Parton Reggeization Approach: prompt-J/ψ pair production at the LHC and developments towards NLO

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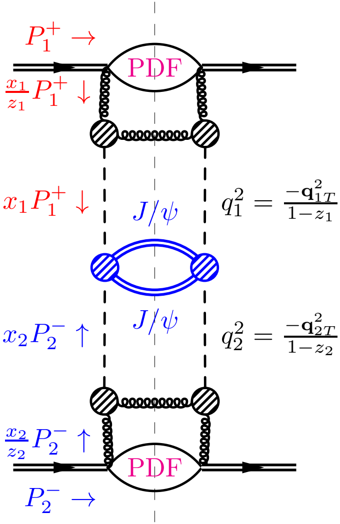
Outline

1. Introduction to PRA
2. Prompt-J/ψ pair production at the LHC in the LO of PRA with BFKL Resummation
3. Towards NLO: rapidity divergences in loop corrections
4. Towards NLO: combining real and virtual corrections and new doubly-logarithmic unintegrated PDF (UPDF)

Introduction to Parton Reggeization Approach

- Any k_T-factorization approach in QCD, which extends beyond $k_T \ll \mu$-region of "standard" TMD-factorization is necessarily based on factorization of QCD amplitudes in Multi-Regge limit.
- Due to large NLO corrections to BFKL kernel [Fadin, Lipatov, 98'] the k_T-factorization with fixed $\log 1/x$-accuracy is not phenomenologically applicable. One has to resum large (collinear, running-coupling, kinematical,...) corrections **in the UPDF**, which leads to variety of approaches to determine the UPDF.
- But the factorization formula and prescription to calculate gauge-invariant k_T-dependent hard-scattering coefficients is fixed by the Multi-Regge limit of QCD scattering amplitudes. This is the essence of PRA.

Derivation of the factorization formula and UPDF



- Factorization formula of PRA is based on *modified-MRK* approximation for matrix element of the **hard process** with emission of two additional partons.
- This *t-channel factorized* approximation is valid in two limits:
  - Collinear limit:

$$|\mathbf{q}_{1,2T}| \ll \mu, \ 0 < z_{1,2} < 1,$$

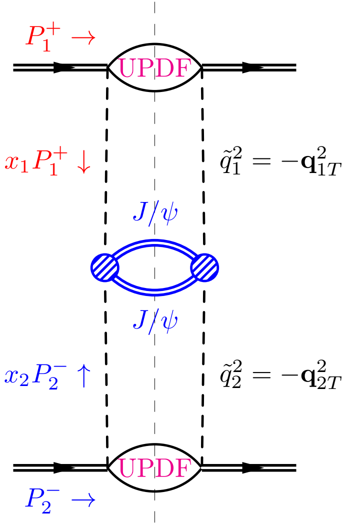
  - Multi-Regge limit:

$$|\mathbf{q}_{1,2T}| \sim \mu, \ z_{1,2} \ll 1.$$

See [Karpishkov, M.N., Saleev 2017] for details.

- The IR divergence at $z_{1,2} \to 1$ is regularized by *rapidity ordering condition* and collinear divergence at $|\mathbf{q}_{1,2T}| \to 0$ is regularized by *Sudakov Formfactor* which resums $\log^2(q_T/\mu)$ and $\log(q_T/\mu)$ corrections with NLL accuracy (for Color-singlet production).

Derivation of the factorization formula and UPDF



- As a result we obtain a usual High-Energy Factorization formula with flux factor $2Sx_1x_2$ for *off-shell initial-state partons*,
- and the KMR(W) UPDF, normalized on the usual PDF as:

$$\int_0^{\mu^2} dt \; \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2).$$

- The initial-state partons in our **hard part** are **Reggeized gluons** (dashed lines) or **Reggeized quarks**. It is calculated using Feynman rules of the **Gauge-Invariant EFT for MRK processes in QCD** [Lipatov, 95'].

This is how LO in α_s is computed in PRA.

Prompt J/ψ pair production.

Based on [Phys. Rev. Lett. 123, 162002 (2019)]

Basics of NRQCD factorization

- In Nonrelativistic-QCD factorization approach [Bodwin, Braaten, Lepage 95'], the hadronization of *perturbatively-produced* $Q\bar{Q}$ pair into heavy quarkonium is considered separately for each possible spin (S), orbital momentum (L) and color ($c = 1, 8$) state of the pair: ${}^{2S+1}L_J^{(c)}$. Transition of each $Q\bar{Q}$ state[4] into physical quarkonium is described by a non-perturbative factor – Long-Distance Matrix Element (LDME).
- Only finite number of intermediate states does contribute to the production, because LDMEs are organized into hierarchy according to their scaling w.r.t. velocity of heavy quarks in the bound state v. For J/ψ or $\psi(2S)$ up to $\mathcal{O}(v^2)$ these are:

$$ {}^3S_1^{(1)}, {}^1S_0^{(8)}, {}^3P_J^{(8)}, {}^3S_1^{(8)}. $$

  For P-wave states (χ_{cJ}) only ${}^3P_J^{(1)}$ and ${}^3S_1^{(8)}$ contribute up to this order.
- CS LDMEs can be related with wave-function of the bound state in the potential models, while CO LDMEs are fitted to data.

[4] As well as higher Fock states, e.g. $Q\bar{Q}g$.

NRQCD factorization: advantages and open problems

(Some) advantages:

- Well-defined factorization at NLO! Solves the problem with un-cancelled IR-divergences in production of ${}^3P_J^{(1)}$-states through mixing with ${}^3S_1^{(8)}$ state.
- Describes inclusive p_T-spectra of charmonia and bottomonia in hadro- and photoproduction [Butenschön, Kniehl 2011, ...]. Solves the "hard-tail" problem of p_T-distribution (mostly) through ${}^3S_1^{(8)}$-contribution.

(Selected) open problems:

- *Polarization puzzle:* for J/ψ, $\psi(2S)$, $\Upsilon(nS)$ at high-p_T, mostly transverse polarization is predicted (due to ${}^3S_1^{(8)}$ again!), while all data are consistent with unpolarized mixture of states. If one tries to fit also polarization data – consistency with photoproduction is lost.
- HQSS relations between LDMEs for J/ψ and η_c seem not to work [Butenschön, Kniehl, He 2014].
- Prompt J/ψ pair production?

J/ψ pair production: (Selected) theory results

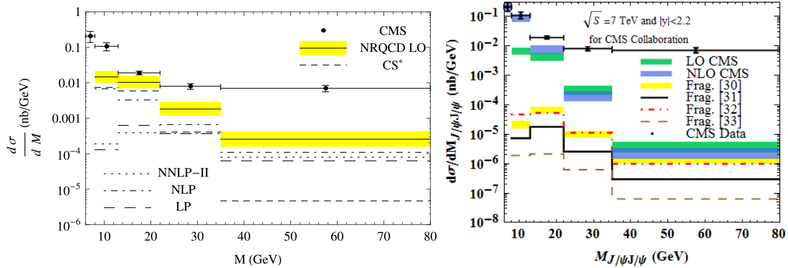
- The total cross-section is dominated by double-${}^3S_1^{(1)}$ contribution [Kartvelishvili, Esakiya 83'; Humpert, Mery 83'; Qiao 2002]
- The CO-states contribute, [Barger et.al. 96'] proposed double-J/ψ production as a test of CO mechanism. Relativistic corrections to $2^3S_1^{(1)}$ and $2^3S_1^{(8)}$-channels where also considered [Li, et.al. 2013].
- The full calculation in the LO of CPM, including all CO states and feed-down, was done by [He, Kniehl 2015]. The double-CO contributions are very important at large-$M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$.
- The Double Parton Scattering (DPS) contributes to the same kinematic region [Lansberg, et.al. 2015] ! But DPS contribution is flat or decreasing with $\Delta Y_{\psi\psi}$.
- The full NLO corrections in CPM for double-${}^3S_1^{(1)}$ channel has been calculated by [Sun, et.al. 2016].
- The CS-model computation in non-gauge-invariant k_T-factorization with CCFM-based UPDFs [Baranov, et.al. 2015] fails to describe data.
- And more...

J/ψ pair production: Experimental data

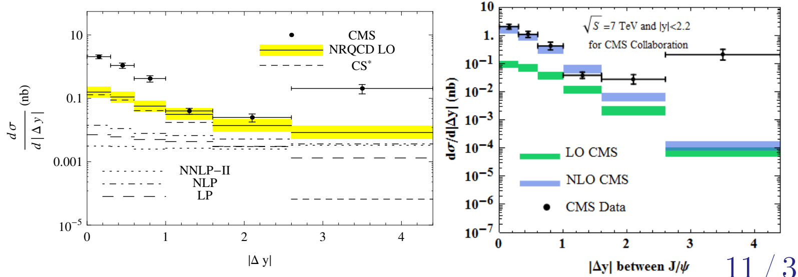
- First measurements of $2J/\psi$ by LHCb (pp @ 7 TeV) [LHCb 2012; 2017]. The p_T^ψ-spectrum at $2 < y^\psi < 4.5$ agrees reasonably with LO CPM+NRQCD [He, Kniehl 2015] with LDMEs fitted for inclusive single-J/ψ hadroproduction.
- Total cross-section measurements by D0 ($p\bar{p}$ @ 1.96 TeV) [D0 2014] are also reproduced in LO CPM + NRQCD.
- We will concentrate on CMS (pp @ 7 TeV) [CMS 2014] and ATLAS (pp @ 8 TeV) [ATLAS 2017] measurements which provide a rich set of spectra vs.: $M_{\psi\psi}$, $\Delta Y_{\psi\psi}$, $p_T^{\psi\psi}$ and $p_{T,\text{ lead.}}^\psi$.

Description in Collinear Parton Model

- The $M_{\psi\psi}$-spectrum (CMS-data, Full LO vs. $2^3S_1^{(1)}$ NLO CPM):



- The $\Delta Y_{\psi\psi}$-spectrum (CMS-data, Full LO vs. $2^3S_1^{(1)}$ NLO CPM):



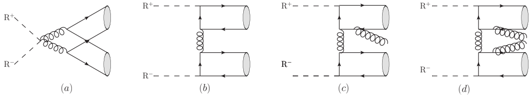
Fixed-order contributions in PRA

We have calculated contributions of **all** diagrams at $\mathcal{O}(\alpha_s^4)$ (LO) to all direct and feed-down partonic channels in PRA:

$$R_+(q_1) + R_-(q_2) \to c\bar{c}[m] + c\bar{c}[n],$$

with $m, n = {}^{2S+1}L_J^{(c)}$.

The dominant *asymptotics* at large $M_{\psi\psi}$ ($\Delta Y_{\psi\psi}$) is provided by diagrams with t-channel (Reggeized) gluon exchange **between** $c\bar{c}$-states. Partonic channels can be classified according to the order in α_s in which the t-channel gluon exchange first occur:



(b) **LT** : $m, n = {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(1,8)}$,

(c) **NLT**: $m = {}^3S_1^{(1)}$ and $n = {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(1,8)}$,

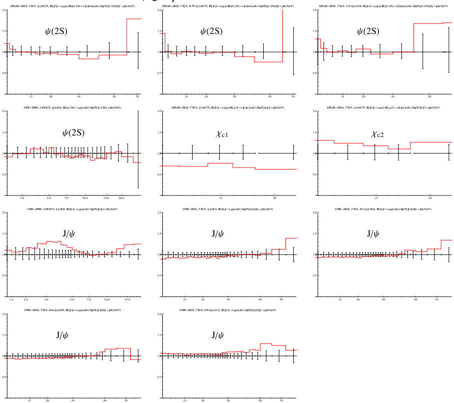
(d) **NNLT** : $m, n = {}^3S_1^{(1)}$.

LDME fit from single charmonium p_T-spectra

The CO LDMEs where fitted on data for inclusive charmonium hadroproduction at *LHC energies* in exactly the same approach as for the double-J/ψ calculation.

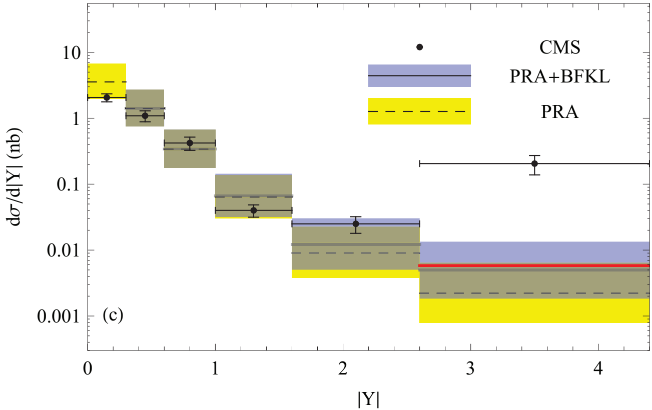
LDME, GeV^3
$\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}] \right\rangle = 1.16$
$M_0^{J/\psi} = 3.61 \times 10^{-2}$
$\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}] \right\rangle = 1.25 \times 10^{-3}$
$\left\langle \mathcal{O}^{\psi'}[^3S_1^{(1)}] \right\rangle = 0.76$
$M_0^{\psi'} = 2.19 \times 10^{-2}$
$\left\langle \mathcal{O}^{\psi'}[^3S_1^{(8)}] \right\rangle = 3.41 \times 10^{-4}$
$\left\langle \mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}] \right\rangle / m_c^2 = 4.77 \times 10^{-2}$
$\left\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \right\rangle = 5.29 \times 10^{-4}$

Only LHC data with $p_T > 10$ GeV where fitted. $\chi^2/\text{d.o.f.} \simeq 1$.



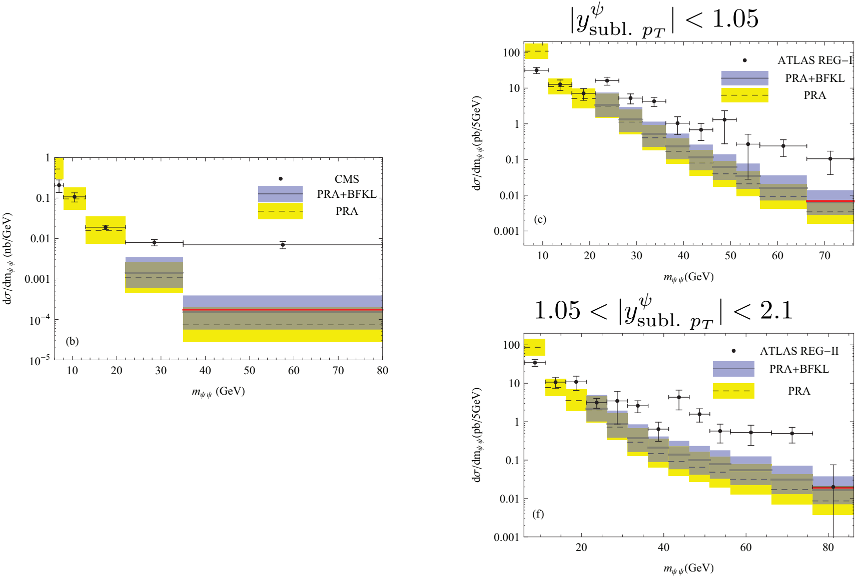
where $M_0^{\mathcal{H}} = \left\langle \mathcal{O}^{\mathcal{H}}[^3S_0^{(8)}] \right\rangle + \frac{R}{m_c^2} \left\langle \mathcal{O}^{\mathcal{H}}[^3P_0^{(8)}] \right\rangle$.

$\Delta Y_{\psi\psi}$ spectrum, CMS

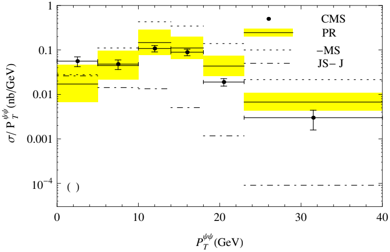


Yellow band and black dashed line – LO PRA.
Unfortunately, ATLAS provides only *fiducial* $\Delta Y_{\psi\psi}$-spectrum which is hard to compare with our predictions.

$M_{\psi\psi}$ spectra, CMS and ATLAS

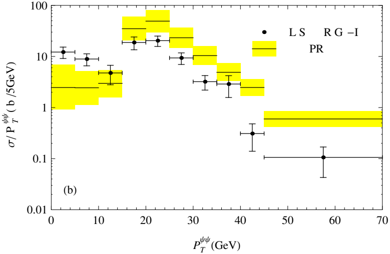


The $p_T^{\psi\psi}$- spectra, CMS and ATLAS

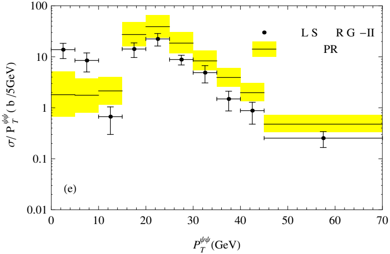


- Solid line – KMR(W) UPDF,
- Dashed line – Blümlein UPDF,
- Dash-dotted line – CCFM-based Jung-Hautmann UPDF (the result from PB UPDF will be similar).

$|y^{\psi}_{\mathrm{subl.}\ p_T}| < 1.05$

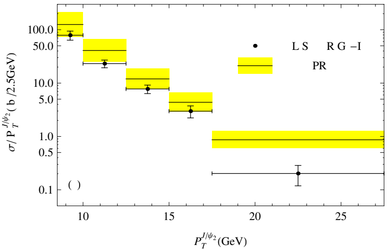


$1.05 < |y^{\psi}_{\mathrm{subl.}\ p_T}| < 2.1$

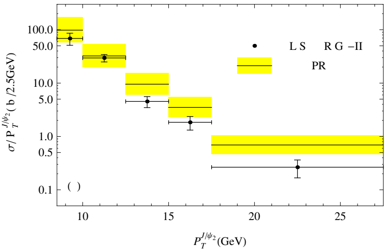


The $p_{T,\text{ lead.}}^{\psi}$ spectra from ATLAS

$|y^{\psi}_{\text{subl. } p_T}| < 1.05$

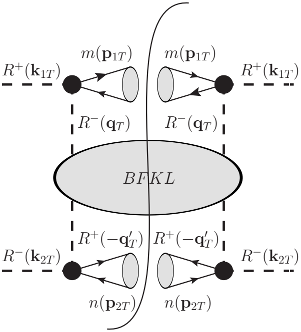


$1.05 < |y^{\psi}_{\text{subl. } p_T}| < 2.1$



BFKL-resummation contribution

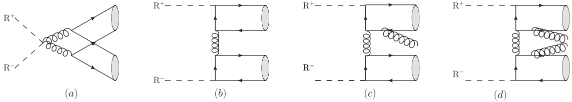
Overall agreement of LO PRA calculation with data is quite reasonable, except large $\mathcal{O}(10-100)$ deficit at large $M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$. But radiative corrections to **LT** and **NLT** contributions could be significant!



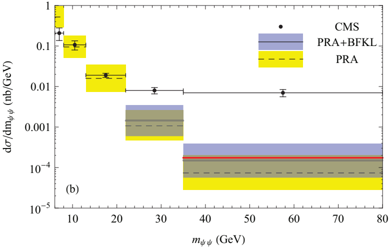
- We resum higher-order corrections $\sim (\alpha_s \Delta Y_{\psi\psi})^n$ to LT-channels using LLA BFKL Green's function with suitable BLM-type renormalization-scale setting [Brodsky, et.al., 99'] to take into account large running-coupling effects.
- Resummation is performed for $\Delta Y_{\psi\psi}$ and $M_{\psi\psi}$-spectra. For other spectra effect is negligible.
- The LO $R_+ R_- \to c\bar{c}[n]$ impact-factors are well-known [Kniehl, Vasin, Saleev 2006].

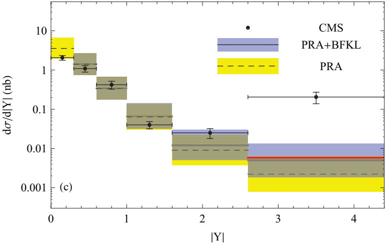
NLT contribution

- Since $\left\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}] \right\rangle \sim (10^2 - 10^3) \times \left\langle \mathcal{O}^{J/\psi}[^3L_J^{(8)}] \right\rangle$, the NLT contribution could be numerically significant.
- The
$$R_+ + R_- \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g \tag{1}$$
amplitude **does not** have any singularities for $E_g \to 0$ or $k_{Tg} \to 0$ since $\mathcal{M}\left(R_+ + R_- \to c\bar{c}\left[{}^3S_1^{(1)}\right]\right) = 0$.
- There is **no** *rapidity divergence* for integration over rapidity of gluon in (1), so no double-counting with BFKL resummation or UPDF.
- So we can construct *gauge-invariant and IR-finite* large-$\Delta Y_{\psi\psi}$ asymptotics for $\mathcal{O}(\alpha_s^5)$ NLT squared amplitudes by replacing ordinary t-channel gluon with Reggeized one in the diagram (c).



Combined effect on $M_{\psi\psi}$ and $\Delta Y_{\psi\psi}$ spectra





- Effect of BFKL-resummation is significant, up to a factor of two.
- Large $\mathcal{O}(100)$ K-factors are found in some NLT channels in the last $\Delta Y_{\psi\psi}$-bin. The effect on direct production is +45%, but after addition of feeddown the overall effect of NLT-contributions reduces to +16% (the thick red line).
- Apparent growth of CMS cross-section with $\Delta Y_{\psi\psi}$ is a complete mystery. One needs enormous Pomeron intercept ($> \alpha_P^{\text{LL BFKL}}$???) to fit this.
- Effects in ATLAS $M_{\psi\psi}$ spectra are roughly the same (see corresponding plots on slide 15).

Loop corrections in Lipatov's EFT.

Mostly based on [Nucl.Phys., B946, 114715 (2019)]

Eikonal denominators in the induced vertices

A closer look at $R_\pm g$-interaction [Lipatov 95'; 97'; Bondarenko, Zubkov 2018]:

$$S_{\rm int.} = \int dx \frac{i}{g_s} \,{\rm tr}\left[R_+(x) \partial_\perp^2 \partial_- \left(W_{x_+}\left[A_-\right] - W^\dagger_{x_+}\left[A_-\right] \right) + (+ \leftrightarrow -) \right],$$

where $\partial_\pm = 2\partial/\partial x_\mp$, $x_\pm = x^\pm = (n_\pm x) = x^0 \pm x^3$, fields $R_\pm$ satisfy MRK constraint $\boxed{\partial_\mp R_\pm(x) = 0}$ and

$$\begin{aligned} W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] &= P \exp\left[\frac{-ig_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T) \right] \\ &= \left(1 + i g_s \partial_\pm^{-1} A_\pm \right)^{-1}, \end{aligned}$$

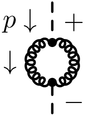
so that $\boxed{\partial_\pm^{-1} \to -i/(k_\pm + i\varepsilon)}$ in the Feynman rules.

$\Rightarrow$ multiple *induced vertices* with light-cone (Eikonal) denominators appear. Pole prescription is fixed by Hermitian form of $R_\pm g$-interaction.

Rapidity divergences and regularization

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, loop integrals in EFT contain the light-cone (Rapidity) divergences:

$$\Pi^{(1)}_{ab} = q\downarrow$$



$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\Pi^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2}\mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

Covariant regularization

To regularize RDs covariantly one have to “tilt” Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis et.al. 2012-2013; Collins 2011]:

$$S_{\text{int.}}^{(\text{reg.})} = \int dx \frac{i}{g_s} \,\text{tr}\left[R_+(x) \partial_\perp^2 \tilde{\partial}_- \left(W_{\tilde{x}_+}\left[\tilde{A}_- \right] - W^\dagger_{\tilde{x}_+}\left[\tilde{A}_- \right] \right) + (+ \leftrightarrow -) \right],$$

where $\boxed{\tilde{x}_\pm = x_\pm + r \cdot x_\mp}$ with $0 < r \ll 1$, and modify the kinematic constraint [M.N. 2019]:

$$\boxed{\tilde{\partial}_\mp R_\pm(x) = 0,}$$

$\Leftrightarrow \tilde{p}_\mp = p_\mp + r \cdot p_\pm$ for $R_\pm$ (Necessary to regularize $R_+ R_+ \to R_- R_-$ Green’s function at one loop!).

Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.

Integrals which do not have log-divergence *before expansion in* ϵ may still contain the power-like dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	...
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	...
3	...	...	...	...

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

State of the art

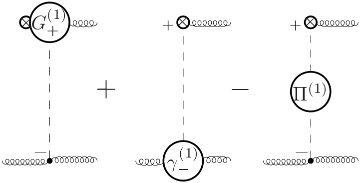
- LO BFKL kernel comes-out as rapidity-divergent part of $R_+ R_+ \to R_- R_-$ Green's function [Bartels, Lipatov, Vacca 2012]
- Known QCD results for one-loop impact-factors of gluon and quark with one scale of virtuality are reproduced [Hentschinski, Sabio Vera, Chachamis et.al. 2012-2013]
- Two-loop Regge trajectory of a gluon is reproduced [Hentschinski, Sabio Vera, Chachamis et.al. 2013]
- Consistency of Reggeized quark formalism is verified at one loop on example of the process $\gamma\gamma \to q\bar{q}$ [M.N., Saleev 2017]
- New one-loop impact-factors $\mathcal{O}(q) + R_+(q_1) \to g(q+q_1)$ (with $\mathcal{O}(x) = \mathrm{tr}[G_{\mu\nu}G^{\mu\nu}]$) and $\gamma^\star(q) + Q(q_1) \to q(q+q_1)$ with additional scale $Q^2 = -q^2$ besides $q_1^2 = -\mathbf{q}_{1T}^2$ are computed [M.N. 2019] and consistency of Regge limits of one-loop amplitudes:

$$g(P) + \mathcal{O}(q) \to g(P-q_1) + g(q+q_1),$$
$$\gamma(P) + \gamma^\star(q) \to q(P-q_1) + \bar{q}(q+q_1),$$

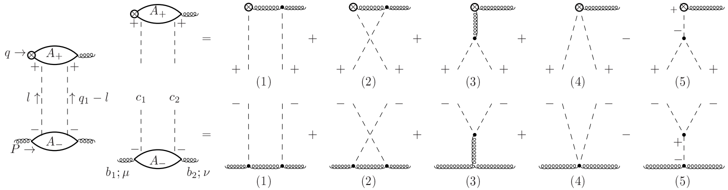
  between EFT and QCD is checked.
- NLO BFKL is in progress...

Contributions in the EFT, gluon case

One-Reggeon contribution (*negative signature*, Re+Im parts @ 1 loop, $\log r$-divergences cancel):



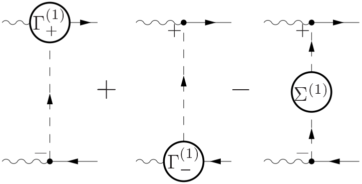
Two-Reggeon contribution (*positive signature*, does not contribute due to color):



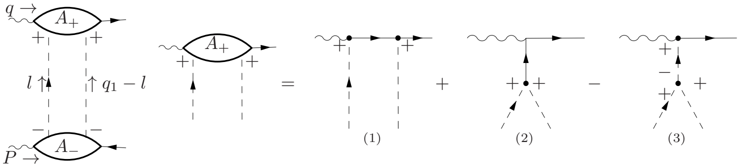
The one-Reggeon contribution reproduces QCD result exactly.

Contributions in the EFT, photon case

One-Reggeon contribution (*positive signature*, Re+Im parts @ 1 loop, $\log r$-divergences cancel):



Two-Reggeon contribution (*negative signature*, Im part @ 1 loop):



Sum of one- and two-Reggeon contributions reproduces QCD result exactly.

Combining real and virtual corrections, new doubly-logarithmic UPDF

Issues in combining real and virtual corrections

- Traditionally in BFKL-physics one ignores longitudinal-momentum conservation and integrates over $q^{\pm}$-components of incoming Reggeons. Then an NLO cross-section or impact-factor becomes:

$$\sigma_{\rm NLO}^{\rm real}(\mathbf{q}_{T1})+\sigma_{\rm NLO}^{\rm virt.}(\mathbf{q}_{T1})=\sigma_{\rm NLO}^{\rm finite}(\mathbf{q}_{T1})+\log r\times K_{\rm BFKL}(\mathbf{q}_{T1},\mathbf{q}'_{T1})\underset{\mathbf{q}'_{T1}}{\otimes}\sigma_{\rm LO}(\mathbf{q}'_{T1})$$

- It is easy to run into very pathological factorization scheme by simply factorizing-out the divergence in arbitrary way. Additional physical insight is needed.
- Using the fact, that rapidity divergences cancel in virtual and real contributions *separately* one can try to introduce an improved treatment of real corrections by relaxing some of BFKL-approximations. The guiding principle is to not to spoil cancellation of IR divergences (c.f. HEJ approach) and *collinear factorization.*
- Very similar ideas lead to reasonably-behaved factorization scheme for the forward hadron production in CGC approach [B. Ducloe et.al. 2017], resolving the problem of negative cross-section at NLO.

Kinematically-improved doubly-logarithmic UPDF

- We where able to restore conservation of large light-cone component of momentum in the usual BFKL-ladder (with usual Lipatov's vertices) and introduce an improved approximation for the virtuality of t-channel parton into propagators: $q_i^2 = -\mathbf{q}_{Ti}^2/(1-z_i)$ in such a way, that evolution is still analytically tractable. In the Mellin space ($\int dx\ x^{-N}\Phi(x)$) ladder with n real emissions is expressed simply as:

$$\frac{1}{N^n}\left(K_{\mathrm{BFKL}}^{\mathrm{real}}\right)^{\otimes_T n}.$$

- $\Rightarrow$ We solve iteratively a usual BFKL-equation in $(N, \mathbf{x}_T)$-space:

$$\Phi(N,\mathbf{x}_T) = 1 + \frac{\bar{\alpha}_s}{N}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon(-\epsilon)}\int d^{2-2\epsilon}\mathbf{y}_T\ \Phi(N,\mathbf{y}_T)\times \left[(\mathbf{x}_T^2)^\epsilon\delta(\mathbf{x}_T-\mathbf{y}_T) - \frac{\epsilon\Gamma(1-\epsilon)}{\pi^{1-\epsilon}((\mathbf{x}_T-\mathbf{y}_T)^2)^{1-2\epsilon}}\right],$$

  where $\bar{\alpha}_s = \alpha_s N_c/\pi$, to obtain a kernel $\Phi(N,\mathbf{x}_T)$, relating PDF and UPDF.

Kinematically-improved doubly-logarithmic UPDF

- Collinear divergences factorize from the transition kernel Φ as in [Catani, Hautmann, 94'] (checked up to $\mathcal{O}(\alpha_s^{10})$!):

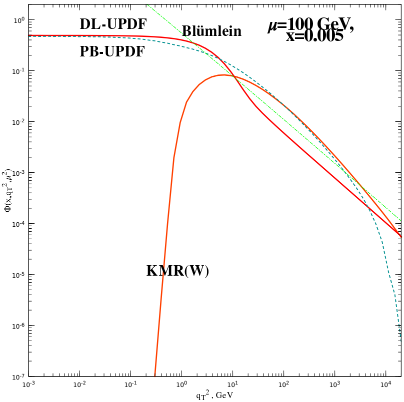
$$Z_{\text{coll.}} = \exp\left[-\frac{1}{\epsilon}\int_0^{\bar{\alpha}_s S_\epsilon} \frac{d\alpha}{\alpha}\gamma_N(\alpha)\right],\ \gamma_N(\alpha) = \gamma_1(N)\alpha + \gamma_2(N)\alpha^2 + \ldots$$

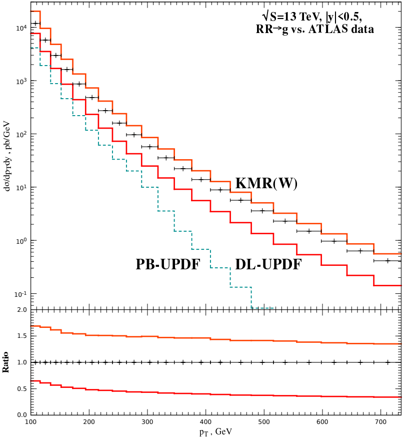
  where $S_\epsilon = \exp[\epsilon(-\gamma_E + \log 4\pi)]$ for $\overline{MS}$-scheme.
- The LL BFKL series [Jaroszewicz 82'] of corrections to DGLAP anomalous dimension γ_N is reproduced: $\gamma_1 = 1/N$, $\gamma_2 = \gamma_3 = 0$, $\gamma_4 = 2\zeta_3/N^4$, $\gamma_5 = 2\zeta_5/N^5, \ldots$
- In *doubly-logarithmic apprimation* (corrections start at $\mathcal{O}(\alpha_s^3)$!), the finite part of Φ can be expressed as:

$$\boxed{\Phi_{\text{ren.}}(N, \mathbf{x}_T, \mu) \underset{\text{DLA}}{\simeq} \exp\left[-\bar{\alpha}_s(\mu)\frac{\log(\mu^2\bar{\mathbf{x}}_T^2)}{N}\right] \times F_{NP}(\mathbf{x}_T),}$$

  where $\bar{\mathbf{x}}_T^2 = \mathbf{x}_T^2 e^{2\gamma_E}/(4\pi)^2$.
- To improve convergence of Fourier-transform to $\mathbf{q}_T$-space we add a non-perturbative factor: $F_{NP} = e^{-\Lambda^2\mathbf{x}_T^2}$. *It has no effect on* $q_T \gg \Lambda$ *or cross-sections.* Possibly the Sudakov FF can be added here as well.

Some numerical results





- LO MSTW-08 PDFs are used to generate DL UPDF, $\Lambda = 1$ GeV.
- UPDF normalization property holds at small-x!
- The slope of DL UPDF at large q_T is the same as that of Blümlein UPDF [Collins, Ellis 91'; Blümlein 94'], which is expectable since their approach is very similar, but fully in q_T-space.

Conclusions

- The loop corrections in Lipatov's EFT are well understood both at one and two-loop level.
- The issue of combining real and virtual corrections requires a lot of attention, longitudinal momentum conservation and collinear factorization are guiding principles
- The doubly-logarithmic approach, resumming $\alpha_s \log(1/x) \log \mathbf{x}_T^2$ is a convenient starting point to define phenomenologically -reasonable UPDF

Thank you for your attention!