Proton-nucleus elastic scattering model

Overview and purpose of the work

The goal of this work is to create a ***simple nucleus-nucleus elastic scattering model***, suitable for ***FRED***, a GPU-accelerated fast-Monte Carlo code for rapid treatment plan recalculation in charged particles therapy.

WE STARTED WITH A SIMPLER CASE STUDY…

PROTON-NUCLEUS ELASTIC SCATTERING

Based on:

J. Ranft's parameterisation of differential elastic cross-section for proton-nucleus collision (J. Ranft, "Estimation of radiation problems around high energy accelerators using calculations of the hadronic cascade in matter," Part. Accel. 3, 129–161 (1972))

Using first: python

And then converting all in: C++

Suitable for FRED

J.Ranft parameterisation

Compute the differential elastic cross-section of a proton-nucleus collision as a function of ***projectile energy*** and ***scattering angle*** **Χ** in Center-of-Mass frame.

$$A < 62 \qquad \frac{d\sigma_{el}}{d\Omega} = A^{1.63} e^{14.5A^{0.66}t} + 1.4A^{0.33}e^{10t} \qquad (mb/Ster)$$

$$A \geq 62 \qquad \frac{d\sigma_{el}}{d\Omega} = A^{1.33} e^{60A^{0.33}t} + 0.4A^{0.40}e^{10t} \qquad (mb/Ster)$$

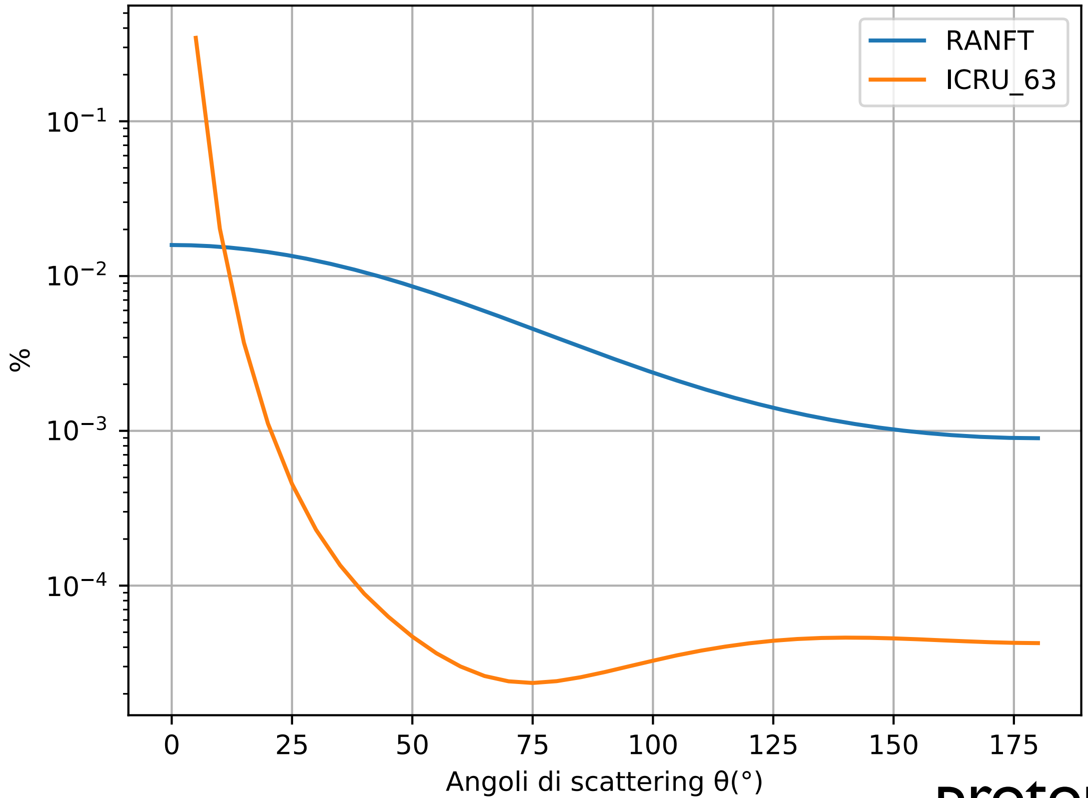
With: $\{\ t = -2p^2(1 - cos\chi)\ \}$ invariant momentum transfer in $(GeV/c)^2$

Where:

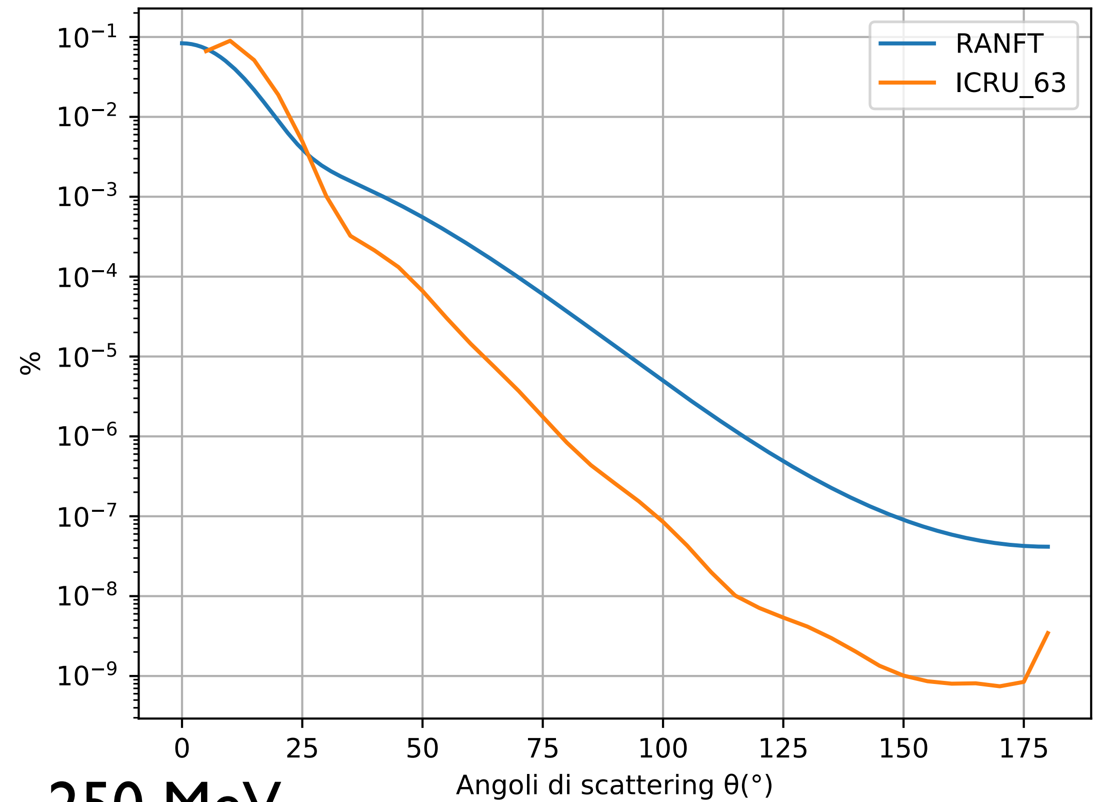
- **p** is momentum of the scattered proton in GeV/c
- **A** is the mass number of the target particle
- **χ** is the scattering angle in Center-of-Mass frame

Comparisons with ICRU-63 tables

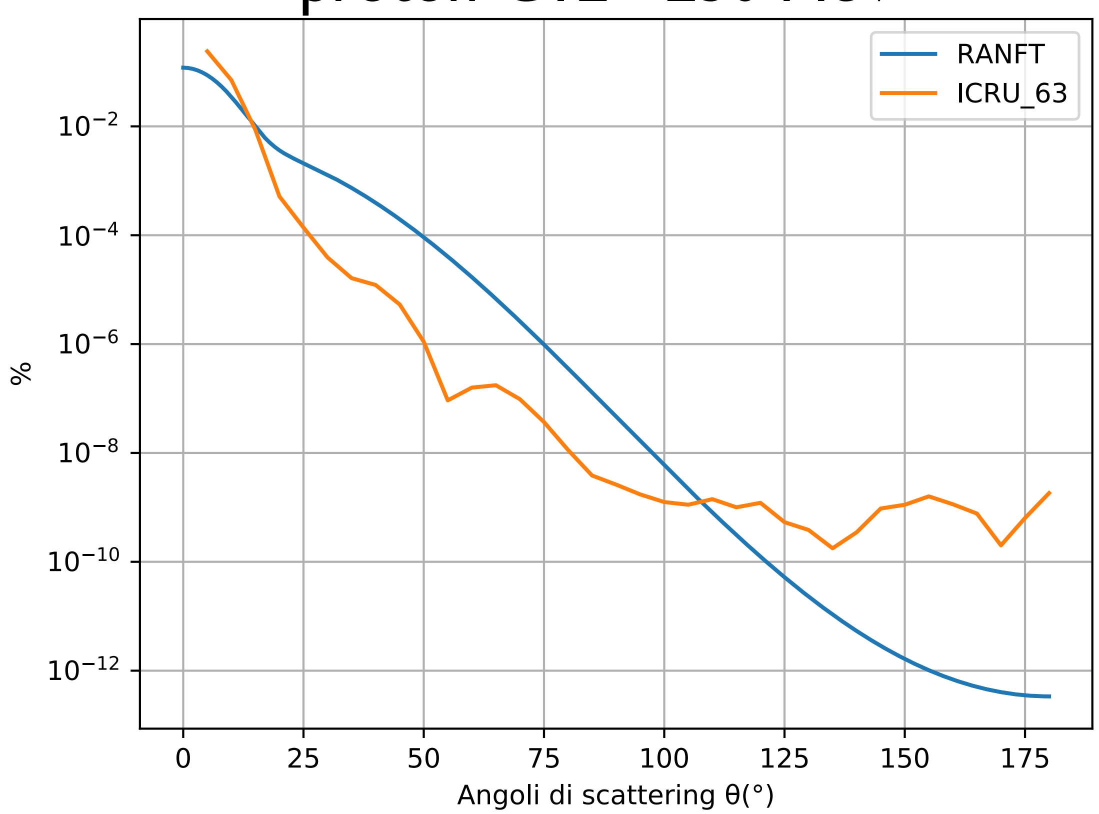
proton-C12 - 5 MeV



proton-C12 - 120 MeV



proton-C12 - 250 MeV



??? WHY DO WE CONSIDER CROSS-SECTION σ? ???

$$\textit{cross} - \textit{section} \propto \textit{scattering angle } \chi \textit{ probability}$$

So, normalizing the differential elastic cross-section computed by Ranft's parameterisation we can obtain the probability density function ***p*(X)*d*X** from which we can ***extract a scattering angle***

Extraction scattering angle χ

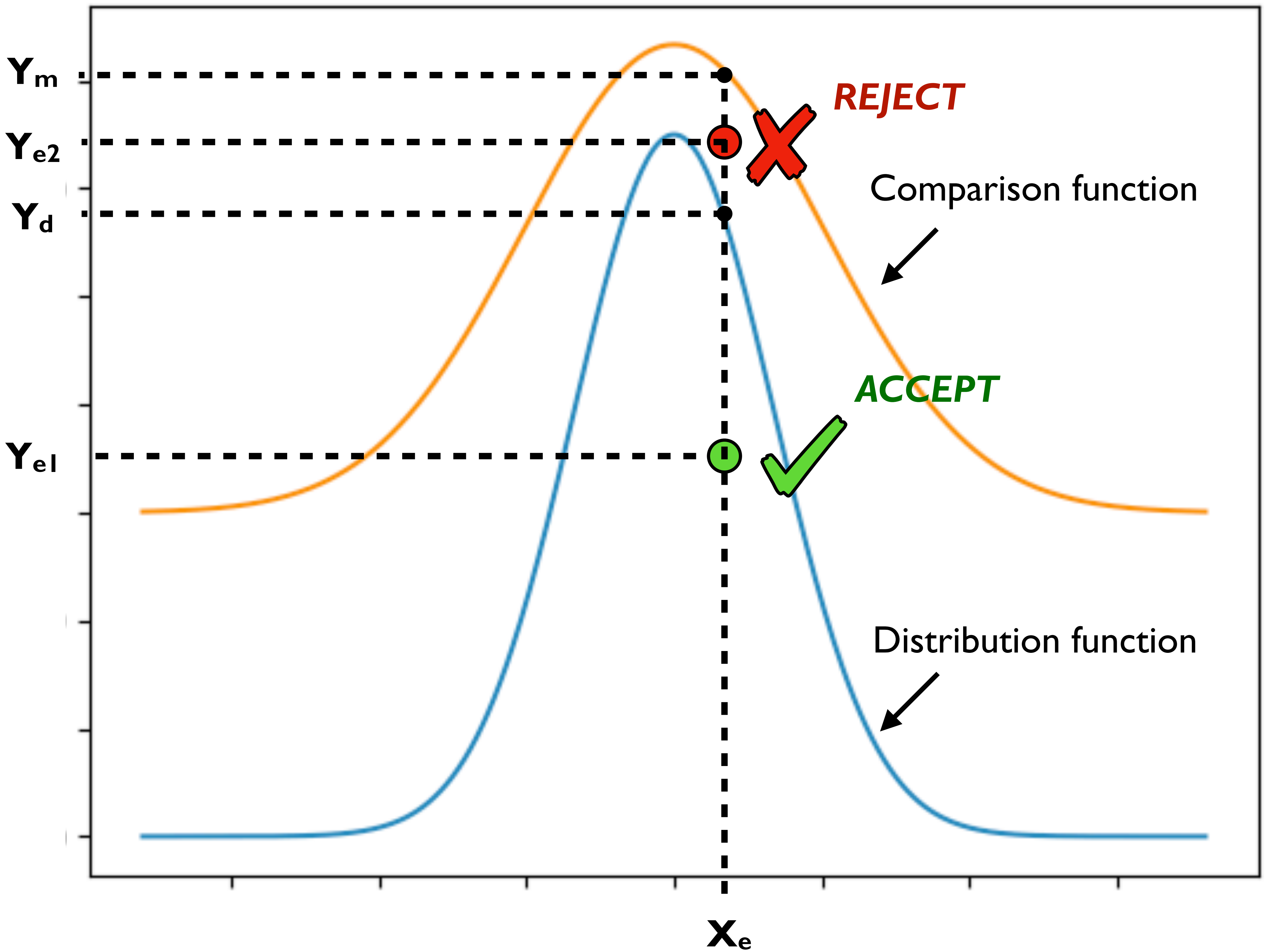
Scattering angle χ allow us to solve collision kinematics because it determines completely the diffusion process in C-o-M frame and, consequently, in any other reference system. Final energies of the two particles E'_1 and E'_2 and scattering angles θ_1 and θ_2 in the Laboratory frame can be found as a ***function of* scattering angle χ**.

??? *HOW TO GENERATE CORRECTLY THE SCATTERING ANGLE X?* **???**

$\Downarrow$

REJECTION METHOD (HIT-OR-MISS)

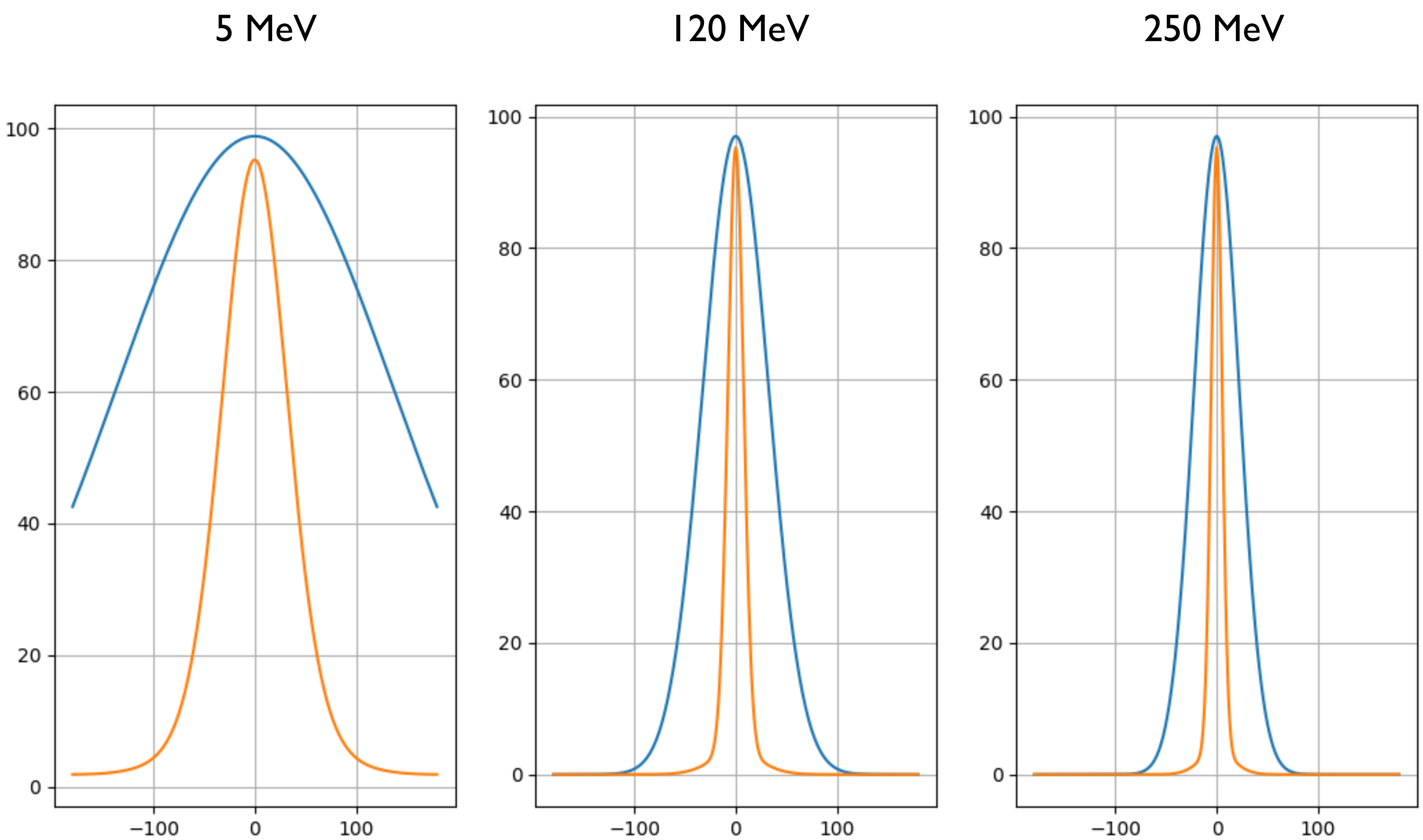
HIT-OR-MISS



Ym
Ye2
Yd
Ye1
REJECT
Comparison function
ACCEPT
Distribution function
Xe

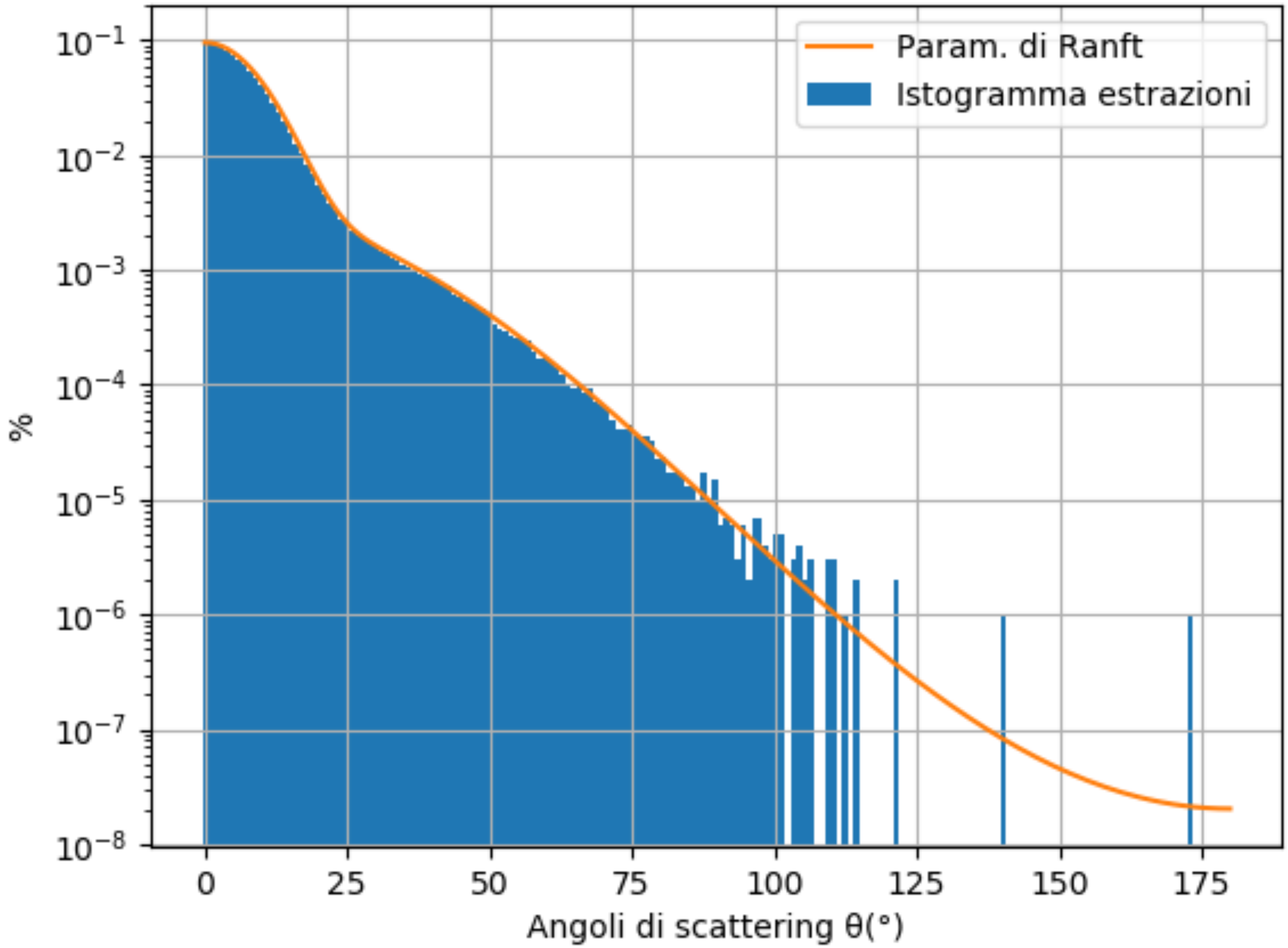
Comparison functions optimization

Example proton-Oxygen16



Hit-or-Miss example

Hit-or-Miss for proton-Oxygen16 collision at 120 MeV -1e06 extractions



Elastic collision kinematics

An elastic collision between two bodies is a process in which both ***conservation of momentum*** and ***conservation of kinetic energy*** are observed.

$$K_1 + K_2 = K_1' + K_2'$$

$$\overrightarrow{p_1} + \overrightarrow{p_2} = \overrightarrow{p_1'} + \overrightarrow{p_2'}$$

By definition of an **elastic** collision the masses of the two bodies are invariant, so the ***total energy is conserved,*** not only kinetic energy of the system.

$$K_1 + m_1c^2 + K_2 + m_2c^2 = K_1' + m_1c^2 + K_2' + m_2c^2 = E_1 + E_2 = E_1' + E_2'$$

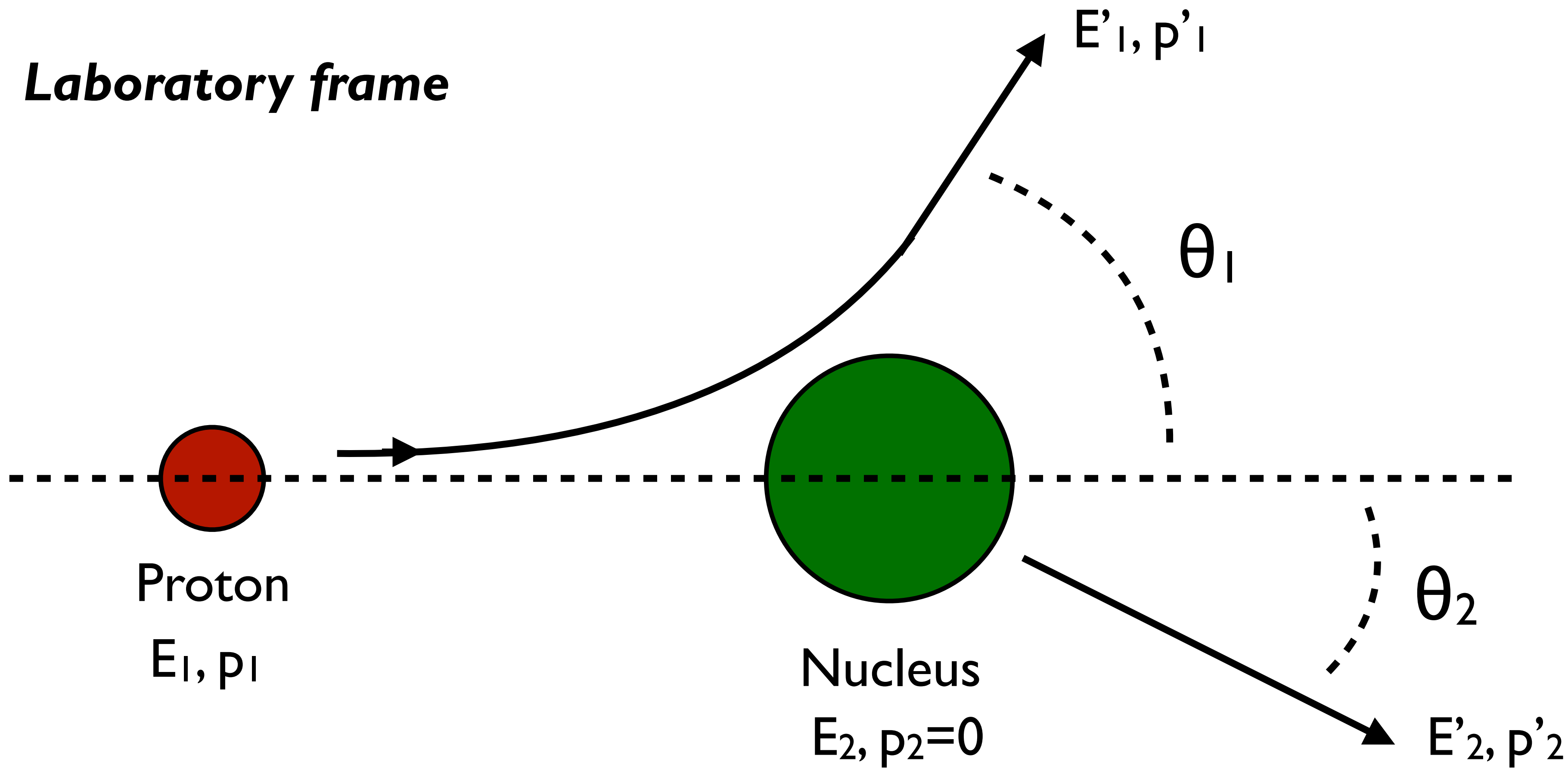
Total energy conservation
+
Total momentum conservation

FOUR-MOMENTUM CONSERVATION

$$\boxed{P_1 + P_2 = P_1' + P_2'}$$

Where:

$$\boxed{P = [E/c, \overrightarrow{p}] = [m\gamma c, m\gamma v_x, m\gamma v_y, m\gamma v_z]}$$



Scattering angles in laboratory frame θ_1 and θ_2:

$$cos\theta_1 = \frac{E_1'(E_1 + m_2) - E_1 m_2 - m_1^2}{p_1 p_1'}$$

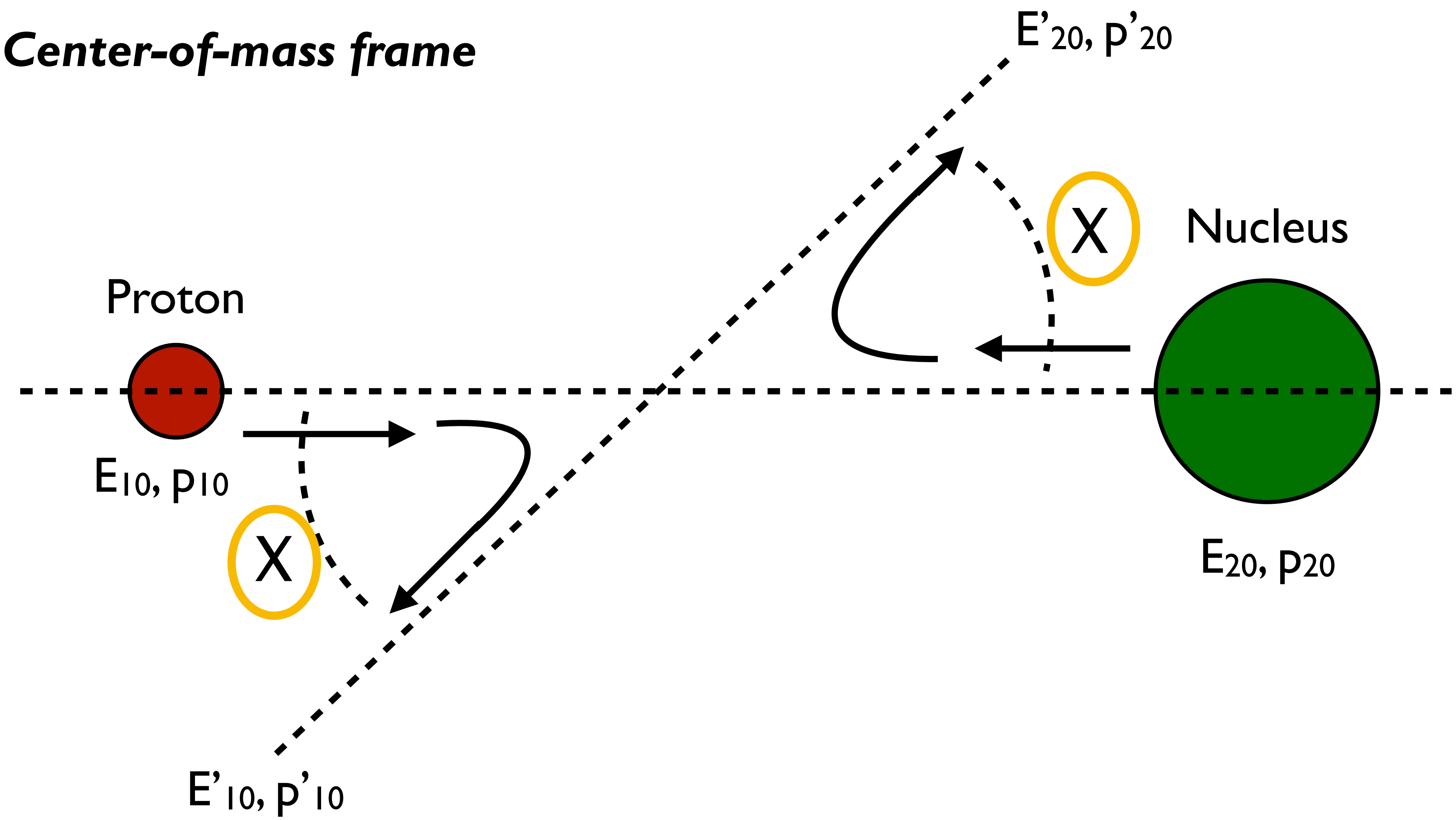
$$cos\theta_2 = \frac{(E_1 + m_2)(E_2' - m_2)}{p_1 p_2'}$$

Where:

$$E_1' = E_1 - \frac{m_2(E_1^2 - m_1^2)}{(m_1^2 + m_2^2 + 2E_1 m_2)}(1 - cos\chi)$$

$$E_2' = E_2 - \frac{m_2(E_1^2 - m_1^2)}{(m_1^2 + m_2^2 + 2E_1 m_2)}(1 - cos\chi)$$

Center-of-mass frame



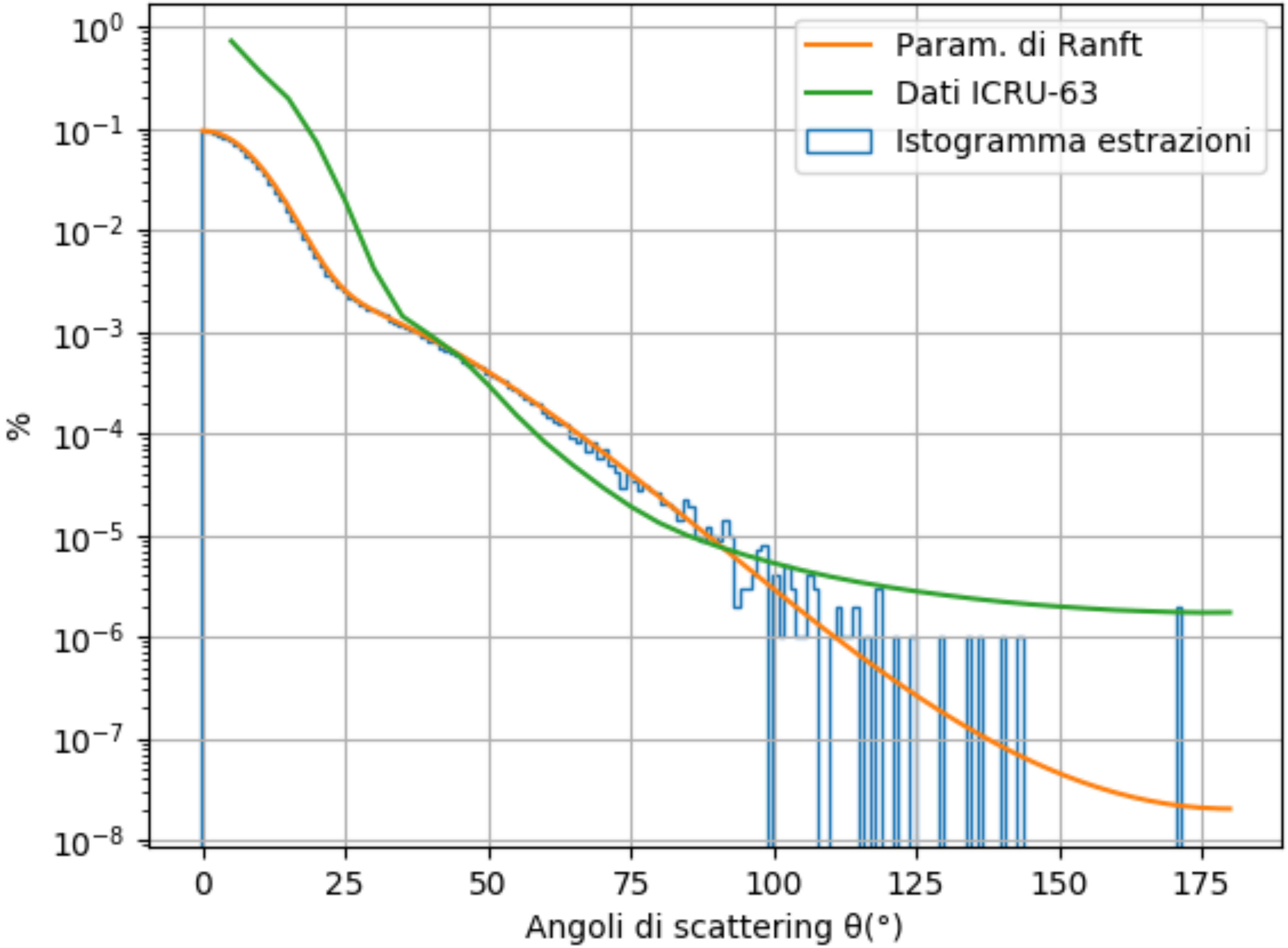
In C-o-M frame, after collision, particles change their direction, while momenta keep to have same modulus with opposite direction; moreover, the energy of each particle doesn't change in the collision and the total momentum is null. In this reference system the ***scattering angles χ are the same.***

$$\overrightarrow{p_{tot}} = 0 \Rightarrow \overrightarrow{p_{10}} + \overrightarrow{p_{20}} = \overrightarrow{p'_{10}} + \overrightarrow{p'_{20}} = 0 \qquad E_{20} = E'_{20}$$

$$|\overrightarrow{p_{10}}| = |\overrightarrow{p'_{10}}| \quad : \quad \overrightarrow{p_{10}} = -\overrightarrow{p_{20}} = \overrightarrow{p_0} \qquad E_{10} = E'_{10}$$

$$|\overrightarrow{p_{20}}| = |\overrightarrow{p'_{20}}| \quad : \quad \overrightarrow{p'_{10}} = -\overrightarrow{p'_{20}} = \overrightarrow{p_0} \qquad E_{10} + E_{20} = E'_{10} + E'_{20}$$

Proton-Oxygen16 collision at 120 MeV



Next steps

- ✦ Modify Ranft's parameterisation in order to have a ***better agreement with ICRU tables***, also making it more accurate and usable for a larger range of energy.

- ✦ Find a model for a ***nucleus-nucleus elastic collision*** that allows to expand this work to several projectile particles, in particular ***Carbon ions***, which are increasingly used for ***Hadron Therapy.***

- ✦ Implement this elastic scattering model in the fast-Monte Carlo FRED, which is currently missing a nuclear elastic scattering model for Carbon ions, in order to allow a ***rapid treatment plan recalculation for Carbon beams*** too.