The HVP contribution to (g_μ-2): lattice QCD+QED calculations

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Napoli

8th – 10th April

OUTLINE

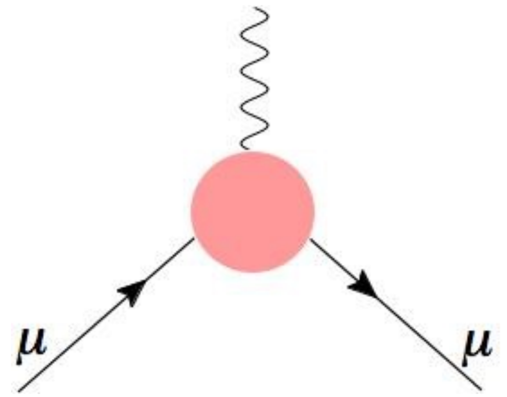
- Leading-order contributions to a_μ^{HVP}
- Isospin-breaking corrections to a_μ^{HVP}

In collaboration with:

V. Lubicz, G. Martinelli, F. Sanfilippo and S. Simula

based on: ArXiv:1707.03019 and ArXiv:1901.10462

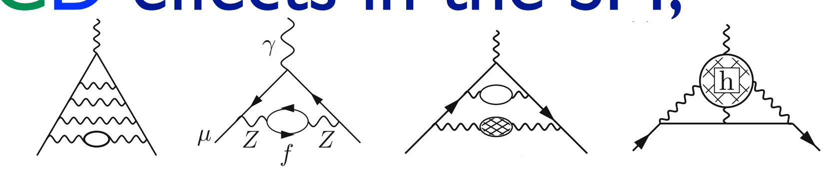
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \dfrac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



PDG 2018

$$a_\mu^{\text{exp}} = 116\ 592\ 091(54)(33)\cdot 10^{-11}$$ 0.5ppm

$$a_\mu^{\text{SM}} = 116\ 591\ 823(1)(34)(26)\cdot 10^{-11}$$ 0.4ppm

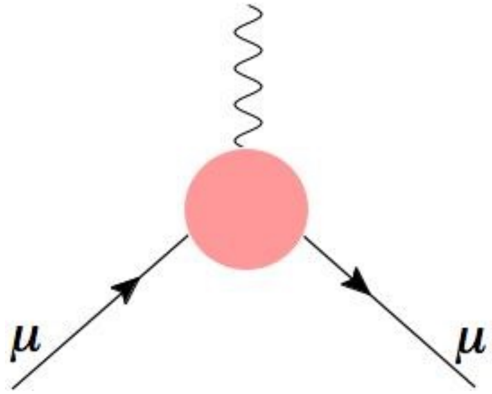
$\sim 3.5\sigma$

QED+EW

LO Had.

NLO/NNLO Had.

Muon magnetic anomaly



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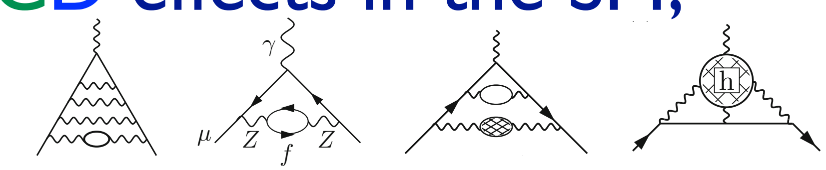
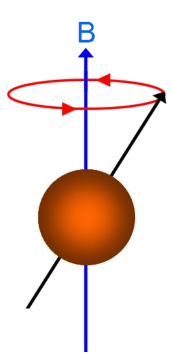


PDG 2018

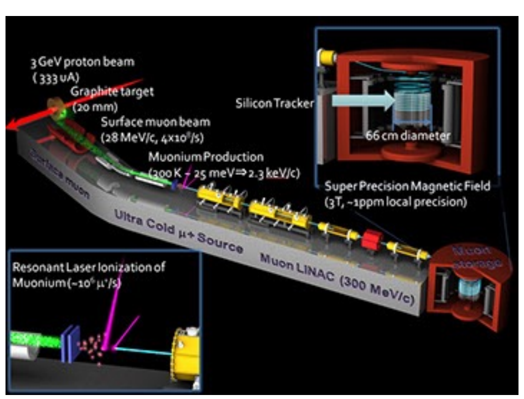
$$a_{\mu}^{\exp} = 116\ 592\ 091(54)(33)\cdot 10^{-11}$$

0.5ppm

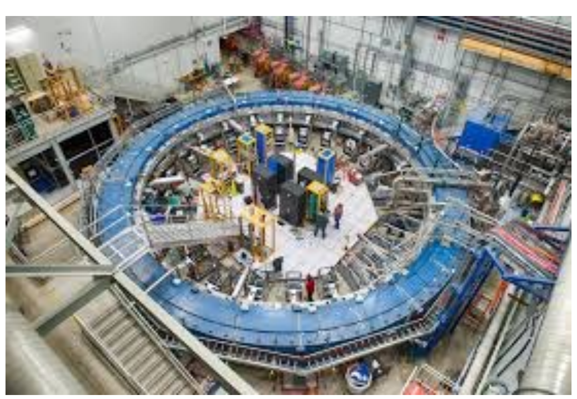
$$\vec{M} = g_{\mu}\frac{e}{2m_{\mu}}\vec{S}$$



target 0.14ppm

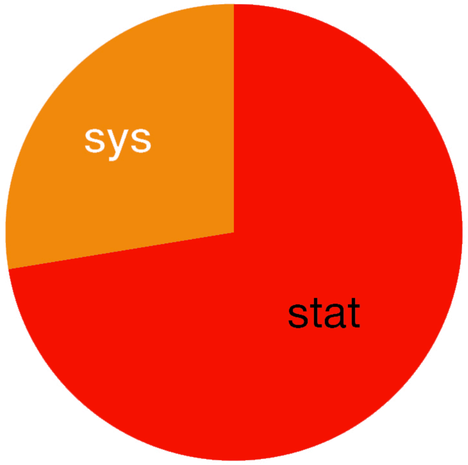


J-PARC (E34)

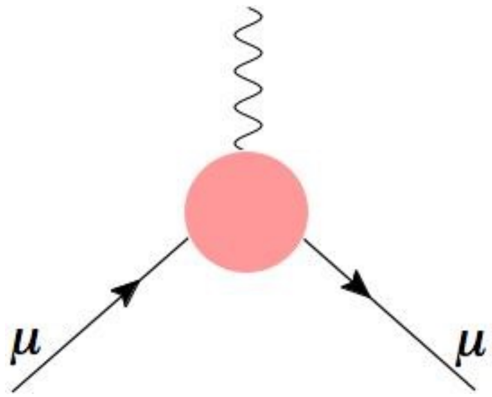


FermiLab (E989)

BNL E821



Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

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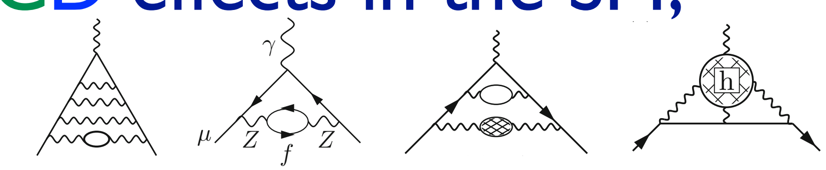
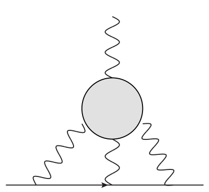


PDG 2018



$$a_\mu^{\mathrm{SM}} = 116\ 591\ 823(1)(34)(26)\cdot 10^{-11}$$

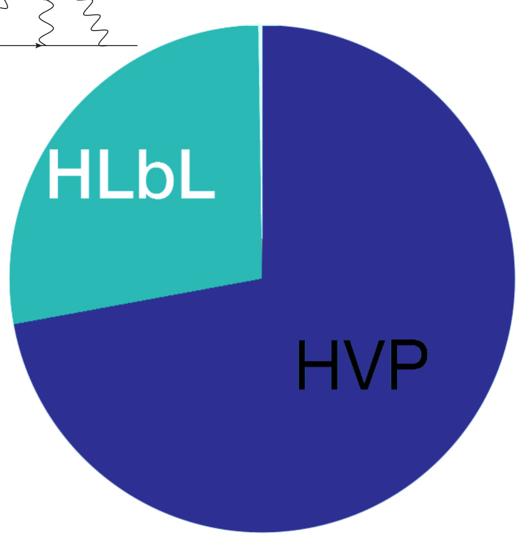
0.4ppm

QED+EW

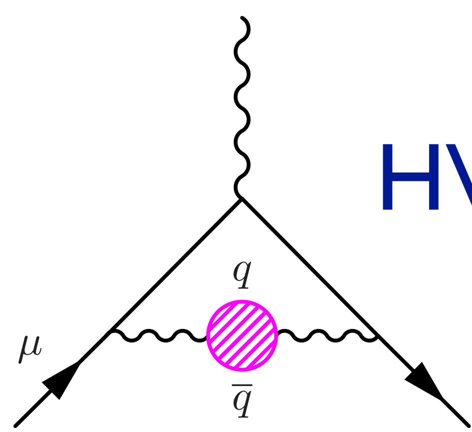
LO Had.

NLO/NNLO Had.

dispersion relations
$e^+e^- \to$ *hadrons*

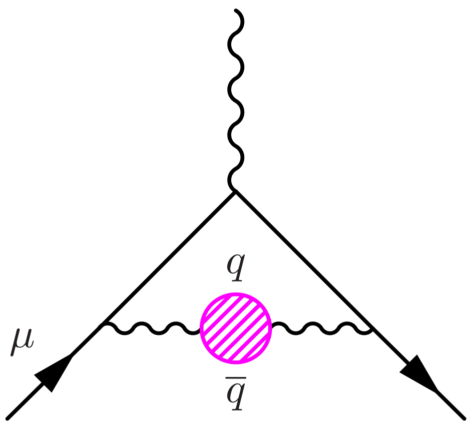


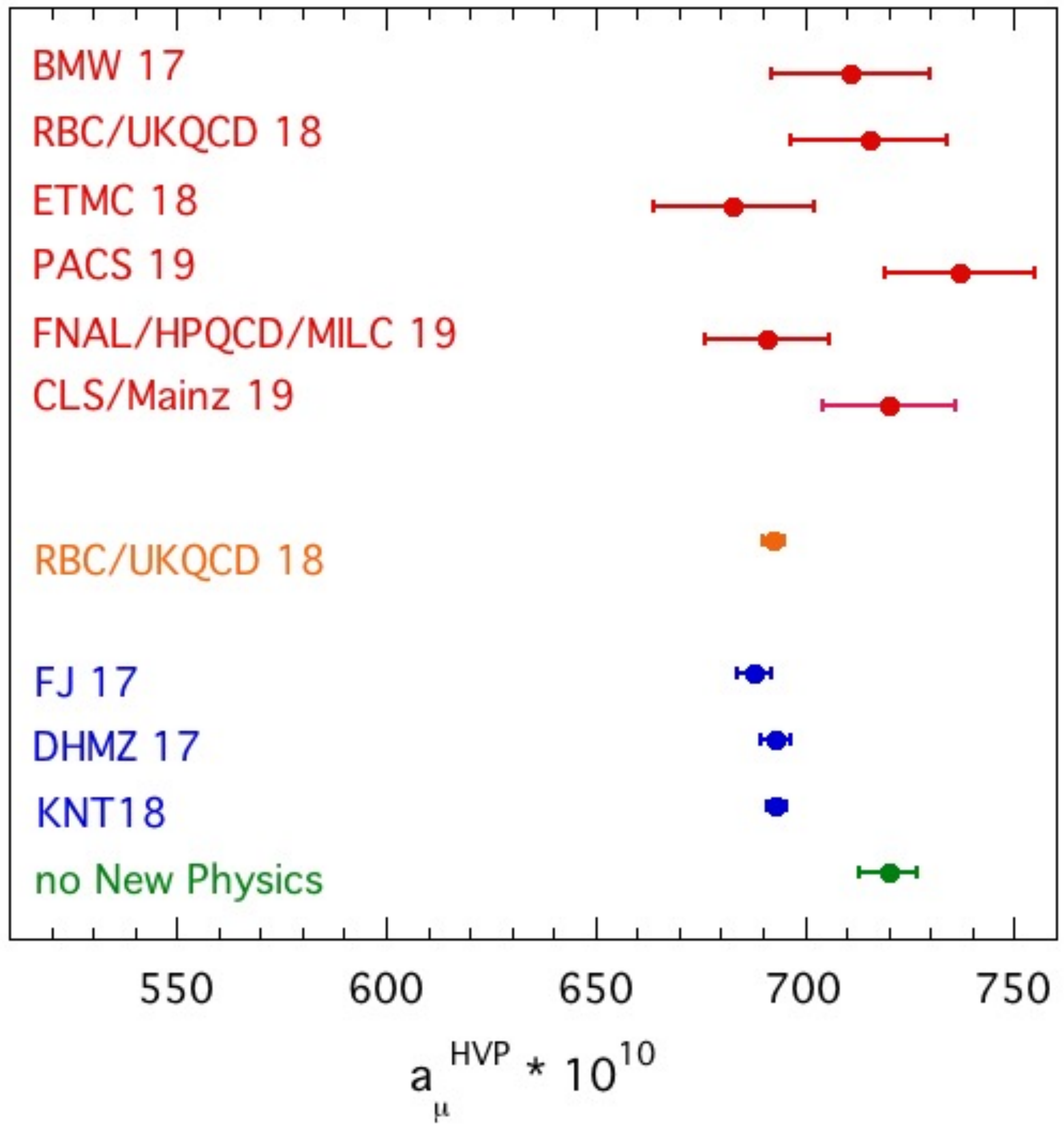
error budget



HVP → *ab-initio* **LQCD**

Hadronic Vacuum Polarisation

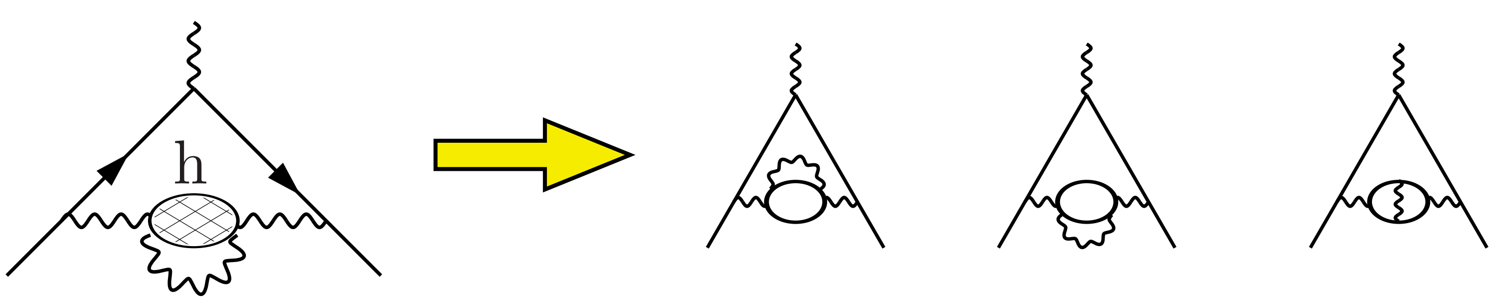




$\gtrsim 2\%$

$\gtrsim 0.4\%$

Given the present exper. and theor. (LQCD) accuracy, an important source of uncertainty are long distance electromagnetic and SU(2) breaking corrections



estimate in sQED

$$\delta a_\mu^{HVP} \sim 39(1) \cdot 10^{-11}$$

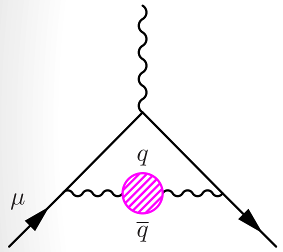
K. Melnikov, 2001

gμ-2 Theory Initiative

Lattice QCD calculations of the HVP

Mattia Bruno,[1] Michele Della Morte,[2] Aida El-Khadra,[3] Davide Giusti,[4] Steven Gottlieb,[5] Vera Gülpers,[6] Gregorio Herdoíza,[7] Taku Izubuchi,[8] Christoph Lehner,[1] Laurent Lellouch,[9] Marina Marinković,[10] Aaron S. Meyer,[1] Kohtaroh Miura,[11] Antonin Portelli,[6] Silvano Simula,[4] Ruth Van de Water,[12] Georg von Hippel,[11] Hartmut Wittig,[11] and . . .

Leading-order contributions to a_μ^{HVP}



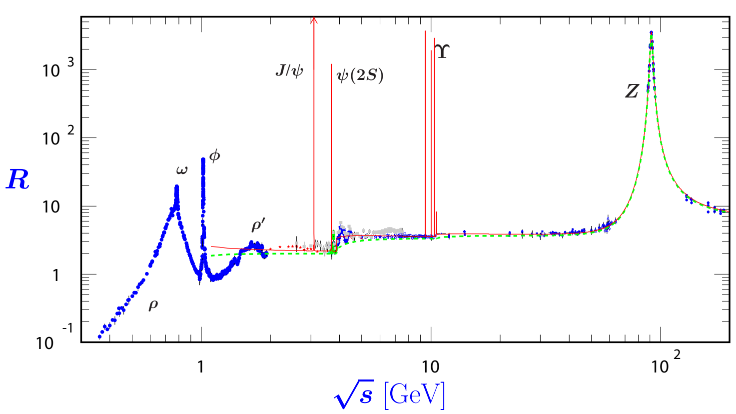
HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \left\langle J_\mu(x) J_\nu(0) \right\rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi(Q^2)$$

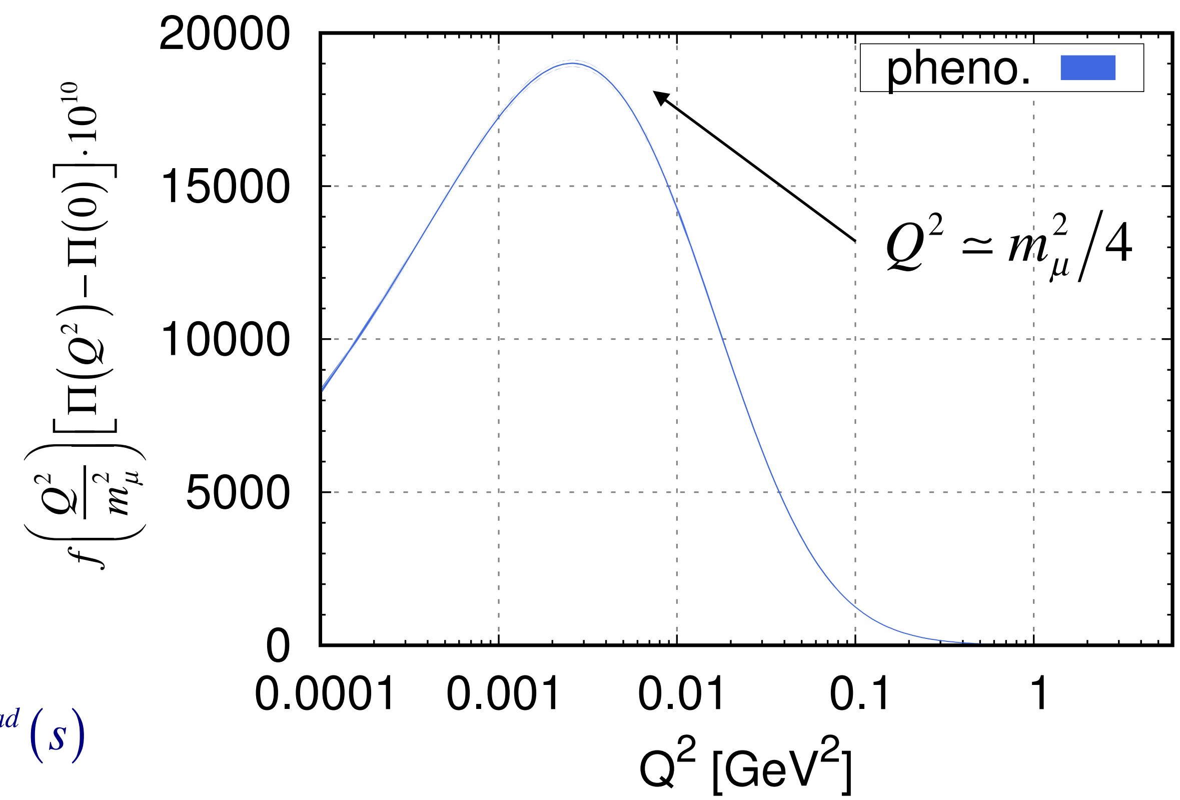
$$a_\mu^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \,\frac{1}{m_\mu^2}\, f\!\left(\frac{Q^2}{m_\mu^2}\right) \left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972

$$\left[\Pi(Q^2) - \Pi(0)\right] = \frac{1}{12\pi^2} \int_{4M_\pi^2}^\infty ds \,\frac{Q^2}{s(s+Q^2)} R^{had}(s)$$



$$\sigma^{had}(s) = \frac{4\pi\alpha_{em}^2}{s} R^{had}(s)$$



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\mu^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dt\ \tilde{f}(t)\, V(t)$$

$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \left\langle J_i(\vec{x},t) J_i(0) \right\rangle$$

D. Bernecker and H. B. Meyer, 2011

Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

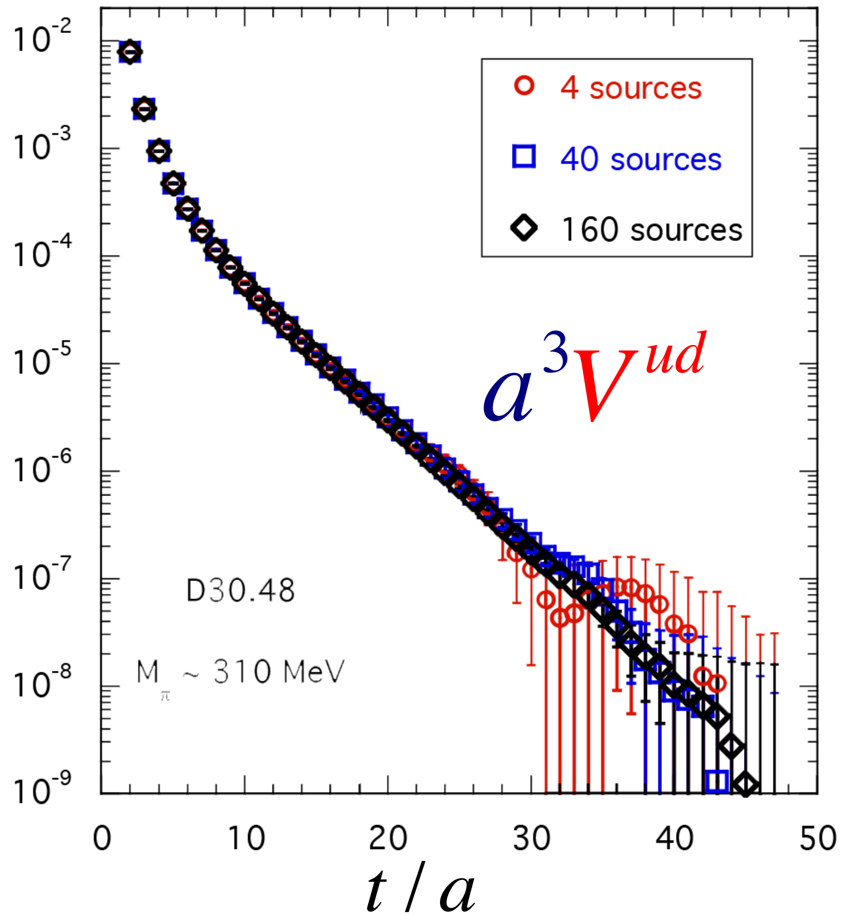
- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)
  OS for *s* and *c* valence quarks

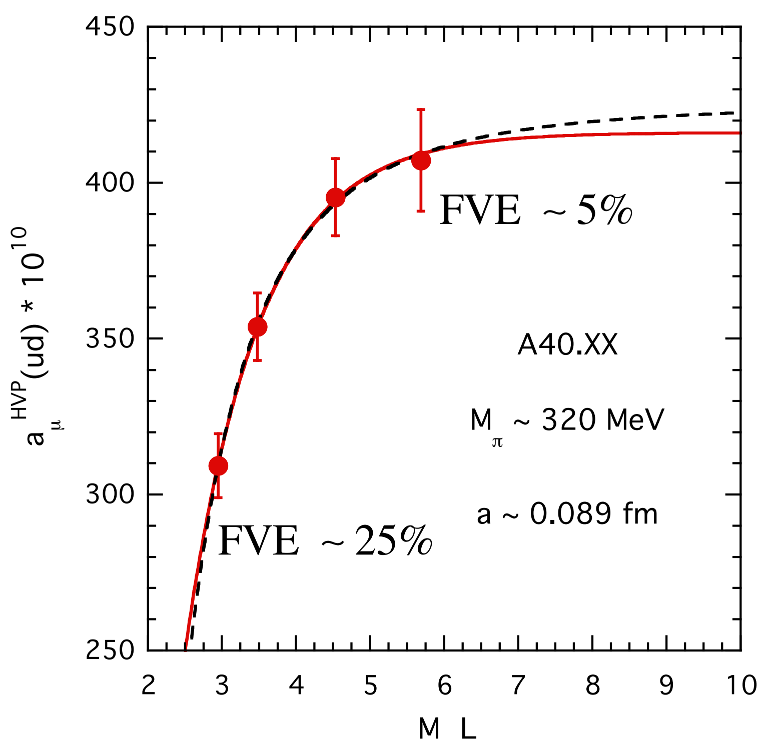
Pion masses in the range 220 - 490 MeV

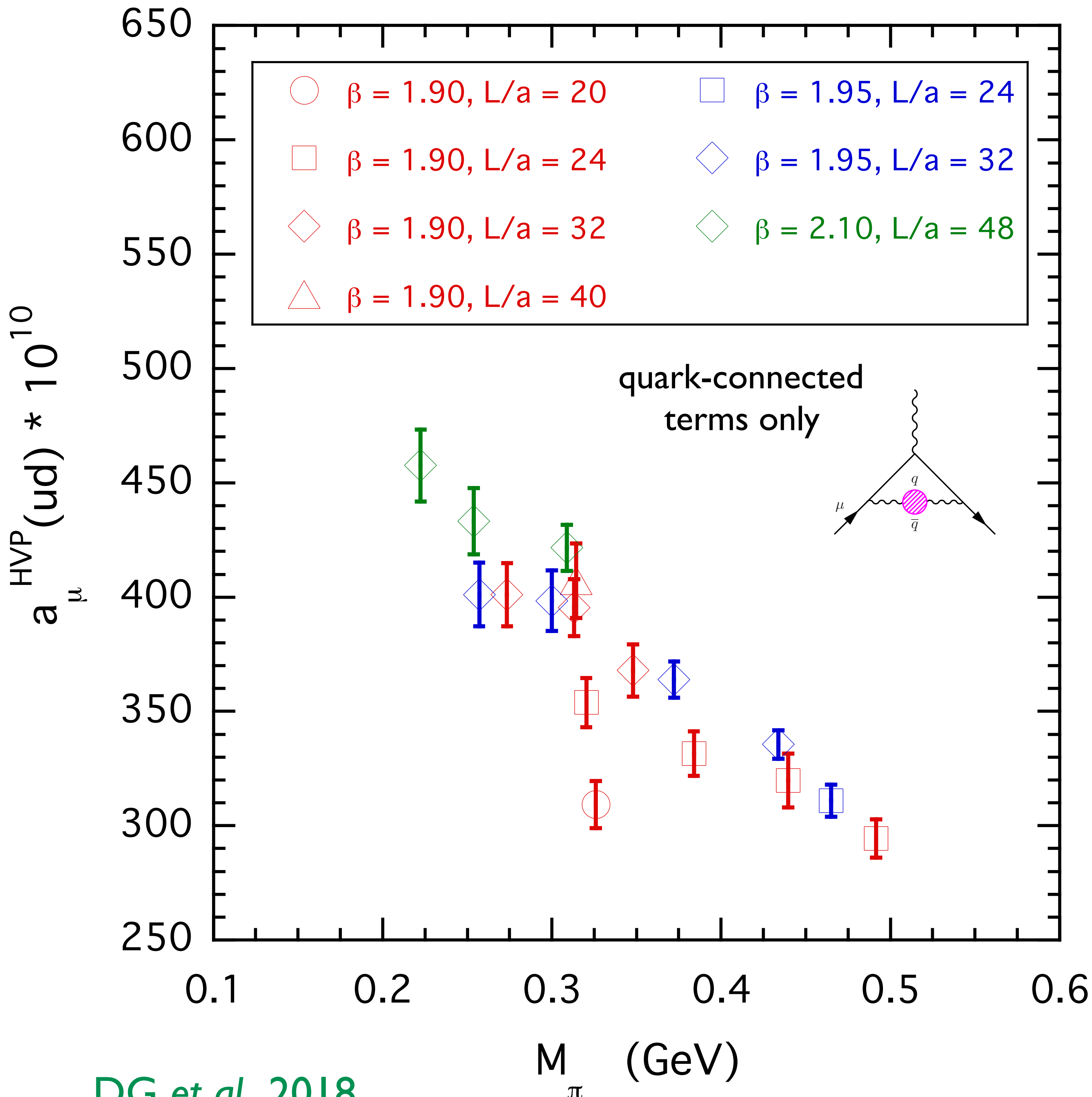
4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

Light quark contribution







DG *et al.*, 2018

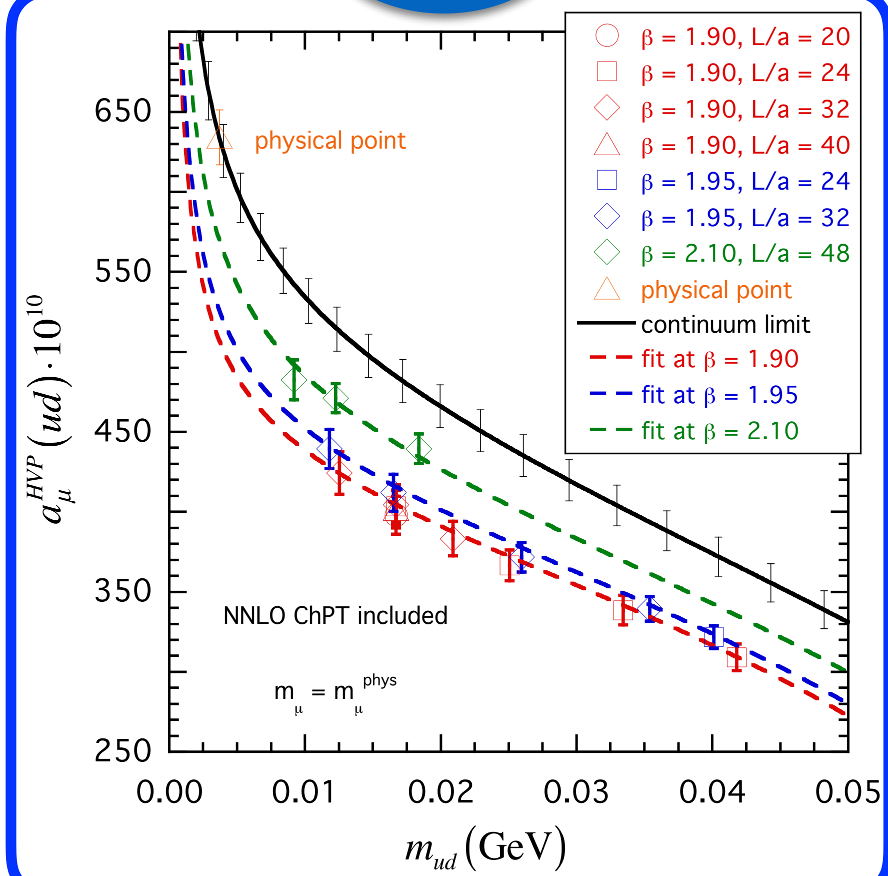
PRD98(2018)114504

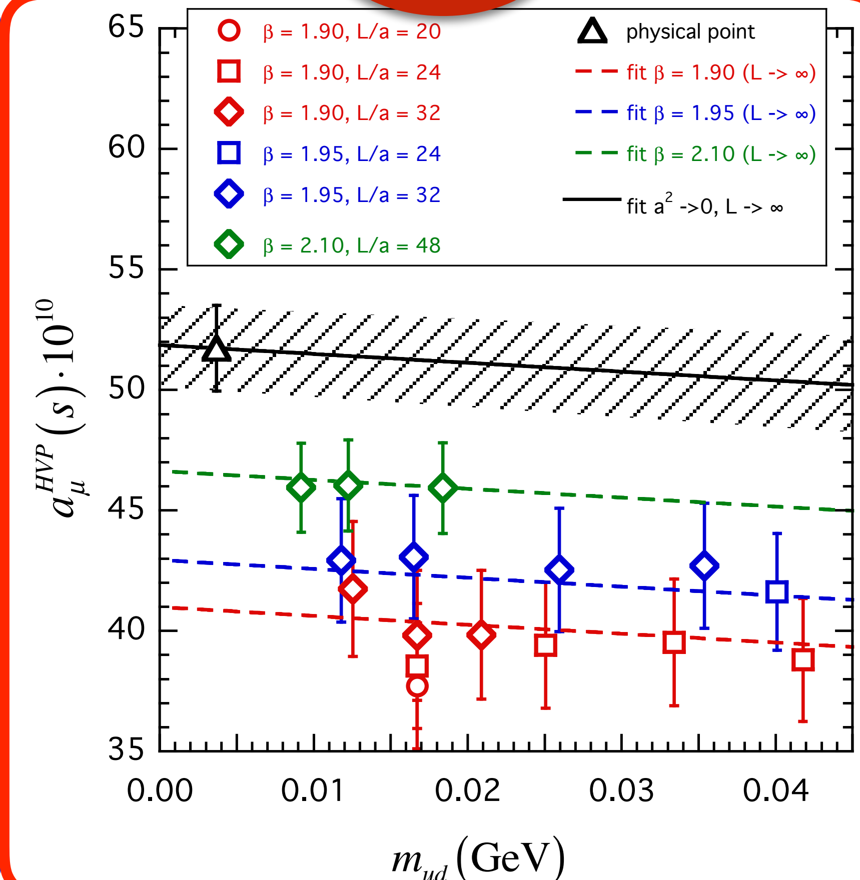
udsc-quark contributions

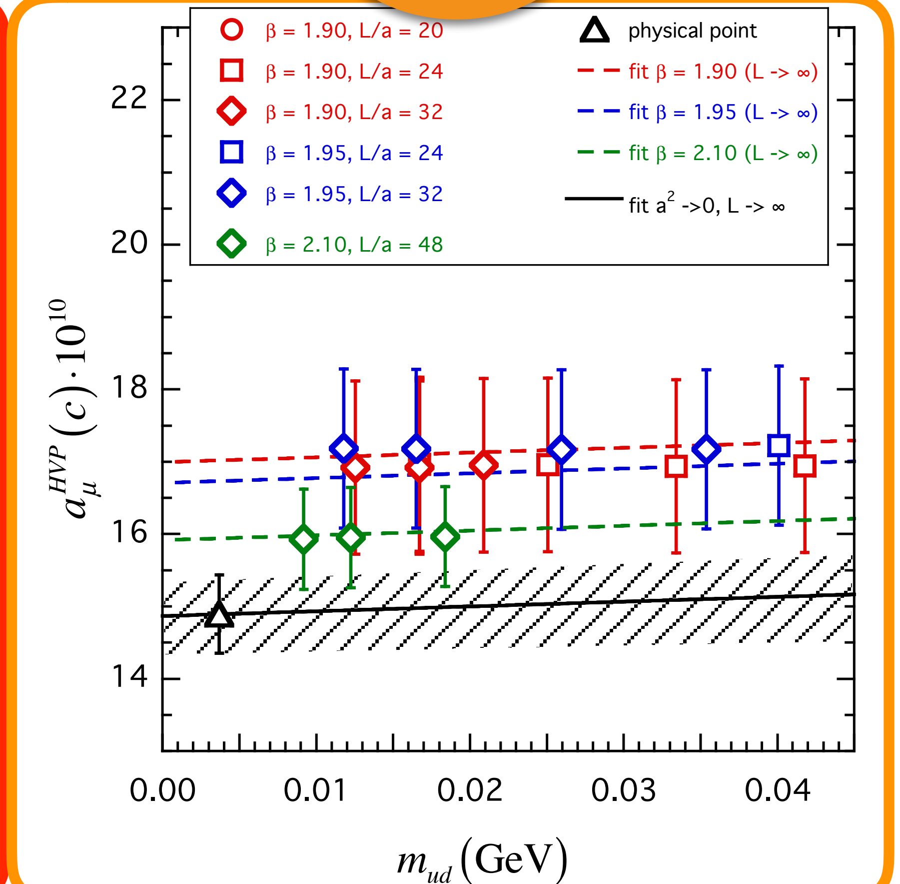
ud

s

c







$$a_\mu^{\rm HVP}(ud) = 619.0(17.8)\cdot 10^{-10}$$

$$a_\mu^{\rm HVP}(s) = 53.1(2.5)\cdot 10^{-10}$$

$$a_\mu^{\rm HVP}(c) = 14.75(56)\cdot 10^{-10}$$

DG *et al.*, 2018
PRD98(2018)114504

DG *et al.*, 2017
JHEP1710(2017)157

quark-connected terms only

Isospin-breaking corrections to a_μ^{HVP}

A strategy for Lattice QCD:

The isospin-breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

JHEP

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ACCEPTED: *April 2, 2*
PUBLISHED: *April 26, 2*

arXiv:1110.6294

Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

G.M. de Divitiis,[a,b] P. Dimopoulos,[c,d] R. Frezzotti,[a,b] V. Lubicz,[e,f] G. Martinelli,[g,d] R. Petronzio,[a,b] G.C. Rossi,[a,b] F. Sanfilippo,[c,d] S. Simula,[f] N. Tantalo[a,b] and C. Tarantino[e,f]

PHYSICAL REVIEW D **87,** 114505 (2013)

Leading isospin breaking effects on the lattice

G. M. de Divitiis,[1,2] R. Frezzotti,[1,2] V. Lubicz,[3,4] G. Martinelli,[5,6] R. Petronzio,[1,2] G. C. Rossi,[1,2] F. Sanfilippo,[7] S. Simula,[4] and N. Tantalo[1,2]

(RM123 Collaboratior

arXiv:1303.4896

[1]*Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy*
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[5]*SISSA, Via Bonomea 265, 34136 Trieste, Italy*
[6]*INFN, Sezione di Roma, Piazzale Aldo Moro 5, I-00185 Rome, Italy*
[7]*Laboratoire de Physique Théorique (Bâtiment 210), Université Paris Sud, F-91405 Orasay-Cedex, France*

(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

The $(m_d - m_u)$ expansion

- Identify the isospin-breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] =$$

$$= \sum_x \left[m_{ud} (\bar{u}u + \bar{d}d) - \Delta m (\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S}$$

$\hat{S} = \Sigma_x(\bar{u}u - \bar{d}d)$

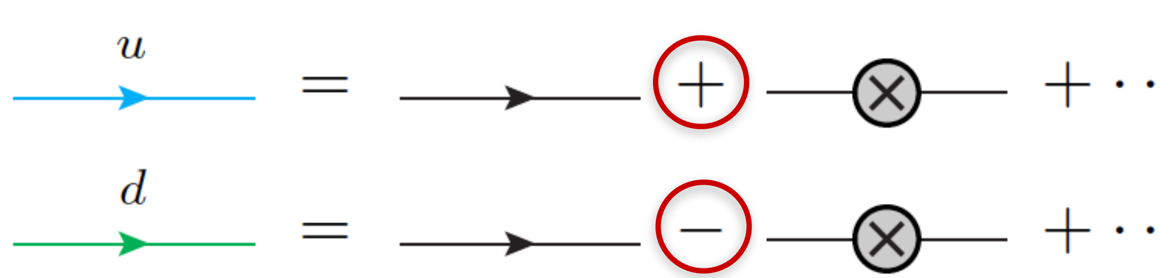
- Expand the functional integral in powers of Δm

Advantage: factorised out

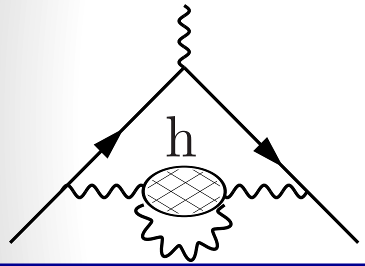
$$\langle O \rangle = \frac{\int D\phi \, O e^{-S_0 + \Delta m \hat{S}}}{\int D\phi \, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi \, O e^{-S_0} (1 + \Delta m \hat{S})}{\int D\phi \, e^{-S_0} (1 + \Delta m \hat{S})} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0}{1 + \Delta m \cancel{\langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0$$

for isospin symmetry

- At leading order in Δm the corrections only appear in the valence-quark propagators:



(disconnected contractions of $\bar{u}u$ and $\bar{d}d$ vanish due to isospin symmetry)



LIB corrections

quark-connected terms only

$$\delta a_\mu^{\mathrm{HVP}} = \delta a_\mu^{\mathrm{HVP}}(QCD) + \delta a_\mu^{\mathrm{HVP}}(QED)$$

photon zero-mode: QED_L

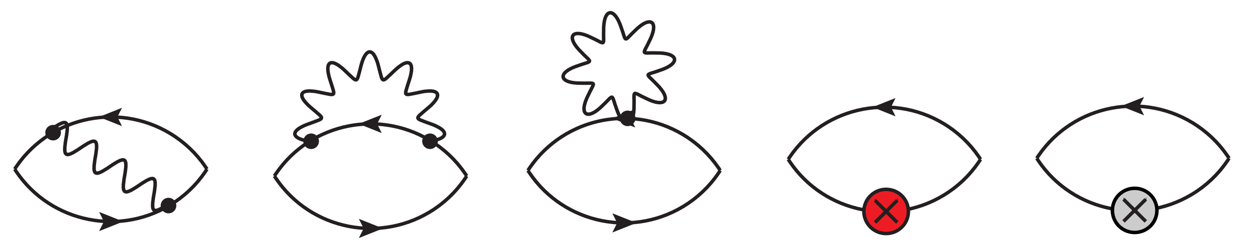
M. Hayakawa and S. Uno, 2008

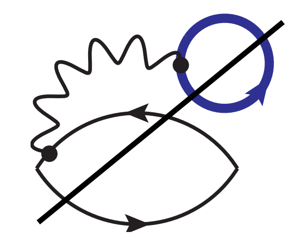
$$\delta a_\mu^{\mathrm{HVP}}(QCD) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QCD}(t)$$

$$\delta a_\mu^{\mathrm{HVP}}(QED) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QED}(t)$$

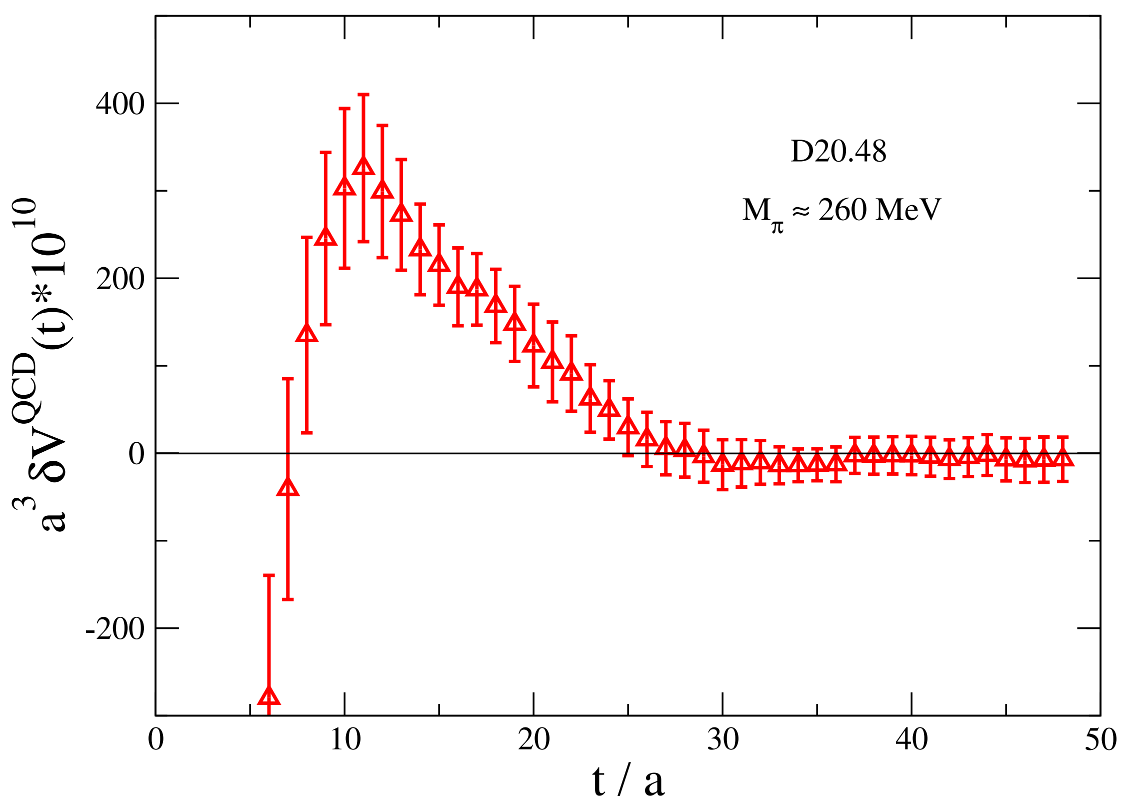
<u>RM123 method</u>

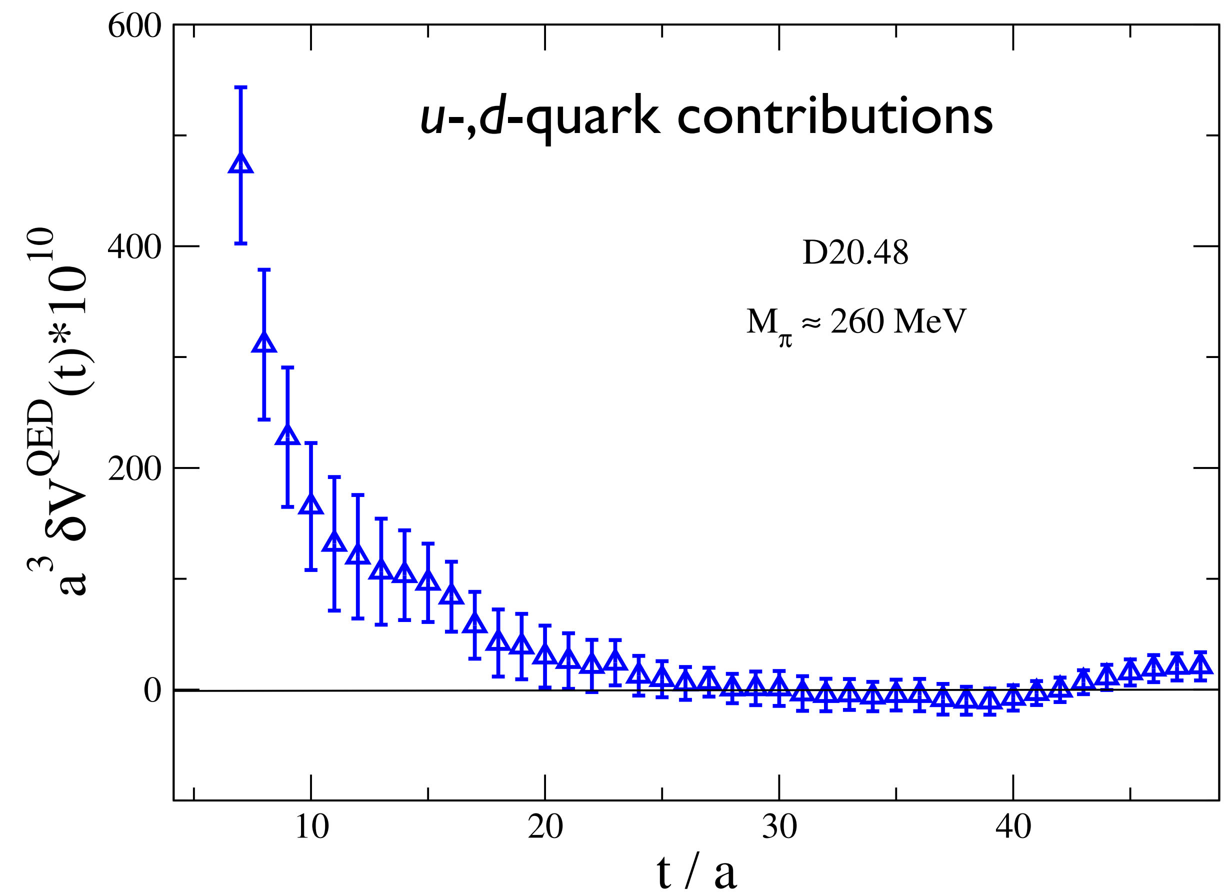
G. M. de Divitiis *et al.*, 2012; 2013





qQED approximation

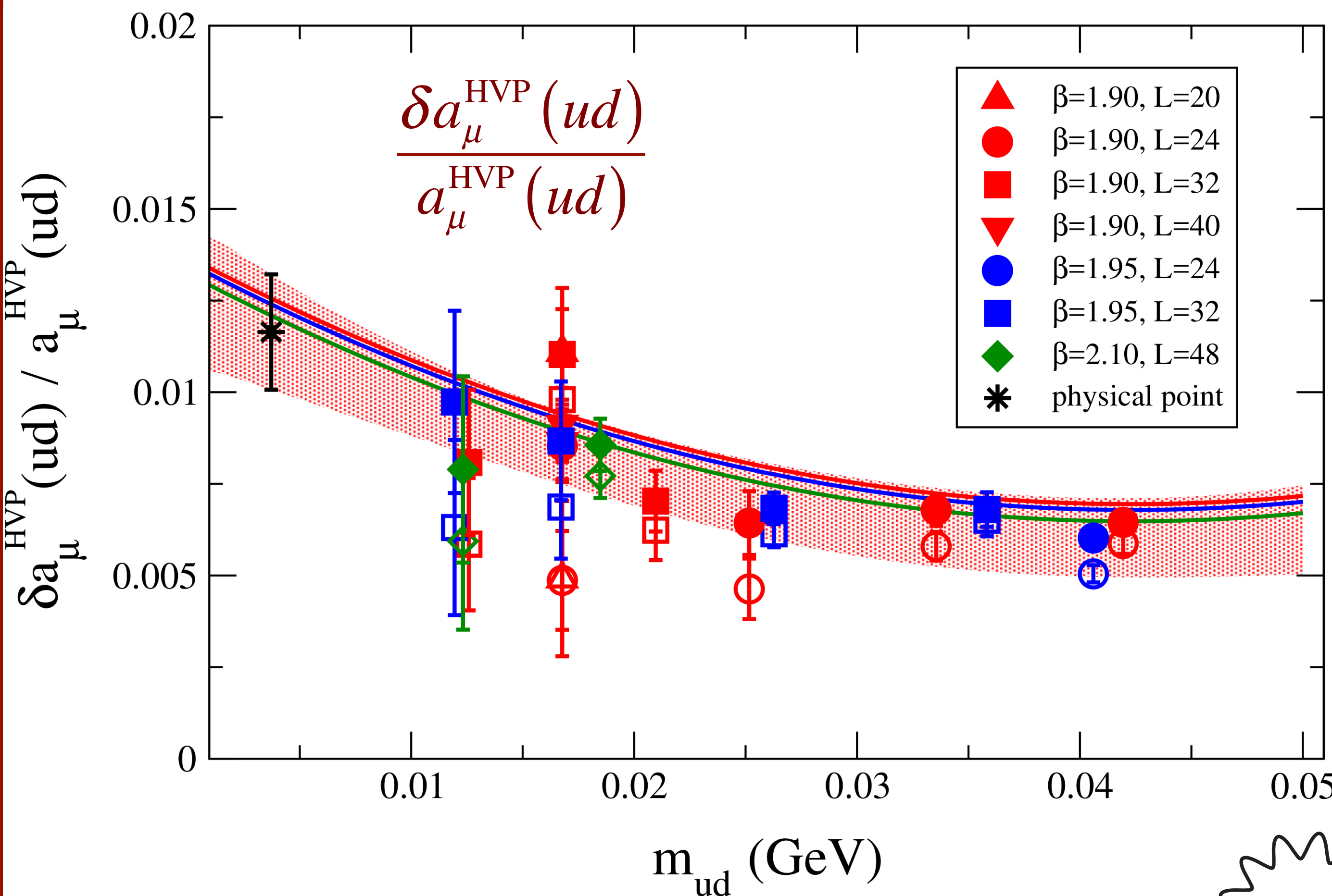




$$[m_d - m_u](\overline{MS}, 2\ \mathrm{GeV}) = 2.38(18)\ \mathrm{MeV}$$

DG *et al.*, 2017

LIB corr.: *udsc*-quark contributions



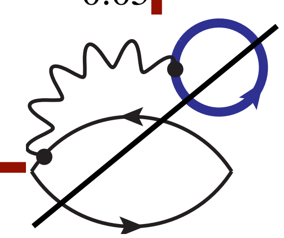
DG *et al.*, 2019; ArXiv:1901.10462

in the ratio various systematics cancel out

$$\frac{\delta a_\mu^{\rm HVP}(ud)}{a_\mu^{\rm HVP}(ud)} = 0.0115(18)_{stat+fit}(21)_{input}(20)_{chir}(19)_{FVE}(9)_{cont}$$
$$= 0.0115(40)$$

$a_\mu^{\rm HVP}(ud) = 619.0(17.8)\cdot 10^{-10}$

$$\boxed{\delta a_\mu^{\rm HVP}(ud) = 7.1(2.5)\cdot 10^{-10}}$$

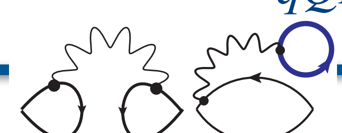


quark-connected terms only

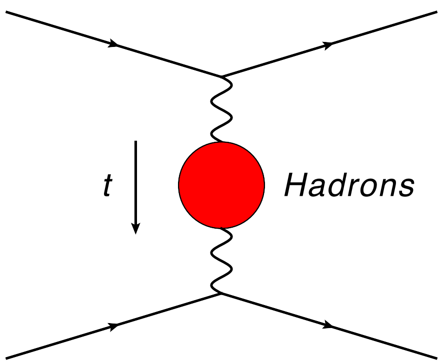
$$\boxed{\delta a_\mu^{\rm HVP}(s) = -0.0053(33)\cdot 10^{-10}}$$

$$\boxed{\delta a_\mu^{\rm HVP}(c) = 0.0182(36)\cdot 10^{-10}}$$

$$\boxed{\delta a_\mu^{\rm HVP} = 7.1(2.6)(1.2)_{qQED+disc}\cdot 10^{-10} = 7.1(2.9)\cdot 10^{-10}}$$



MUonE



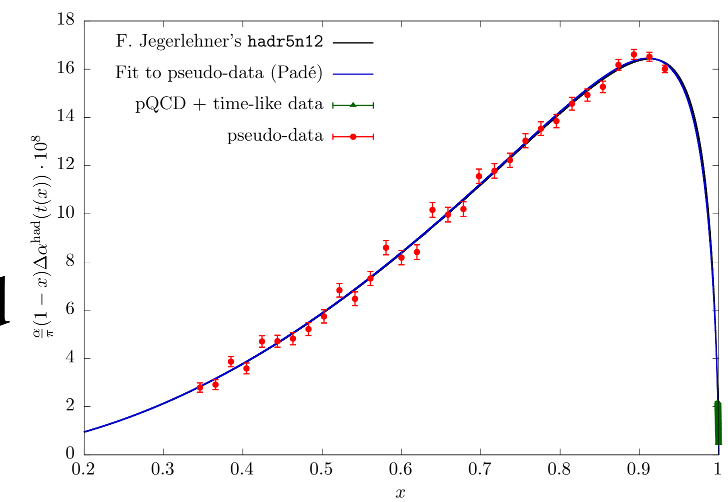
B. E. Lautrup *et al.*, 1972

$$\boxed{a_\mu^{\mathrm{HVP}} = \frac{\alpha_{em}}{\pi}\int_0^1 dx(1-x)\,\Delta\alpha_{em}^{\mathrm{HVP}}\left[t(x)\right]}$$

$$\sigma(\mu e \to \mu e)$$

$$t(x) \equiv -\frac{x^2}{1-x}m_\mu^2$$

$x \in [0.93,\ 1]$ not experimentally reached



LQCD

preliminary (including IB corr.)

$$\left[a_\mu^{\mathrm{HVP}}\right]_> = 4\alpha_{em}^2\int_0^\infty dt\, f_>(t)V(t) \quad \Rightarrow \quad \left[a_\mu^{\mathrm{HVP}}\right]_> = 92(2)\cdot 10^{-10}$$

quark-connected terms only

Uncertainty $(\simeq 2\cdot 10^{-10})$ close to the experimental statistical target $(\simeq 0.3\%)$ of $\left[a_\mu^{\mathrm{HVP}}\right]_<$

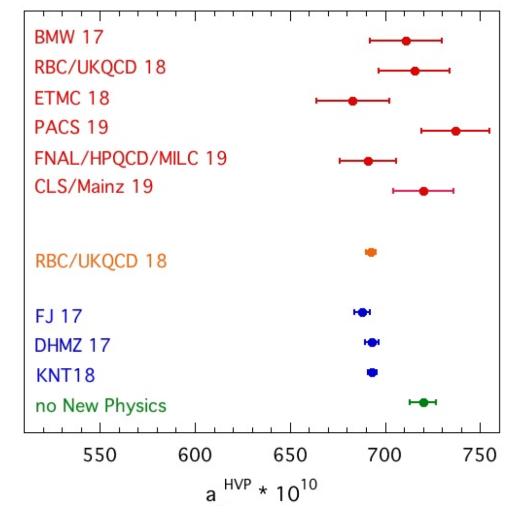
Conclusions

- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) → LQCD

- We have performed a first-principles lattice QCD+QED calculation of a_μ^{HVP}. Our result agrees with recent determinations based on dispersive analyses.

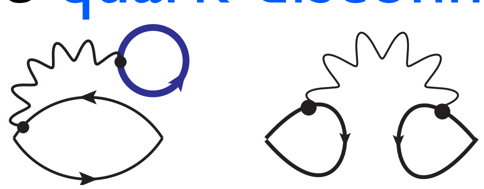
RM123 method

$$a_\mu^{\mathrm{HVP}} = 682(19)\cdot 10^{-10} \qquad \delta a_\mu^{\mathrm{HVP}} = 7.1(2.9)\cdot 10^{-10}$$



In progress...

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation



- study of the electron and tau (g-2)

- use of the new ETMC lattice setup @ the physical pion point

Backup slides

Correlator representation

$$\boxed{V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)}$$

low and intermediate time distances

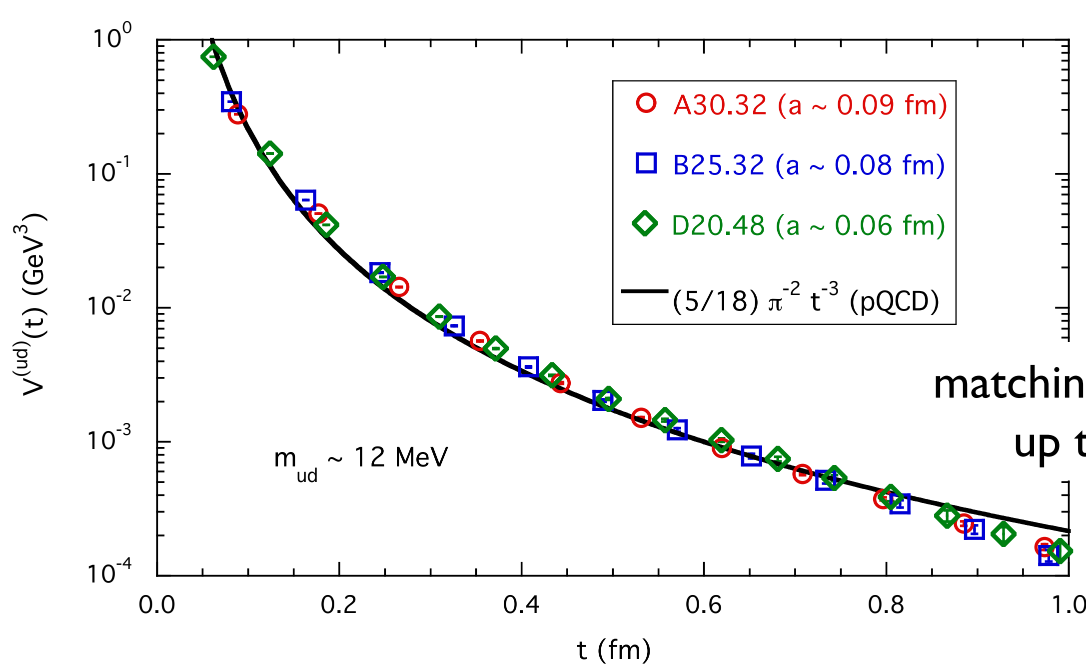
$$V_{dual}(t) \equiv \frac{1}{24\pi^2}\int_{s_{dual}}^{\infty} ds\sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

$$s_{dual} = \left(M_\rho + E_{dual}\right)^2 \qquad R_{dual} = 1 + O\left(\frac{m_{ud}^4}{s_{dual}^2}\right) + O(\alpha_s) + O(a)$$

$$V_{dual}(t) = \frac{5}{18\pi^2}\frac{R_{dual}}{t^3} e^{-\left(M_\rho + E_{dual}\right)t}\left[1 + \left(M_\rho + E_{dual}\right)t + \frac{1}{2}\left(M_\rho + E_{dual}\right)^2 t^2\right]$$

quark-hadron duality *à la SVZ*

SVZ, 1979



matching with pQCD
up to *t* ~1 fm

long time distances

$$V_{\pi\pi}(t) = \sum_n \nu_n \left|A_n\right|^2 e^{-\omega_n t}$$

M. Lüscher 1991

$$\omega_n = 2\sqrt{M_\pi^2 + k_n^2}$$

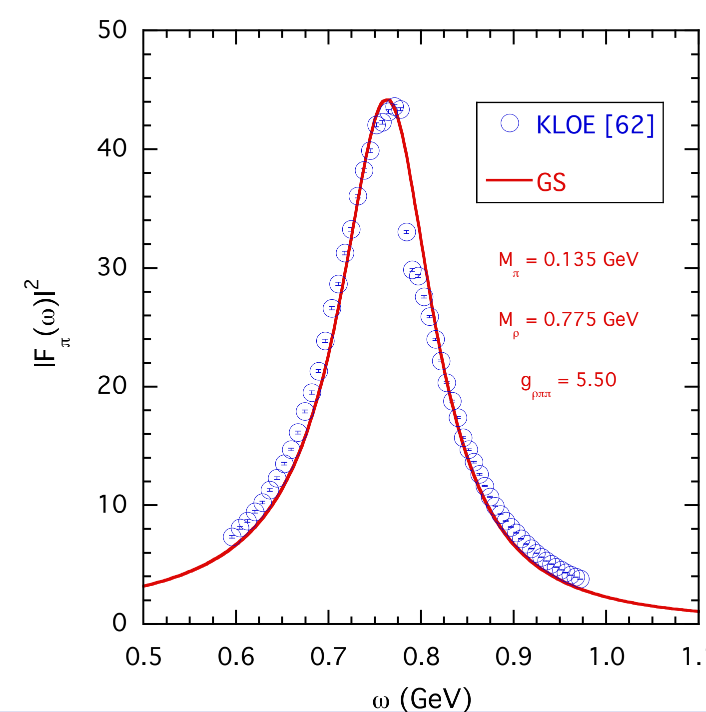
Lüscher condition

L. Lellouch and M. Lüscher, 2001

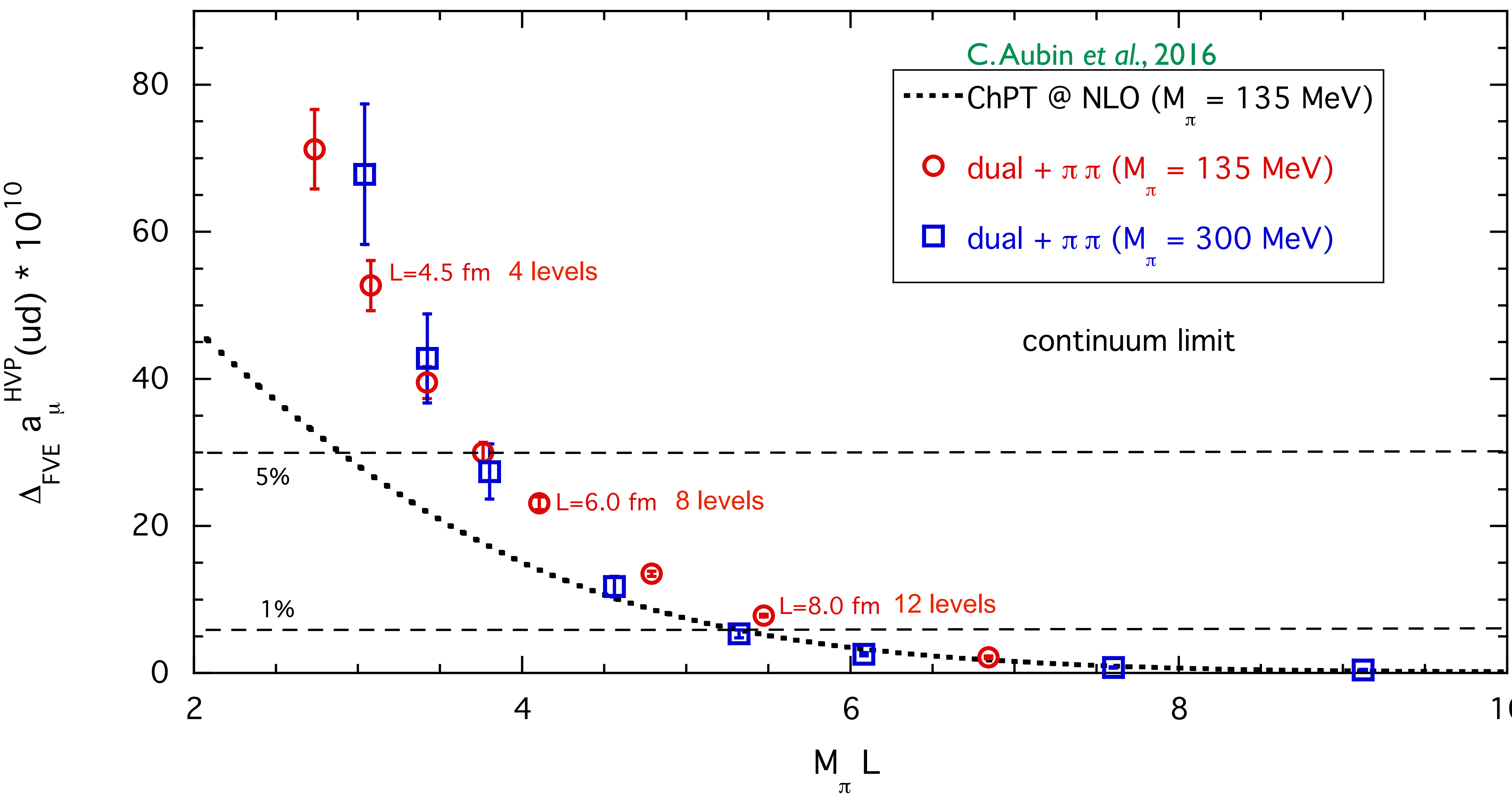
$$\left|A_n\right|^2 \rightarrow \left|F_\pi(\omega_n)\right|^2$$

Gounaris-Sakurai parameterization

$M_\rho \quad g_{\rho\pi\pi}$ GS, 1968



FVEs correction @ $a^2 \to 0$



NLO ChPT does not take into account the resonant interaction in the two-pion system