel of the same set of data which are used in the BHLS/BHLS$_2$ fits.

HVP: new developments from other groups

Benayoun+DelBuono+ Jegerlehner:

arXiv:1903.11034

- Their preferred mean value is for total LOHVP in (10^{-10}): 687.1±4.0 [vs. KNT's 693.3±2.5]
- Adding all contributions, they then >~ 4.2**σ**.

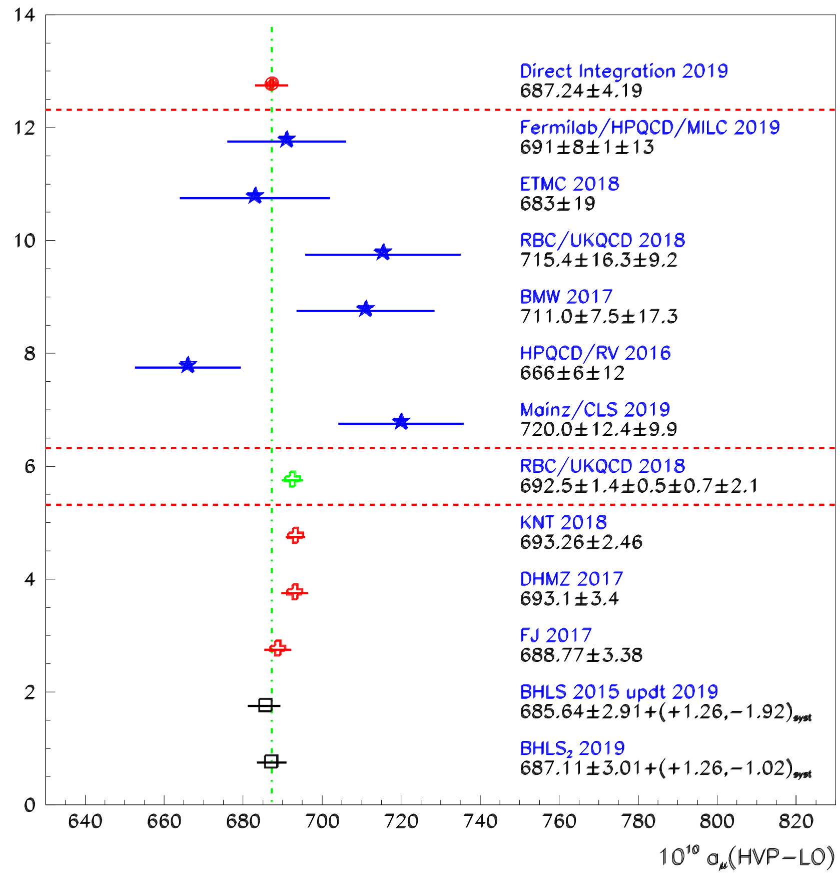


Figure 11: Recent evaluations of $10^{10} \times a_\mu^{\mathrm{HVP-LO}}$: On top, the result derived by direct integration of the data combined with perturbative QCD; the next six points display some recent evaluations derived by LQCD methods and reported in resp. [124], [125], [126], [127], [11] and [128] with $N_f = 2+1+1$. The second point from [126] displayed has been derived by supplementing lattice data with some phenomenological information. These are followed by the evaluations from [71],[129] and [4]. The value derived using BHLS [27] – updated with the presently available data – and the evaluation from BHLS$_2$ are given with their full systematic uncertainties (see text).

Alternatively: HVP from the lattice

One-page summary, for details see the lattice talks at the TGm2 plenary meeting in Mainz, June 2018: https://indico.him.uni-mainz.de/event/11/

- Complementary to data-driven (`pheno') DR.
- Need high statistics, and control highly non-trivial systematics:
  - need simulations at physical pion mass,
  - control continuum extrapol. limit and Finite Volume effects,
  - need to include full QED and Strong Isospin Breaking effects,
    i.e. full QED+QCD including $m_u \neq m_d$ & disconnected diagrams

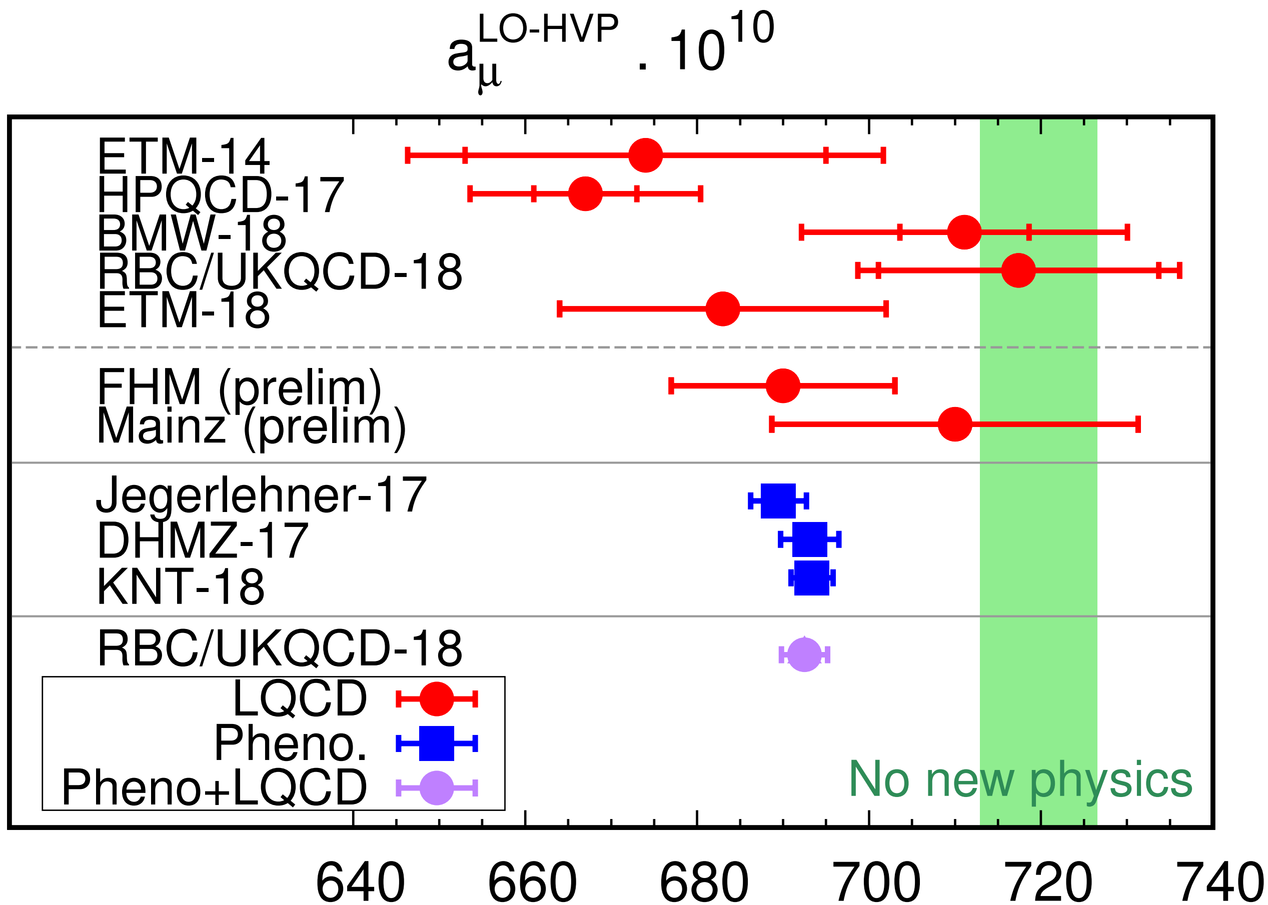
- There has been a lot of activity on the lattice, for HVP (& HLbL):
  - Budapest-Marseille-Wuppertal (staggered q's, also moments)
  - RBC / UKQCD collaboration (Time-Momentum-Representation, DW fermions, window method to comb. `pheno' with lattice)
  - Mainz (CLS) group (O(a) improved Wilson fermions, TMR)
  - HPQCD & MILC collaborations (HISQ quarks, Pade fits)

HVP from the lattice

Christoph Lehner at the recent meeting of the Theory Initiative for g-2, Mainz, June 2018:

`We need to improve the precision of our pure lattice result so that it can distinguish the ”no new physics” results from the cluster of precise R-ratio results.’

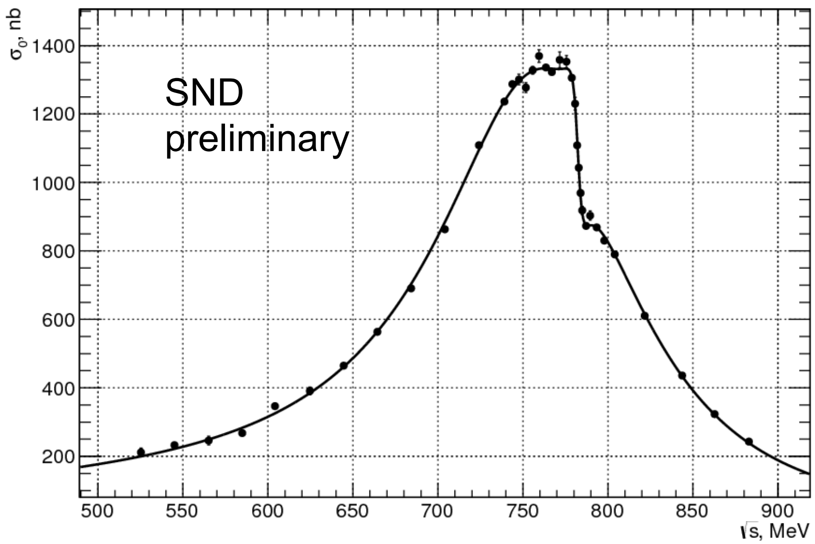
K.Miura, LATTICE2018 proceedings, arXiv:1901.09052



Outlook: prel. news from SND (1) [~15 procs. under analysis]

V. Druzhinin, EPS 2019

$$e^+e^- \to \pi^+\pi^-$$



Systematic uncertainty on the cross section (%)

Source	**< 0.6 GeV**	**0.6 - 0.9 GeV**
Trigger	0.5	0.5
Selection criteria	0.6	0.6
e/π separation	0.5	0.1
Nucl. interaction	0.2	0.2
Theory	0.2	0.2
Total	**0.9**	**0.8**

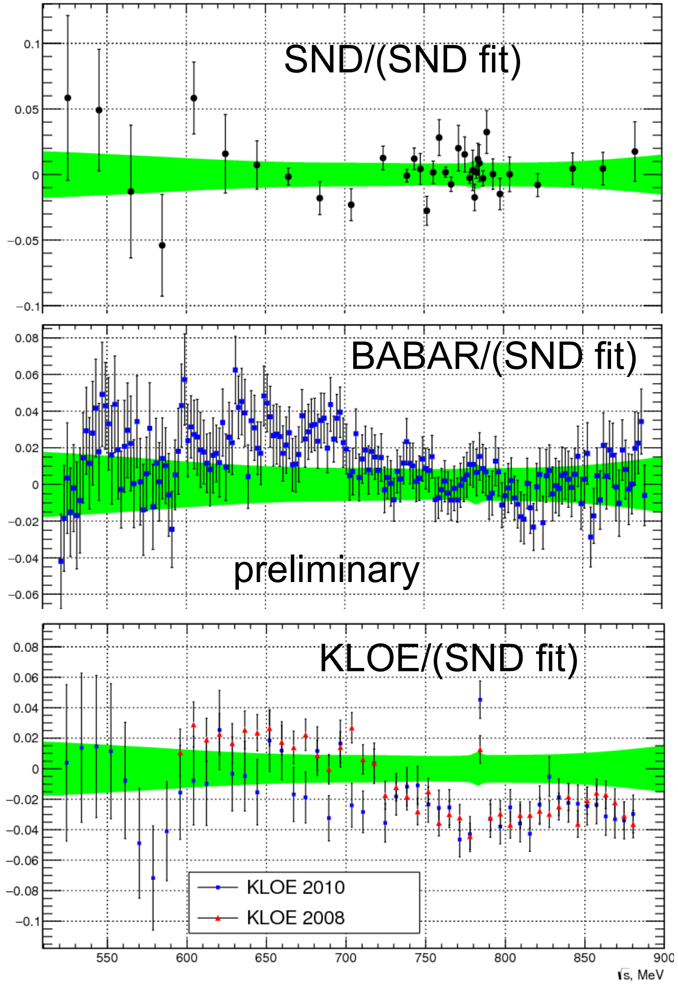
	SND @ VEPP-2000	**SND @ VEPP-2M**	**PDG**
M_ρ, MeV	775.4±0.5±0.4	775.6±0.4±0.5	775.3±0.3
Γ_ρ, MeV	145.7±0.7±1.0	146.1±0.8±1.5	147.8±0.9
$B_{\rho ee}\times 10^5$	4.89±0.2±0.4	4.88±0.2±0.6	4.72±0.5
$B_{\omega\pi\pi}$, %	1.77±0.08±0.02	1.66±0.08±0.05	1.53±0.06

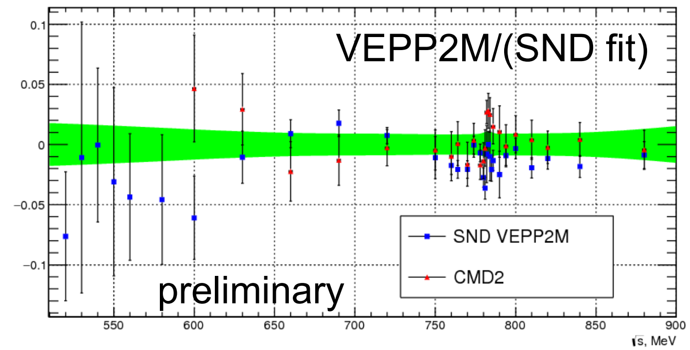
The analysis is based on 4.7 pb^{-1} data recorded in 2013, ~1/10 full SND data set.

Outlook: prel. news from SND (2)   Will this become a mediator?

V. Druzhinin, EPS 2019

$$e^+e^- \rightarrow \pi^+\pi^-$$





$$0.53 < \sqrt{s} < 0.88 \text{ GeV}$$

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$411.8 \pm 1.0 \pm 3.7$
SND & VEPP-2M	$408.9 \pm 1.3 \pm 5.3$
BABAR	$414.9 \pm 0.3 \pm 2.1$

HVP determinations: Outlook

- Next big step as more data in the pipeline;
  - in the 2π channel from BaBar, CMD-3, SND,
  - in subleading channels, 3π, 4π, KK
  - in the inclusive region from BESIII and KEDR,
  - BELLE II will be able to contribute with ISR measurements.

- If new data produce no new tensions/puzzles, further improvement should be significant within few years
  (but ultimately hit a limit with experimental systematics)

- Lattice expected to become a competitive alternative and check/challenge direct data-driven analyses;
- combined methods may provide the best HVP predictions.

- Still room for global combined fits with with more TH input?
- Long term: a direct measurement in the space-like: MuonE

Extras

Data Combination

To evaluate the vacuum polarization contribution, we have to combine lots of experimental data.

To do so, we usually construct a χ^2 function and find the value of $R(s)$ at each bin which minimizes χ^2.

Naively, the χ^2 function defined as

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} (R_i^{(n)} - \overline{R}_i)(V_n^{-1})_{ij}(R_j^{(n)} - \overline{R}_j) ,$$

where V_n is the cov. matrix of the n-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

may seem OK, but when there are non-negligible normalization uncertainties in the data, we have to be more careful.

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1.
Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$\text{(cov.)} = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} \ ,$$
$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \text{(cov.)}^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \ .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} .$$

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} ,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!

In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization.

R.D.Ball et al, JHEP 1005 (2010) 075.

KNT: further ongoing improvements compiled by Alex, no new release at this stage

There have been some notable data updates from SND and BaBar:

SND (arXiv:1809.07631)

- $e^+e^- \to \pi^0\gamma, 1.075 \leq \sqrt{s} < 2$ GeV

It extends the upper border of the pi0 gamma data from 1.35 GeV to 1.935 GeV.

KNT18: $a_\mu^{\pi^0\gamma} = 4.46 \pm 0.08$, $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.} = 1.44$

Now [preliminary]: $a_\mu^{\pi^0\gamma} = 4.46 \pm 0.08$, $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.} = 1.41$

→ Negligible changes, consolidation of previous estimate.

BaBar (arXiv:1810.11962)

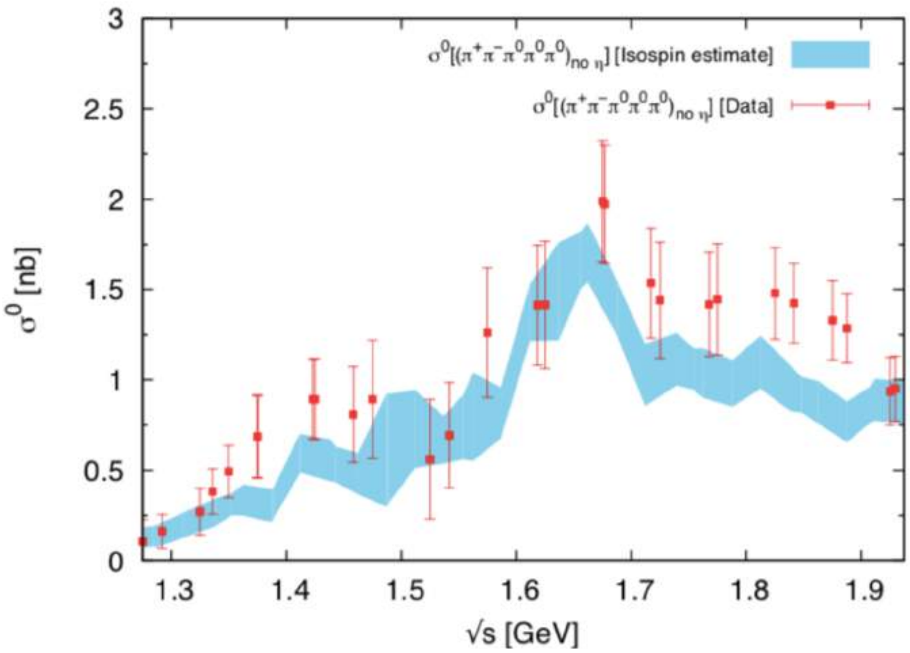
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\pi^0, 1.125 \leq \sqrt{s} \leq 4.325$ GeV
- $e^+e^- \to \pi^+\pi^-\eta, 1.075 \leq \sqrt{s} \leq 3.025$ GeV
- $e^+e^- \to \omega\pi^0\pi^0, 1.125 \leq \sqrt{s} \leq 4.325$ GeV
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\eta, 1.625 \leq \sqrt{s} \leq 4.325$ GeV
- $e^+e^- \to \omega\eta\pi^0, 1.525 \leq \sqrt{s} \leq 4.325$ GeV

→ The BaBar updates in particular as they update modes/final states that were previously estimated via isospin relations.

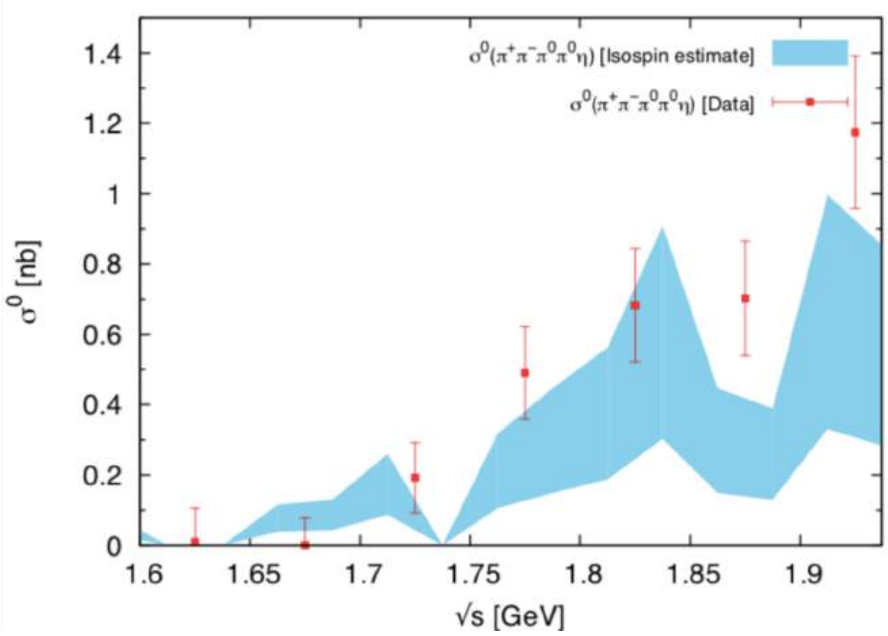
KNT: further ongoing improvements compiled by Alex, no new release at this stage

BaBar (arXiv:1810.11962)

$\pi^+\pi^-\pi^0\pi^0\pi^0$



$\pi^+\pi^-\pi^0\pi^0\eta$



KNT18: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\pi^0} = 0.50 \pm 0.04$

Now: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\pi^0} = 0.64 \pm 0.11$

[preliminary]

KNT18: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\eta} = 0.08 \pm 0.04$

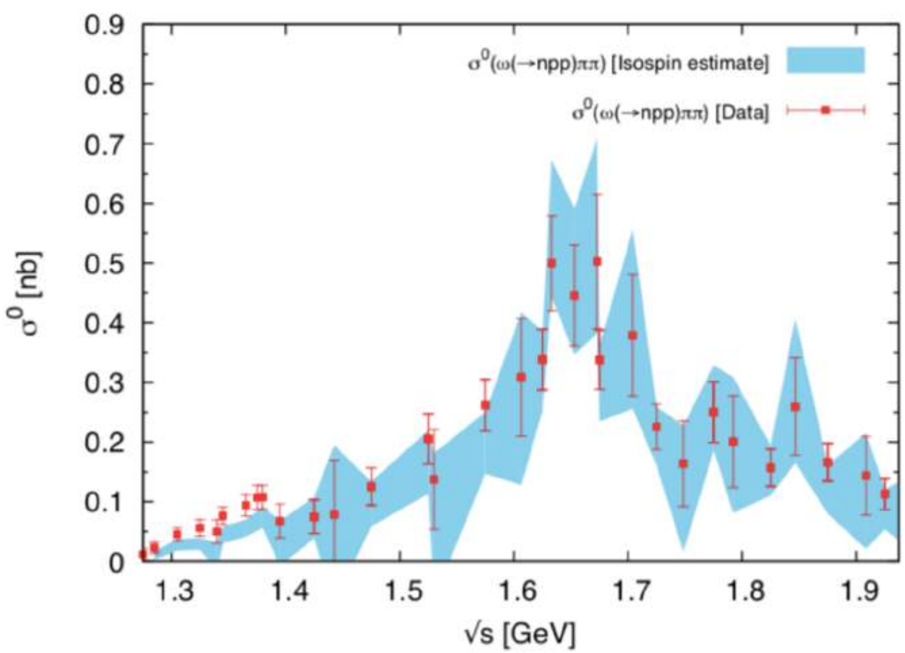
Now: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\eta} = 0.12 \pm 0.02$

[preliminary]

KNT: further ongoing improvements compiled by Alex, no new release at this stage

BaBar (arXiv:1810.11962)

$\omega(\rightarrow \text{npp})\pi\pi$



KNT18: $a_\mu^{\omega(\rightarrow\text{npp})\pi\pi} = 0.10 \pm 0.02$

Now: $a_\mu^{\omega(\rightarrow\text{npp})\pi\pi} = 0.11 \pm 0.01$

[preliminary]

$\pi^+\pi^-\eta$

KNT18: $a_\mu^{\pi^+\pi^-\eta} = 1.29 \pm 0.06$

Now: $a_\mu^{\pi^+\pi^-\eta} = 1.30 \pm 0.06$

[preliminary]

$\omega\eta\pi^0$

KNT18: $a_\mu^{\omega\eta\pi^0} = 0.35 \pm 0.09$

Now: $a_\mu^{\omega\eta\pi^0} = 0.24 \pm 0.05$

[preliminary]

→ These changes have a minor effect overall:

KNT18: $a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46$

Now: $a_\mu^{\text{had, LOVP}} = 693.23 \pm 2.46$

[preliminary]

But, good that isospin estimates are further consolidated…