GR *or not* GR?

Testing General Relativity through gravitational wave observations

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Outlook

Introduction to gravitational waves

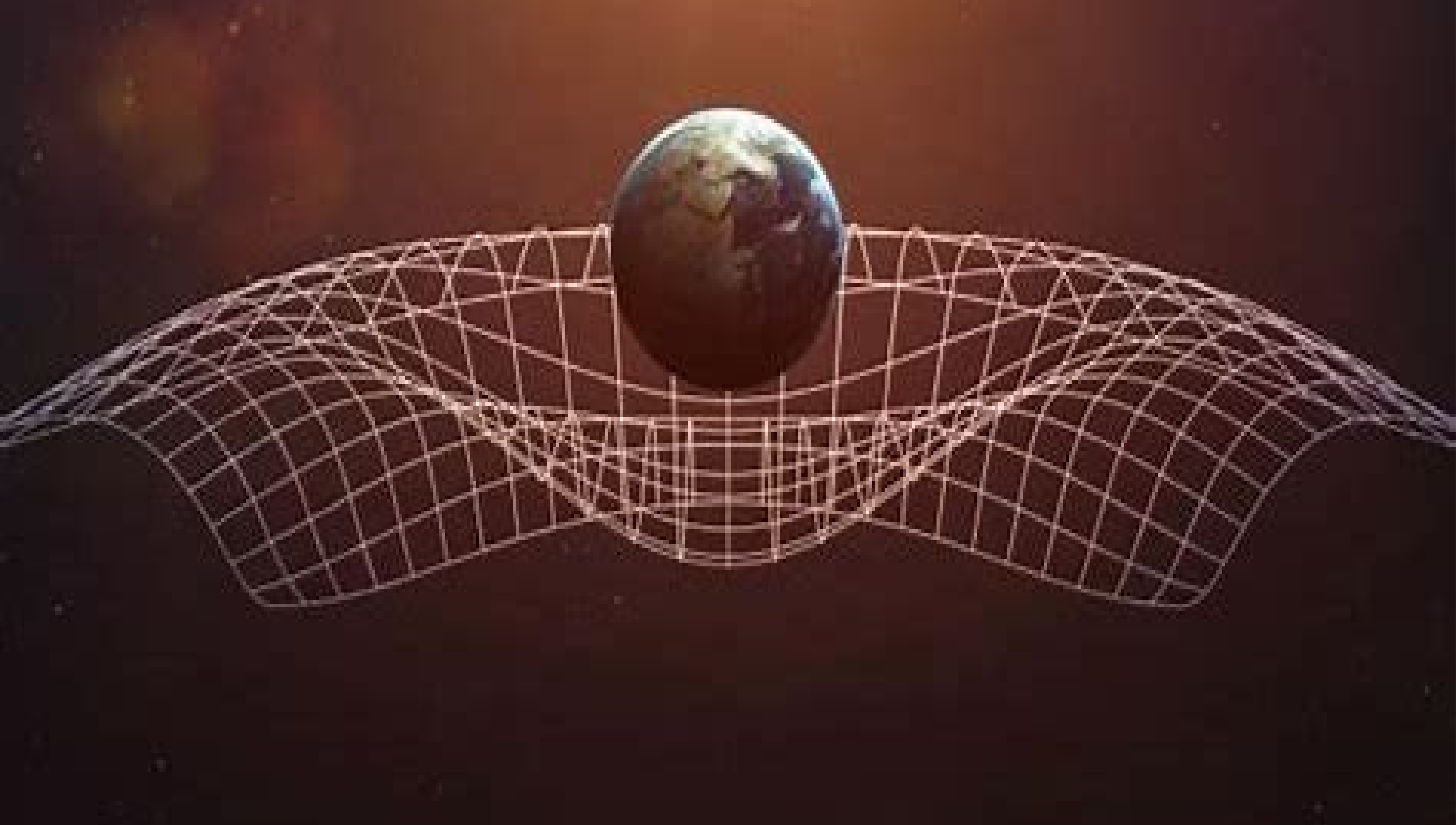
- GWs in general relativity
- Detection and data analysis

Tests of General Relativity

- **Consistency tests**
  - Compatibility of residuals with noise
  - IMR consistency test
  - Ringdown test
- **Production effects**
  - Waveform expansion
  - Polarisation tests
- **Propagation effects**
  - Modified dispersion relation
  - Higher dimesions

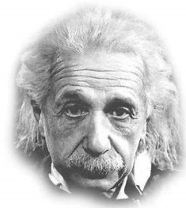
Conclusions and perspectives

Introduction to gravitational waves

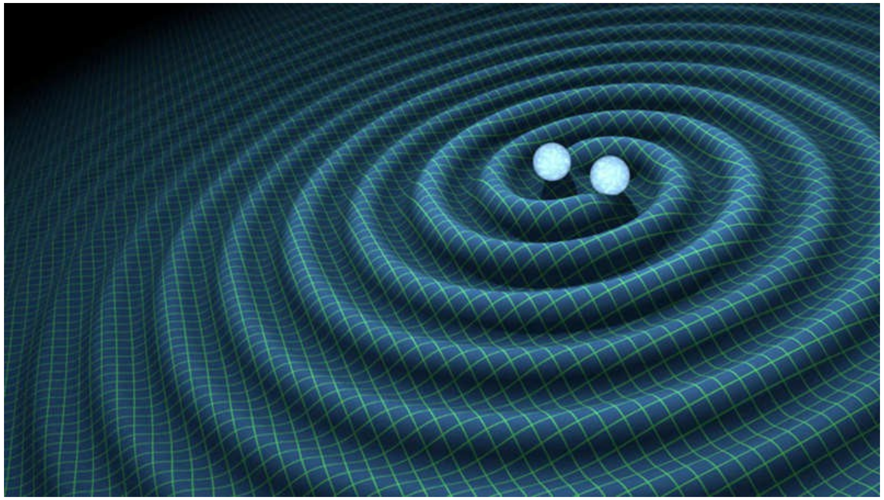


Getty images

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$



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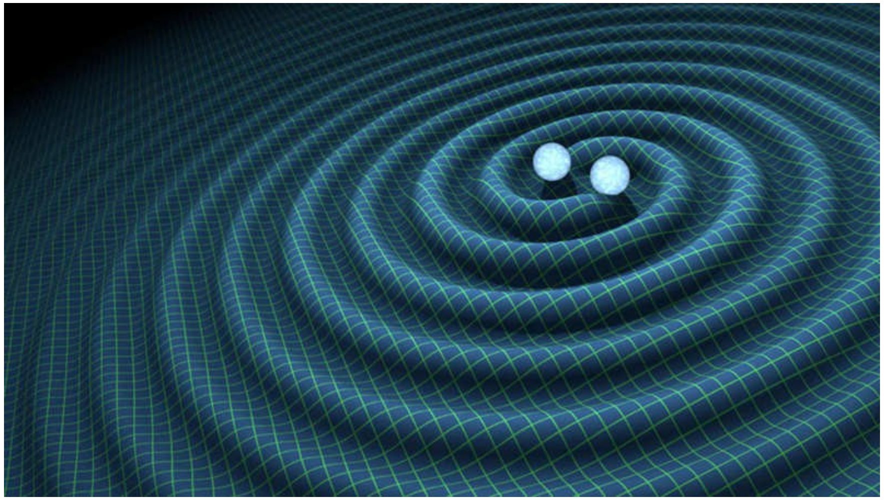

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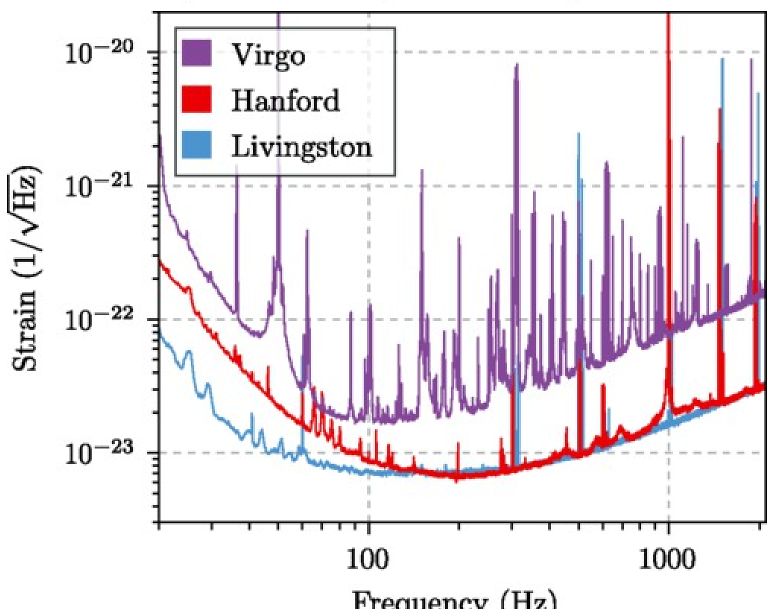
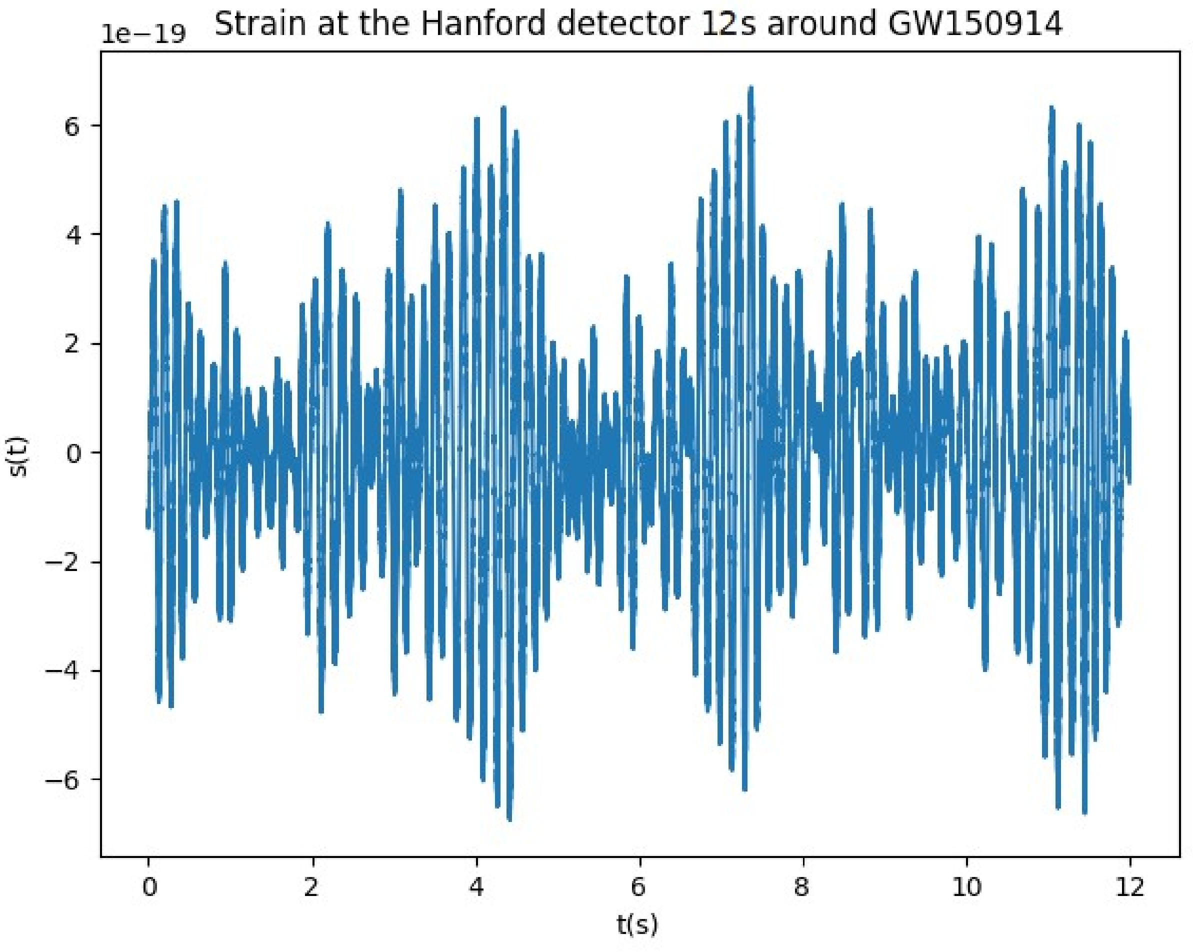


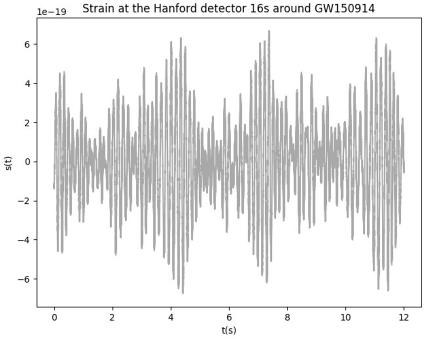
Phys. Rev. Lett., 119, 141101 (2017)

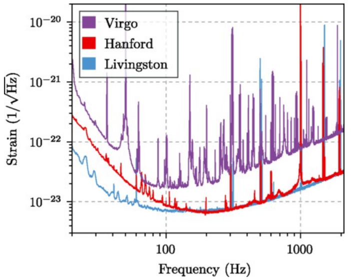
Buried in noise



Buried in noise

$$d(t) = n(t) + h(t, \vec{\theta})$$





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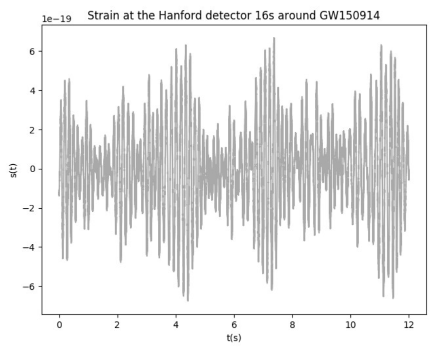

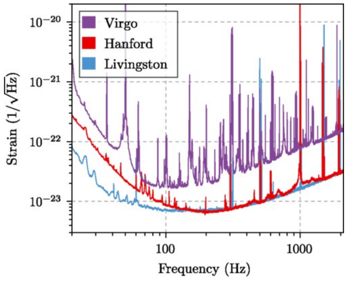

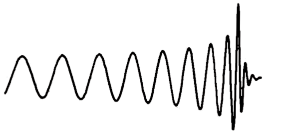

GR

⇩

Prediction

Buried in noise

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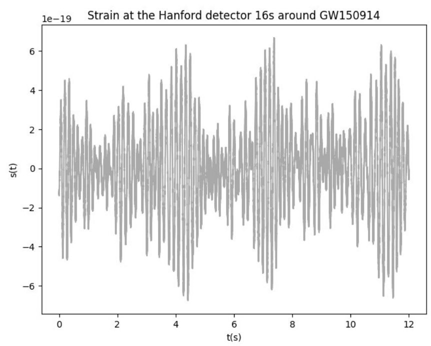

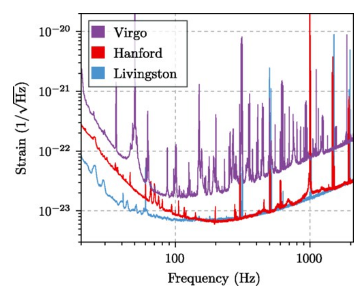

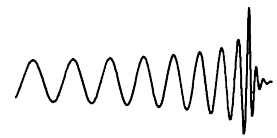

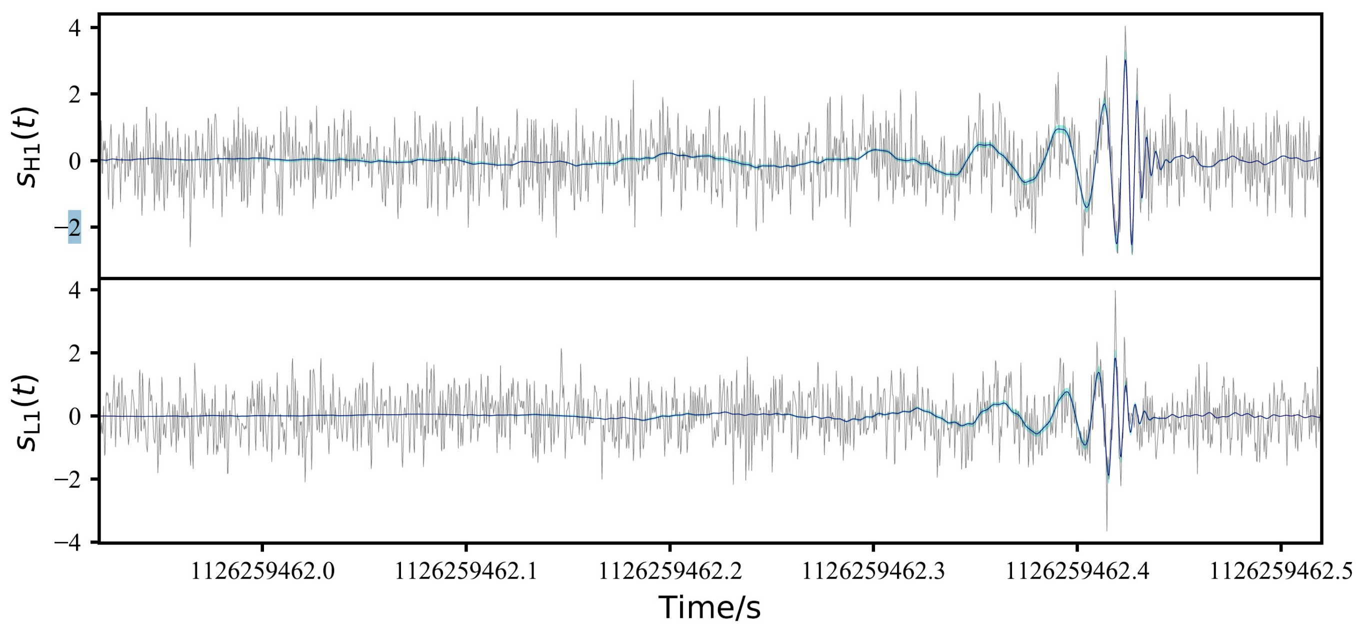

Nagar et al., arXiv:1806.01772v2

GR

⇩

Prediction

⇩

Match with data

GR tests: basics

- How to test for the presence of a **GR** waveform in the data?
- How to test for **violations**?

$$d(t) - h(t, \vec{\theta}) = n(t)$$

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$\Longrightarrow$ $h(t, \vec{\theta})$

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$\Rightarrow$ $h(t, \vec{\theta})$ $\Rightarrow$ Test residuals against the **distribution of noise**

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$$d(t) - h(t, \vec{\theta}) = n(t)$$

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$h(t, \vec{\theta})$ ⇒ Test residuals against the **distribution of noise**

- Constrain/measure **parameters**
- Model **selection**

GR tests: Bayesian inference

- **Probability distributions** are recovered **for each parameter**
- Uses **Bayes' theorem**

Bayes' theorem

$$p(\vec{\theta}|dHI) = p(\vec{\theta}|HI) \cdot \frac{p(d|\vec{\theta}HI)}{p(d|HI)}$$

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Bayes' theorem

$$p(\vec{\theta}|d\,H\,I) = p(\vec{\theta}|H\,I) \cdot \frac{p(d|\vec{\theta}\,H\,I)}{p(d|H\,I)}$$

The data

$d_H(t)$, $d_L(t)$, $d_V(t)$

The model

GR

non **GR**

Prior information

Any **information** available **before** analysing the **data**

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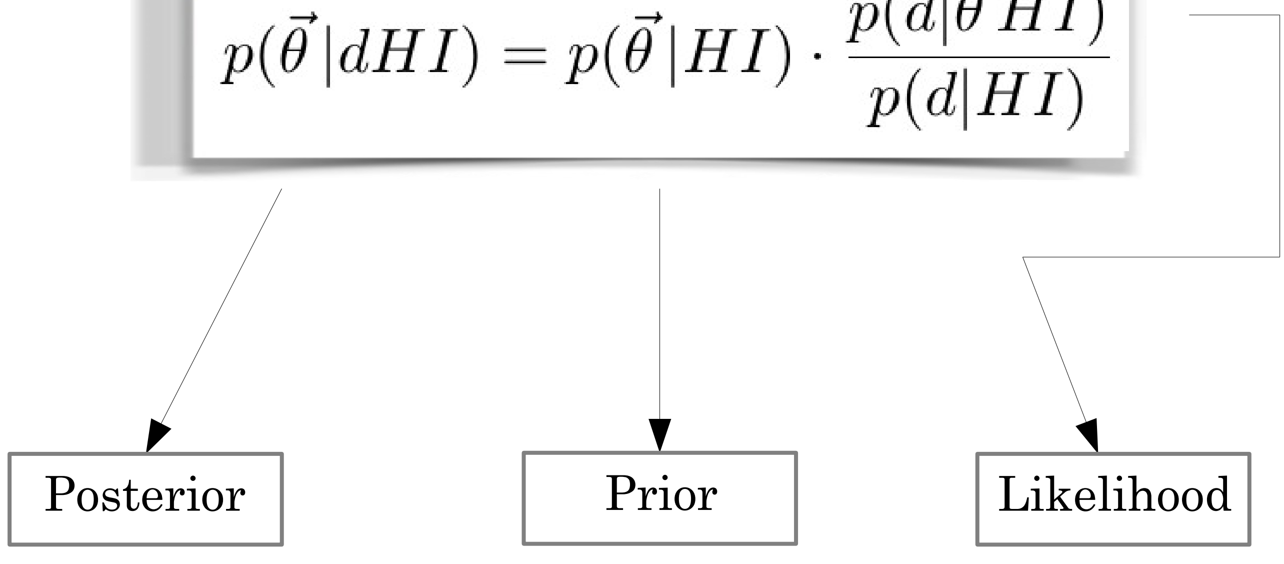
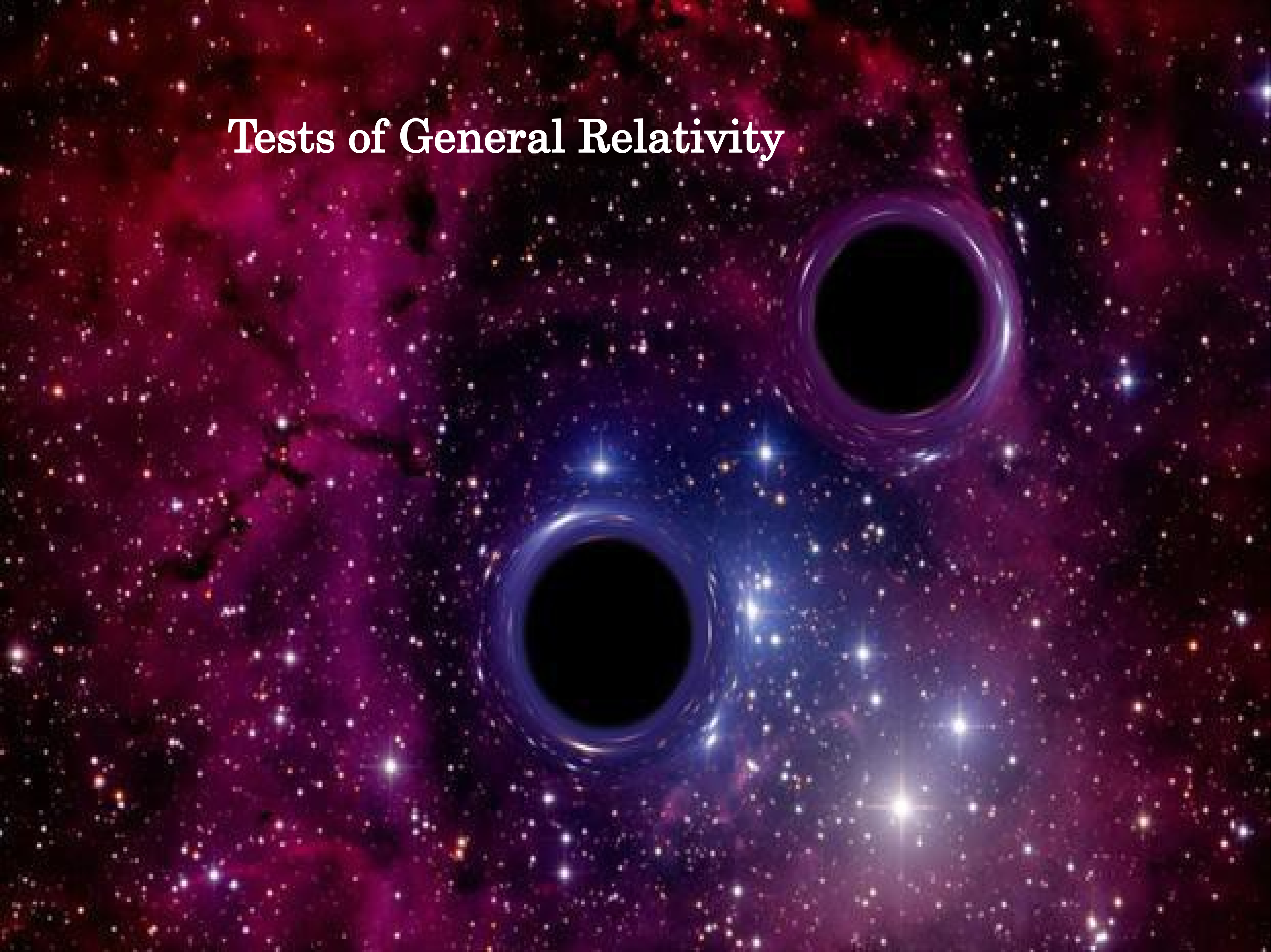


Tests of General Relativity



Consistency tests

Distribution of the residuals

- Subtract the **most probable GR waveform** from the data
- Analyse the residuals **only** looking for **coherence** (BayesWave)

Bayes' theorem

$$p(H_1|DI) = p(H_1|I)\frac{p(D|H_1I)}{p(D|I)}$$

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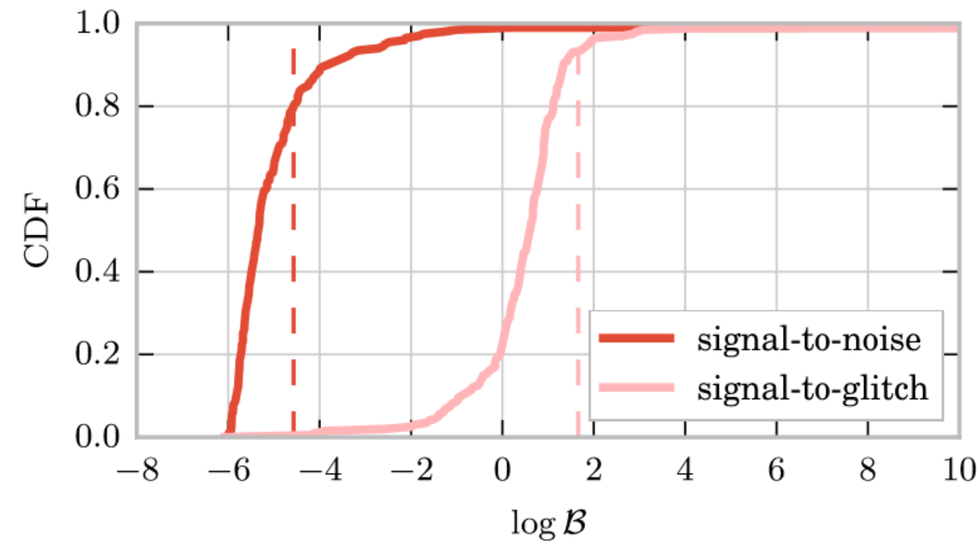
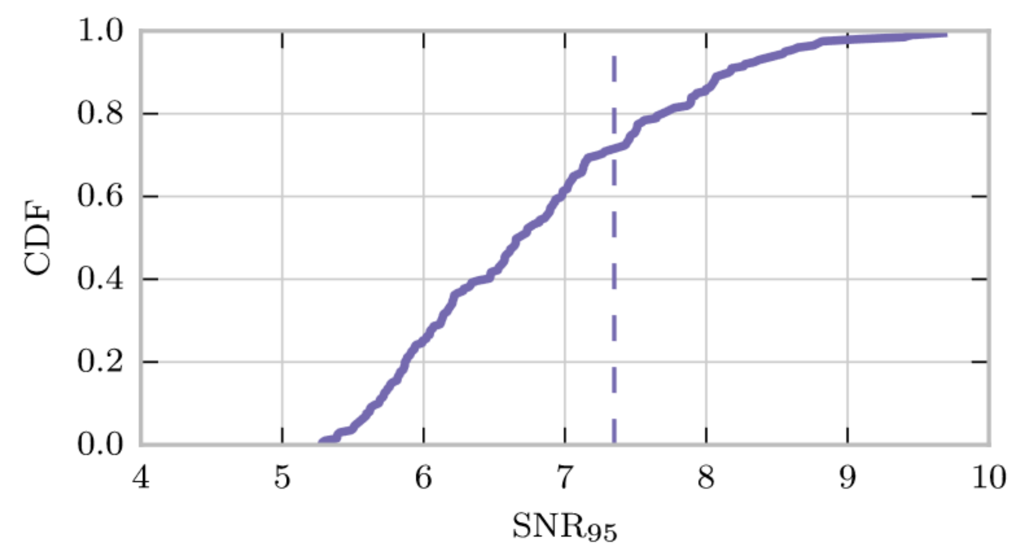
Odds ratio and Bayes factor

$$O_{1,2} \equiv \frac{p(H_1|I)}{p(H_2|I)}\frac{p(D|H_1I)}{p(D|H_2I)}$$

GW150914





Phys. Rev. Lett. 116, 221101 (2016)

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GW150914

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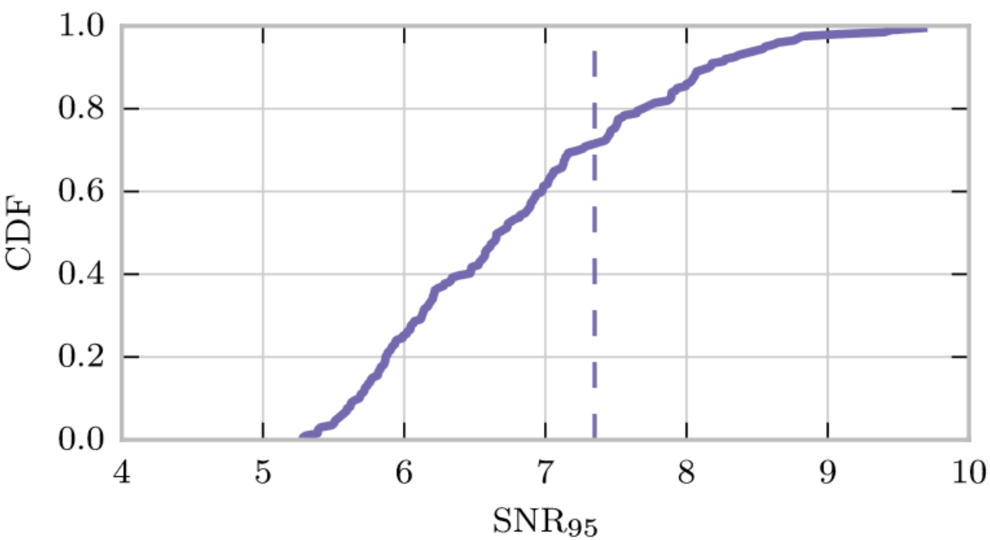

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GW150914

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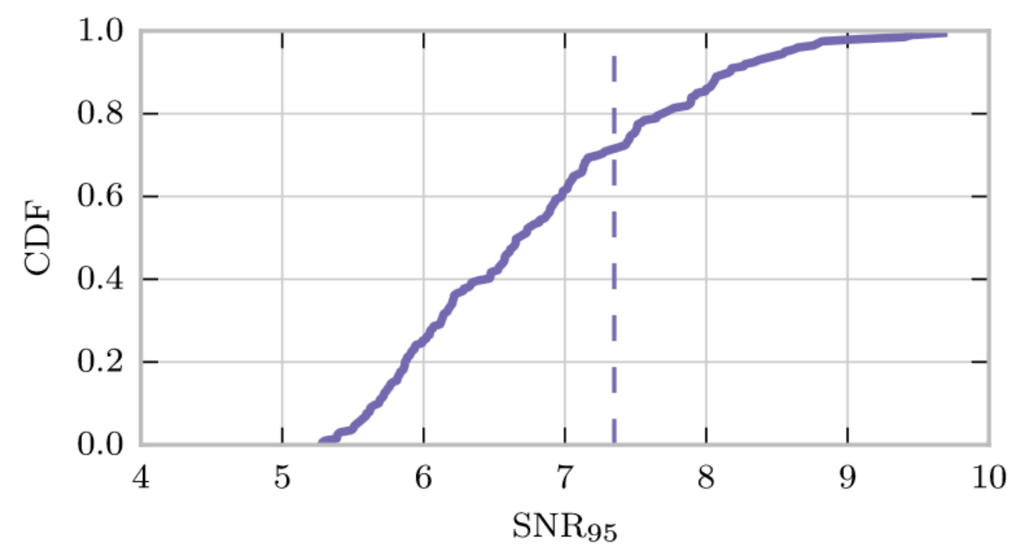
$\mathrm{SNR}_{\mathrm{res}} \leq 7.3$ ⇒ $\mathrm{FF} \geq 0.96$

GR deviations less than 4%



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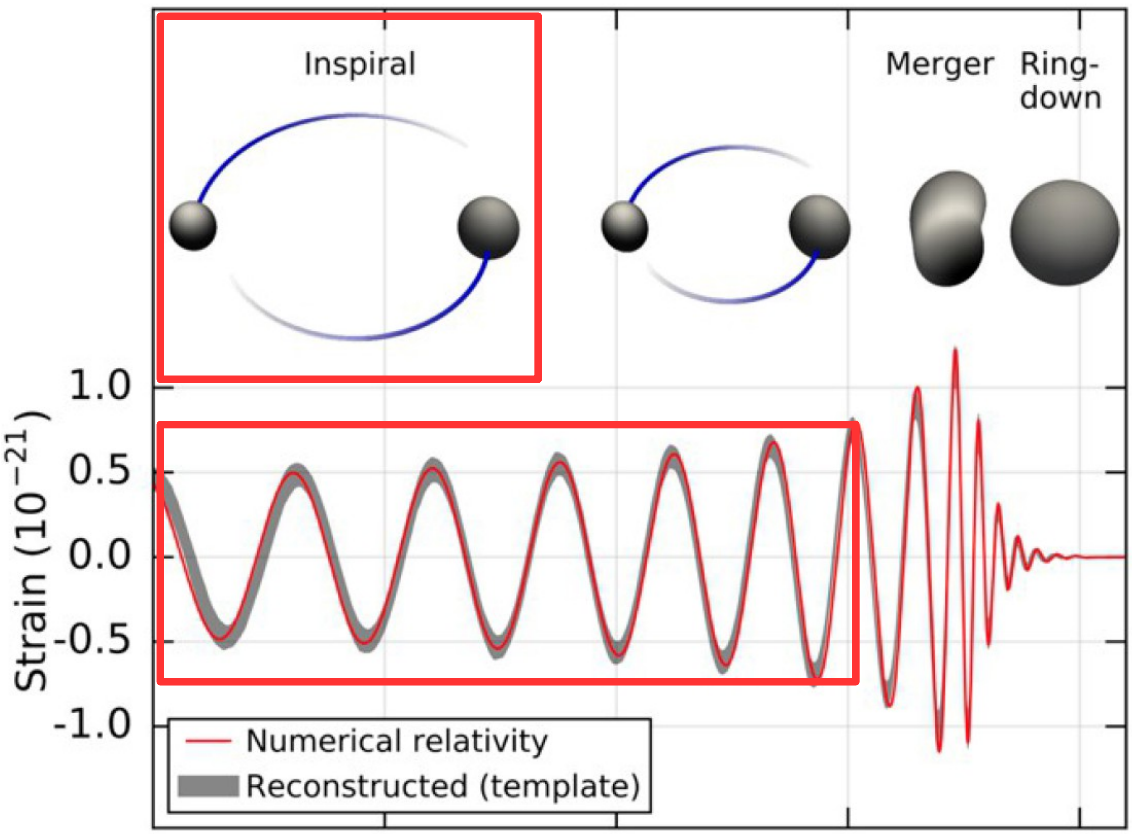
Consistency tests

Inspiral-Merger-Ringdown (IMR)

- Test that the **entire coalescence** does not deviate from **GR predictions**



Phys. Rev. Lett. 116, 221101 (2016)

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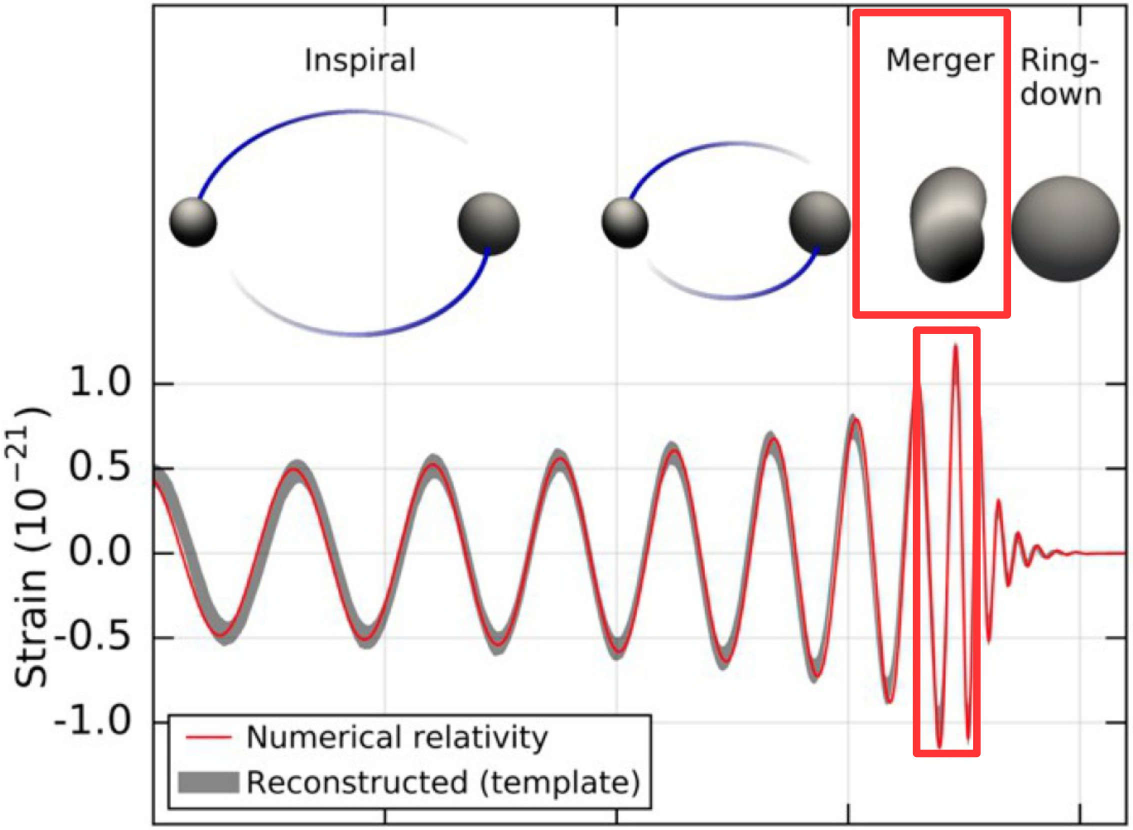

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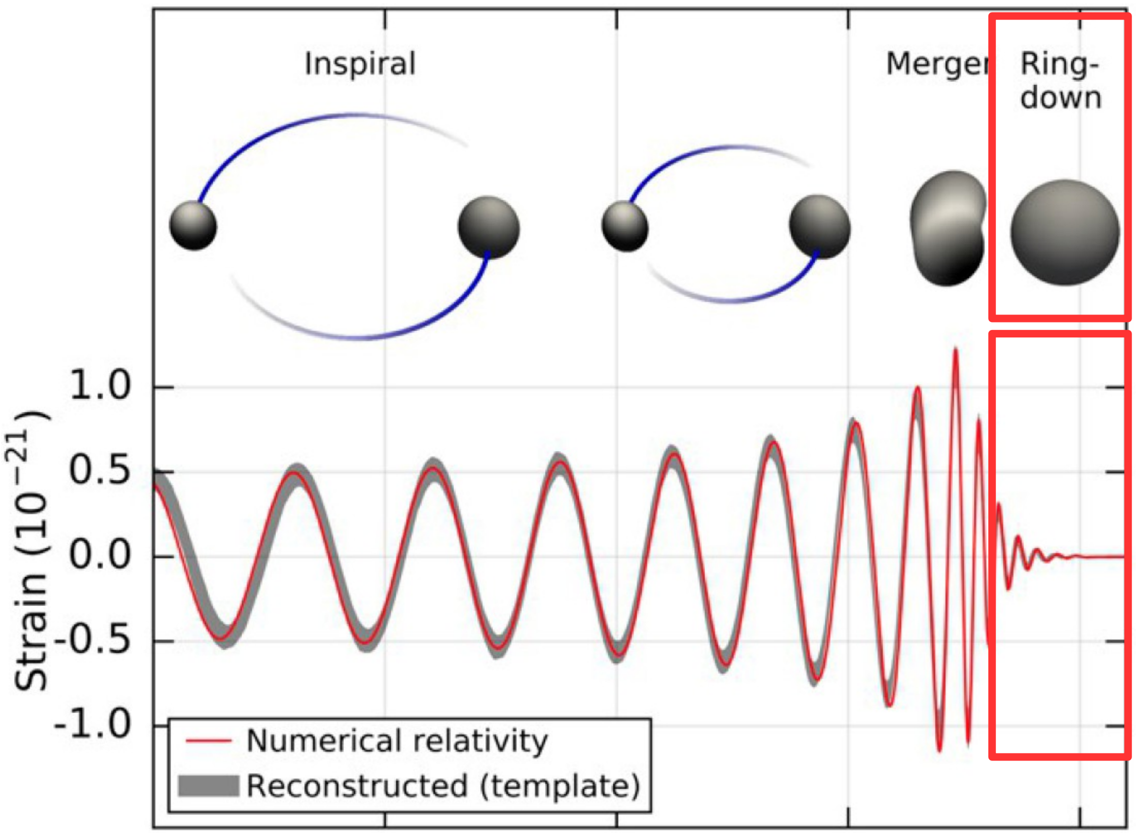

Phys. Rev. Lett. 116, 221101 (2016)

- **Post Newtonian** expansion **(PN)** of the phase in $v/c \sim f^{1/3}$ up to $(v/c)^7$
- **Numerical relativity (NR)** regime
- **Perturbation theory**

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Inspiral-Merger-Ringdown (IMR)

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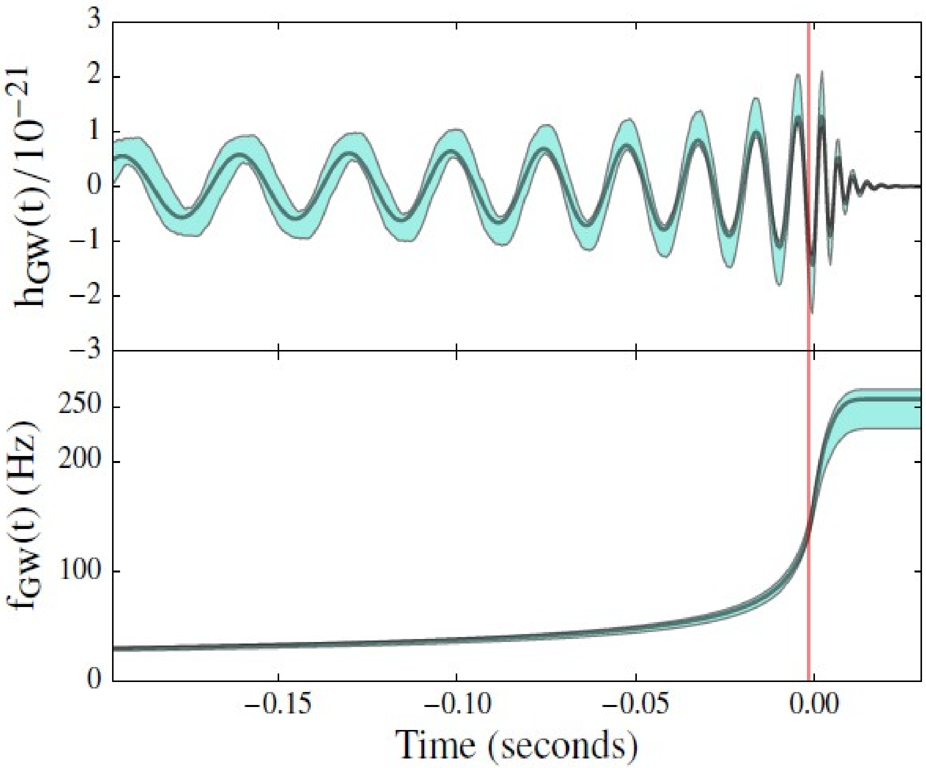

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- Infer m_1, m_2 and a_1, a_2 from the **low frequency** and the **high frequency part** of the waveform (**Kerr ISCO** cutoff)
- Obtain two **independent estimates** for M_f, a_f through **NR fits**

BUT

- **GR** predicts a **unique solution**
- **Estimates** must be **compatible**

Consistency tests

Inspiral-Merger-Ringdown (IMR)

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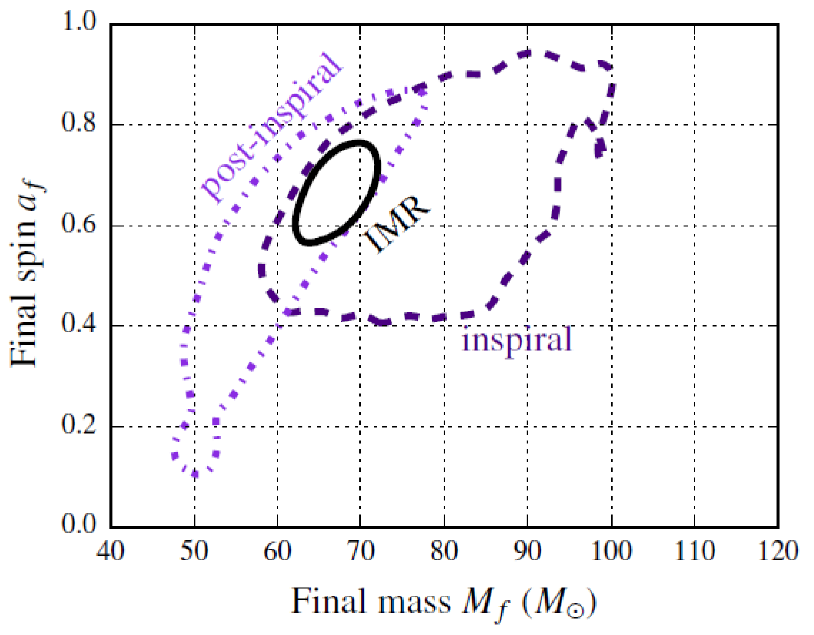
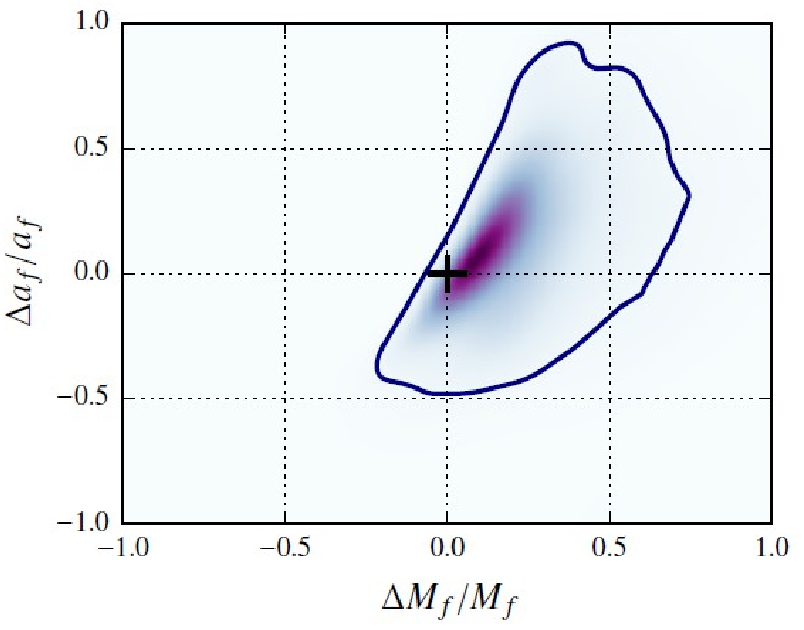
GW150914





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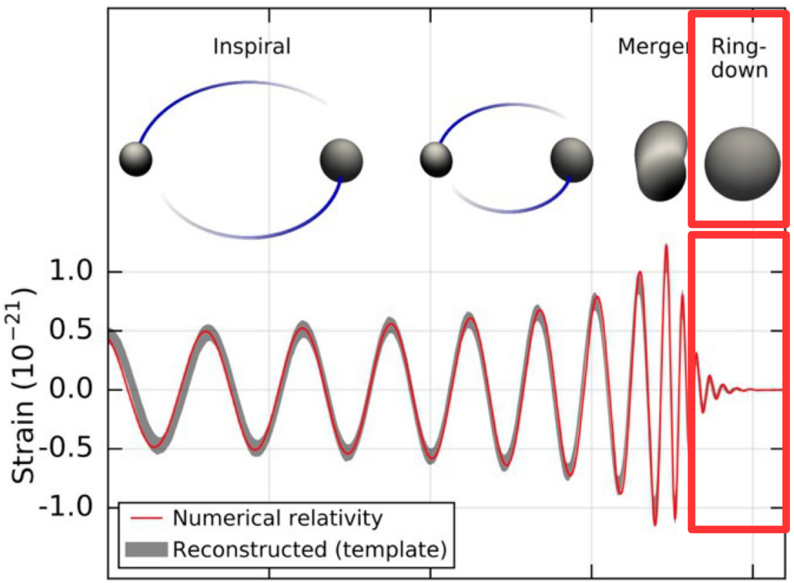
Consistency tests

Ringdown test

- Superposition of **damped sinusoids** when the **linear regime** takes over after the merger



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- $l = m = 2$ (**spherical harmonics** indices) $n = 0$ (**overtone**) mode is the **least damped**

$$h(t \geq t_0) = A\, e^{-(t-t_0)/\tau} \cos\left[2\pi f_0 (t - t_0) + \phi_0\right]$$

Consistency tests

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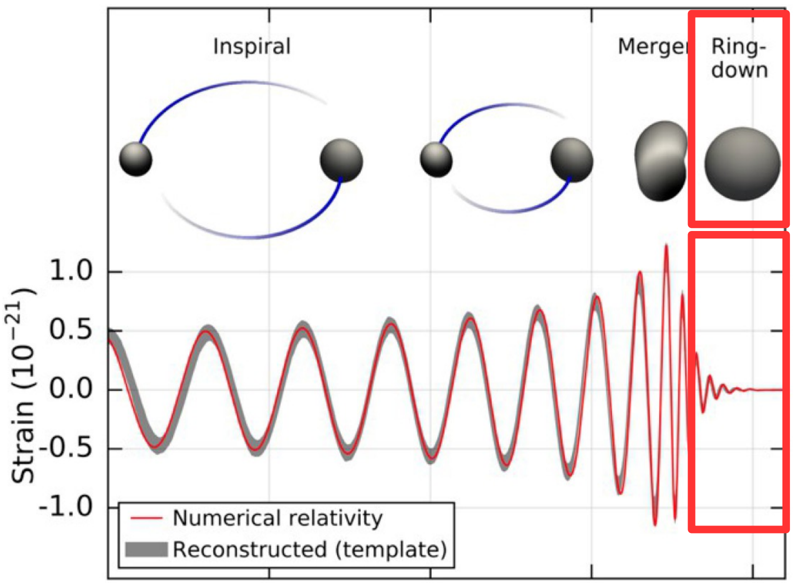

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predicted by the remnant's **mass** and **spin**

Consistency tests

Ringdown test

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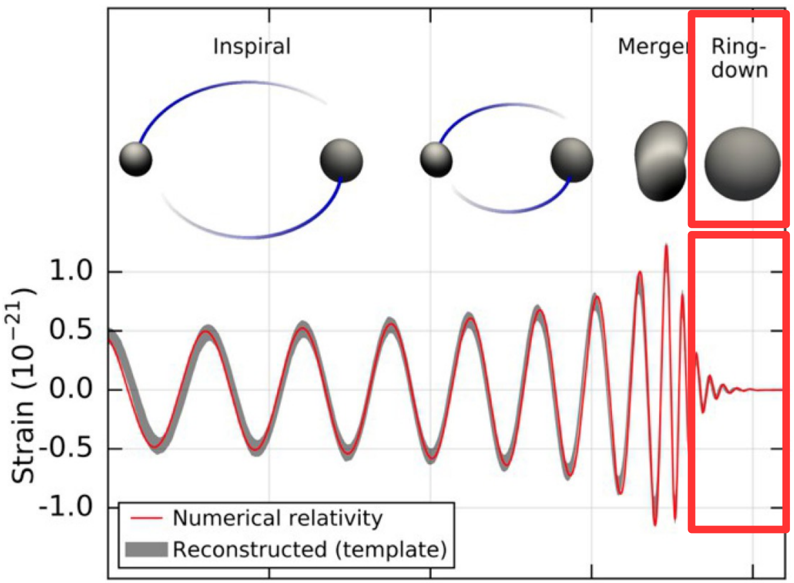

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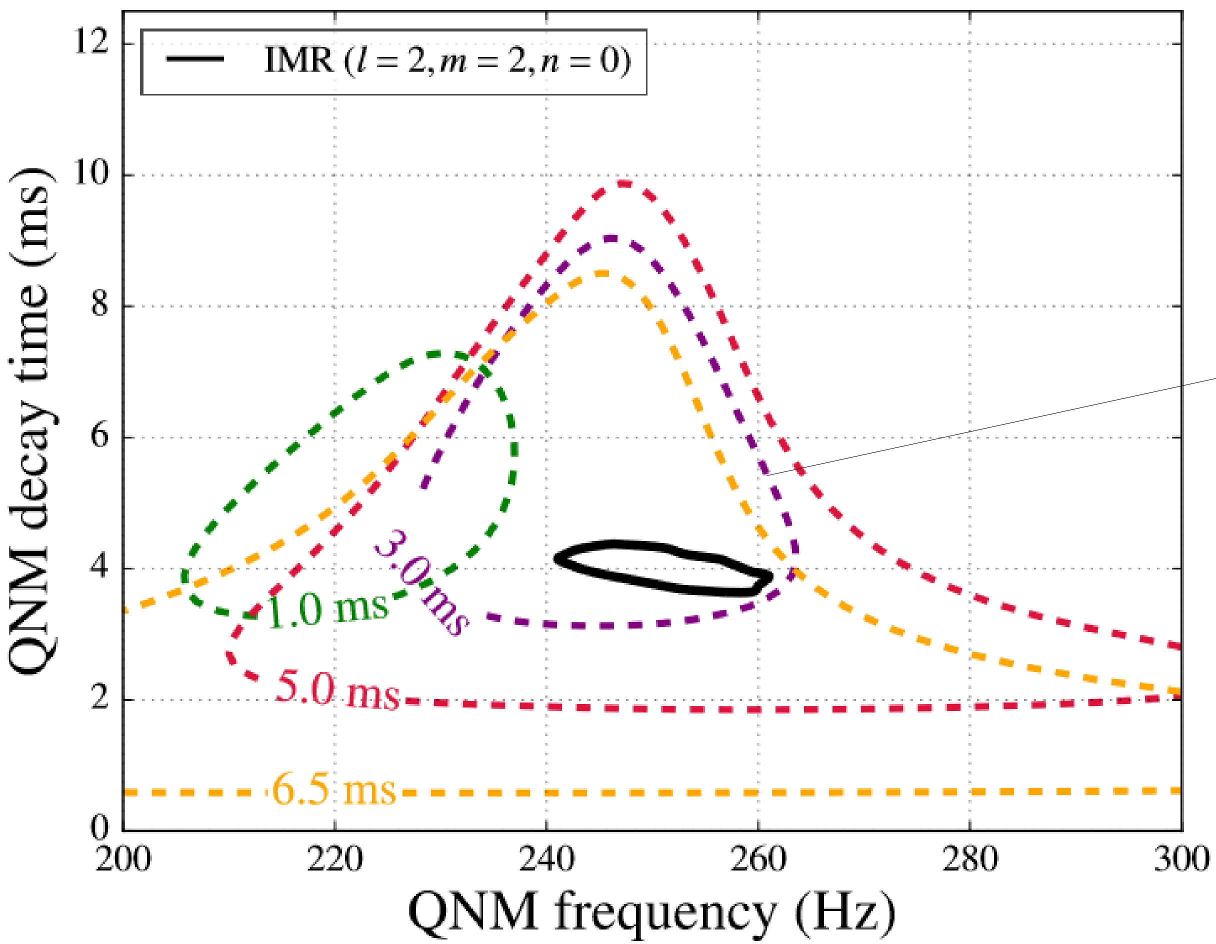
- Measure its **frequency** and **damping time**
- **Compare** with the **GR prediction** given by M_f, a_f inferred from **IMR**

Consistency tests

Ringdown test

GW150914



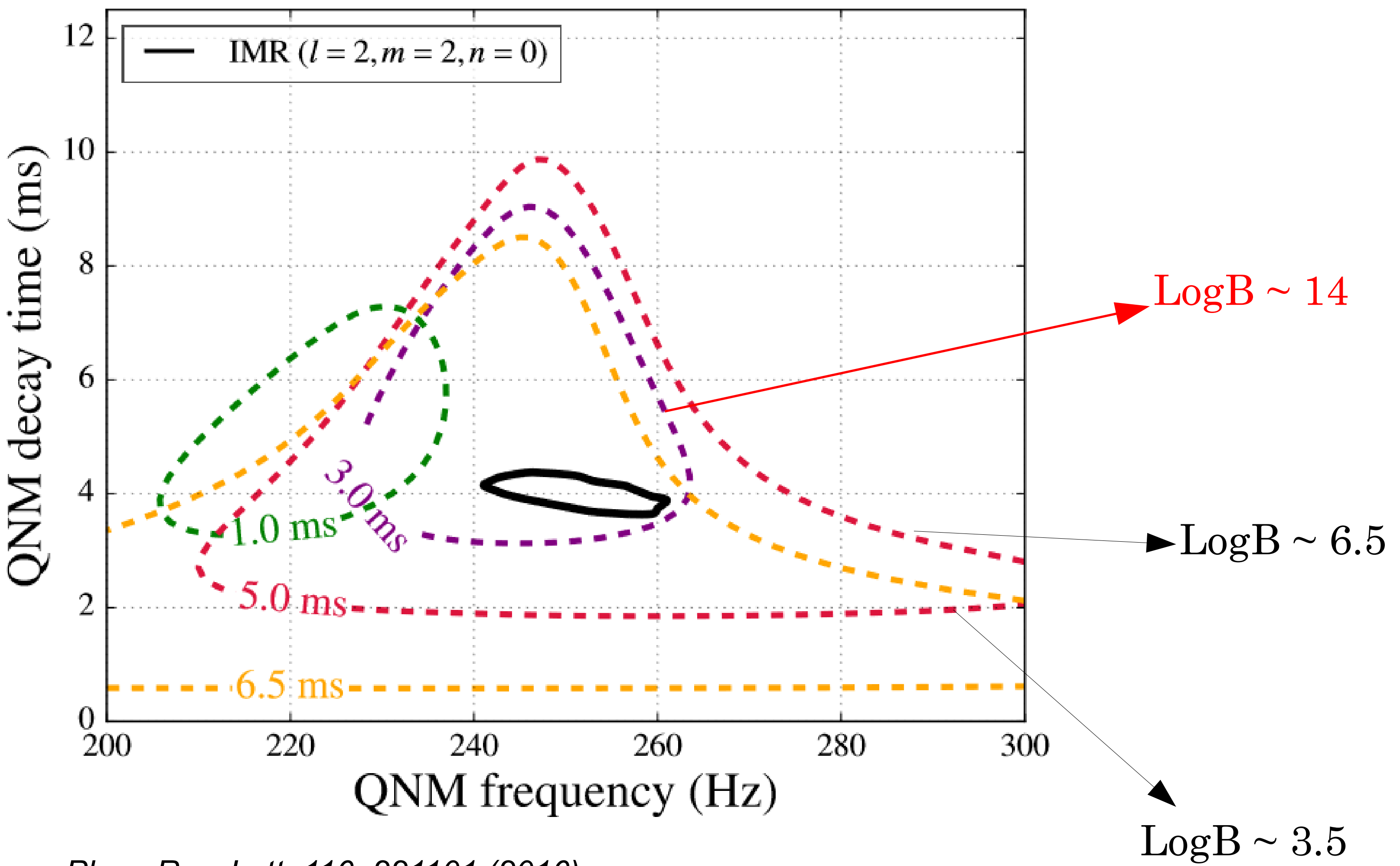
Starts to be consistent after
$3ms \sim 10M$

Phys. Rev. Lett. 116, 221101 (2016)

Consistency tests

Ringdown test

GW150914



Phys. Rev. Lett. 116, 221101 (2016)

Production effects

Test **alternative models** which **change the waveform at** its **generation** (source)

Waveform expansion

- Consider an analytical, **parametric waveform**
- Treat the **GR predicted coefficients as free parameters**
- **Measure** and **compare** with GR predictions

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Post Newtonian (PN) expansion of the inspiral phase + effective terms

- Expansion in $v/c \sim f^{1/3}$, **analytically known** up to $(v/c)^7$ for the **inspiral phase**
- **Effective waveform** for the **other stages**, **calibrated** on NR/EOB

$$\psi_{3.5}(f) = \sum_{i=0}^{7} \varphi_i f^{(i-5)/3} + \varphi_{5l} \ln(f) + \varphi_{7l} f^{1/3} \ln(f) + eff.$$

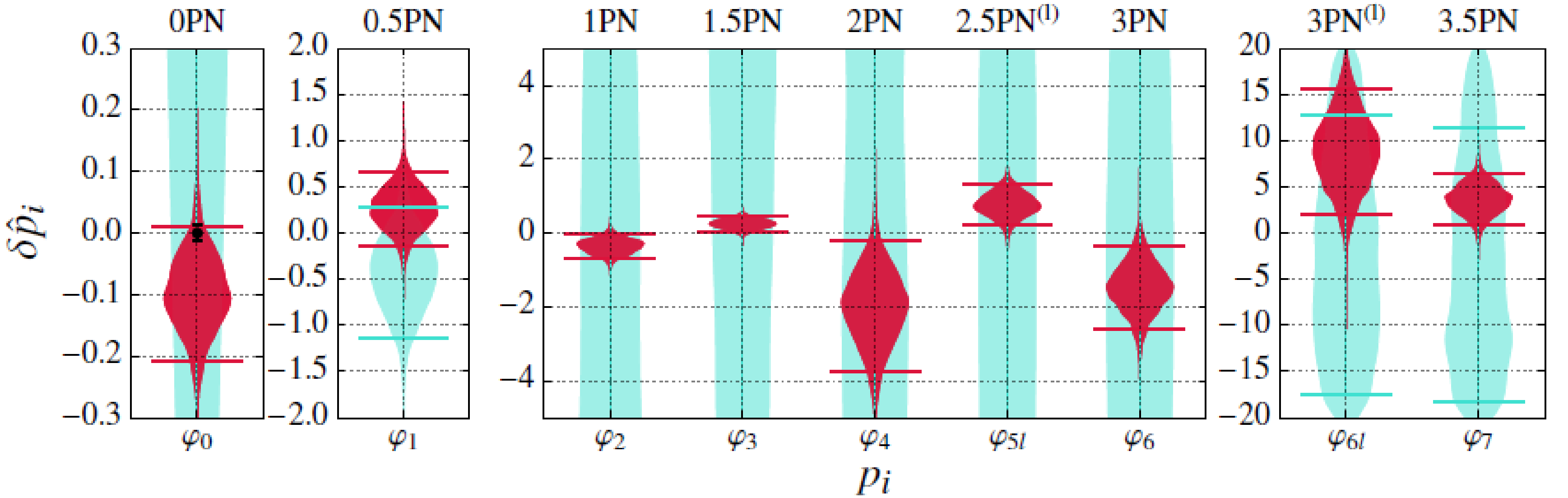
$$\varphi_i(m_1, m_2, a_1, a_2)$$

Production effects

Waveform expansion

- Add a **fractional deviation** and infer its value from **data**
- $\varphi_i \longrightarrow \varphi_i(1 + \delta\hat{\varphi}_i)$

GW150914



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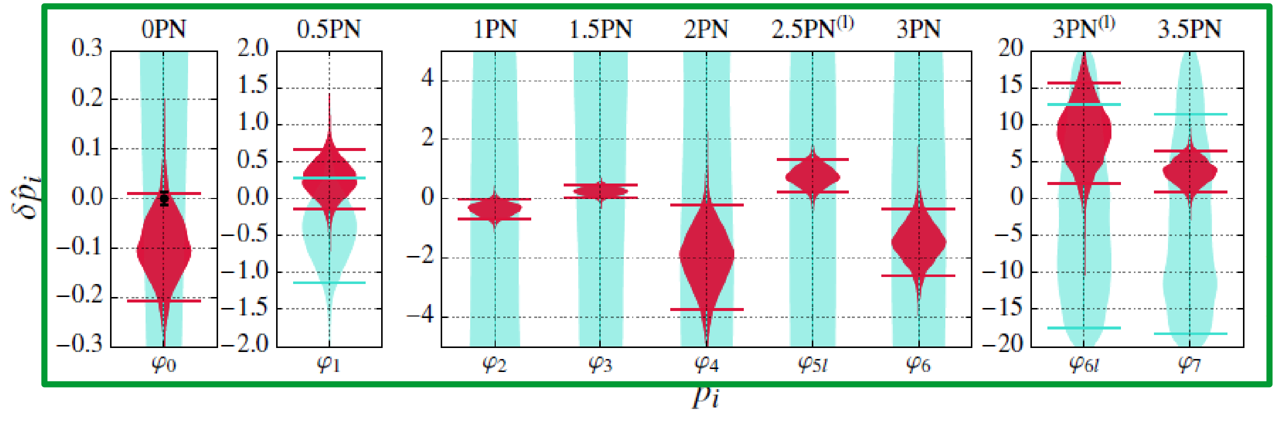

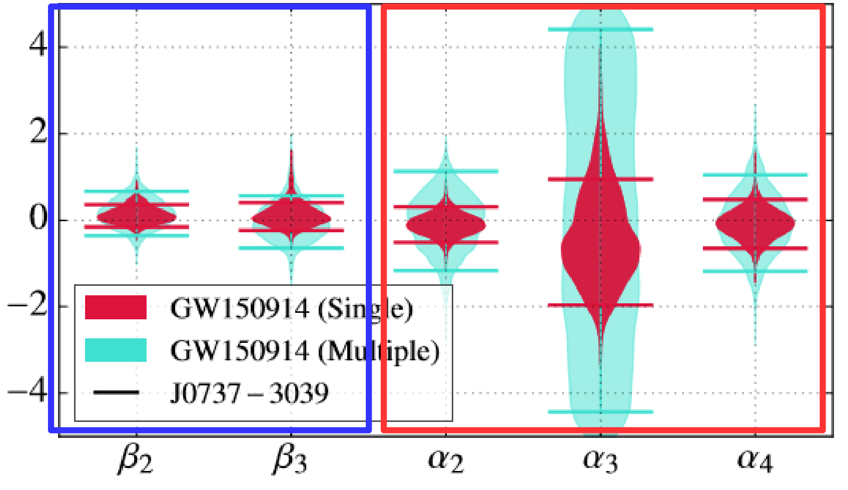

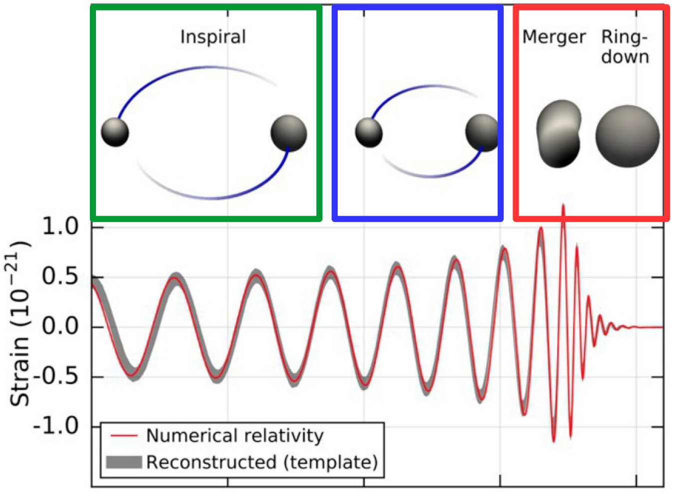

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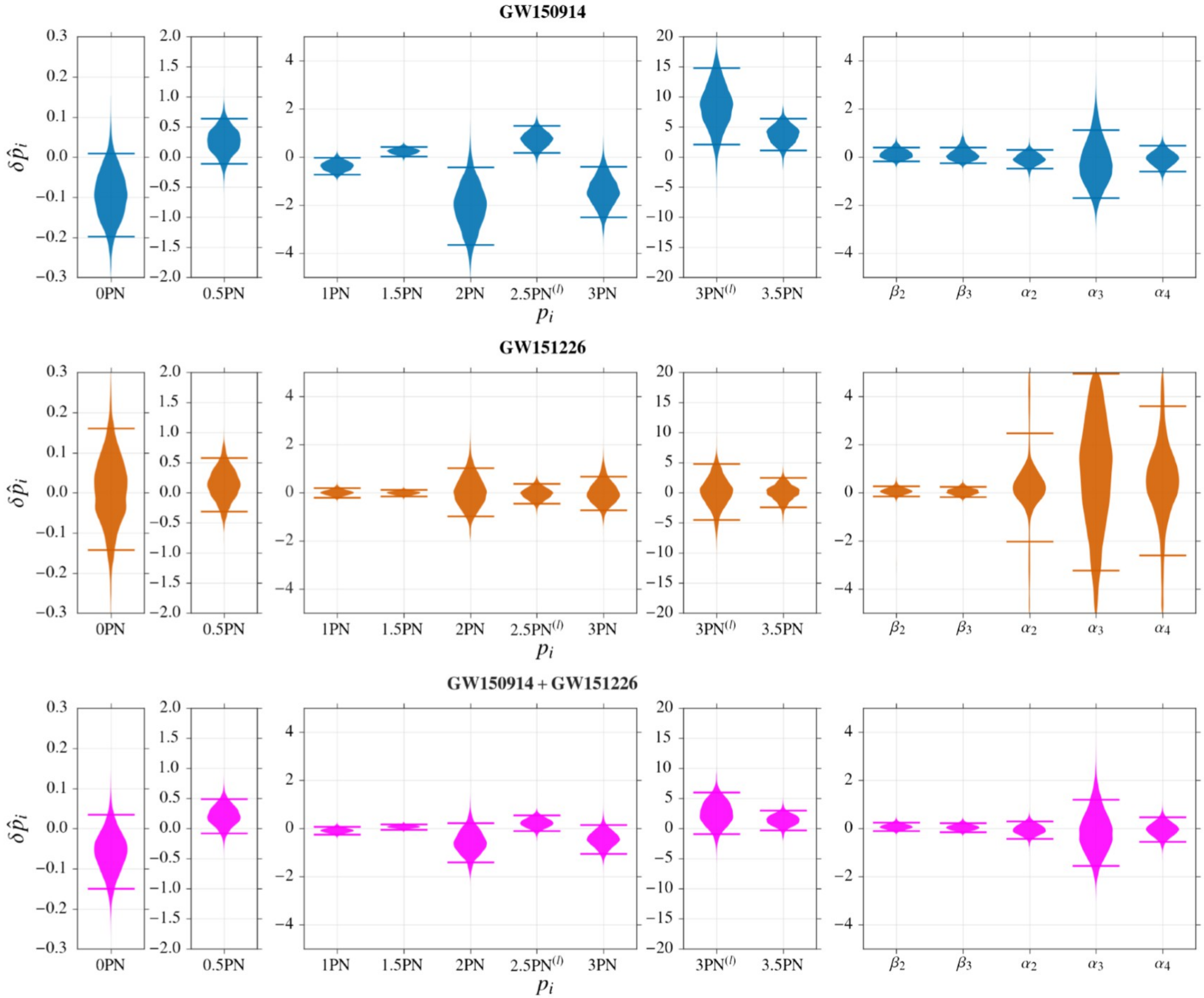
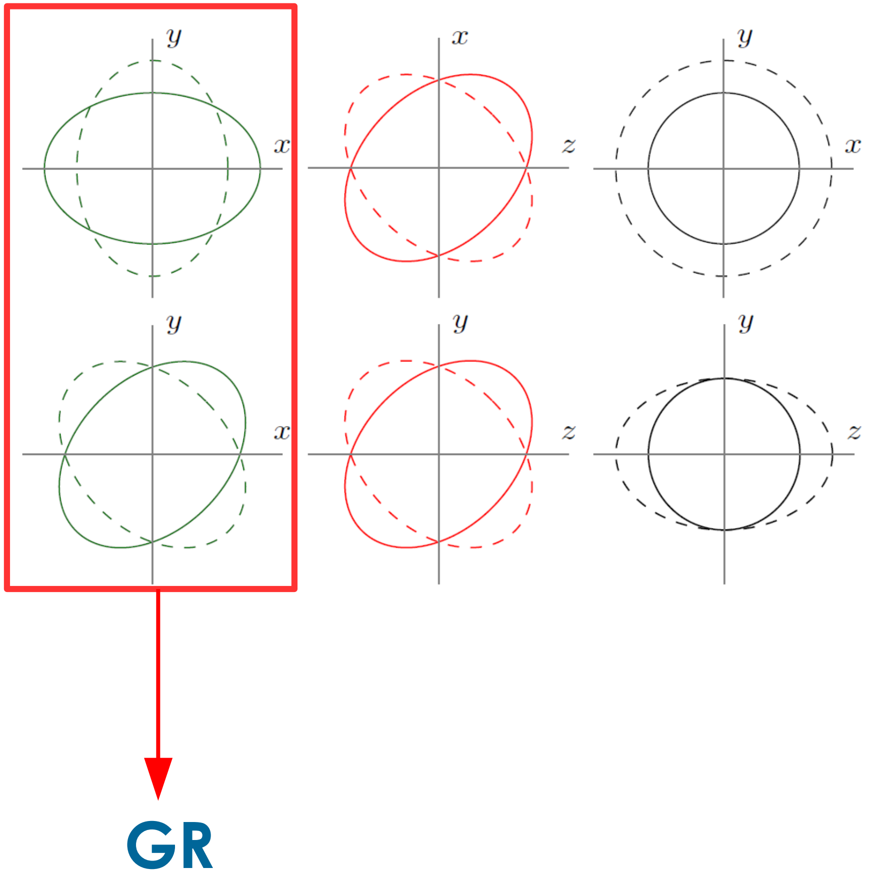


Phys. Rev. X. 4 041015 (2016)

Production effects

Polarisation tests

Polarisations of a **generic theory** of gravity



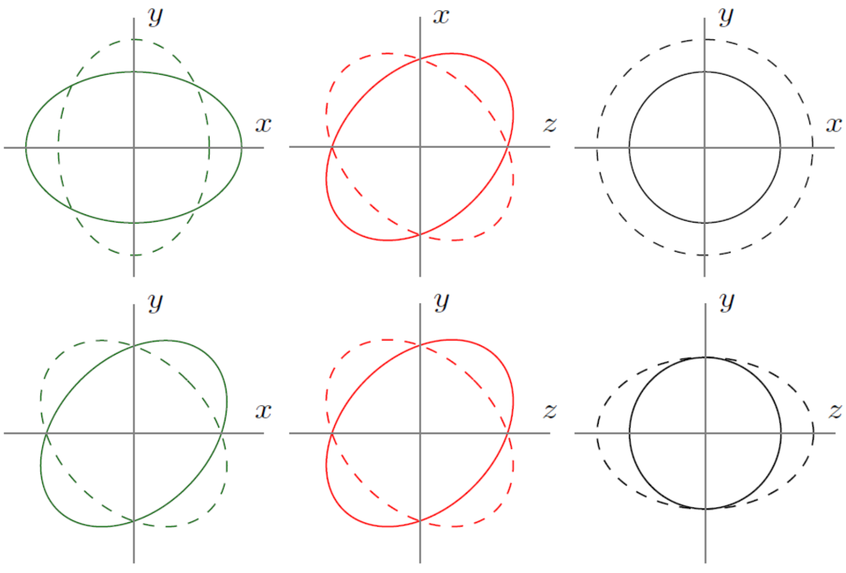
$$h(t,\vec{\theta}) = F^A(\hat{\Omega}) \cdot h_A(t,\vec{\theta})$$

- The polarisation content enters the **projection into the detector**
- Need **more than 3 detectors** to test all the combinations

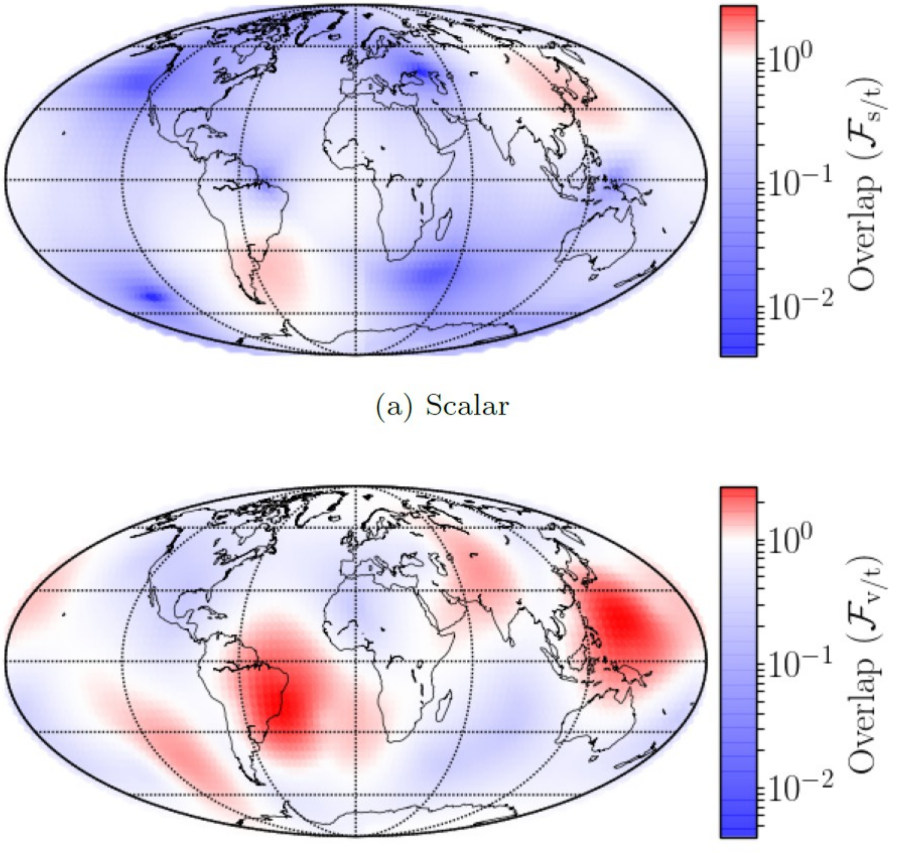
Production effects

Polarisation tests

Polarisations of a **generic theory** of gravity



Sensitivity of the **LIGO/Virgo network**



(a) Scalar

(b) Vector

Isi&Weinstein, arXiv:1710.03794v1

Production effects

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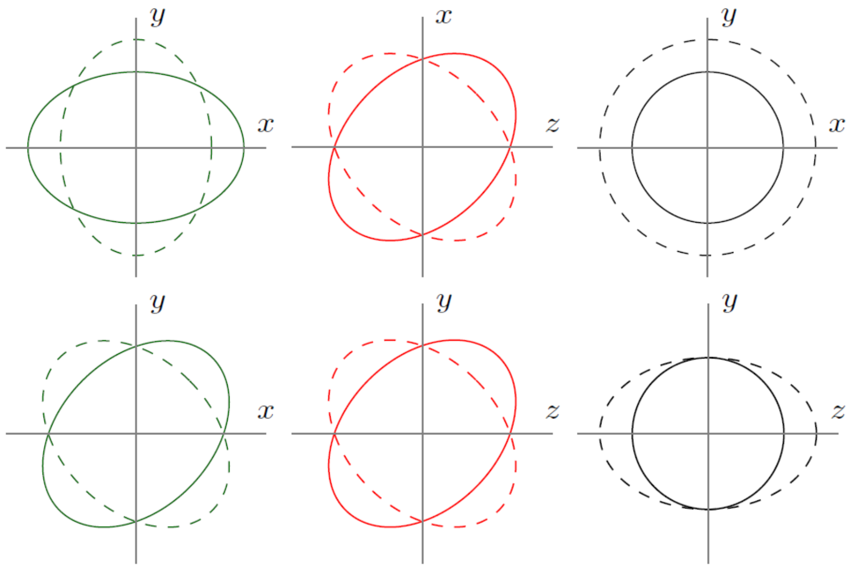


	T/V	T/S
Bayes factor	200	1000

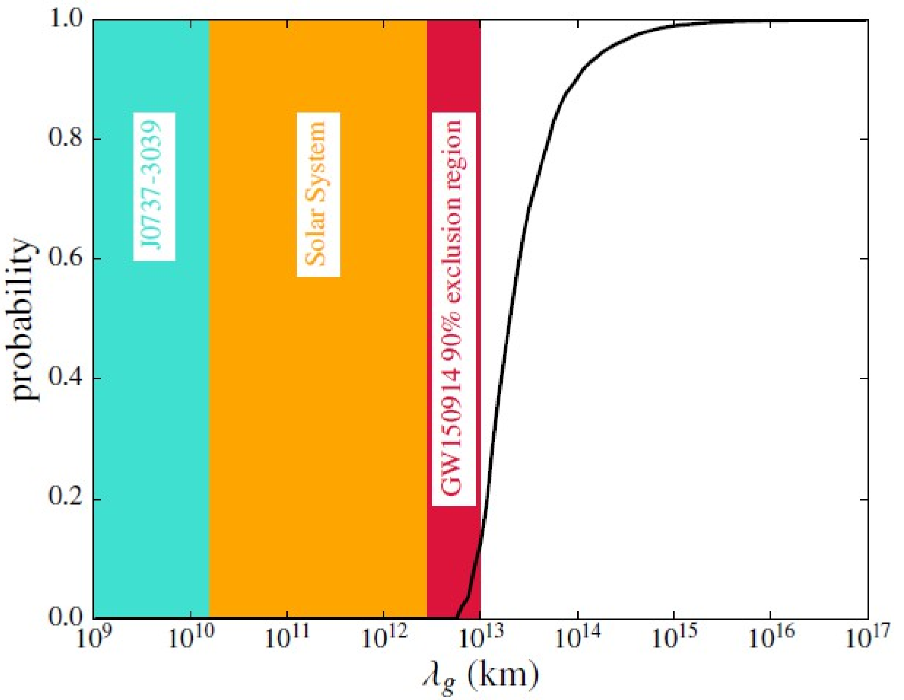
NO vector/scalar polarisations in GW170814

Propagation effects

Modified dispersion relation

- Consider a **massive graviton**: $E^2 = p^2c^2 + m_g^2c^4$ *(Will, 2014)*
- The **group velocity** gets modified by $v_g^2/c^2 = 1 - h^2c^2/(\lambda_g^2E^2)$, $\lambda_g = h/(m_g c)$
- Which **adds a 1PN term** to the **phase** of the waveform: $\Phi_m = -(\pi D c)/\left[\lambda_g^2(1+z)f\right]$

GW150914



Phys. Rev. Lett. 116, 221101 (2016)

GW150914

$$\lambda_g > 10^{13} km$$

$$m_g < 1.2 \cdot 10^{-22} eV/c^2$$

GW170817

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\rm EM}} \leqslant +7 \times 10^{-16}$$

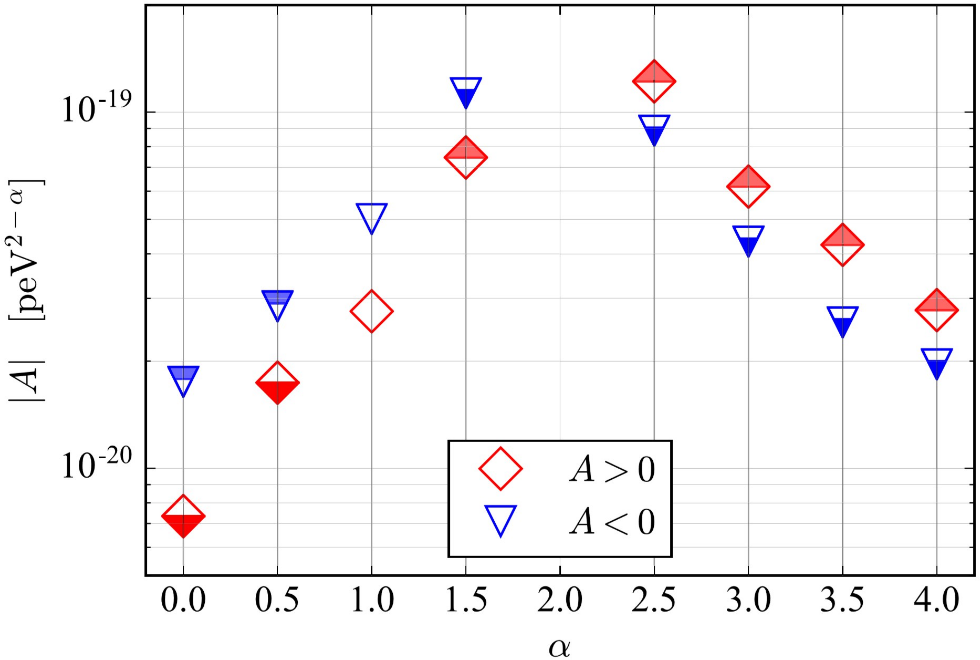
Propagation effects

More generally

- $E^2 = p^2c^2 + Ap^\alpha c^\alpha,\ \alpha \geq 0$
- $v_g/c \simeq 1 + (\alpha - 1)AE^{\alpha-2}/2$

Massive graviton	Multifractal spacetime	Doubly special relativity	Extra dimensions
$\alpha = 0,\ A > 0$	$\alpha = 2.5$	$\alpha = 3$	$\alpha = 4$

GW150914 + GW151226 + GW170104



- **Weeker consraints** than **photon/neutrino** observations
- Some theories predict violations in just **one sector**
- **First bound** in the **gravitational sector**

Phys. Rev. Lett. 118, 221101 (2017)

Propagation effects

Higher dimensions

- A **leakage** of GWs in **higher dimensions** cause them to travel for a **greater distance** w.r.t. electromagnetic waves

$$h \propto \frac{1}{d_L^{\mathrm{GW}}} = \frac{1}{d_L^{\mathrm{EM}}} \left[1 + \left(\frac{d_L^{\mathrm{EM}}}{R_c} \right)^n \right]^{-(D-4)/(2n)}$$

Propagation effects

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Propagation effects

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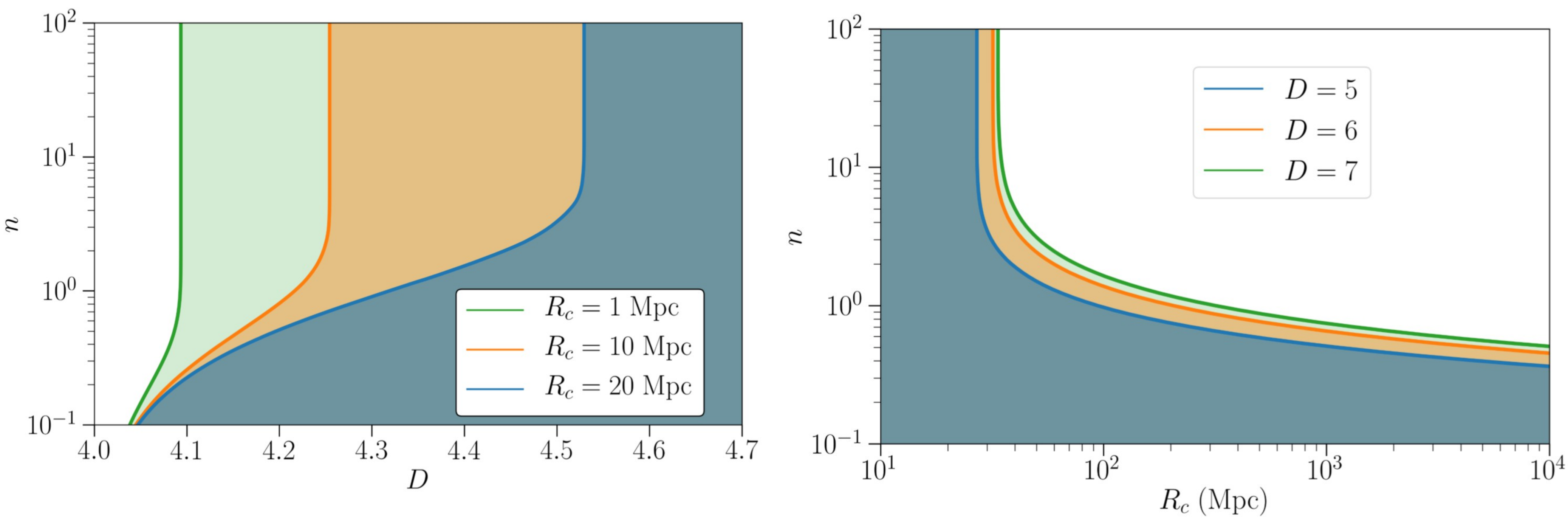
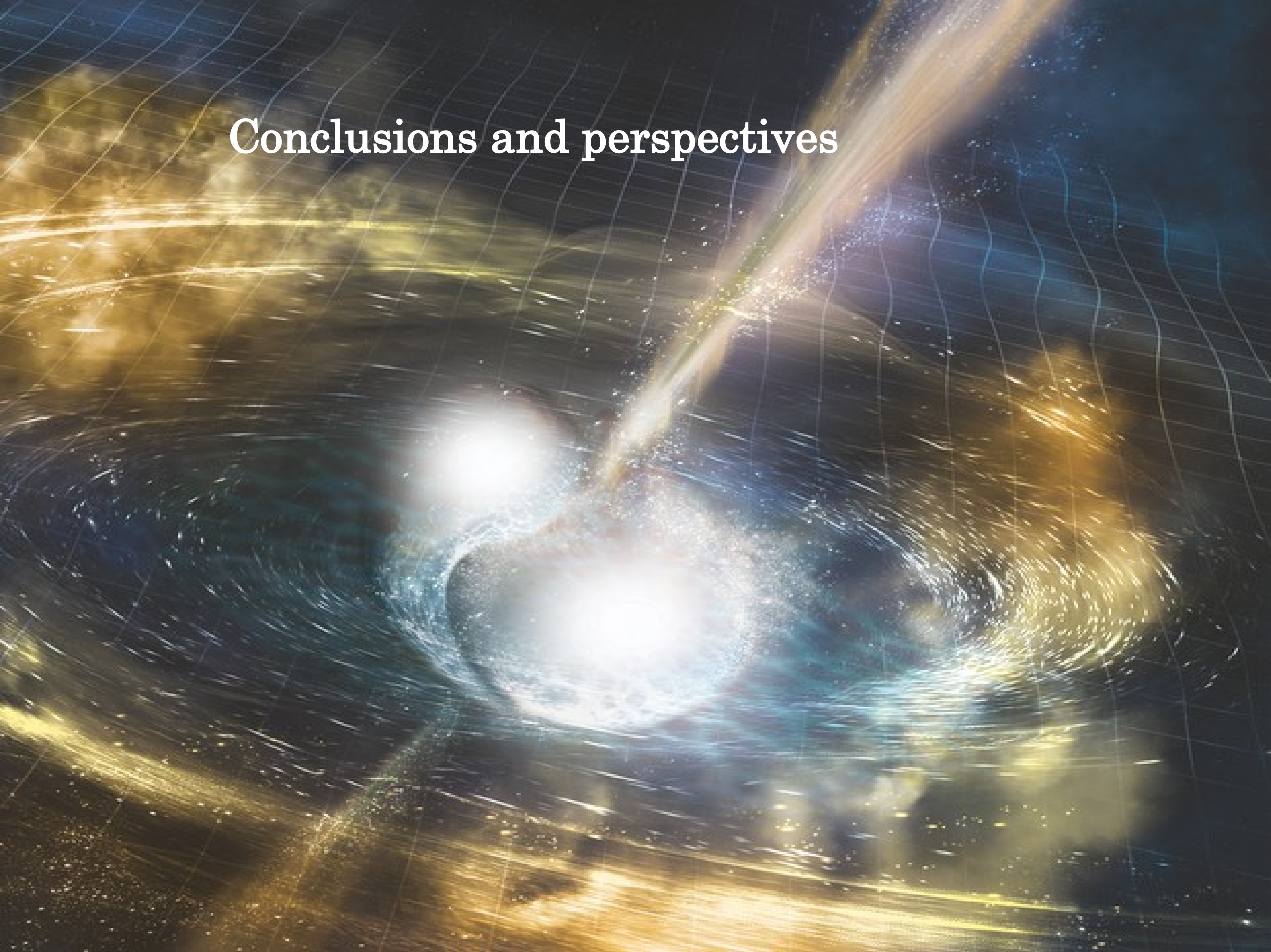
GW170817: a binary neutron star signal



Abbott et al., arXiv 1811.00364v2

Conclusions and perspectives



Conclusions and perspectives

- General Relativity tested in the **strong field**, **non linear regime** thanks to **BBHs** and **BNSs**
- **Generation** and **consistency** of the signal **as expected** from Einstein's theory
  - The **power** accumulates as predicted (**residuals** compatible with **noise**)
  - **Consistency** of the whole **IMR** waveform
  - **Ringdown** regime **consistent** with GR for BBHs
  - Ony **tensor degrees of freedom**
- **Propagation** of gravitational waves as expected. **For now** compatible with:
  - **Massless** graviton
  - **Light speed** propagation
  - **4 dimensions**

Conclusions and perspectives

….as many more detections are expected in the upcoming months

- As **statistics** accumulates, **posteriors** will be **combined** and **higher precision** will be reached in the tests presented
- **Higher SNR** signals will allow for more **refined tests** (multimodal ringdown tests, spin interactions, polarisations etc.)
- ?

Far future

- Possible detection of a **stochastic background**
  - The **cosmological background** is a window on **Planck scale energies** and on the **early universe**, where an **extended theory of gravity** could come into play