General Discussion on $g-2$

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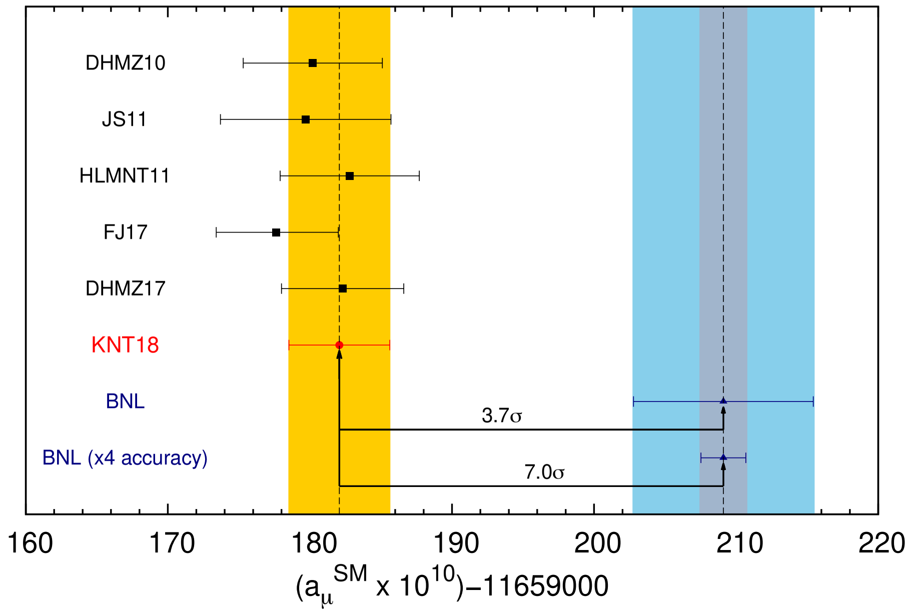
3rd Workshop on flavour changing and conserving processes (FCCP2019) – Anacapri, August 29 - 31, 2019

Present situation



A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 97, 114025 (2018)

$$a_\mu^{\text{exp}} = 116\,592\,09.1(5.4)(3.3) \cdot 10^{-10} \quad [0.54\text{ppm}]$$

G. W. Bennett *et al.* [Muon $g-2$ collaboration, BNL E821], Phys. Rev. D 73, 072003 (2006)

M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001 (2018)

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim +27(7) \cdot 10^{-10} \quad [\sim 3.5\sigma]$$

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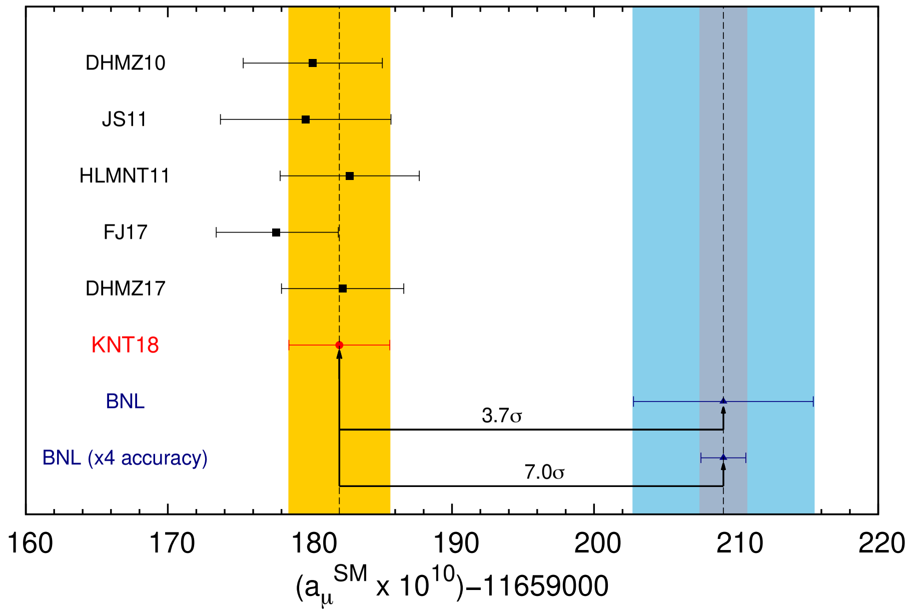

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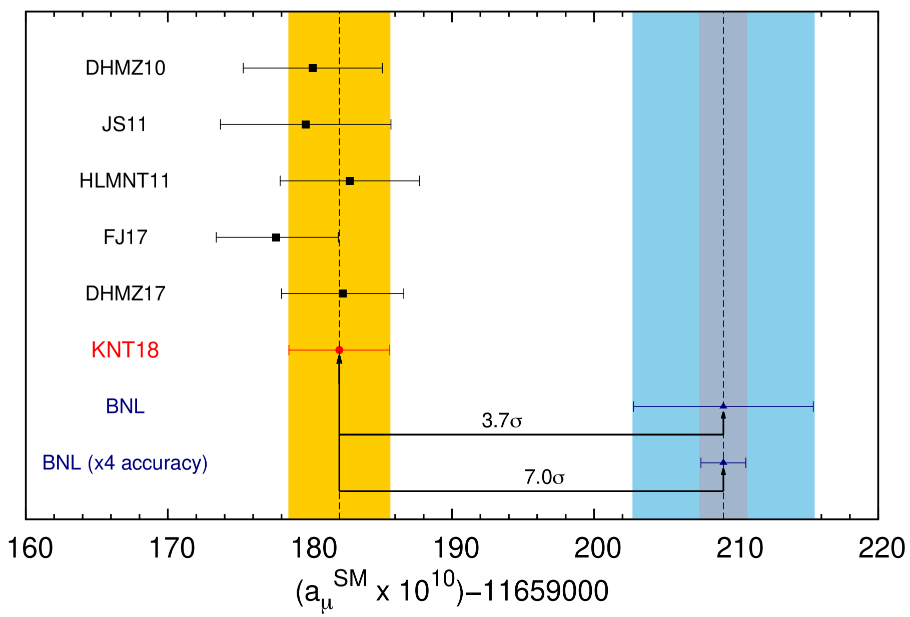

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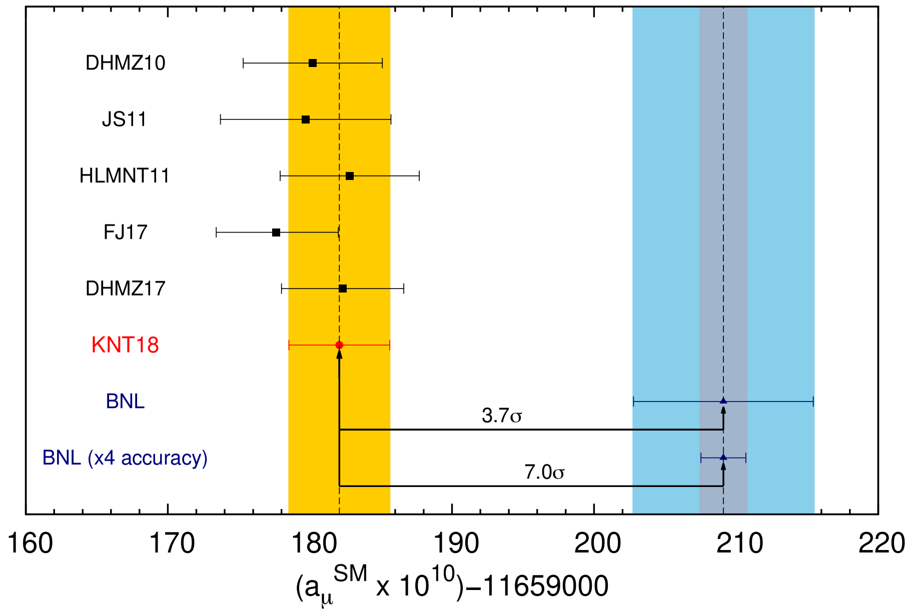
Origin of the discrepancy? (if experimentally confirmed)

$\longrightarrow$ BSM physics! e.g. LFUV,... (but situation kind of weird...)

Present situation



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Origin of the discrepancy? (if experimentally confirmed)

$\longrightarrow$ scrutinize SM contributions (and possibly try to improve them...)

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim \left\{ \begin{array}{r} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{array} \right.$$

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of course, could be a combination of several effects...

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Experimental situation

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
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This is not the end of the story!...

Experimental situation and prospects

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FNAL E989	2019	μ^+	???	~ 0.35
FNAL E989	2022?	μ^+	???	~ 0.14
J-PARC E34	???	μ^+	???	~ 0.45

A. Keshavarzi, EPJ Web Conf. **212**, 05003 (2019)
M. Abe *et al.*, Prog. Theor. Exp. Phys. 2019, 053C02 (2019)

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$\longrightarrow$ cf. talks by D. Počanić and K. Ishida

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim \begin{cases} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{cases}$$

Uncertainties on a_μ^{HVP} or a_μ^{exp} reliable ?

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)
C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)
A. Petermann, Helv. Phys. Acta 30, 407 (1957)
H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)
S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)
S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$A_1^{(8)}$ has also been evaluated! (a_e)

S. Laporta, Phys. Lett. B 772, 232 (2017)

$$A_1^{(8)} = -1.912\,245\,764\,926\,445\,574\,152\,647\,167\,439\,830\,054\,060\,873\,390\,658\,725\,345\,171\,329\ldots$$

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$, $A_1^{(8)}$

$\longrightarrow$ precision $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$

$\longrightarrow$ for a_μ^{QED} mainly $A_2^{(2n)}(m_\mu/m_e)$ matter

order $(\alpha/\pi)^4$: 891 diagrams

Mass-dependent contributions matter for a_μ

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012)

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T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012)

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)
A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)
A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015)

Agreement at the level of accuracy required by present and future experiments for a_μ

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)
J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

$$A_1^{(10)} = 9.168(571)$$

$$A_2^{(10)}(m_\mu/m_e) = 742.18(87) \quad A_2^{(10)}(m_\mu/m_\tau) = -0.068(5)$$

$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011(10)$$

No systematic cross-checks even for mass-dependent contributions

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}$$

- No sign of substantial contribution to a_μ from higher order QED
- $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} = 737.0(6.3) \cdot 10^{-10}$

QED provides more than 99.99% of the total value, without theory uncertainties at this level of precision

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Uncertainties on a_μ^{HVP} or a_μ^{exp} reliable ?

Weak contributions : W, Z,... loops

$$\begin{aligned}a_\mu^{\text{weak(1)}} &= \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]\\ &= 19.48\times 10^{-10}\end{aligned}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)
G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)
R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)
I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)
M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

Complete three-loop short-distance leading logarithms

G. Degrassi and G. F. Giudice, Phys. Rev. D 58, 053007 (1998)

Updated a few years ago: $a_\mu^{\text{weak}} = 15.36(10)\cdot 10^{-10}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

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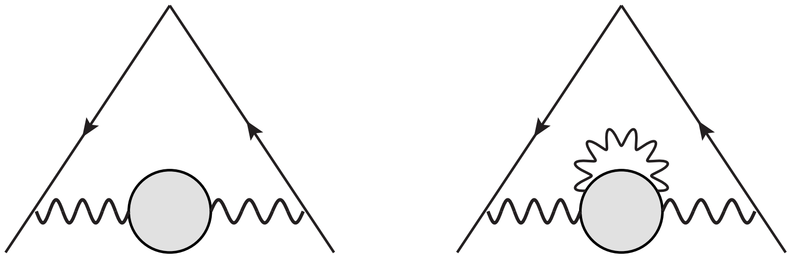
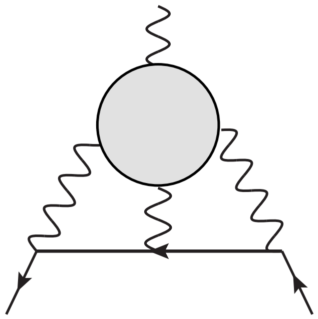
Uncertainties on a_μ^{HVP} or a_μ^{exp} reliable ?

non-perturbative hadronic contributions

hadronic vacuum polarization (HVP)



(virtual) hadronic light-by-light (HLxL)



Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

$\longrightarrow$ {
- traditional approach: $e^+e^- \to \mathrm{had}$ cross section
- more recently: lattice QCD
- future? $e\mu$ scattering

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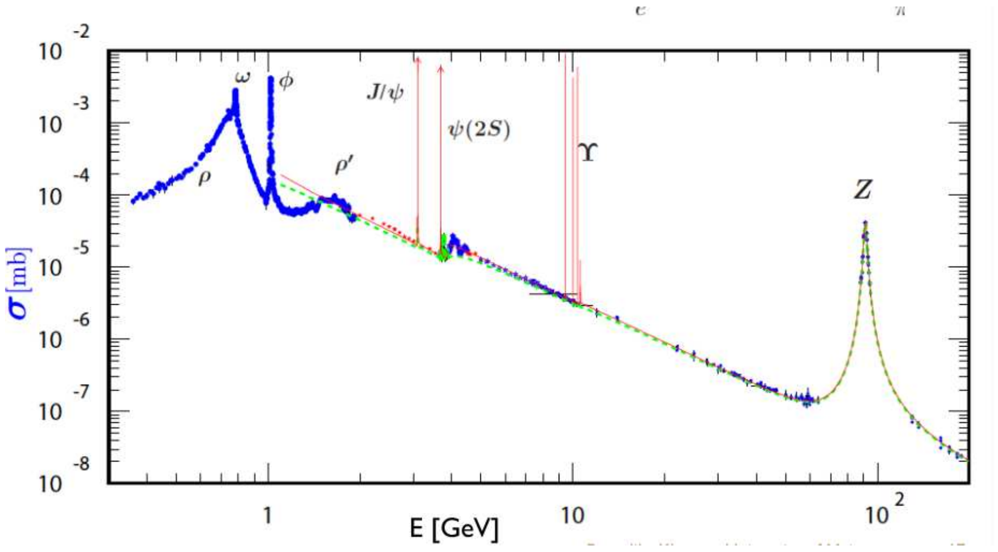
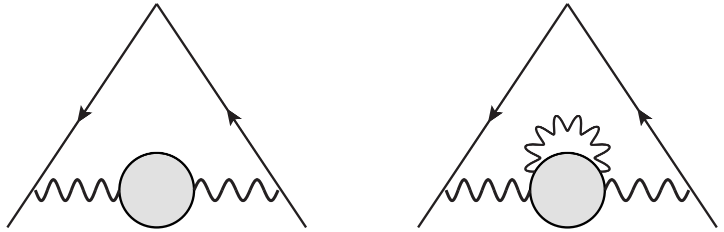
$$\longrightarrow \left\{ \begin{array}{l} \text{- traditional approach: } e^+e^- \to \mathrm{had} \text{ cross section} \quad \longrightarrow \text{ cf. talks by A. Denig and T. Teubner} \\ \text{- more recently: lattice QCD} \quad \longrightarrow \text{ cf. talk by H. Wittig} \\ \text{- future? } e\mu \text{ scattering} \quad \longrightarrow \text{ cf. talks by U. Marconi, C. M. Carloni Calame and J. Ronca} \end{array} \right.$$

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Combination of ~ 39 exclusive channels

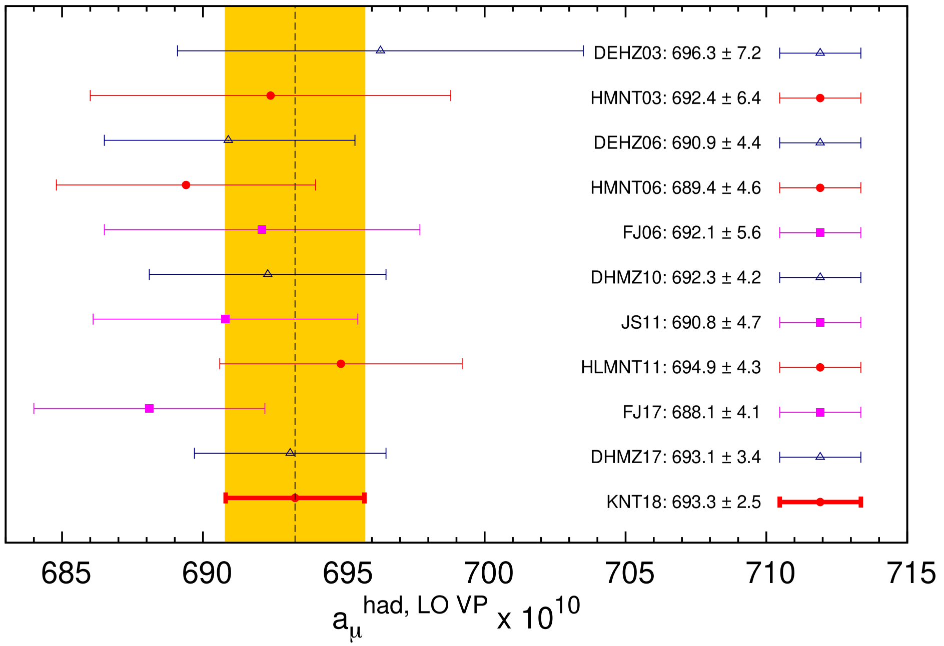
$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Some recent evaluations

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)



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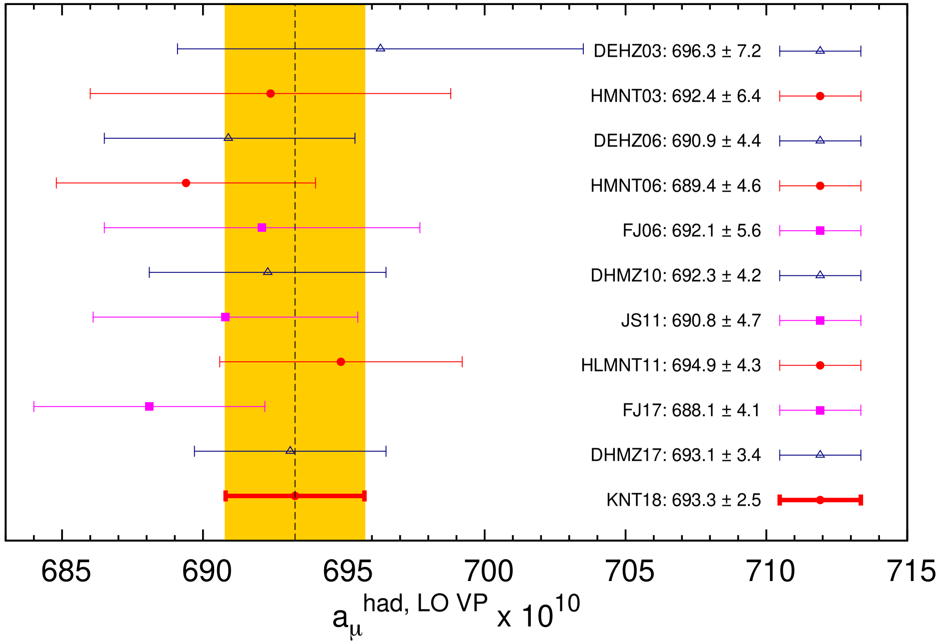
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)	
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)	$\sim 0.4\%$
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)	



A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 97, 114025 (2018)

Some recent evaluations

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}, e^+e^-$

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)	
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)	$\sim 0.6\%$
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$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)	
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$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}, e^+e^-$

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}, e^+e^-$

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

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- Some tension, for instance, in the region of the ρ resonance

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600 - 900\ \mathrm{MeV})$
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)
KLOE comb.	366.9(2.1)
BaBar 09	376.7(2.0)(1.9)
BESIII 16	368.2(2.5)(3.3)
SND 04	371.7(5.0)
CMD-2 03,06	372.4 (3.0)

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- Dispersive analyses of the $\pi\pi$ and $\pi\pi\pi$ channels provide cross-checks

$$a_\mu^{\pi\pi}|_{\leq 1GeV} = 495.0(1.5)(2.1) \cdot 10^{-10}$$

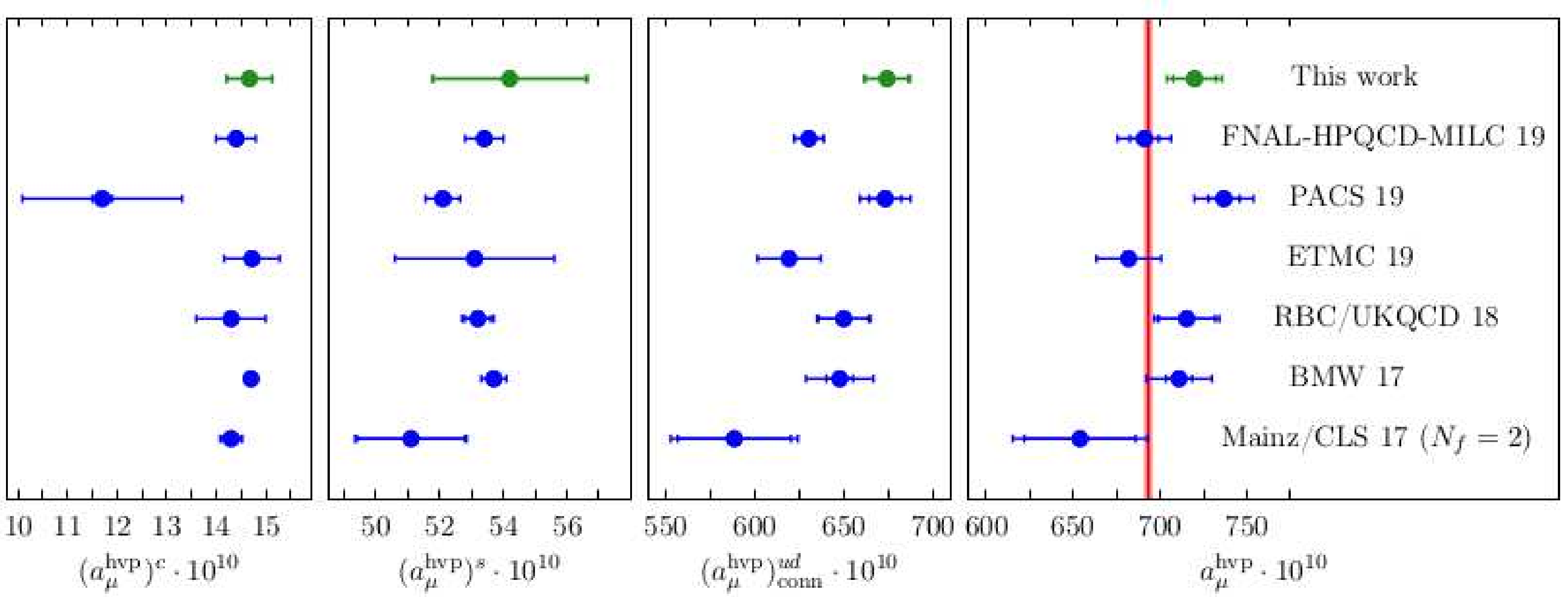
G. Colangelo, M. Hoferichter, P. Stoffer, JHEP 1902,006 (2019)

$$a_\mu^{\pi\pi\pi}|_{\leq 1.8GeV} = 46.2(6)(6) \cdot 10^{-10} \qquad a_\mu^{\text{HVP-LO}} = 692.3(3.3) \cdot 10^{-10}$$

M. Hoferichter, B.-L. Hoid, B. Kubis, arXiv:1907.01556 [hep-ph]

⟶ cf. talk by B.-L. Hoid

HVP from Lattice QCD

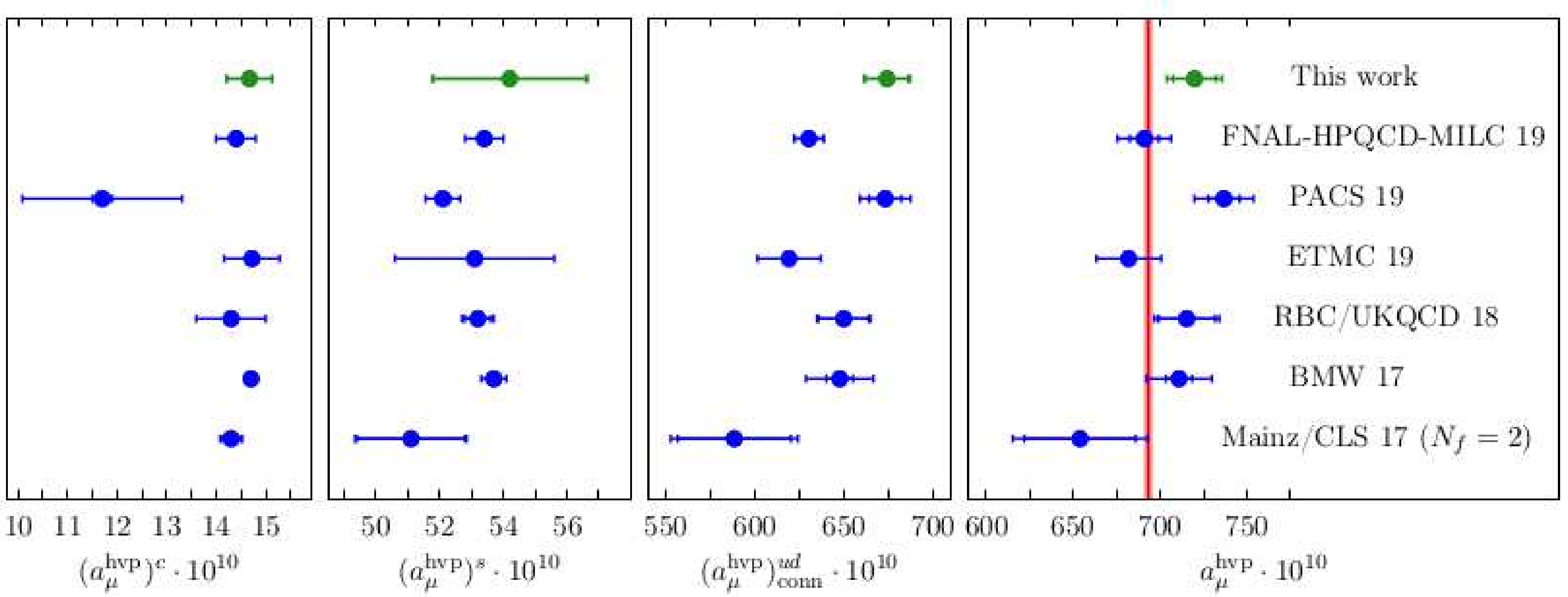


A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

present precision $\sim 2.5\%$
work in progress

Review of lattice calculations for a_μ: H. B. Meyer and H. Wittig, Prog. Part. Nucl. Phys. 104, 46 (2019)

HVP from Lattice QCD



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M. Della Morte *et al.*, JHEP 10, 020 (2017)

present precision $\sim 2.5\%$
work in progress

could also be combined with Mellin-Barnes approach (expansion in moments, class of interpolation functions)

E. de Rafael, Phys. Lett. B 736, 522 (2014)
E. de Rafael, Phys. Rev. D 96, 014510 (2017)
J. Charles, E. de Rafael and D. Greynat, Phys. Rev. D 97, 076014 (2018)

$\longrightarrow$ cf. talk by D. Greynat

Hadronic vacuum polarization

Possibility to measure HVP in the space-like region (from Bhabha or μe scattering)?

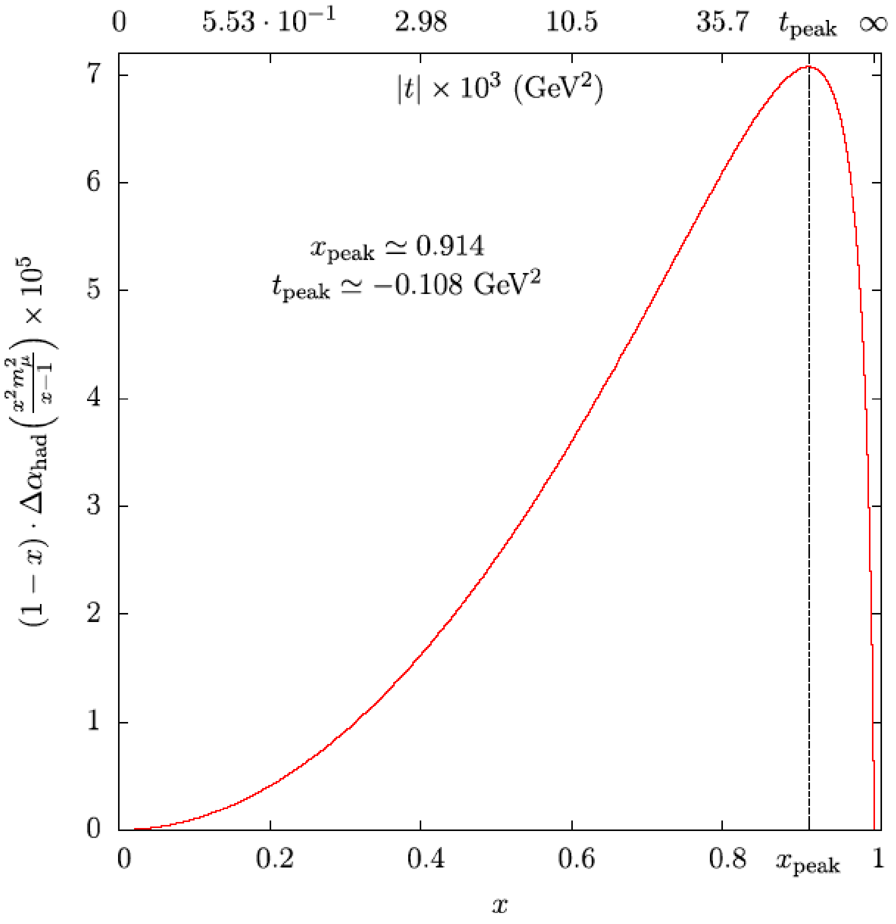
C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\rm HVP} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\rm had}(\frac{x^2}{x-1}m_\mu^2)$

$t = \frac{x^2 m_\mu^2}{x-1} \quad 0 \le -t < +\infty \; 0 \le x < 1$

- $a_\mu^{\rm HVP}$ given by the integral
- measurement of $\Delta\alpha_{\rm had}$ in the space-like region
- contribution at small t enhanced
- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 $\mu/{\rm s}$ (CERN)
- control of systematics challenging
- requires QED calculations at NNLO



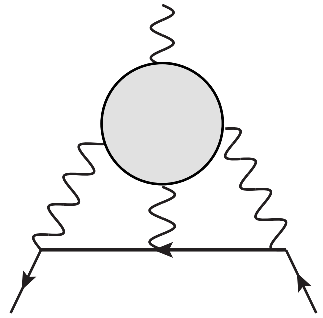
status of MUonE proposal $\longrightarrow$ data taking in 2021-2024 (?) G. Venanzoni, arXiv:1811.11466 [hep-ex]

Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

- Not related, as a whole, to an experimental observable...

? $\longrightarrow$



- Involves the fourth-rank vacuum polarization tensor

$$\mathrm{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

- Present estimates rely mainly on two model-dependent calculation

$$a_\mu^{\text{HLxL}} = +(8.3\pm 3.2)\cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\text{HLxL}} = +(89.6\pm 15.4)\cdot 10^{-11}$$

M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)

M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

that turn out to be positive M.K. and A. Nyffeler, Phys. Rev. D 65, 073034 (2002)

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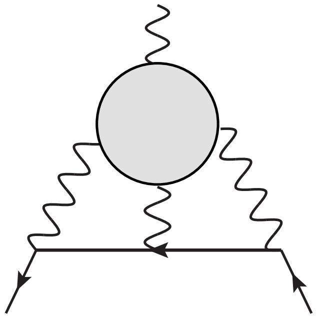
? $\longrightarrow$ 

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M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

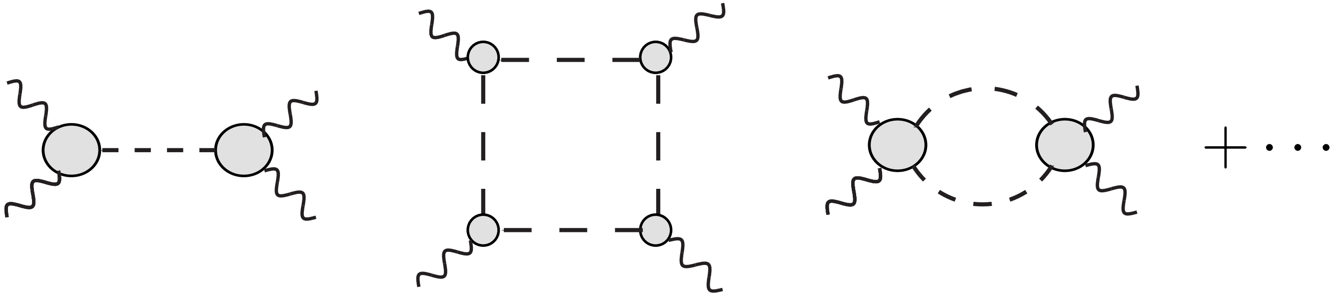
Provide useful hints on the expected sizes of various contributions

Other approaches have been developed:

- dispersive methods

G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 1409, 091 (2014)
G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 1509, 074 (2015)

$$\Pi_{\mu\nu\rho\sigma} = \Pi^{\pi^0,\eta,\eta' \text{ poles}}_{\mu\nu\rho\sigma} + \Pi^{\pi,K\text{box}}_{\mu\nu\rho\sigma} + \Pi^{\pi\pi}_{\mu\nu\rho\sigma} + \Pi^{\pi\pi\pi}_{\mu\nu\rho\sigma} + \cdots$$



require input for form factors

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura and P. Stoffer, Phys. Lett. B 738, 6 (2014)

either from experiment ⟶ cf. talk by A. Kupsc

A. Nyffeler, Phys. Rev. D 94, 053006 (2016)

or from lattice QCD

A. Gerardin, H. B. Meyer, A. Nyffeler, Phys. Rev. D 94 074507 (2016)

Other approaches have been developed:

- lattice QCD

HLxL calculations in lattice QCD: M. Hayakawa *et al.*, PoS LAT2005, 353 (2006)

$$a_\mu^{\mathrm{HLxL}} = 53.5(13.5) \cdot 10^{-11}$$

T. Blum *et al.*, Phys. Rev. Lett. 118, 022005 (2017)

- statistical errors only
- only two classes of Wick contractions:
  $a_\mu^{\mathrm{HLxL}(4)} = 116.0(9.6) \cdot 10^{-11}$, $a_\mu^{\mathrm{HLxL}(2+2)} = -62.5(8.0) \cdot 10^{-11}$
- FV and FS effects still to be evaluated

$$a_\mu^{\mathrm{HLxL}} = 74.1(63.3) \cdot 10^{-11}$$

T. Blum *et al.*, arXiv:1907.00864 [hep-lat]

- only two classes of Wick contractions:
  $a_\mu^{\mathrm{HLxL}(4)} = 276.1(31.2)(3.2) \cdot 10^{-11}$, $a_\mu^{\mathrm{HLxL}(2+2)} = -202.0(56.5) \cdot 10^{-11}$
- FV and FS effects accounted for

Work in progress from the Mainz group N. Asmussen *et al.*, EPJ Web Conf. 179, 01017 (2018)

Goal: lattice evaluation of a_μ^{HLxL} with ∼10% accuracy within a few years from now...

Reevaluation of single meson exchanges (axials, scalars, tensors)

V. Pauk, M. Vanderhaeghen, Eur. Phys. J C 74, 3008 (2014)
F. Jegerlehner, EPJ Web Conf. 118 (2016)
M.K, S. Narison, A. Rabemananjara, D. Rabetiarivony, Phys. Lett. B 787, 111 (2018)

- ...or of one- and two-pion exchanges (S-wave $\pi\pi$ rescattering) contributions (but only pion contribution to left-hand cut $\rightarrow$ incomplete description of $S \rightarrow \gamma^*\gamma^*$)

$$a_\mu^{\pi\ \mathrm{box}} + a_\mu^{\pi\pi;\pi\ \mathrm{pole\ lhc}} = 24(1)\cdot 10^{-11}$$

G. Colangelo *et al.*, Phys. Rev. Lett. 118, 232001 (2017); JHEP 1704 (2017)

$$a_\mu^{\pi^0,\eta,\eta'\ \mathrm{poles}} = 62.6^{+3.0}_{-2.5}\cdot 10^{-11}$$

M. Hoferichter *et al.*, Phys. Rev. Lett. 121, 112002 (2018)

Updated estimate

$$a_\mu^{\mathrm{HLxL}} \quad = \quad +(10.3 \pm 2.9)\cdot 10^{-10}$$

F. Jegerlehner, arXiv:1705.002633 [hep-ph]

Other approaches

— Dyson-Schwinger/Bethe-Salpeter equations

G. Eichman et al, arXiv:1903.10844 [hep-ph]

— Non-local quark model

A. E. Dorokhov, A. E. Radzhabov and A. S. Zhevlakov, Eur.Phys.J. C75, 417 (2015)

— Holographic QCD $\longrightarrow$ cf. talk by L. Cappiello

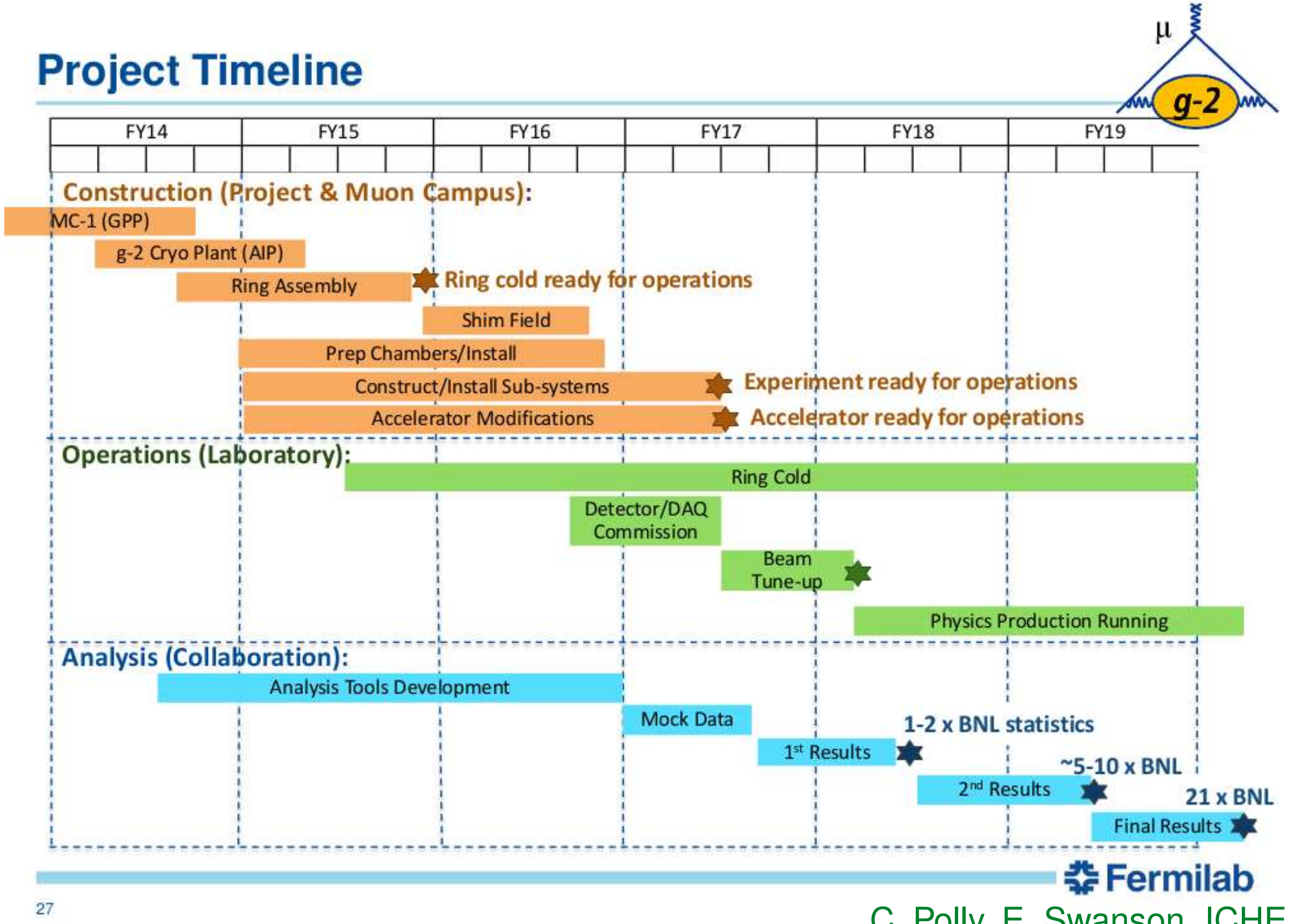
Goal: evaluation of HLxL with a reliable uncertainty of $\sim 10\%$

Conclusions and perspectives for FCCP2021

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim \left\{ \begin{array}{r} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{array} \right.$$

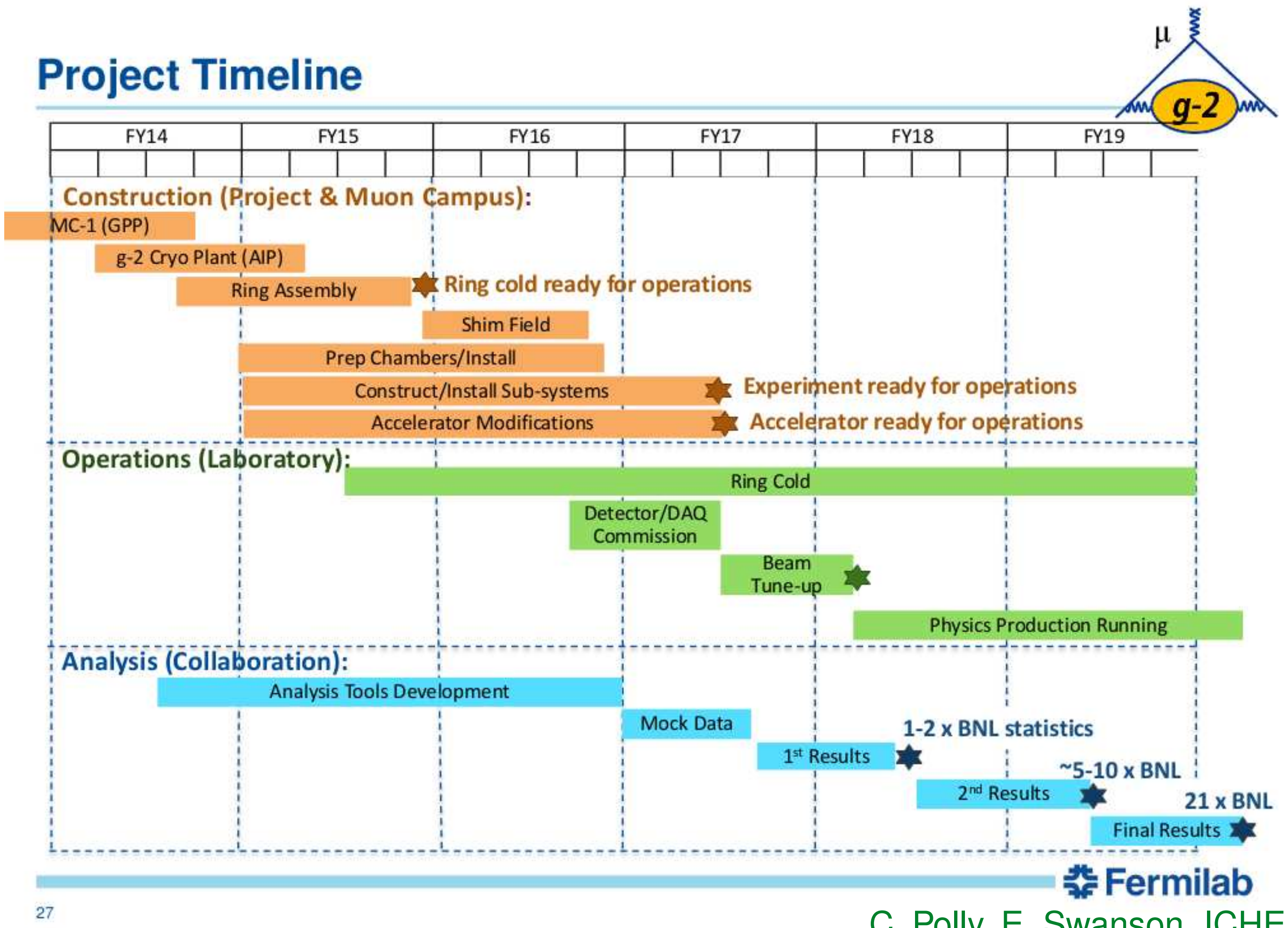
Uncertainties on a_μ^{HVP} or a_μ^{exp} reliable ?

What became of the FCCP2017 wish list for FCCP2019?


Project Timeline

C. Polly, E. Swanson, ICHEP, Aug. 2016

Results with statistics comparable to BNL E821 available by the end of 2018?



C. Polly, E. Swanson, ICHEP, Aug. 2016

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Maybe at the end of 2019

Experiment E34 at J-PARC in progress (full approval and funding for FCCP2019?)

Experiment E34 at J-PARC in progress (full approval and funding for FCCP2019?) $\longrightarrow$ cf. talk by K. Ishida

Operates under completely different experimental conditions

Important to provide independent cross-check of BNL E821 and FNAL E989 results

Complete cross-check of α^4 QED contributions...

Next steps?

- $A_2^{(10)}(m_\mu/m_e)$ for a_μ
- $A_1^{(10)}$ for a_e

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Next steps?

- $A_2^{(10)}(m_\mu/m_e)$ for a_μ ☹
- $A_1^{(10)}$ for a_e ☹

HVP

New estimates based on 39 measured exclusive channels

Precision below the 0.5% level in relative terms

Some tensions between data remain
$\longrightarrow$ would be interesting to see analysis for the $\pi^+\pi^-$ channel of the data collected at VEPP at FCCP2019 (or before...)

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Interesting possibilities for cross-checks, either from
- Bhabha or $e\mu$ scattering

$\longrightarrow$ first data at FCCP2019?

- Lattice QCD

(several groups, different strategies to overcome challenging difficulties)
$\longrightarrow$ determination at 2% -3% with controlled systematics (incl. IB) at FCCP2019?

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HLxL

Dispersive evaluation

- implementation of short-distance constraints
- computation of other contributions than one- and two-pion states? Cf. axial vectors (leading in large-N_c) $\longrightarrow 3\pi$ channel
- form factors to be provided from data and/or lattice QCD

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$\longrightarrow$ determination of HLxL at $\sim 10\%$ with controlled systematics at FCCP2019?

HLxL

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* for a preliminary study, see however J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, arXiv:1908.03331 [hep-ph]

Wish list for FCCP2021?

Wish list for FCCP2021?

turn all remaining $\ddot{\frown}$ into $\ddot{\smile}$

Thanks for your attention!