V_{cb} dai decadimenti inlcusivi del B: un metodo alternativo

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in collaborazione con T. Mannel and K. Vos

JHEP 02 (2019) 177

UNIVERSITÄT SIEGEN

Problema: come misurare V_{cb}?

Decadimenti Inclusivi

- $\bar{B} \to X_c \ell \bar{\nu}$
  con $X_c = D, D^*, D\pi, DKK, \ldots$

$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$

Gambino et al, PRL 114 (2015) 061802

Decadimenti Esclusivi

- $\bar{B} \to D\, \ell \bar{\nu}$
- $\bar{B} \to D^* \ell \bar{\nu}$

$|V_{cb}| = (39.2 \pm 0.7) \times 10^{-3}$ (LQCD, CLN)

HFLAV '17, EPJ C 77, 895

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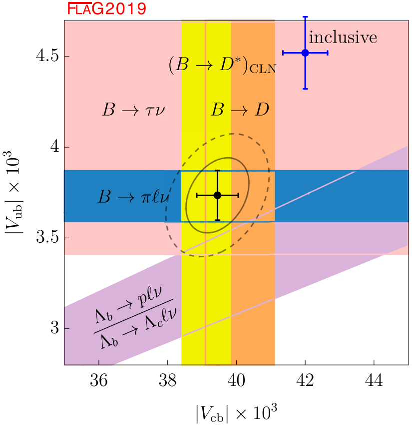

Fit FLAG2019:

$|V_{cb}|_{\mathrm{exc,CLN}} = (39.45 \pm 0.60) \times 10^{-3}$

Belle, arXiv:1702.01521 [hep-ex];
Grinstein, Kobach, PLB 771 359 (2017);
Bigi, Gambino, Schacht, PLB 769 441 (2017),
Bernlochner, Ligeti, Robinson, hep-ph/1902.09553.

Decadimenti Inclusivi

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Gambino et al, PRL 114 (2015) 061802

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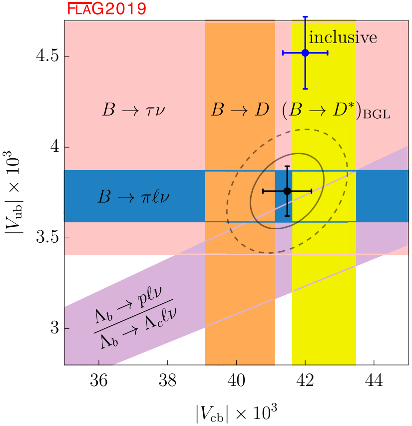
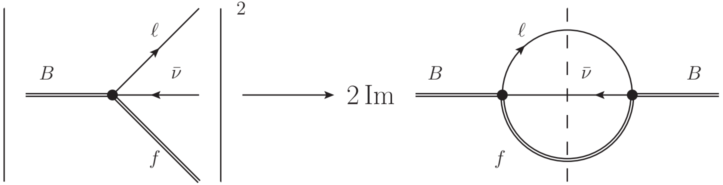
HFLAV '17, EPJ C 77, 895



Fit FLAG2019:
$|V_{cb}|_{\rm exc,BGL} = (41.47 \pm 0.70) \times 10^{-3}$

Belle, arXiv:1702.01521 [hep-ex];
Grinstein, Kobach, PLB 771 359 (2017);
Bigi, Gambino, Schacht, PLB 769 441 (2017),
Bernlochner, Ligeti, Robinson, hep-ph/1902.09553.

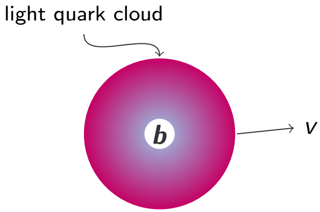
- Teorema Ottico



$$\sum_f |\langle f|\, \mathcal{H}_{\text{eff}}(0)\, |B\rangle|^2 = 2\,\text{Im} \int d^4x\, e^{-iq\cdot x} \langle B|\, T\{\mathcal{H}^\dagger_{\text{eff}}(x), \mathcal{H}_{\text{eff}}(0)\}\, |B\rangle$$

Decadimenti Inclusivi

- Teorema Ottico
- Espansione nella massa del quark pesante (HQE)



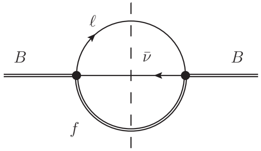
- Per il mesone B:
  $p_B = m_B v$ con $v^2 = 1$

- Per il quark b:
  $p_b = m_b v + k$ con $k \ll m_b$

Decadimenti Inclusivi

- Teorema Ottico
- Espansione nella massa del quark pesante (HQE)
- Espansione di Operatori Bilocali (OPE)

$2\,\mathrm{Im}$



$$= \sum_i \mathcal{C}_i(\mu, \alpha_s) \langle B| \mathcal{O}_i |B\rangle_\mu$$

$\boldsymbol{\mu}$: scala di matching.

$\boldsymbol{\mathcal{C}_i(\mu, \alpha_s)}$: effetti (perturbativi) a breve distanza.

$\boldsymbol{\langle B| \mathcal{O}_i |B\rangle_\mu}$: effetti non perturbativi.

Decadimenti Inclusivi

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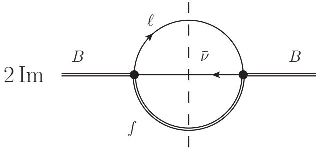
- Espansione nella massa del quark pesante (HQE)
- Espansione di Operatori Bilocali (OPE)



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$\boldsymbol{\langle B | \mathcal{O}_i | B \rangle_\mu}$: effetti non perturbativi.

Quanti sono i parametri della HQE?

Parametri dell'HQE

- $1/m_b^2$

$$\text{Energia cinetica:} \quad 2m_B\mu_\pi^2 = -\langle B|\,\bar{b}_v (iD)^2 b_v\,|B\rangle$$

$$\text{Momento cromomagnetico:} \quad 2m_B\mu_G^2 = \langle B|\,\bar{b}_v (iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu}) b_v\,|B\rangle$$

- $1/m_b^3$

$$\text{Termine di Darwin:} \quad 2m_B\rho_D^3 = \langle B|\,\bar{b}_v (iD_\mu)(ivD)(iD^\mu) b_v\,|B\rangle$$

$$\text{Interazione Spin-Orbita:} \quad 2m_B\rho_{LS}^3 = \langle B|\,\bar{b}_v (iD_\mu)(ivD)(iD_\nu)(-i\sigma^{\mu\nu}) b_v\,|B\rangle$$

- $1/m_b^4$: 9 parametri (tree level);
- $1/m_b^5$: 18 parametri (tree level).

Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087;
Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109;
Kobach, Pal, hep-ph/1810.02356.

Decadimenti Inclusivi

Possiamo predire un'osservabile come:

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Reviews:

Benson, Bigi, Mannel, Uraltsev, Nucl.Phys. B665 (2003) 367;
Dingfelder, Mannel, Rev.Mod.Phys. 88 (2016) 035008.

Momenti della distribuzione in energia dell'elettrone

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

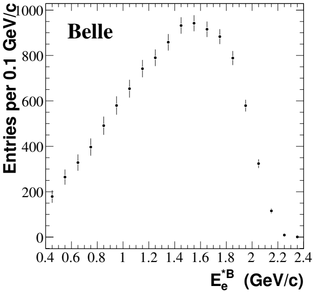


Fig: Belle, PRD 75 (2007) 032001;

BABAR, PRD 69 (2004) 111104; BABAR, PRD 81 (2010) 032003.

Momenti della distribuzione della massa invariante adronica

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

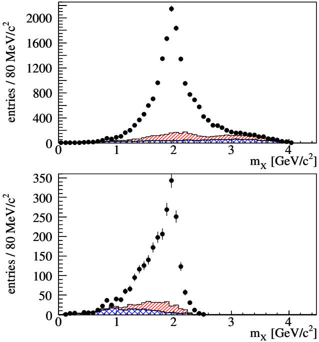


Fig: BABAR, PRD 81 (2010) 032003

Belle, PRD 75 (2007) 032005.

Rapporto di ramificazione in funzione di $E_{\rm cut}$

$$R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

$$\Delta \mathrm{Br}(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\Gamma_B}$$

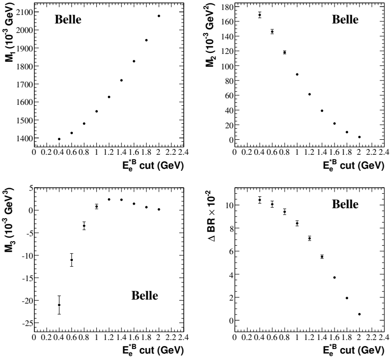


Fig: Belle, PRD 75 (2007) 032005

BABAR, PRD 81 (2010) 032003

$$R^*(E_{\rm cut}) \quad \langle E^n \rangle_{\rm cut} \quad \langle (M_X^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_\pi, \mu_G, \rho_D, \rho_{LS}, m_b, (m_c)$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} \right]$$

$$\downarrow$$

$$V_{cb} = (42.21 \pm 0.78) \times 10^{-3}$$

vedi: Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802

- Problema, input non perturbativi a ordini superiori:
  - 4 fino $1/m_b^3$
  - 13 fino $1/m_b^4$
  - 31 fino $1/m_b^5$
- Approssimazione: Lowest State Saturation (LSSA)

$$\langle B|\,\bar{b}_v(iD)^2(iD)^2 b_v\,|B\rangle \sim \langle B|\,\bar{b}_v(iD)^2\,|B\rangle\,\langle B|\,(iD)^2 b_v\,|B\rangle$$

Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46.

- LSSA + Fit dei termini $1/m_b^4$ e $1/m^5$:
  spostamento di $|V_{cb}|$ del -0.25%.

Healey, Turczyk, Gambino, PLB 763 (2016) 60.

Come ridurre il numero di parametri?

Invarianza per Riparametrizzazione

- Impulso del quark b

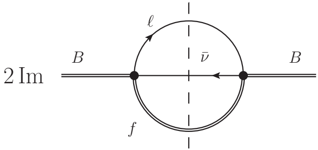
  $$p_b = m_b v + k \text{ con } k \ll m_b$$

- Questa separazione non è unica:

  $$\begin{cases} v \to v + \delta v / m_b \\ k \to k - \delta v \end{cases}$$

- Riparametrizzazione dell'OPE:
  - $\delta_{\mathrm{RP}} v_\mu = \delta v_\mu$ con $v \cdot \delta v = 0$
  - $\delta_{\mathrm{RP}} iD_\mu = -m_b \delta v_\mu$
  - $\delta_{\mathrm{RP}} b_v(x) = i m_b (x \cdot \delta v) b_v(x)$, quindi $\delta_{\mathrm{RP}} b_v(0) = 0$

Luke, Manohar, PLB 286, 348 (1992);
Manohar, PRD 82, 014009 (2010);
Heinonen, Hill, Solon, PRD 86 094020 (2012).



$$= \sum_n \mathcal{C}^{(n)}_{\mu_1 \dots \mu_n}(v)\, \bar{b}_v (iD^{\mu_1} \dots iD^{\mu_n}) b_v$$

- $\delta_{\rm RP} \Gamma_{\rm tot} = 0$
- Relazione tra i coefficienti di Wilson:

$$\delta_{\rm RP}\, C^{(n)}_{\mu_1 \cdots \mu_n}(S) = m_b\, \delta v^\alpha \left[C^{(n+1)}_{\alpha \mu_1 \cdots \mu_n}(S) + C^{(n+1)}_{\mu_1 \alpha \mu_2 \cdots \mu_n}(S) + \cdots + C^{(n+1)}_{\mu_1 \cdots \mu_n \alpha}(S) \right]$$

Mannel, Vos, JHEP 1806 (2018) 115

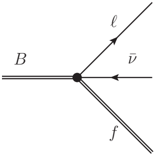
Insieme Ristretto di Parametri per Γ_{tot}

- 1
  - $2m_B\mu_3 = \langle \bar{b}_v b_v \rangle = 2m_B \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b}\right)$
- $1/m_b^2$
  - $2m_B\mu_G^2 = \langle \bar{b}_v (iD_\alpha)(iD_\beta)(-i\sigma^{\alpha\beta}) b_v \rangle$
- $1/m_b^3$
  - $2m_B\tilde{\rho}_D^3 = \frac{1}{2}\langle \bar{b}_v \left[(iD_\mu)\,,\,\left[\left(ivD + \frac{1}{2m}(iD)^2\right)\,,\,(iD^\mu)\right]\right] b_v \rangle$
- $1/m_b^4$
  - $2m_B r_G^4 = \langle \bar{b}_v \left[(iD_\mu)\,,\,(iD_\nu)\right]\left[(iD^\mu)\,,\,(iD^\nu)\right] b_v \rangle$
  - $2m_B r_E^4 = \langle \bar{b}_v \left[(ivD)\,,\,(iD_\mu)\right]\left[(ivD)\,,\,(iD^\mu)\right] b_v \rangle$
  - $2m_B s_B^4 = \langle \bar{b}_v \left[(iD_\mu)\,,\,(iD_\alpha)\right]\left[(iD^\mu)\,,\,(iD_\beta)\right](-i\sigma^{\alpha\beta}) b_v \rangle$
  - $2m_B s_E^4 = \langle \bar{b}_v \left[(ivD)\,,\,(iD_\alpha)\right]\left[(ivD)\,,\,(iD_\beta)\right](-i\sigma^{\alpha\beta}) b_v \rangle$
  - $2m_B s_{qB}^4 = \langle \bar{b}_v \left[iD_\mu\,,\,\left[iD^\mu\,,\,\left[iD_\alpha\,,\,iD_\beta\right]\right]\right](-i\sigma^{\alpha\beta}) b_v \rangle$

Mannel, Vos, JHEP 1806 (2018) 115

Quali Osservabili sono RPI?

$$d\Gamma = \int w(v, p_e, p_\nu) \left| \quad \right|^2 d\Phi_n$$



- Un'osservabile $d\Gamma$ è RPI se $\delta_{\mathrm{RP}} w(v, p_e, p_\nu) = 0$:

$d\Gamma$	$w(v, p_e, p_\nu)$	RPI
Larghezza totale	1	✓
Momenti energia elettrone	$(v \cdot p_e)^n$	✗
Momenti massa invariante adronica	$(M_B v - q)^{2n}$	✗
Momenti massa invariante leptonica	$(q^2)^n$	✓

MF, Mannel, Vos, JHEP 02 (2019) 177

- Momenti della distribuzione della massa invariante leptonica (q^2):

$$\left\langle (q^2)^n \right\rangle_{\mathrm{cut}} = \int_{q^2 > q^2_{\mathrm{cut}}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2 > q^2_{\mathrm{cut}}} dq^2 \, \frac{d\Gamma}{dq^2}$$

- Rapporto tra la larghezza totale con e senza taglio:

$$R^*(q^2_{\mathrm{cut}}) = \int_{q^2 > q^2_{\mathrm{cut}}} dq^2 \, \frac{d\Gamma}{dq^2} \Bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

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Per i momenti del q^2

serve solo

l'insieme ristretto di parametri

$R^*(q^2_{\rm cut}) \quad \langle (q^2)^n \rangle_{\rm cut}$

$\downarrow$

$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c$

$\downarrow$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \Bigg[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \Bigg]$$

$\downarrow$

$V_{cb} = ?$

MF, Mannel, Vos, hep-ph/1812.07472

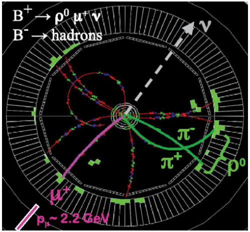
Conclusioni

- Fit per V_{cb} considerano HQE fino a $1/m_b^3$.
- Per $1/m_b^4 + 1/m_b^5$ serve una stima preliminare (LSSA)

- RPI connette l'HQE a ordini differenti in $1/m_b$.
- La larghezza totale e i momenti del q^2 sono RPI: bastano 8 param. invece che 13 (fino a $1/m_b^4$).
- Nuovo metodo: misurare V_{cb} da $\Gamma_{\rm tot}$, $\Delta Br(q^2_{\rm cut})$ e $\langle (q^2)^n \rangle_{\rm cut}$ senza alcun input esterno fino a $1/m_b^4$.
- Momento del neutrino può essere ricostruito alle B-factories basandosi sul $B_{\rm tag}$.
- I dati di Belle e BABAR possono essere già analizzati, Belle II ha appena cominciato ...

Backup

Experimental Perspectives



- Need to reconstruct neutrino momentum
  - BABAR
  - Belle, Belle-II

Dingfelder, Mannel, Rev. Mod. Phy. 88 035008

- BABAR: 433 fb^{-1} of data at $\Upsilon(4S)$

  Note: $\langle E_e^n \rangle$ & $\langle M_X^n \rangle$ obtained with 210 fb^{-1}

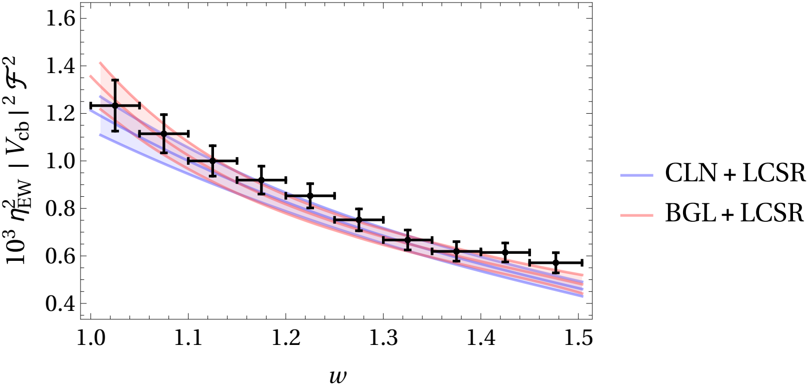
  BABAR, PRD 69 (2004) 111104; PRD 81 (2010) 032003.

- Belle: 711 fb^{-1} of data at $\Upsilon(4S)$

  Note: $\langle E_e^n \rangle$ & $\langle M_X^n \rangle$ obtained with 140 fb^{-1}

  Belle, PRD 75 (2007) 032005; PRD 75 (2007) 032001

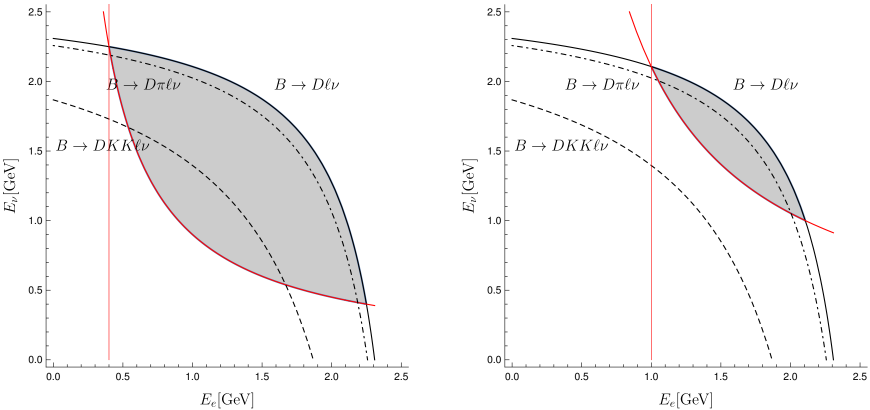
- Belle-II $=$ Belle $\times$ 50



Bigi, Gambino, Schacht, PLB 769 (2017) 441

Belle, arXiv:1702.01521 [hep-ex]

$q^2 > 3.6\ \text{GeV}^2$ $\quad$ $q^2 > 8.4\ \text{GeV}^2$



MF, Mannel, Vos, JHEP 02 (2019) 177

OPE for Semileptonic Decays

- The Effective Hamiltonian

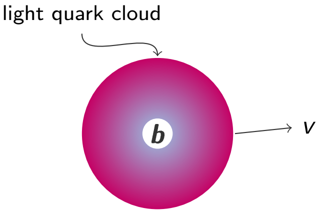
$$\mathcal{H}_{\text{eff}} = \frac{4 G_F V_{\text{cb}}}{\sqrt{2}} j_q^{\mu} L_{\mu}$$

- The decay rate: $\Gamma(B \to X_c \ell \bar{\nu}) \propto 2\,\mathrm{Im}\, T_{\mu\nu} L^{\mu\nu}$

$$T^{\mu\nu} = i \int dx^4 e^{-iqx} \langle B | \, T \left\{ \bar{b}(x) \gamma^{\mu} P_L c(x), \bar{c}(0) \gamma^{\nu} P_L b(0) \right\} | B \rangle$$

Heavy Quark Expansion

- B meson:
  $p_B = m_B v$ with $v^2 = 1$
- b quark:
  $p_b = m_b v + k$ with $k \ll m_b$

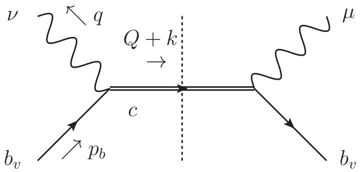


"Rephase" the field $b(x)$:

$$b(x) = \exp(-i m_b v \cdot x) b_v(x)$$

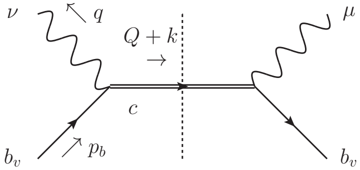
$$iD_\mu b(x) = e^{-i m_b v \cdot x} \left(m_b v_\mu + iD_\mu \right) b_v(x)$$

$$T^{\mu\nu} = i \int dx^4 e^{i(m_b v - q)\cdot x} T\left\{\bar{b}_v(x)\gamma^\mu P_L c(x), \bar{c}(0)\gamma^\nu P_L b_v(0)\right\}$$



with $Q = m_b v - q$

- Take matrix elements with quarks and gluons



$$= \bar{b}_v \gamma_\mu P_L \left[\frac{i}{\not{Q} + \not{k} - m_c} \right] \gamma_\nu P_L b_v$$

- Now expand ...

$$S = \frac{i}{\not{Q} + \not{k} - m_c}$$
$$= \frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c}(-\not{k})\frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c}(-\not{k})\frac{i}{\not{Q} - m_c}(-\not{k})\frac{i}{\not{Q} - m_c} + \dots$$

Technical Ingredients

We need to calculate:

$$\bar{b}_v \Gamma^\dagger \left[\frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c}(-i\not{D})\frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c}(-i\not{D})\frac{i}{\not{Q} - m_c}(-i\not{D})\frac{i}{\not{Q} - m_c} + \dots \right] \Gamma b_v$$

- Reduce all possible matrix elements to scalar operators (also redundant ones):

$$\bar{b}_v (iD_{\mu_1})(iD_{\mu_2})\dots(iD_{\mu_n})\Gamma b_v$$

  with $\Gamma = 1, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma^5, -i\sigma^{\mu\nu}$
- Use equation of motion:
  - $\not{v} b_v = b_v - \frac{i\not{D}}{m_b} b_v$
  - $(iv \cdot D) b_v = -\frac{1}{2m_b}(i\not{D})(i\not{D}) b_v$
- Example:

  $\langle \bar{b}_v (iD^\mu)(iD^\nu) b_v \rangle = 2m_B \left[\frac{1}{3}\mu_\pi (v^\mu v^\nu - g^{\mu\nu}) + \frac{1}{3}\sigma_{G1}(4v^\mu v^\nu - g^{\mu\nu}) \right]$

Phase space integration

- $\mathrm{Im}\left(\frac{1}{(p^2 - m^2 + i\varepsilon)^n}\right) = -\pi\frac{(-1)^n}{n!}\delta^{(n)}(p^2 - m^2)$
- Integrate:

$$\int dq^2 dv{\cdot}q dE_e \theta(q^2)\theta(4E_e^2 + 4E_e v{\cdot}q - q^2) f(q^2, v{\cdot}q, E_\ell)\delta^{(n)}(q^2 + m_b^2 - 2v{\cdot}q - m_c^2)$$

- q^2-specturm:

$$\frac{d\Gamma}{d\hat{q}^2} = f_{\mathrm{reg}}(\hat{q}^2) + \delta(z(\hat{q}^2))f_1(\hat{q}^2) + \delta'(z(\hat{q}^2))f_2(\hat{q}^2) + \ldots$$

with $z(\hat{q}^2) = 1 - 2\sqrt{\hat{q}^2} + \hat{q}^2 - \rho$