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Workshop on Resummation, Evolution, Factorization (REF 2019)

Universality and universality breaking effects in Factorized e^+e^- hadroproduction cross sections

CSS Factorization: general features

1. Process with HARD SCALE Q

2. Find and weigh
   IR singular contributions

   - Landau equations
   - Power Counting

3. **Sum over**
   all the regions and all the graphs

   Validity to **EVERY order** in perturbation theory

4. Use Gauge Invariance (Ward Identities)

5. **FACTORIZATION** of
   - HARD
   - COLLINEAR(S)
   - SOFT

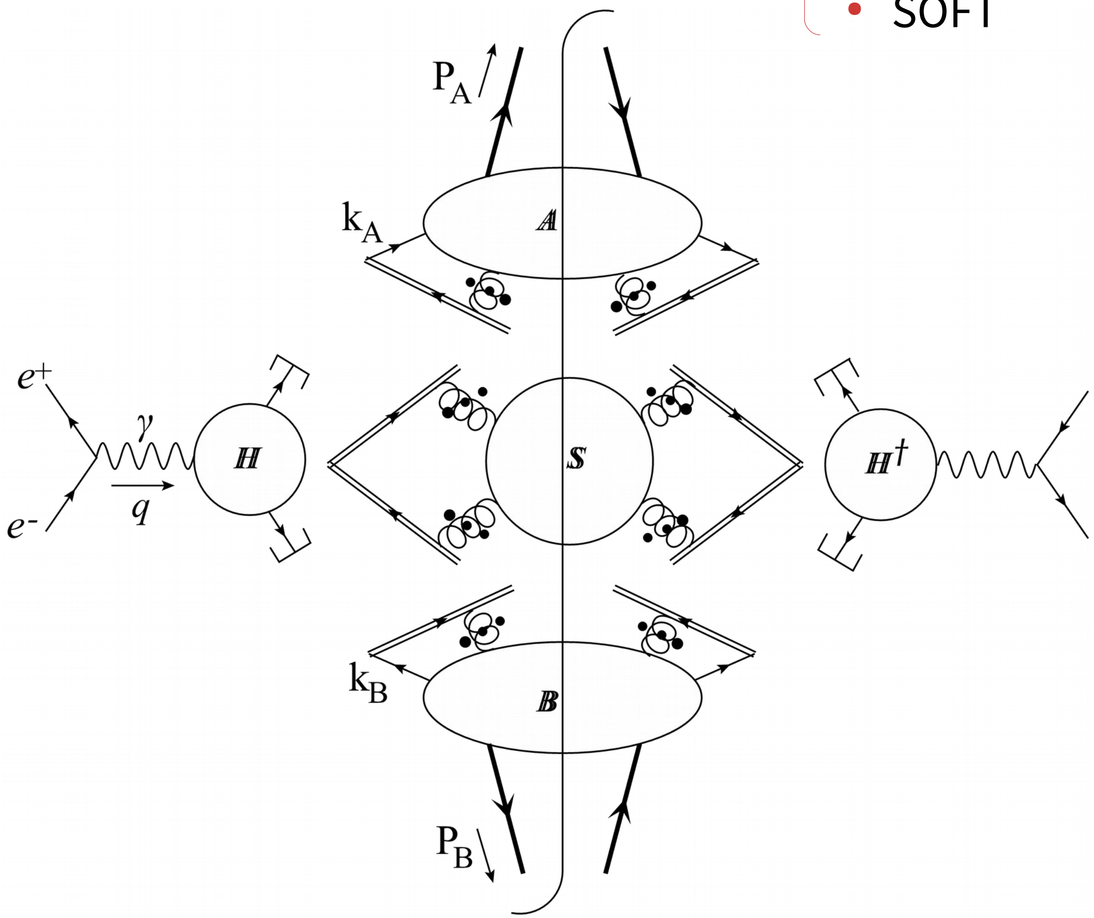
   contributions.

CSS Factorization: e^+e^- to 2 back-to-back hadrons

1. HARD SCALE Q is the CM energy

5. **FACTORIZATION** of
   - HARD
   - COLLINEAR-to-A and COLLINEAR-to-B
   - SOFT

   contributions.



KINEMATICS:
In *CM-frame* the two detected hadrons are (almost) **back-to-back**

In the *hadron-frame* the transverse momentum q_{hT} of the photon is small compared to Q:

$$\frac{q_{h,T}}{Q} << 1$$

CSS Factorization: e^+e^- to 2 back-to-back hadrons

The hadronic part of the cross section, differential in q_{hT}, is given by:

Hard Part
specific of the process

$$W^{\mu\nu}(p_A, p_B, q) = \frac{8\pi^3 z_A z_B}{Q} \sum_f \boxed{\mathrm{Tr}_D \left[k^+_{A,\gamma} \gamma^- H^\nu_f(Q)\, k^-_{B,\gamma} \gamma^+ \overline{H}^\mu_f(Q) \right]}$$

$$\int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{-i\vec{q}_{h,T}\cdot\vec{b}_T} \boxed{\widetilde{\mathbb{S}}(b_T)} \boxed{\widetilde{D}_{1,h_A/f}(z_A, b_T)\, \widetilde{D}_{1,h_B/\overline{f}}(z_B, b_T)} + \text{polarized terms}$$

[John Collins. *Foundations of perturbative QCD*, 2011]

Unpolarized TMD FFs
relative to the hadrons A and B

Soft Factor

- Contains non-perturbative effects → NOT COMPUTABLE
- Always appears multiplied by collinear objects → NOT EXTRACTABLE FROM DATA

Classification of processes

I will use an improper definition of jet:

A **jet** of a process is a **collinear region**, separated out through the factorization procedure.

Each collinear region is always associated to a hadron

- final state → FFs
- initial state → PDFs

JET CLASS
A process with N-jets belongs to the N-jet class of processes.

Relevant examples:

$$\text{1-jet class} = \{\text{DIS},\ e^+e^- \to H X\}$$

$$\text{2-jet class} = \{\text{Drell Yan},\ \text{SIDIS},\ e^+e^- \to H_A\, H_B X\}$$

A closer look to the Soft Factor

When kinematics allow for TMD-factorization, the various jets can exchange information ONLY through soft gluons

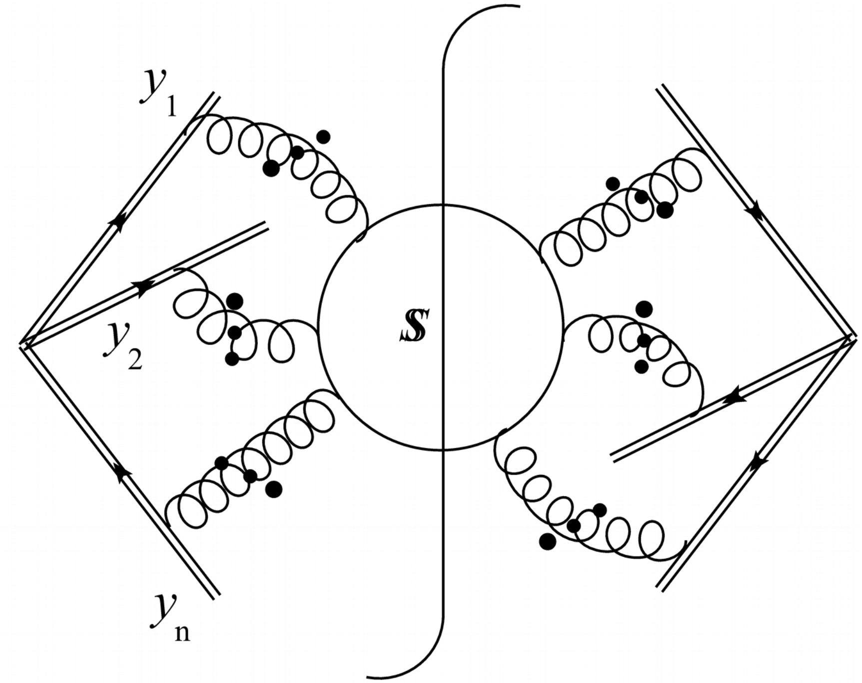
A non-trivial Soft Factor appears in the cross section.

$\mathbb{S}$ depend on:

- The total transverse soft momentum $k_{S,T}$.
- The RG scale μ.
- **The number of jets (n)** in the process.
  Each jet is replaced in $\mathbb{S}$ by a Wilson Line slightly-space-like thanks to a rapidity cut-off y_k ($1 \leq k \leq n$).

$$\mathbb{S}_{\text{n-jets}}\Big(\vec{k}_{S,T};\, \mu,\, \underbrace{\{y_1,\, y_2 \ldots y_n\}}_{\text{Lorentz-invariant combination of the rapidity cut-offs.}}\Big) \longleftrightarrow \mathrm{k_S} \longrightarrow$$

Lorentz-invariant combination of the rapidity cut-offs.



A closer look to TMDs

Each TMD is associated to a certain collinear region.
They are equipped with subtractions in order to avoid the double counting of the soft sub-divergences.

TMDs **depend only on their own jet** variables:

- The light-cone fraction of momentum z
- The total transverse momentum k_T of the (fragmenting) parton.
- The RG scale μ.
- The rapidity cut-off y_1 introduced by the subtraction procedure.

The resulting definition comes directly from the factorization procedure and hence I will call it **Factorization Definition (FD)**:

$$\widetilde{D}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_1\Big) = \lim_{y_{u_2}\to -\infty} \frac{\widetilde{D}^{\text{unsub}}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_{u_2}\Big)}{\widetilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_{u_2}\Big)}$$

A Hierarchy of Universality

There is a **hierarchy** of universality for the objects appearing in the cross section:

UNIVERSAL

TMDs belong to this category.
They are sensitive **ONLY** to their own jet.
The rest of the process is replaced (before subtractions) by a (light-like) Wilson Line in the opposite jet-direction.

JET-CLASS UNIVERSAL

The **Soft Factor** (of the process) belongs to this category.
It is blind to the rest of the process **APART FROM** its number of jets, each one appearing as a slightly space-like Wilson Line.

PROCESS DEPENDENT

The **Hard Factor** belongs to this category.
It totally depends on the process in exam.
However, it can be computed in perturbation theory at the desired accuracy.

2-jet class of processes

The 2-jet class of process has a **very special property**:

The Soft Factor in the cross section **is the same object** that appears in the subtraction mechanism implemented inside the Factorization Definition of the TMDs.

$$\widetilde{D}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_A - y_1\Big)\, \widetilde{D}_{H_B/\overline{f}}\Big(z_B,\, b_T;\, \mu,\, y_2 - y_B\Big)\, \widetilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_2\Big) =$$
$$= \lim_{\substack{y_{u_1}\to+\infty \\ y_{u_2}\to-\infty}} \frac{\widetilde{D}^{\text{unsub}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_P - y_{u_2}\Big)\, \widetilde{D}^{\text{unsub}}_{H_B/\overline{f}}\Big(z_B,\, b_T;\, \mu,\, y_{u_1} - y_B\Big)\, \widetilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_2\Big)}{\widetilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_{u_2}\Big)\, \widetilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_{u_1} - y_2\Big)}$$

In the 2-jet class it is possible to **reorganize the soft factors** inside the TMDs definitions

2-jet class of processes: Square Root Definition

The standard definition for TMDs in the 2-jet class is the **Square Root Definition (SRD)**:

$$\tilde{D}^{\text{sqrt}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_A - y_n\Big) =$$

$$= \lim_{\substack{y_{u_1}\to+\infty \\ y_{u_2}\to-\infty}} \tilde{D}^{\text{unsub}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_P - y_{u_2}\Big) \sqrt{\frac{\tilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_n\Big)}{\tilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_1 - y_2\Big)\, \tilde{\mathbb{S}}_{\text{2-jets}}\Big(b_T;\, \mu,\, y_n - y_2\Big)}}$$

[John Collins. *Foundations of perturbative QCD*, 2011]

Now the cross section is "**Parton Model-like**":

$$d\sigma \sim \mathbb{H}\, \tilde{D}_{H_A/f}\Big(y_A - y_1\Big)\, \tilde{D}_{H_B/\overline{f}}\Big(y_2 - y_B\Big)\, \tilde{\mathbb{S}}_{\text{2-jets}}\Big(y_1 - y_2\Big) =$$

$$= \mathbb{H}\, \tilde{D}^{\text{sqrt}}_{H_A/f}\Big(y_A - y_n\Big)\, \tilde{D}^{\text{sqrt}}_{H_B/\overline{f}}\Big(y_n - y_B\Big)$$

The Soft Factor has disappeared in the cross section!

2-jet class of processes: Square Root Definition

Square Root Definition (SRD)

Advantages:

- It is compatible with Factorization.
- It solves the problem of the Soft Factor in the 2-jet class.
- The new TMDs depend only on one rapidity cut-off y_n (symmetry in the evolution equations).
- Perturbative computations are easier.
- Gauge invariance is more explicit.

Disadvantage:

- It reduces the universality of TMDs. The new TMDs are universal ONLY inside the 2-jet class.

The SRD is OPTIMAL for the 2-jet class

However, it becomes problematic if one has to deal with N-jet class processes, where N ≠ 2.

e^+e^- in 1 hadron jet: BELLE data

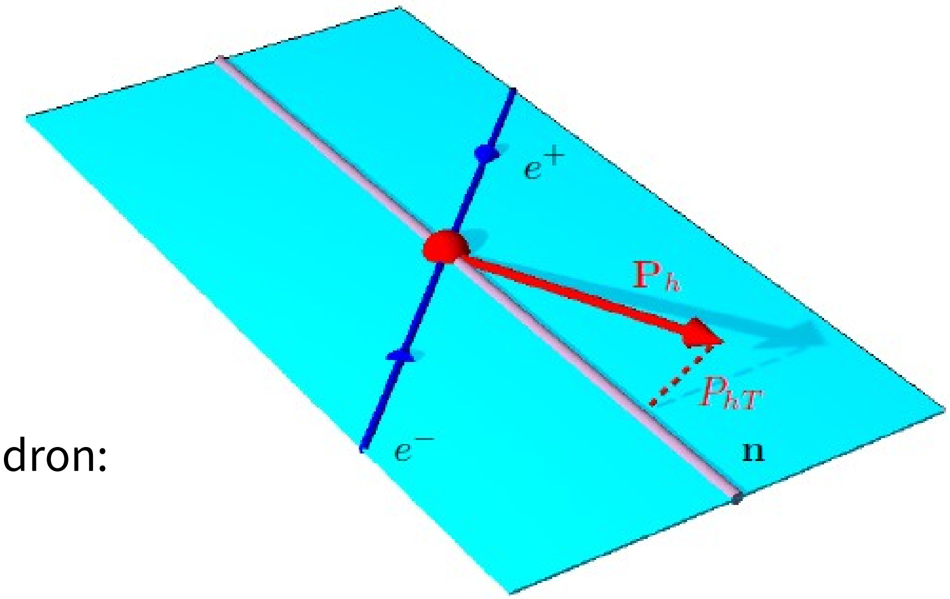
$$e^+e^- \to HX$$

Belle Collaboration provides the cross section differential in:

- CM-energy Q
- Fractional energy z of the detected hadron:

$$z = \frac{E_H}{Q/2}$$

- Transverse Momentum P_T of the detected hadron with respect the thrust axis of the jet.



[The Belle Collaboration. *Transverse momentum dependent production cross sections of charged pions, kaons and protons produced in inclusive e+e- annihilation at √s = 10.58 GeV*, 2019]

This extra data allows to extract information about TMD (unpolarized) FFs from the collinear factorized cross section

e^+e^- in 1 hadron jet: cross section

For spin-less detected hadrons (pions, kaons) the cross section (for ideal thrust T = 1) is:

$$\frac{d\sigma}{dQ\,dz\,dP_T} = \frac{2}{z^2}\left(\frac{P_T}{Q}\right)^2 \sum_j \int_z^1 d\hat{z}\,\hat{z}^3 \boxed{\frac{d\widehat{\sigma}_j}{dQ\,d\hat{z}}}$$

$$\int \frac{d^2\vec{b}_T}{(2\pi)^2} \boxed{\widetilde{D}_{H/j}\left(\hat{z},\, b_T\right)} \frac{J_1\left(b_T\,\frac{P_T}{b_T}\right) - z J_1\left(b_T\,P_T\right)}{b_T}$$

Partonic cross section

Unpolarized TMD FF associated to the detected hadron

NOT defined by using the SRD.

This object is not the same function appearing in e^+e^- to 2 back-to-back hadrons.

Notice that in this case, **the Soft Factor of the process is unity**, since collinear factorization holds:

$$\mathbb{S}_{(\mathrm{coll.})} = 1$$

e^+e^- in 1 hadron jet: cross section

By using **evolution equation for TMDs** we can re-write the cross section as:

$$
\frac{d\sigma}{dQ\,dz\,dP_T} = \frac{2}{z^2}\Big(\frac{P_T}{Q}\Big)^2 \sum_j \int_z^1 d\hat{z}\;\hat{z}^3\,\frac{d\widehat{\sigma}_j}{dQ\,d\hat{z}}
$$

$$
\int \frac{d^2\vec{b}_T}{(2\pi)^2}\,\Big[\frac{J_1\big(b_T\,\frac{P_T}{b_T}\big) - zJ_1\big(b_T\,P_T\big)}{b_T}\,\sum_k \int_{\hat{z}}^1 \frac{dw}{w^3}\,d_{H/k}(w,\mu_b)\,\widetilde{C}_j^k\big(\frac{\hat{z}}{w}\big)\times
$$

$$
\times\,\exp\Big\{\frac{1}{4}\,\widetilde{K}(b_T^*,\,\mu_b)\,\log\big(\frac{Q^2}{\mu_b^2}\big) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\,\Big[\gamma_D(a(\mu'),\,1) - \frac{1}{4}\gamma_K(a(\mu'))\,\log\big(\frac{Q^2}{\mu'^2}\big)\Big]\Big\}\Big]\times
$$

$$
\times\,\Big[M_j(b_T)\,\exp\Big\{-\frac{1}{4}\,g_K(b_T)\,\log\big(\frac{Q^2\,\hat{z}^2}{M_H^2}\big)\Big\}\Big]
$$

These functions **should be extracted from data (Belle)**.
Everything else is computable in perturbation theory.

Combining e^+e^- in 1-jet and 2-jets

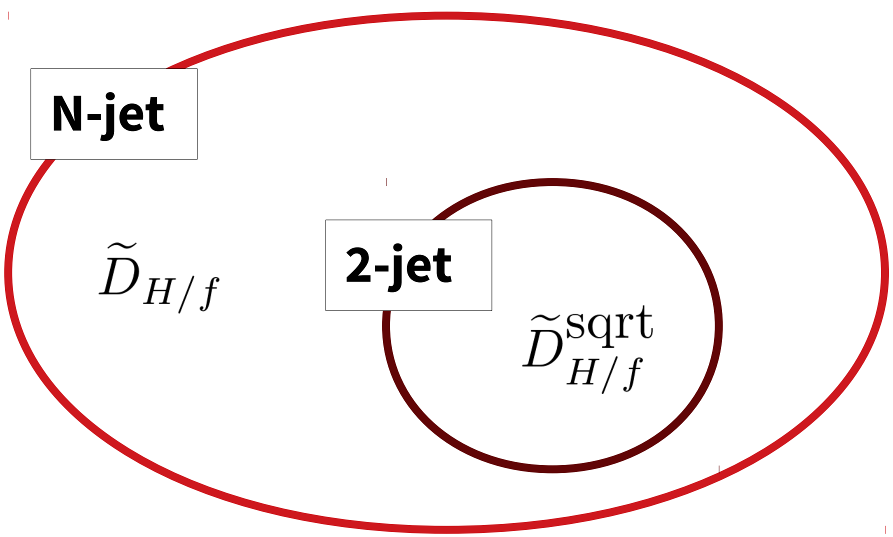
Now, the situation is the following

- From $e^+e^- \to H\,X$: ⟶ Extraction of $\widetilde{D}_{H/f}$

The **only possible** definition in 1-jet class

- From $e^+e^- \to H_A\,H_B\,X$: ⟶ Extraction of $\widetilde{D}^{\text{sqrt}}_{H/f}$

The **optimal** definition in 2-jet class



Combining e^+e^- in 1-jet and 2-jet means extracting the same TMD from both the processes.

Since the **FD** is more universal then the **SRD**, we should adopt it also in the 2-jet class.

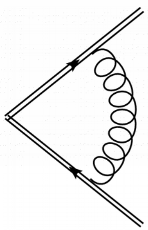
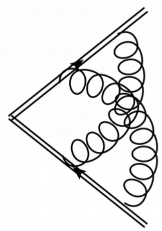
We have to deal directly with the 2-jet Soft Factor appearing in the cross section

A more closer look to 2-jet Soft Factor

The 2-jet Soft Factor is not entirely a black box. We can infer:

- How it behaves in the perturbative regime (when b_T is small):

$$\widetilde{\mathbb{S}}_{2\text{-jet}}(b_T^*;\, \mu, y_1 - y_2) = C \exp\Big\{ \text{Webs}(b_T^*;\, \mu, y_1 - y_2) \Big\}$$

$= C \exp\Big\{ \Big|$  $+$  $+ \ldots \Big|^2 \Big\}$

The strictly non-perturbative part of the Soft Factor **has to be modeled**

Then:

$$\widetilde{\mathbb{S}}_{2\text{-jet}}(b_T;\, \mu, y_1 - y_2) = C \exp\Big\{ \text{Webs}(b_T^*;\, \mu, y_1 - y_2) \Big\} \times \boxed{M_S(b_T;\, \mu, y_1 - y_2)}$$

- Its **evolution equation** in the limit of infinite rapidity cut-offs:

$$\widetilde{\mathbb{S}}_{2\text{-jet}}(b_T;\, \mu, y_1 - y_2) \sim \widetilde{\mathbb{S}}_{2\text{-jet}}(b_T;\, \mu_0, 0) \exp\Big\{ \frac{y_1 - y_2}{2}\, \widetilde{K}(b_T;\, \mu) \Big\} + \mathcal{O}\Big(e^{-\frac{y_1 - y_2}{2}}\Big)$$

Comparison between the two TMD definitions

Combining this informations we can **directly compare** the FD and the SRD:

$$\frac{\widetilde{D}^{\text{sqrt}}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_n\Big)}{\widetilde{D}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_1\Big)} = \sqrt{\widetilde{\mathbb{S}}_{\text{2-jet}}(b_T;\, \mu_0, 0)}\, \exp\Big\{\frac{y_1 - y_n}{2}\, \widetilde{K}(b_T;\, \mu)\Big\} =$$

$$= \underbrace{C \exp\Big\{\frac{y_1 - y_n}{2}\, \widetilde{K}(b_T^*;\, \mu)\Big\}}_{\text{Perturbative Deviation}} \times \underbrace{\sqrt{M_S(b_T)}\, \exp\Big\{-\frac{y_1 - y_n}{2}\, g_K(b_T)\Big\}}_{\text{Non-Perturbative Deviation}}$$

Setting the cut-offs to the same value we have the simple relation:

$$\widetilde{D}^{\text{sqrt}}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_n\Big) = \widetilde{D}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_n\Big) \times \sqrt{M_S(b_T)}$$

The two definitions differ **ONLY IN THEIR NON PERTURBATIVE PART**, for a square root of the **SOFT MODEL M_s**

Strategy

We have to extract from the experimental data **3 non-perturbative functions**:

- The TMD model $M_j(b_T)$
- The Soft Kernel model (TMD evolution) $g_K(b_T)$
- The Soft Model $M_S(b_T)$

Collinear (Jet) Physics

Soft Physics

Once they are known, we should be able to describe **both** $e^+e^- \to H X$ and $e^+e^- \to H_A\, H_B\, X$data.

HOW?

1. Extract the UNPOLARIZED TMD FFs from 1-jet data, i.e. the functions $M_j(b_T)$ and $g_K(b_T)$.

2. Extract the Soft Model $M_S(b_T)$ from the 2-jet cross section, which now it is the only remaining unknown funtion.

3. Now everything is known. In principle, we can extract all the TMDs appearing up to 2-jet.

Andrea Simonelli

Summary

1. The processes can be classified in jet classes.

2. The Soft Factor that appears in the cross sections involving the TMDs is less universal than TMDs themselves. Furthermore, it is neither totally computable in perturbation theory, nor extractable independently from experimental data.

3. The Square Root Definition solves the Soft Factor problem in the 2-jet class.

4. In $e^+e^- \to HX$ the TMDs can only be defined through the Factorization Definition.

5. Adopting the Factorization Definition also in $e^+e^- \to H_A\, H_B\, X$ implies that we have to extract another non-perturbative function, the Soft Model.

6. The 1-jet and 2-jet class data can be combined using a well defined strategy.