Discussion session: benchmark of different formalisms

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3DSPIN
MAPPING THE PROTON IN 3D

Resummation formalisms

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{q_T-\text{res.}}{\underset{\text{PB}}{\propto}} e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right]$$

$$\overset{\text{TMD}}{\propto} H \times F_1 \times F_2 \qquad + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

$$\overset{\text{SCET}}{\propto} H \times B_1 \times B_2 \times S$$

- Dictionary:

$$\mathcal{H} = H C_1 C_2$$

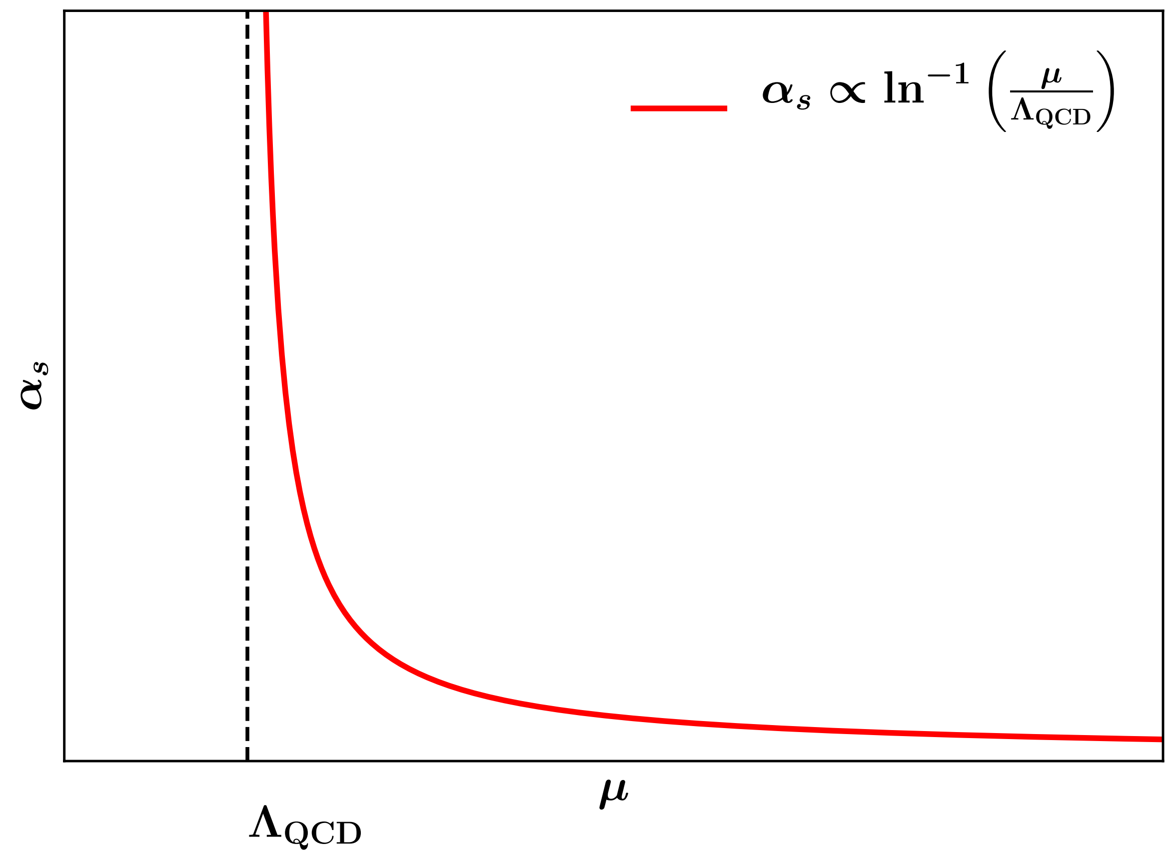
$$F_i = e^S C_i \otimes f_i$$

$$F_i = \sqrt{S} \times B_i$$

- All **equivalent** for factorising processes such as Drell-Yan.

Non-perturbative region

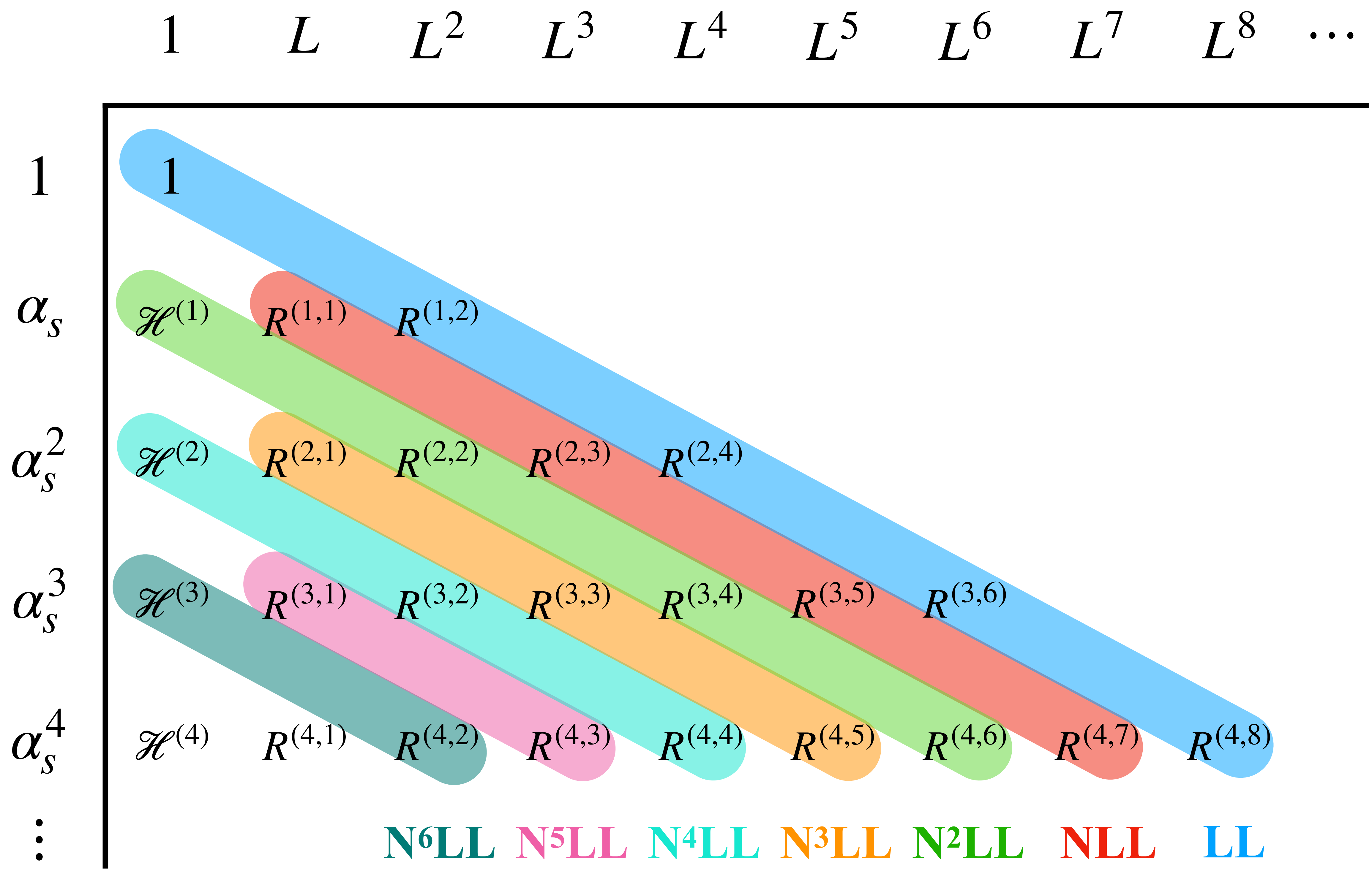
$$\sigma \propto \int_0^\infty db_T\, \alpha_s^p\left(\frac{1}{b_T}\right)\cdots \sim \int_0^\infty dk_T\, \alpha_s^p\left(k_T\right)\ldots$$



- **Prescriptions** to avoid integrating over the **Landau pole**:
  - minimal prescription,
  - ***b**** or ***k*****T*** prescription.
- Non-perturbative effects are thus ***intrinsically present***:
  - whether large or small depends on the experimental/theoretical uncertainties.

Logarithmic counting (1)

$$\frac{d\sigma}{dq_T} \propto \left(1 + \sum_{m=1}^{\infty} \alpha_s^m \mathcal{H}^{(m)}\right)\left(1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{2n} R^{(n,k)} L^k\right) \qquad \alpha_S L^2 \sim 1$$



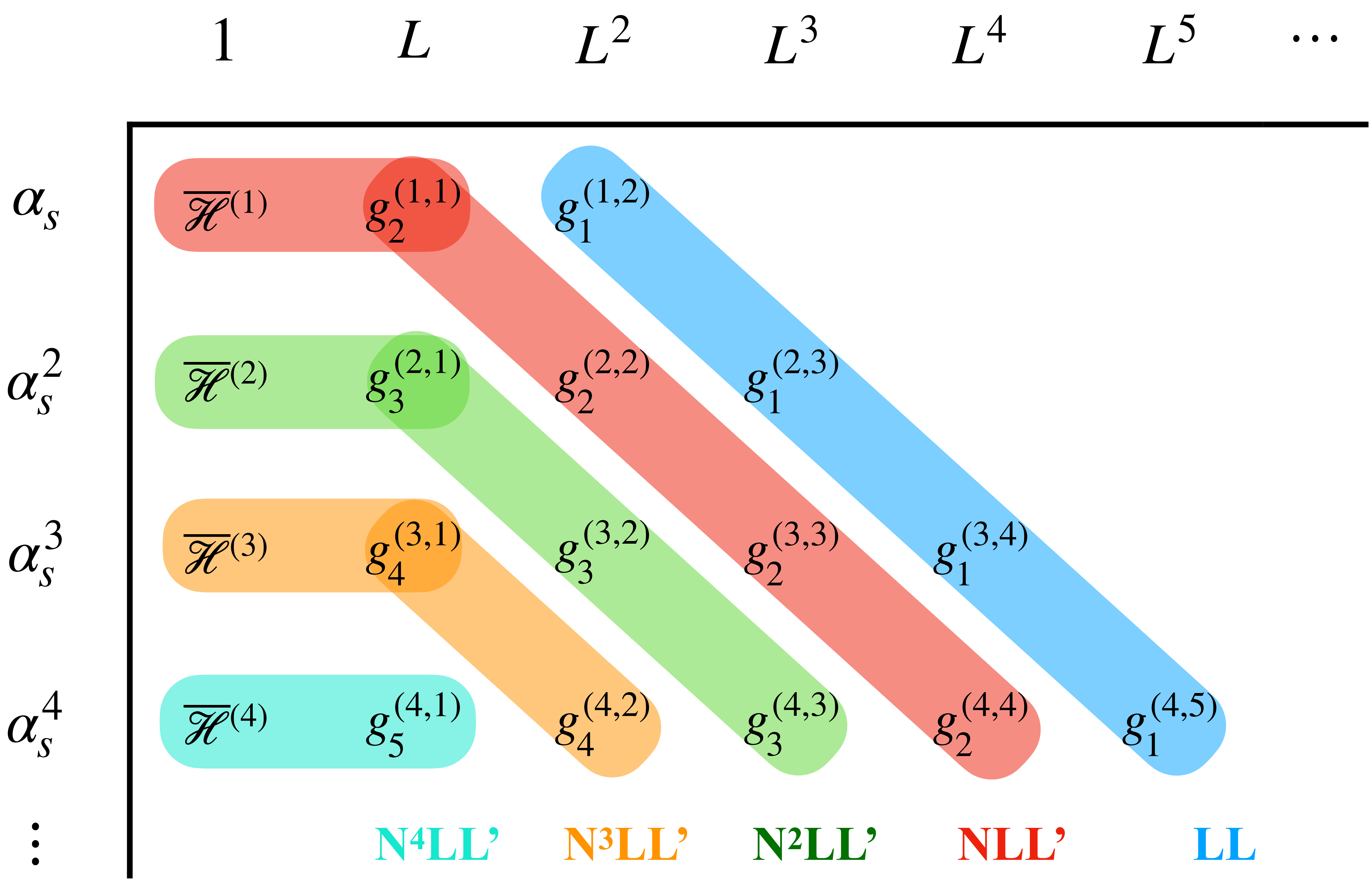
Logarithmic counting (2)

$$\ln\left(\frac{d\sigma}{dq_T}\right) \propto \ln\left(1+\sum_{m=1}^{\infty}\alpha_s^m \mathcal{H}^{(m)}\right)+\sum_{n=1}^{\infty}\alpha_s^n \sum_{k=1}^{n+1} g^{(n,k)} L^k \qquad \alpha_S L \sim 1$$

	1	L	L^2	L^3	L^4	L^5	$\cdots$
α_s	$\overline{\mathscr{H}}^{(1)}$	$g_2^{(1,1)}$	$g_1^{(1,2)}$				
α_s^2	$\overline{\mathscr{H}}^{(2)}$	$g_3^{(2,1)}$	$g_2^{(2,2)}$	$g_1^{(2,3)}$			
α_s^3	$\overline{\mathscr{H}}^{(3)}$	$g_4^{(3,1)}$	$g_3^{(3,2)}$	$g_2^{(3,3)}$	$g_1^{(3,4)}$		
α_s^4	$\overline{\mathscr{H}}^{(4)}$	$g_5^{(4,1)}$	$g_4^{(4,2)}$	$g_3^{(4,3)}$	$g_2^{(4,4)}$	$g_1^{(4,5)}$	
$\vdots$		N^{4}LL	N^{3}LL	N^{2}LL	NLL	LL	

Logarithmic counting (3)

$$\ln\left(\frac{d\sigma}{dq_T}\right) \propto \ln\left(1+\sum_{m=1}^{\infty}\alpha_s^m \mathcal{H}^{(m)}\right)+\sum_{n=1}^{\infty}\alpha_s^n\sum_{k=1}^{n+1} g^{(n,k)}L^k \qquad \alpha_S L \sim 1$$



Logarithmic counting

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2)$$

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_T; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b)$$

$$\times \exp\left\{K(b_T; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'}\right]\right\}$$

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	$\alpha_s{}^2$	α_s	α_s	1	1
NLL’	$\alpha_s{}^2$	α_s	α_s	α_s	α_s
N²LL	$\alpha_s{}^3$	$\alpha_s{}^2$	$\alpha_s{}^2$	α_s	α_s
N²LL’	$\alpha_s{}^3$	$\alpha_s{}^2$	$\alpha_s{}^2$	$\alpha_s{}^2$	$\alpha_s{}^2$
N³LL	$\alpha_s{}^4$	$\alpha_s{}^3$	$\alpha_s{}^3$	$\alpha_s{}^2$	$\alpha_s{}^2$

Codes

SCETlib, CuTe	SCET
DYRes/DYTURBO, ReSolve	q_T-res.
RadISH, PB-TMD	PB
NangaParbat, arTeMiDe	TMD

Basic ingredients common to all codes

Main differences:

- working space (b_T or q_T)
- NP physics (prescription/cutoff and intrinsic k_T)
- matching with fixed order

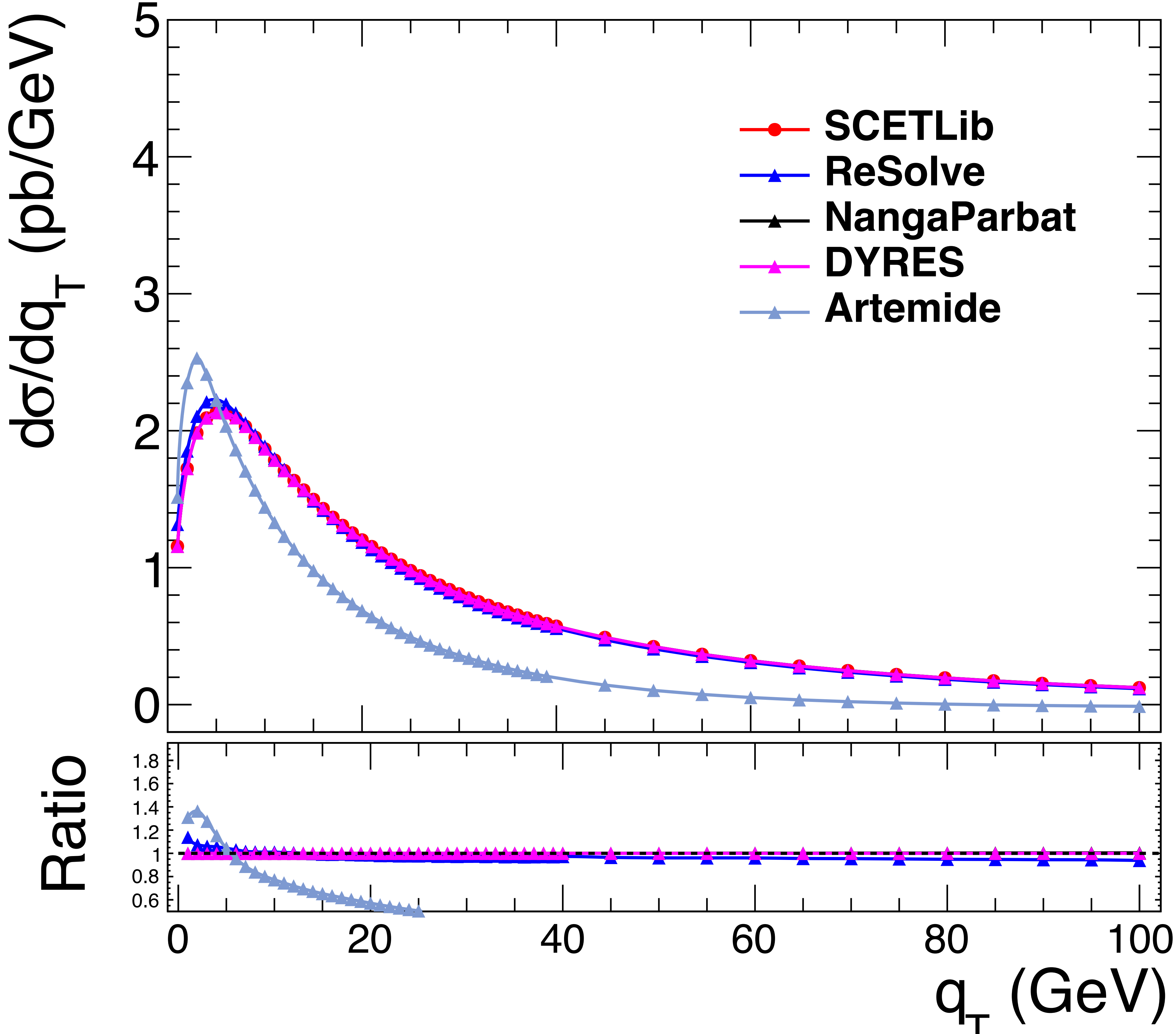
Differences

- **NP-physics (1): avoiding Landau pole**
  - q_T-space: low-q_T cutoff
  - b_T-space: b^* or “minimal prescription” (complex-b plane)
- **NP-physics (2): intrinsic-k_T**
  - NP form factor fitted to data (in principle, x and flavour dependent)
- **matching with fixed order**
  - multiplicative or additive
  - damping function to switch off resummation/evolution
  - unitarity enforcing, i.e., modified logs (NP may spoil it)
- **lepton cuts**

The benchmark settings

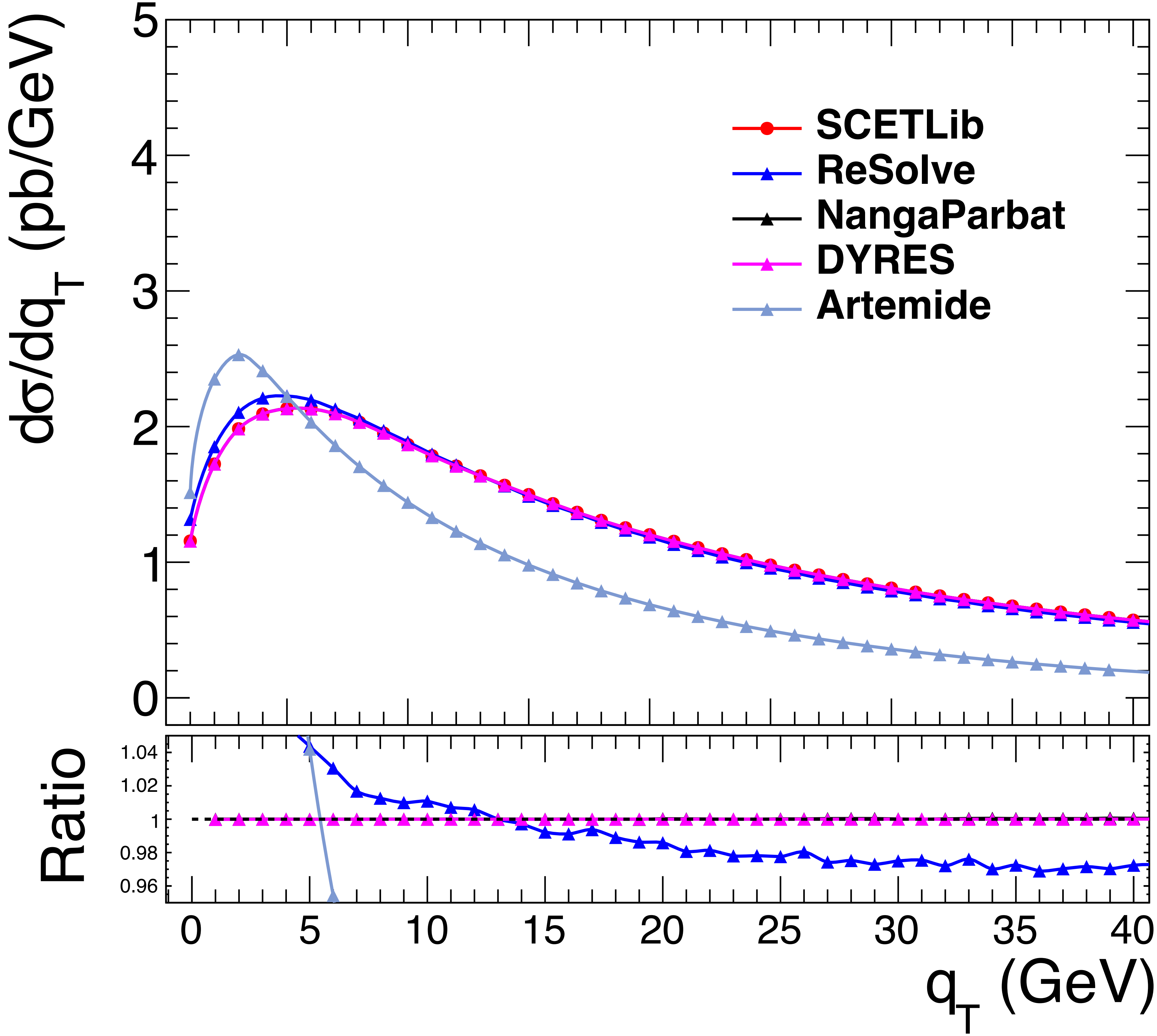
- Z/γ^* production at $\sqrt{s}$ = 13 TeV,
- **Resummation** only (no matching to fixed order yet),
- A number of values of Q and y:
  - we will only show results at $Q = M_Z$ and $y = 0$.
- Consider all possible logarithmic orders:
  - up to N^3LL.
- Favourite Landau-pole regularisation procedure:
  - b^*/k_T^* or "minimal prescription".
- Only standard logs:
  - no modified logs to enforce unitarity.
- q_T distribution from 1 to 100 GeV:
  - we are aware that for resummation breaks down well before,
  - benchmark exercise aimed at checking the consistency of codes/formalisms.

LL



SCETLib
ReSolve
NangaParbat
DYRES
Artemide
$d\sigma/dq_T$ (pb/GeV)
Ratio
q_T (GeV)
0
1
2
3
4
5
0.6
0.8
1
1.2
1.4
1.6
1.8
0
20
40
60
80
100

LL



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ReSolve
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DYRES
Artemide
dσ/dq_T (pb/GeV)
Ratio
q_T (GeV)
0
1
2
3
4
5
1.04
1.02
1
0.98
0.96
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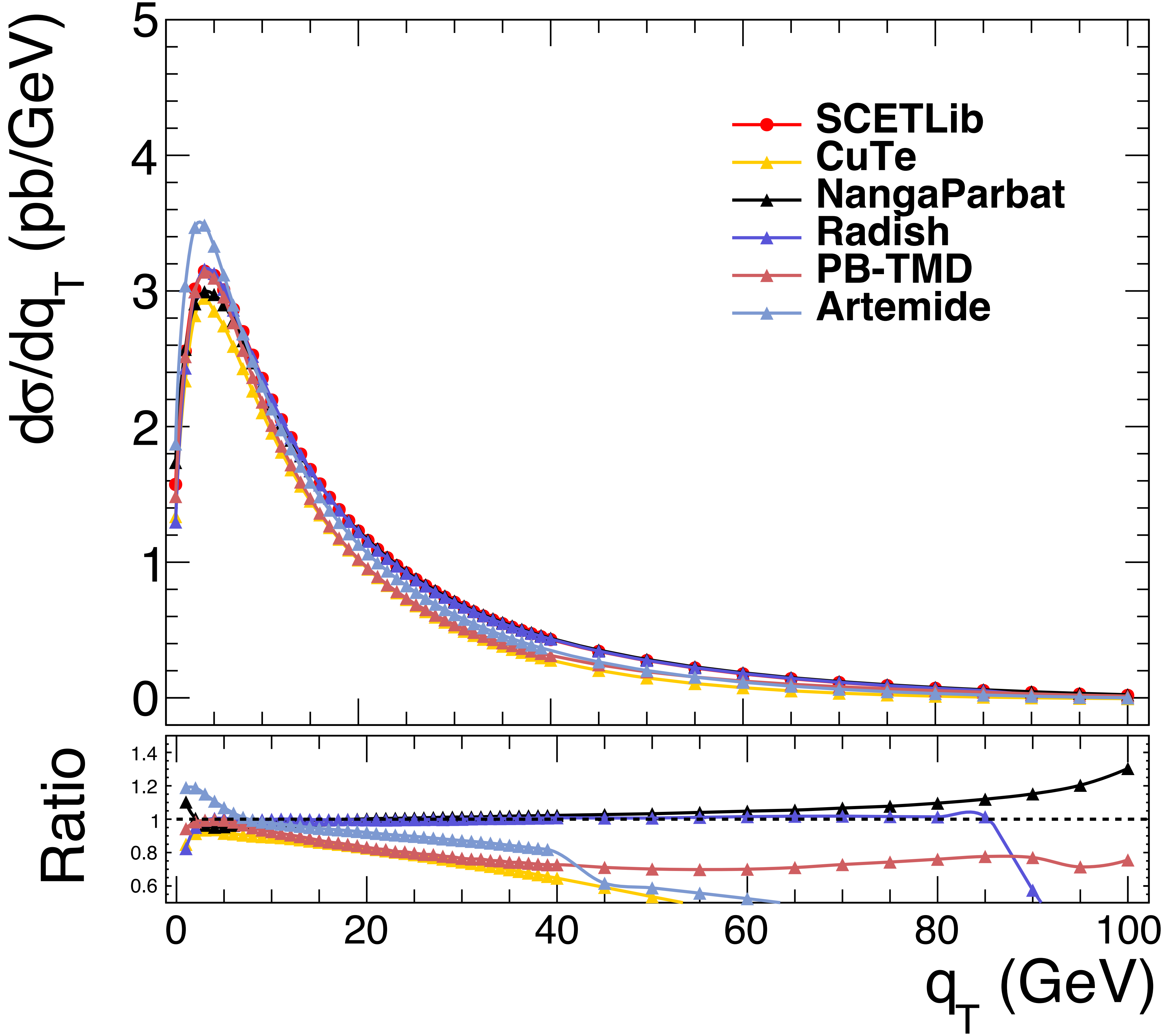
25
30
35
40

NLL



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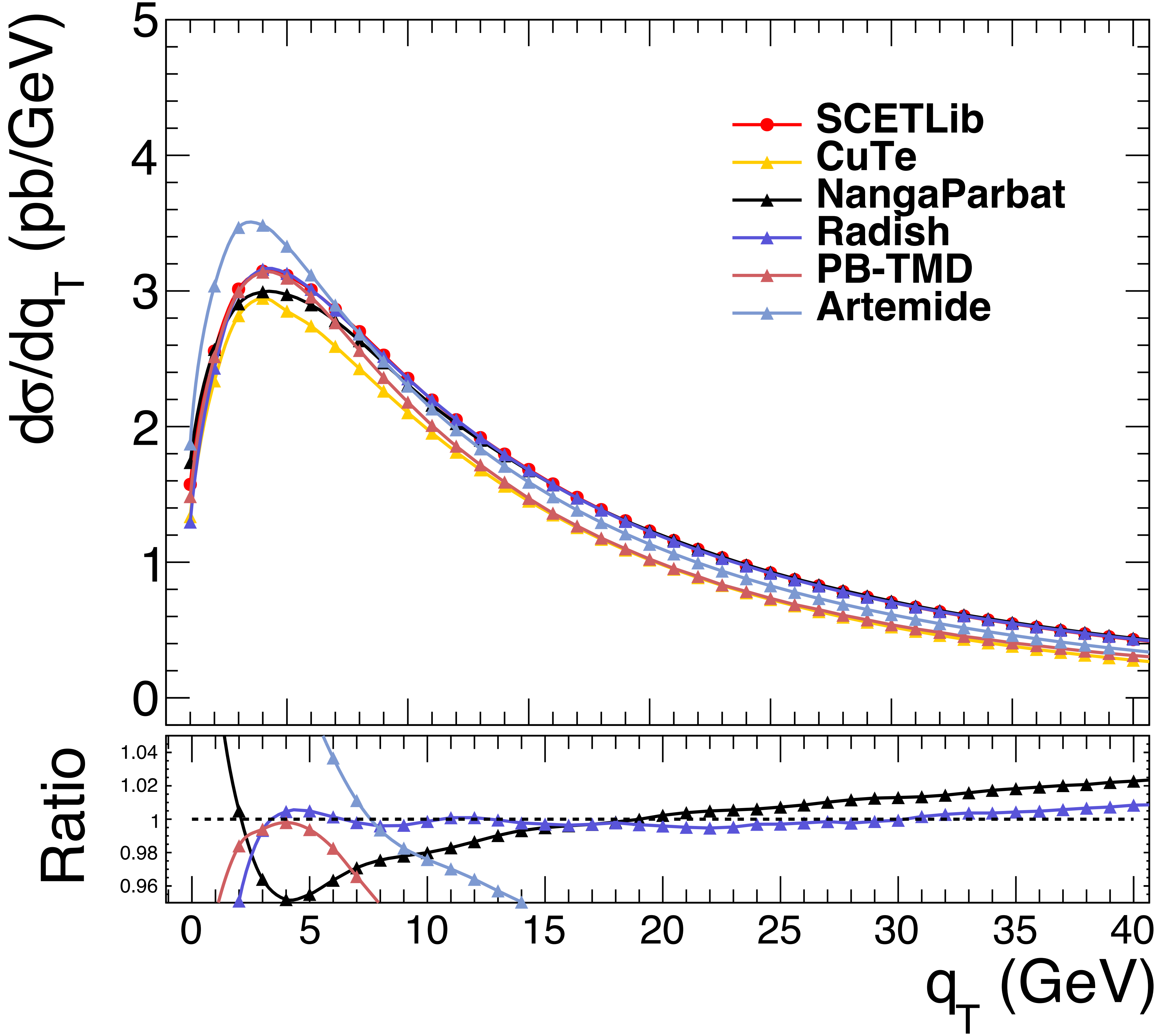

SCETLib
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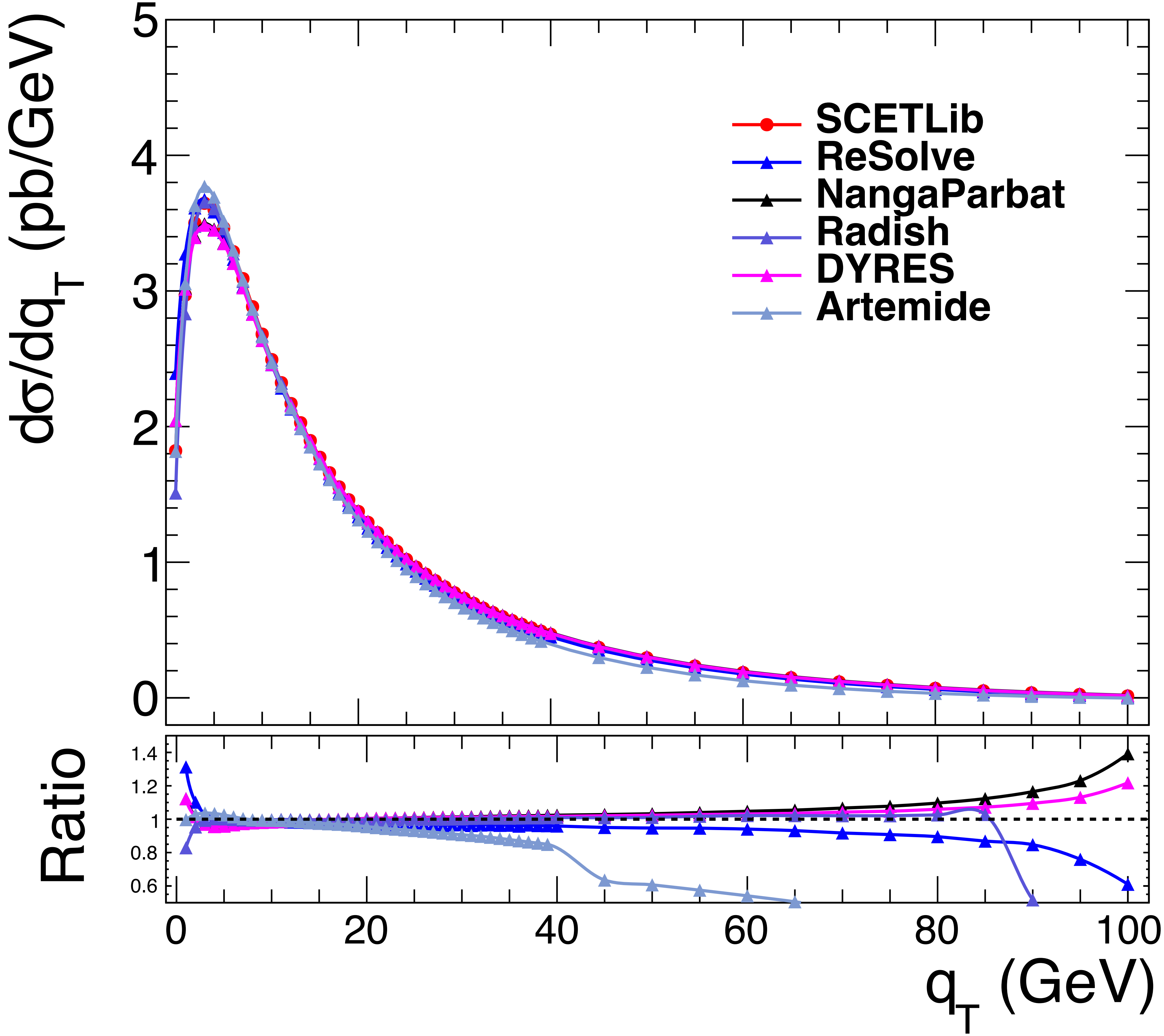
NLL’



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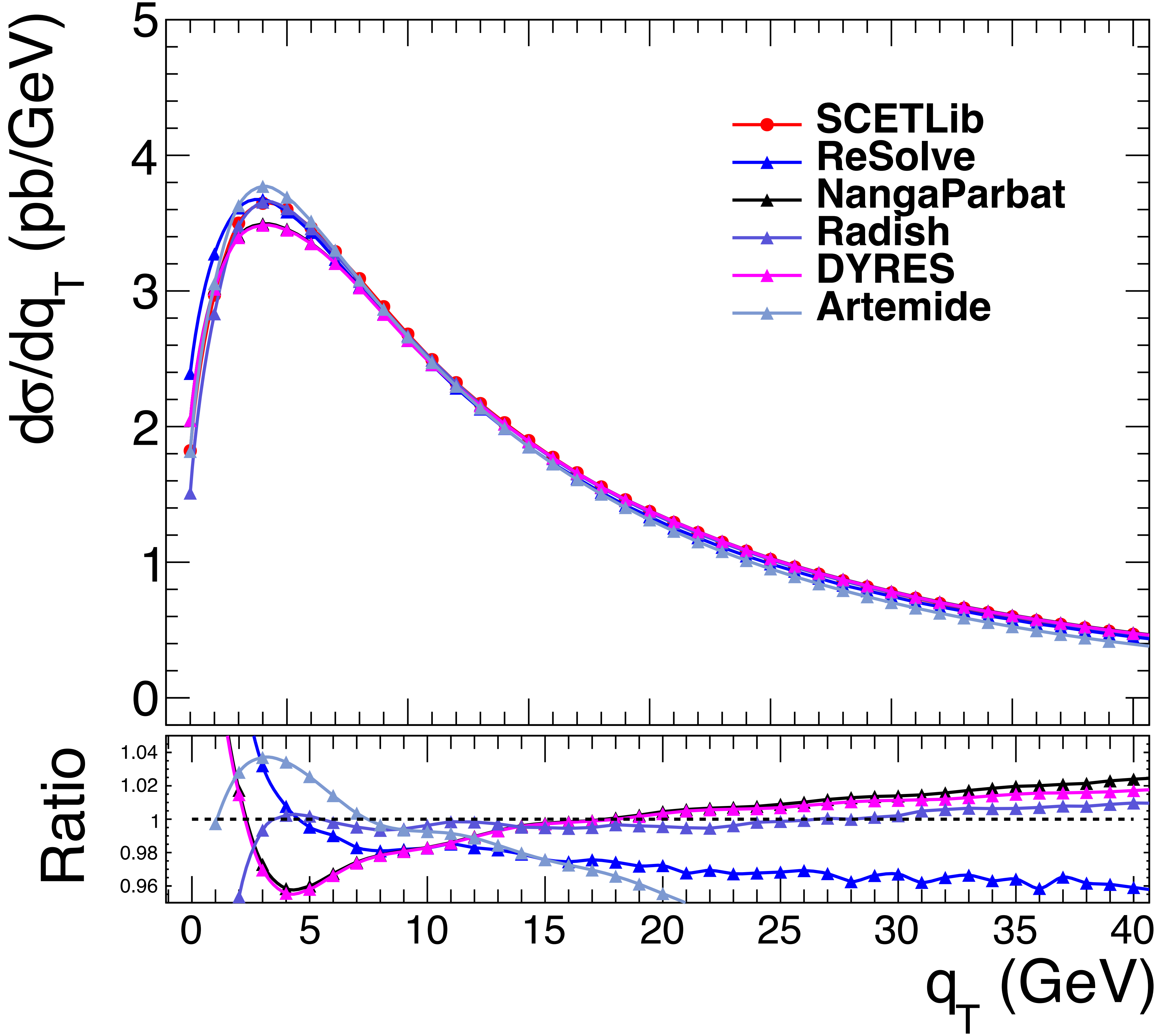
NLL'



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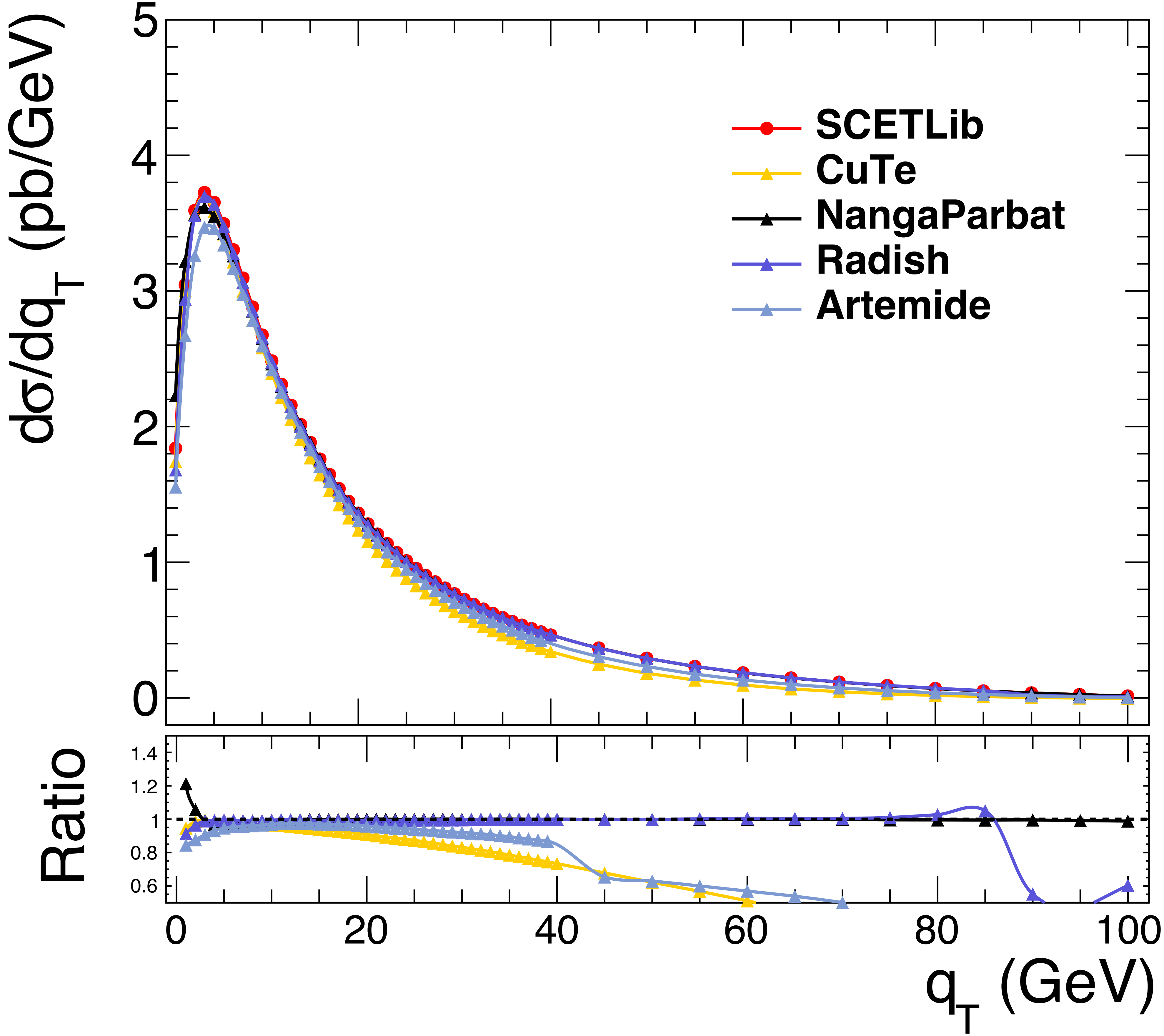

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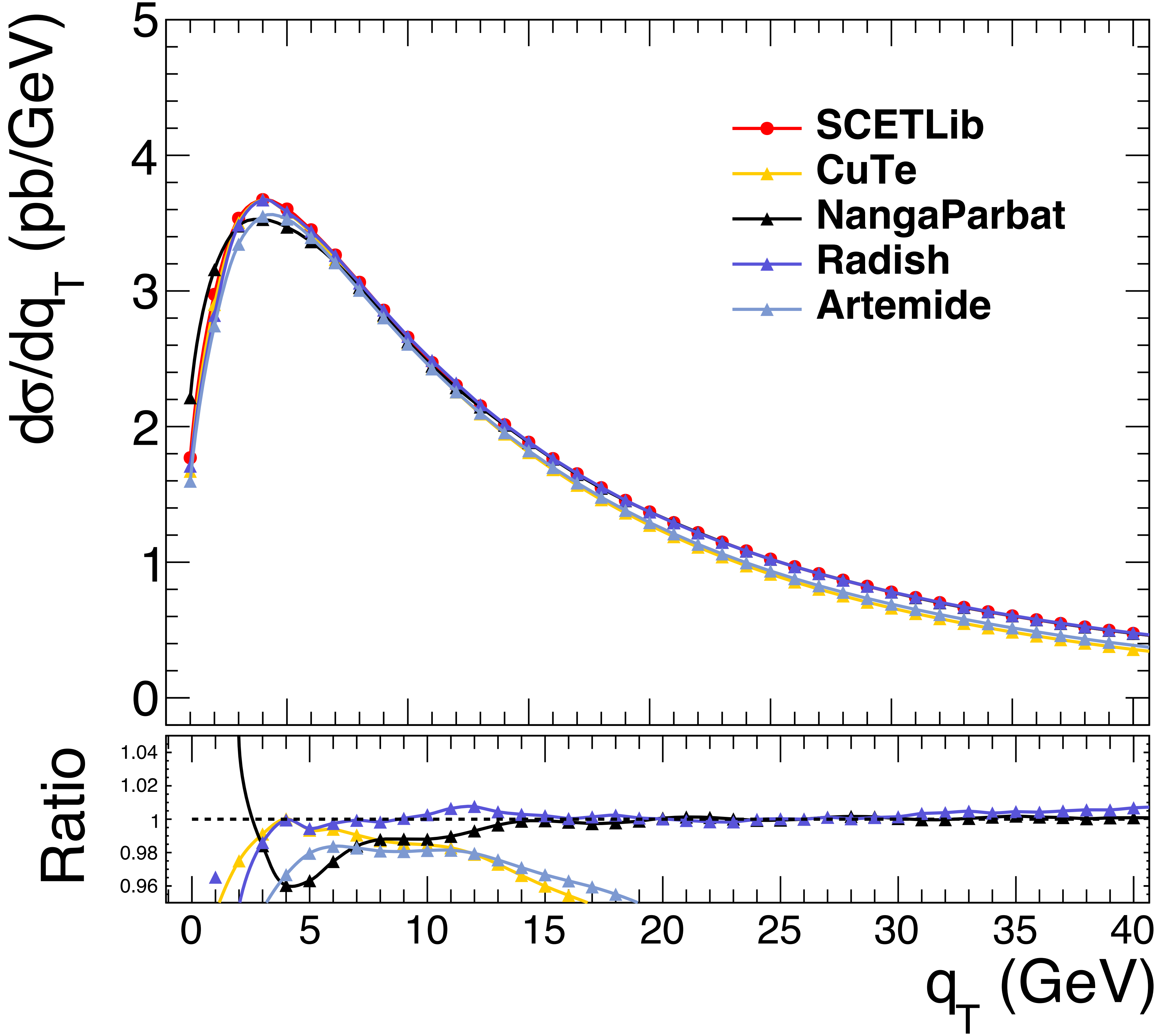

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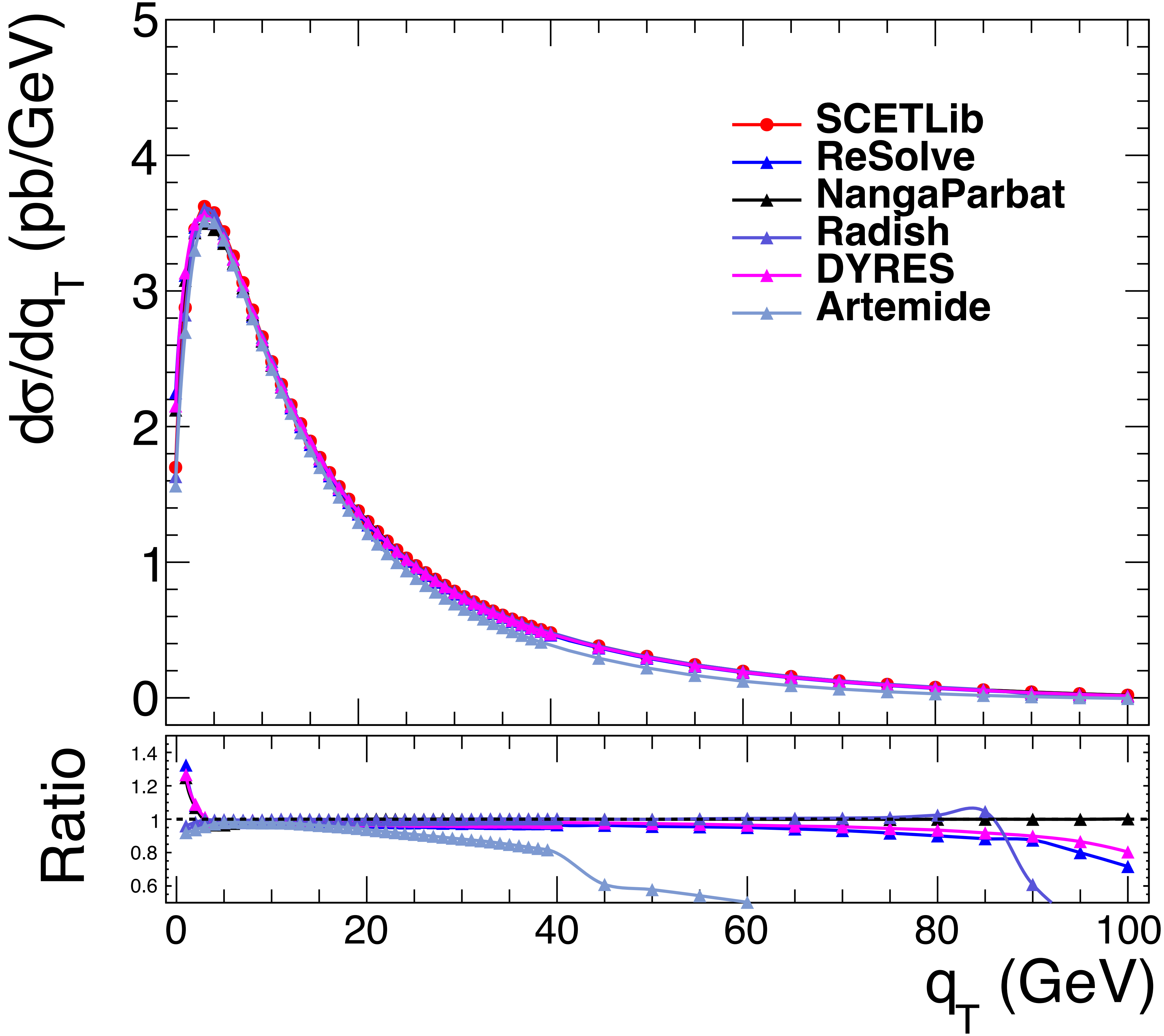
Ratio
q_T (GeV)

NNLL’



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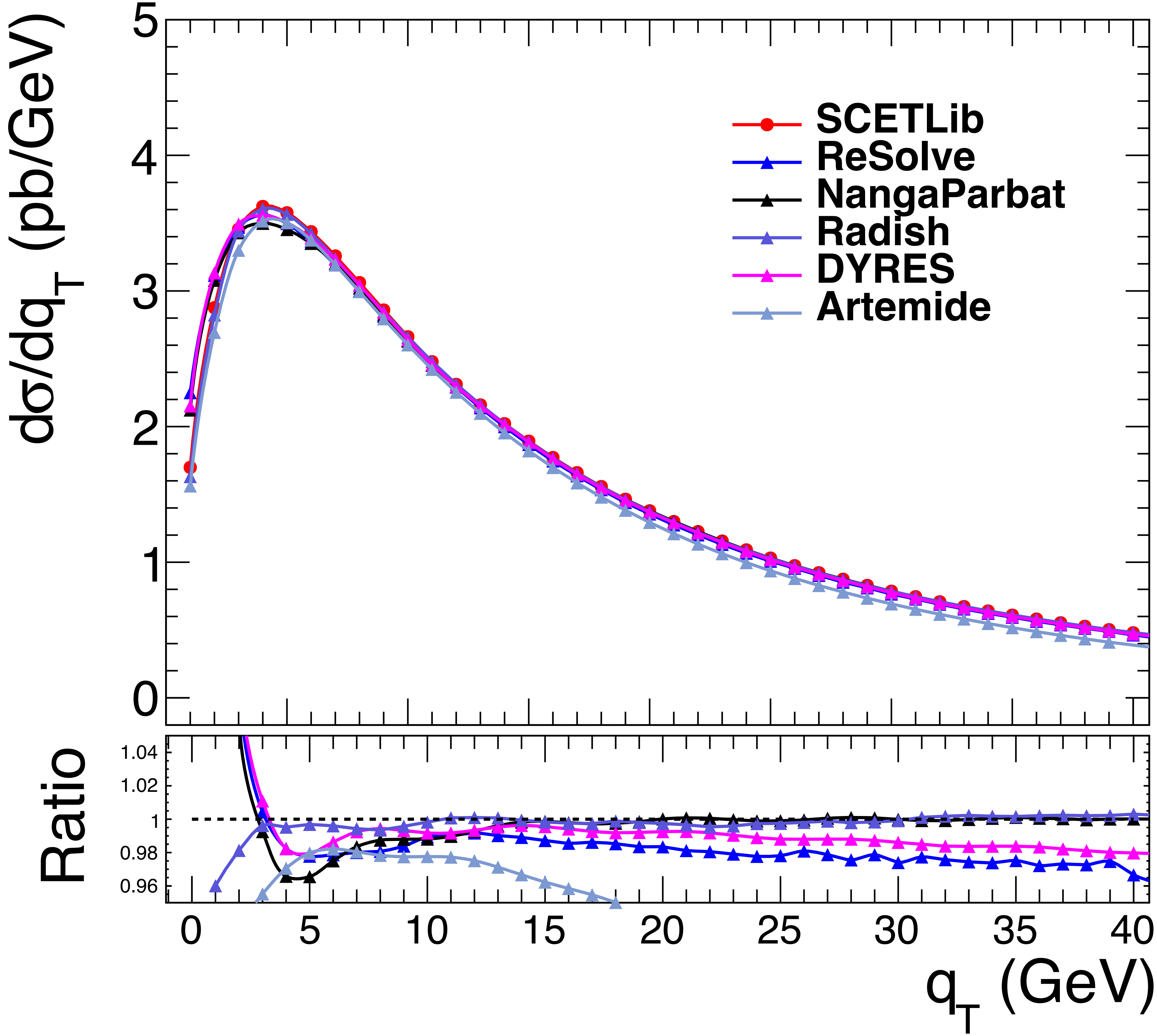
NNLL’



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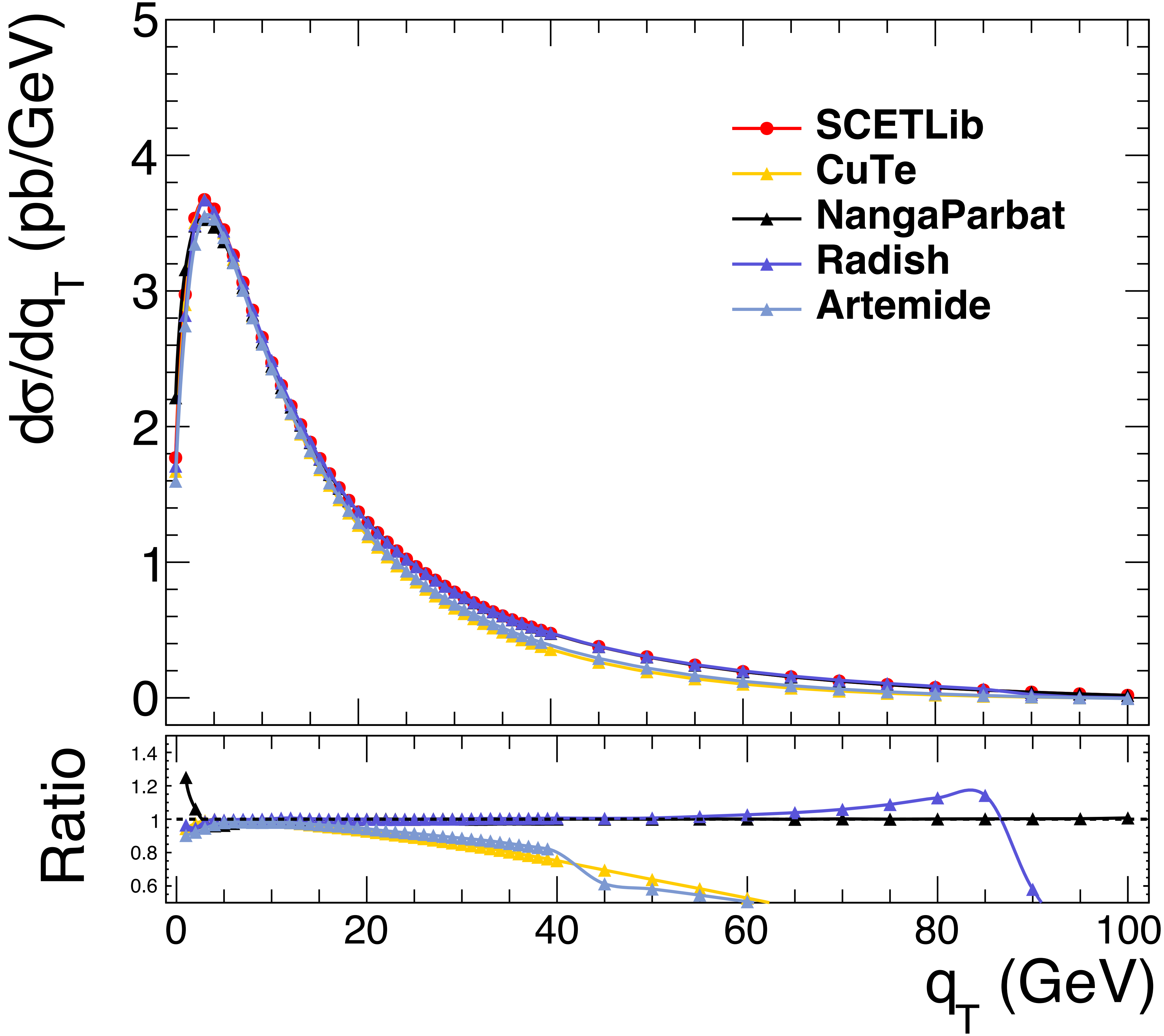
40

N3LL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Ratio
q_T (GeV)
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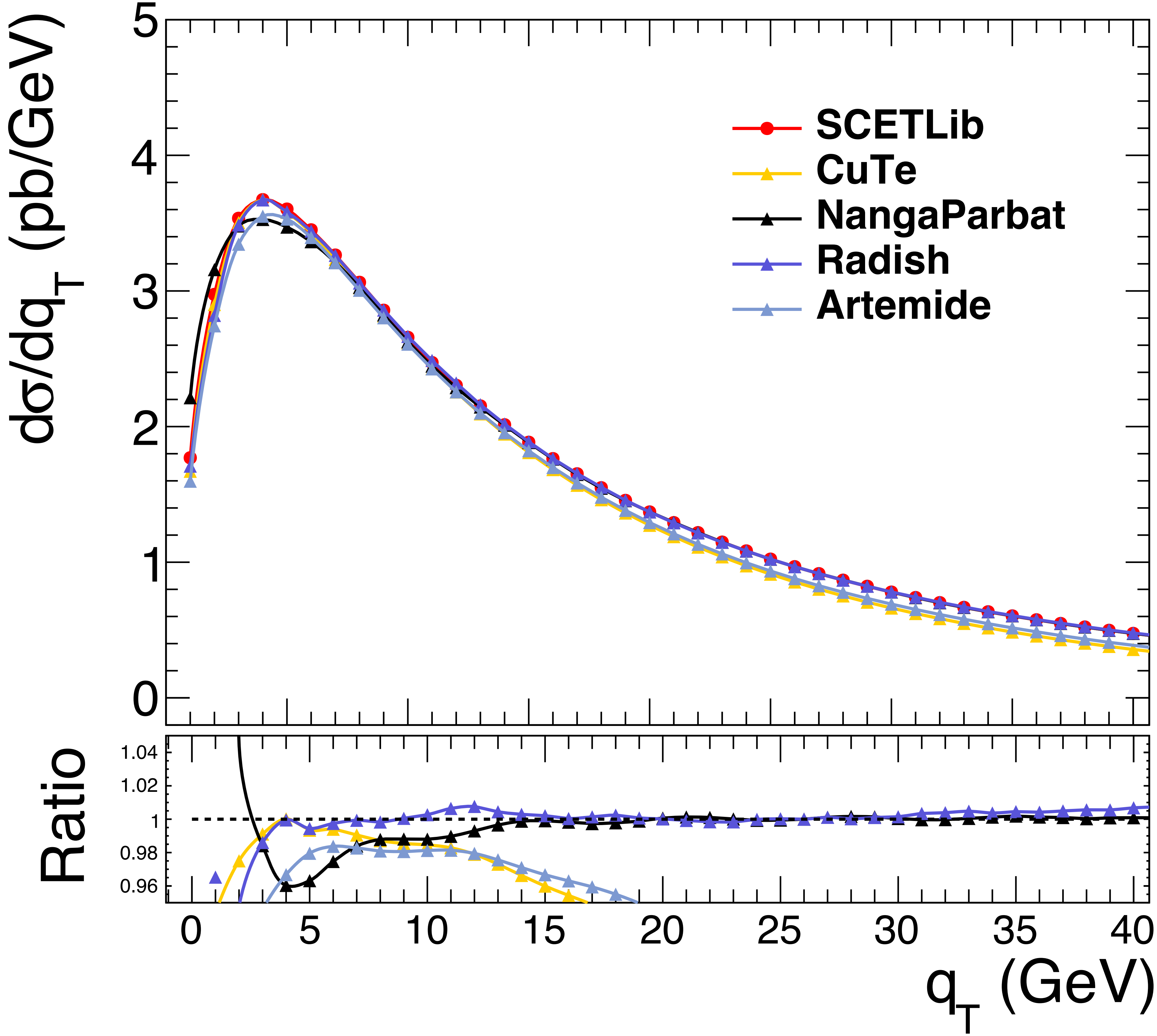
N3LL



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q_T (GeV)

Regularising strategies

b^* Prescription(s).

Basic idea: Replace

$$b_T \to b^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{\max})^2}} \quad \Rightarrow \quad \begin{cases} b^*(b_T \to 0) \to b_T \\ b^*(b_T \to \infty) \to b_{\max} \end{cases}$$

- $b_0/b^*(b_T \to \infty) \to b_0/b_{\max}$, so take $b_0/b_{\max} \sim 1\,\mathrm{GeV}$ as cutoff

Different options:

- "Global b^*": original, most often used

$$\tilde{\sigma}(b_T) \equiv \tilde{\sigma}[b_T, \mu_i(b_T)] \to \tilde{\sigma}(b^*) \equiv \tilde{\sigma}[b^*, \mu_i(b^*)]$$

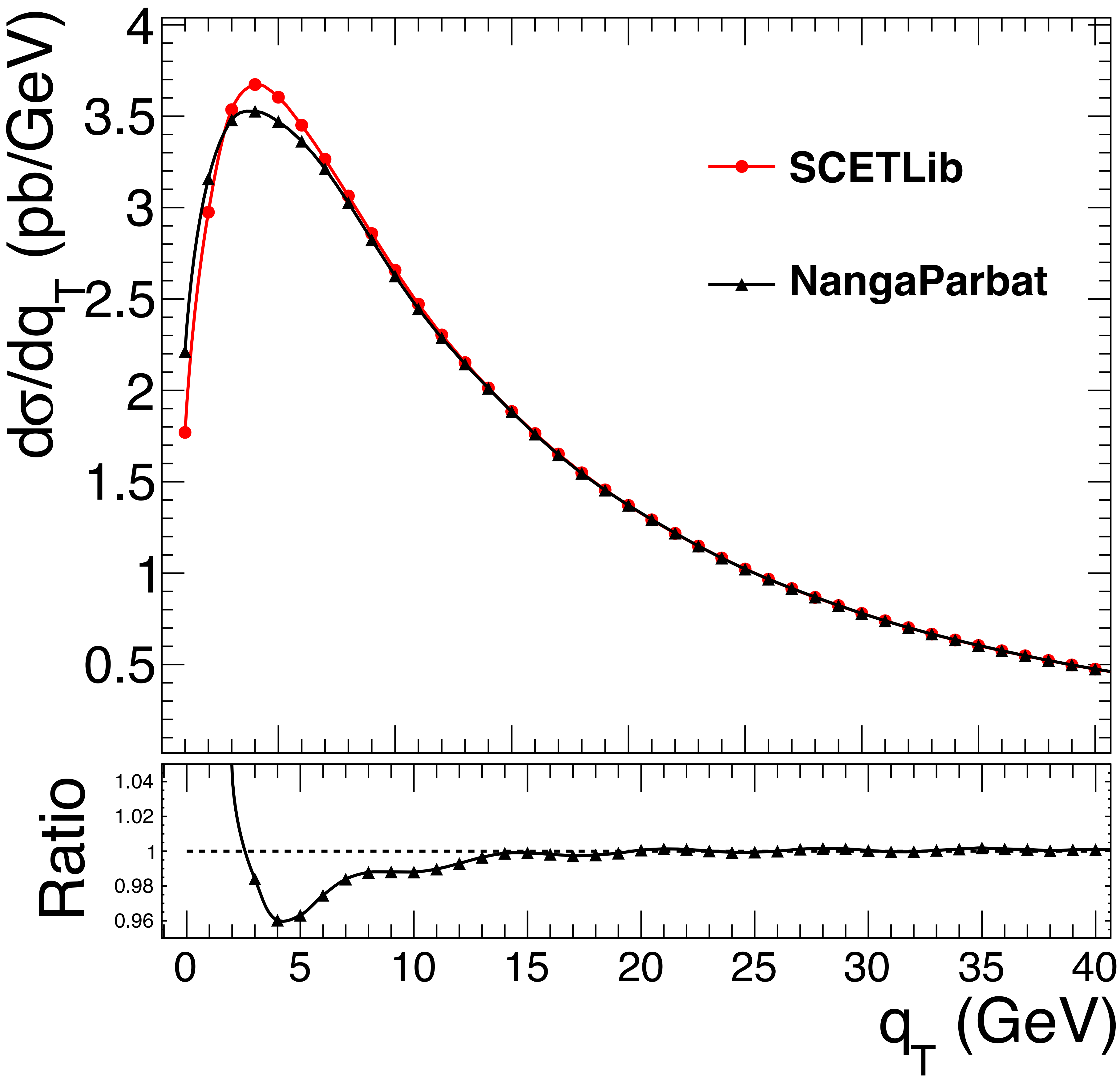
- "Local b^*": when keeping all RGE scales explicit
  [for more details see Lustermans, Michel, FT, Waalewijn, 1901.03331]

$$\tilde{\sigma}[b_T, \mu_i(b_T)] \to \tilde{\sigma}[b_T, \mu_i(b^*)]$$

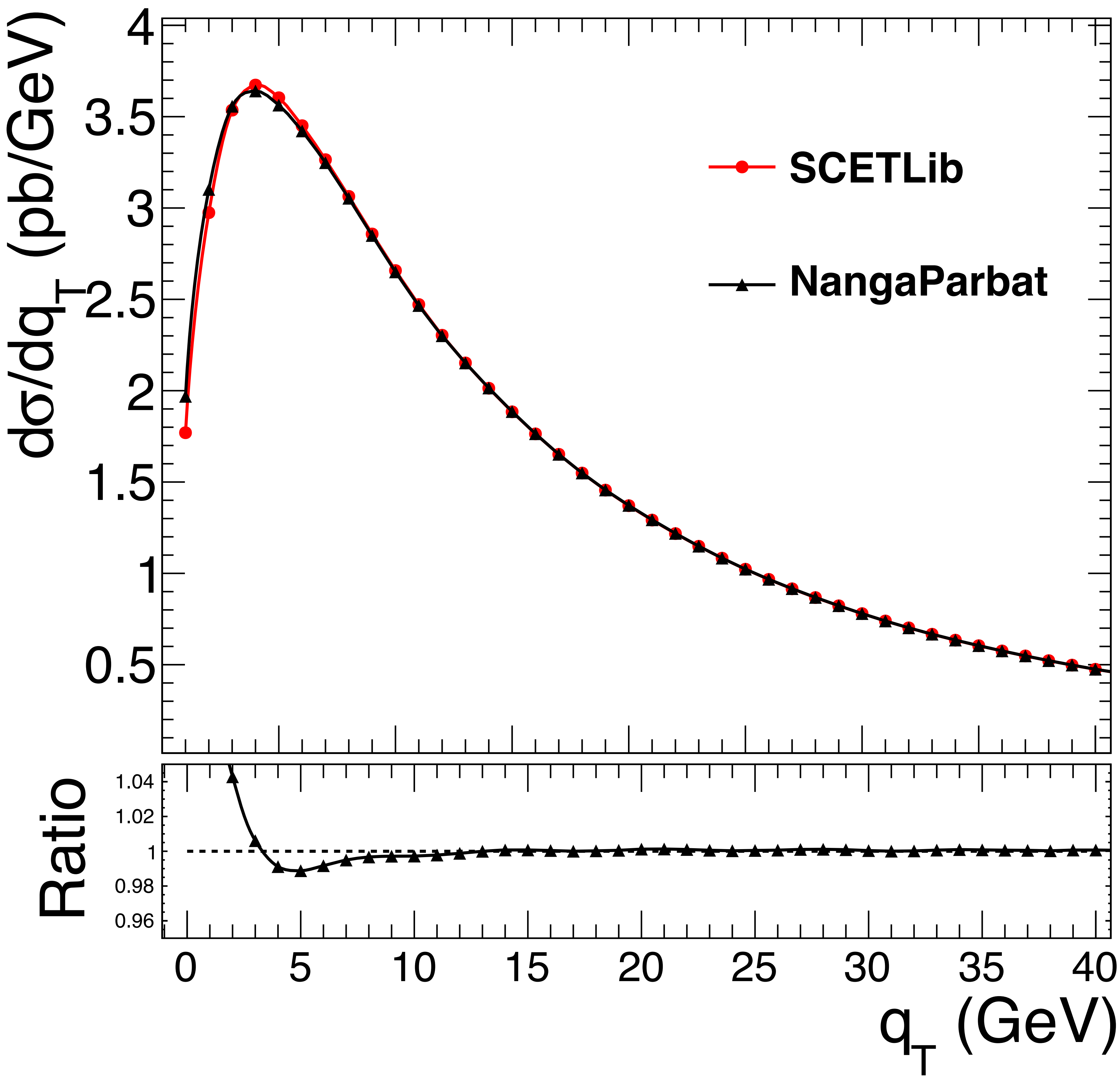
- Intermediate versions are possible as well
  - All amount to factorizing pert. from nonpert. contributions, "ad-hocness"

Borrowed from F. Tackmann's presentation at the EWWG

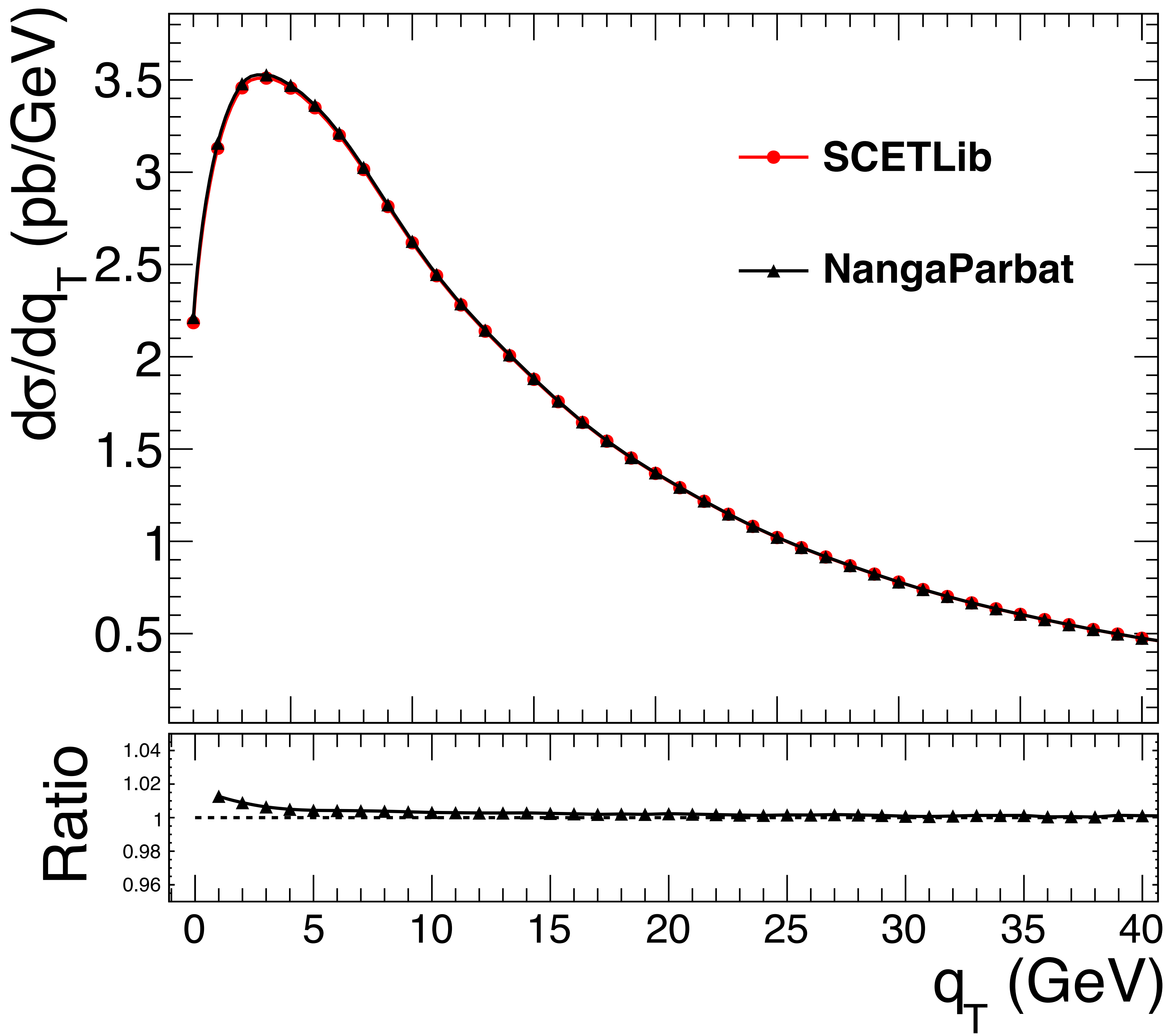
Regularising strategies



Regularising strategies

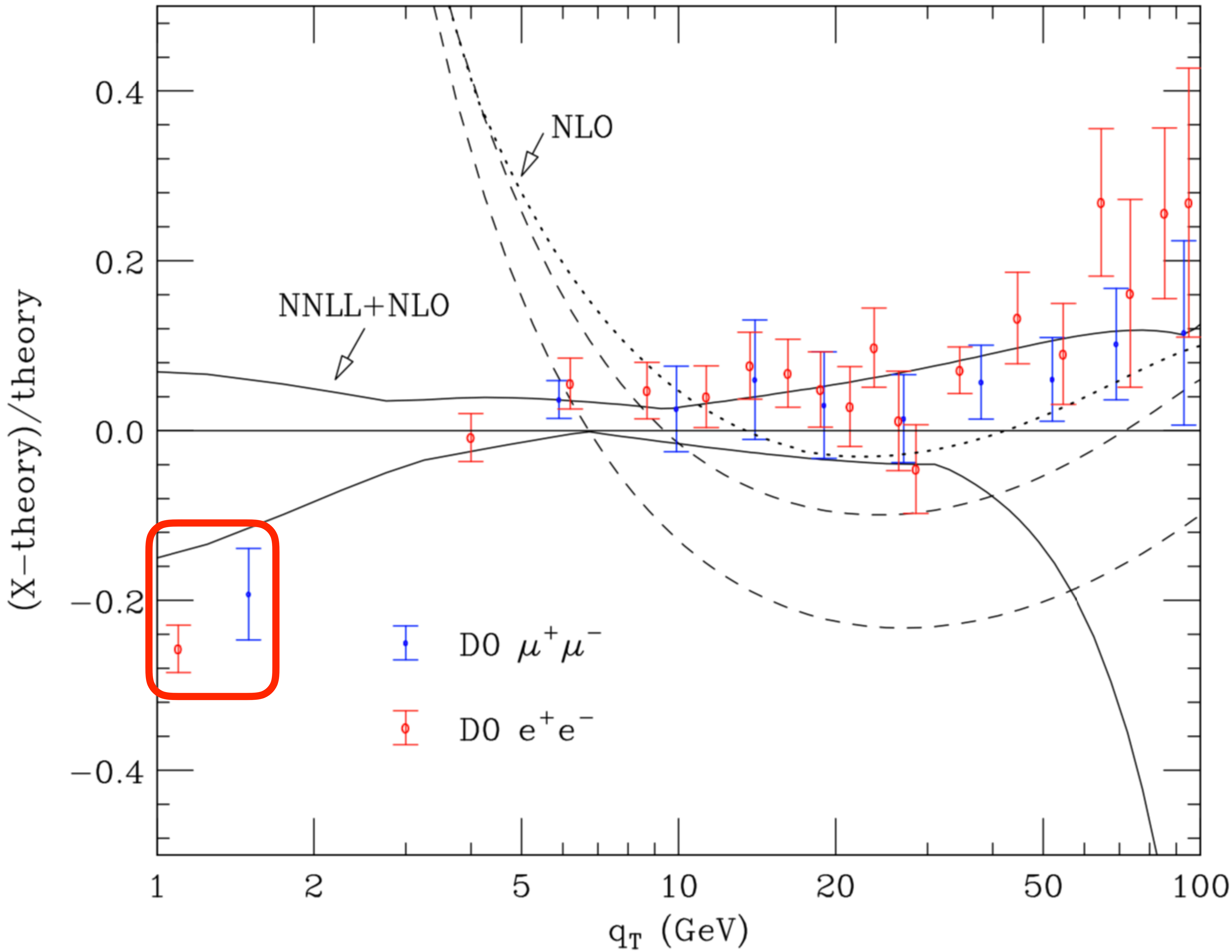


Regularising strategies

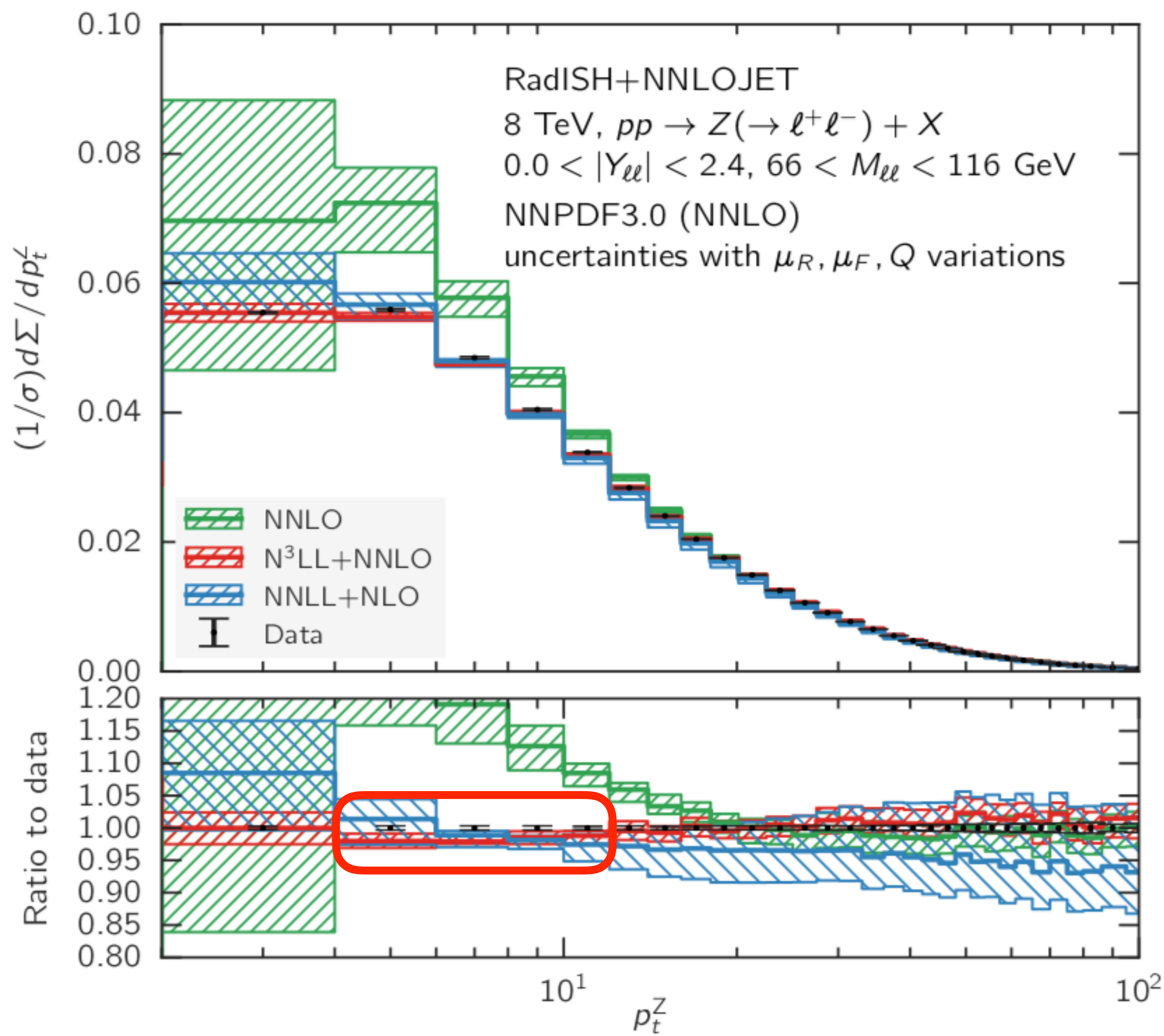


Backup

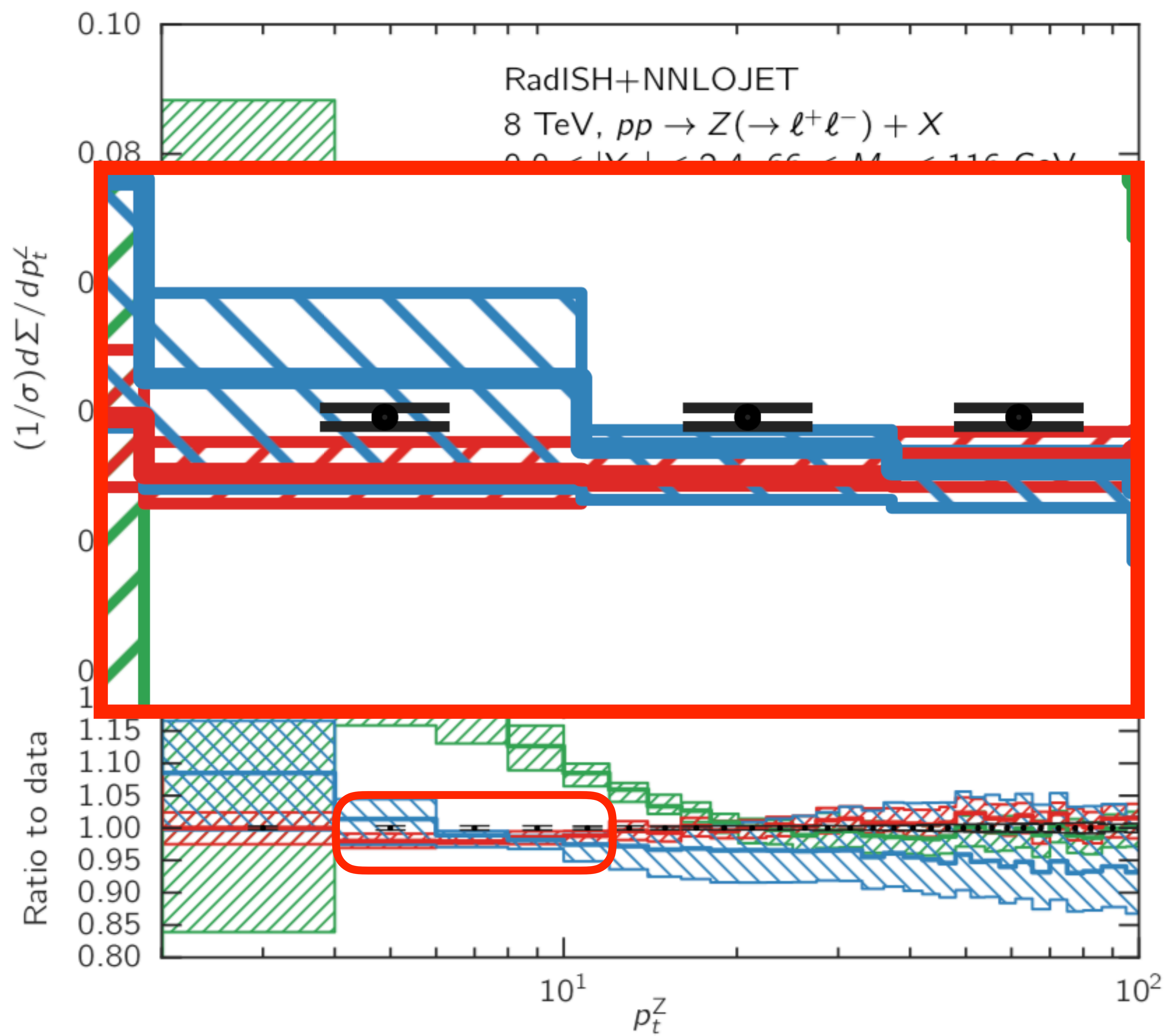
Non-perturbative region



Non-perturbative region



Non-perturbative region



Logarithmic counting

- TMD factorisation provides **resummation** of large logs $L = \log(q_T/Q)$:
  - implemented through the **Sudakov** form fact R.
- A **perturbative expansion** in powers of α_s of R would give:

One Sudakov for each TMD

$$R^2 = \sum_{n=0}^{\infty} a_s^n \sum_{k=1}^{2n} \widetilde{S}^{(n,k)} L^k$$

Double-log expansion

- that can be rearranged as:

$$R^2 = \sum_{m=0}^{\infty} R^2_{\mathrm{N^mLL}} \quad \text{with} \quad R^2_{\mathrm{N^mLL}} = \sum_{n=[m/2]}^{\infty} \widetilde{S}^{(n,2n-m)} a_s^n L^{2n-m}$$

Integer part of m/2

- Therefore, multiplying R by a power p of α_s gives:

$$a_s^p R^2_{\mathrm{N^mLL}} = \sum_{j=[(m+2p)/2]}^{\infty} \widetilde{S}^{(j-p,2j-(m+2p))} a_s^j L^{2j-(m+2p)} \sim R^2_{\mathrm{N^{m+2p}LL}}$$

- Bottom line: any additional power of α_s causes a shift of **two units** in the logarithmic ordering.

Matching TMD to collinear

- Accurate predictions for all q_T's by **additive matching**, order by order in perturbation theory, of collinear and TMD calculations:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathrm{add.match.}} = \left(\frac{d\sigma}{dq_T}\right)_{\mathrm{res.}} + \left(\frac{d\sigma}{dq_T}\right)_{\mathrm{f.o.}} - \left(\frac{d\sigma}{dq_T}\right)_{\mathrm{d.c.}}$$

- In order for the match to actually take place:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathrm{res.}} \xrightarrow[\mathrm{f.o.}]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathrm{d.c.}} \xleftarrow[q_T \ll Q]{} \left(\frac{d\sigma}{dq_T}\right)_{\mathrm{f.o.}}$$

- Therefore, the "fixed-order" parts have to match in the relevant limits:

Log Accuracy	Minimal f.o. accuracy
NLL'	α_s (LO)
N^{2}LL	α_s (LO)
N^{2}LL'	α_s^2 (NLO)
N^{3}LL	α_s^2 (NLO)