Transversal momentum dependence and di-jets in p-p, p-Pb and Pb-Pb collisions

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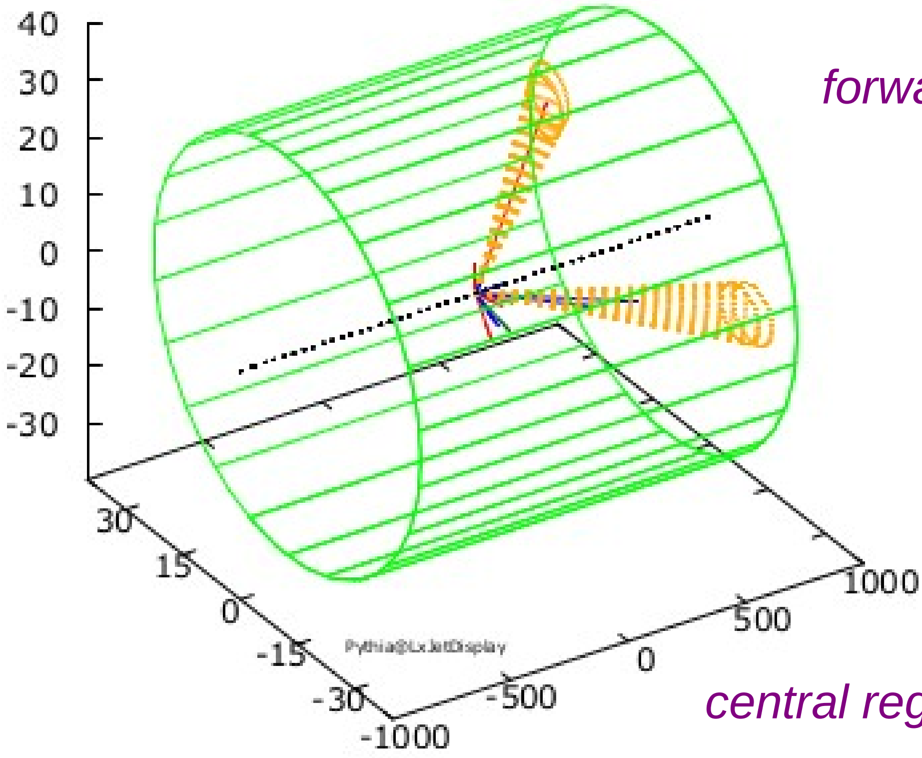
HORIZON 2020

NCN

p-p and p-Pb

$p - A$ (dilute-dense) forward-forward di-jets



forward region

central region

The collisions we consider:

e.g minimal p_T 28 GeV

both jets are forward → $4 > y > 2.7$

From: Piotr Kotko
LxJet

There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not. Kinematics relevant for saturation

J. Albacete, C, Marquet
Phys.Rev.Lett. 105 (2010) 162301

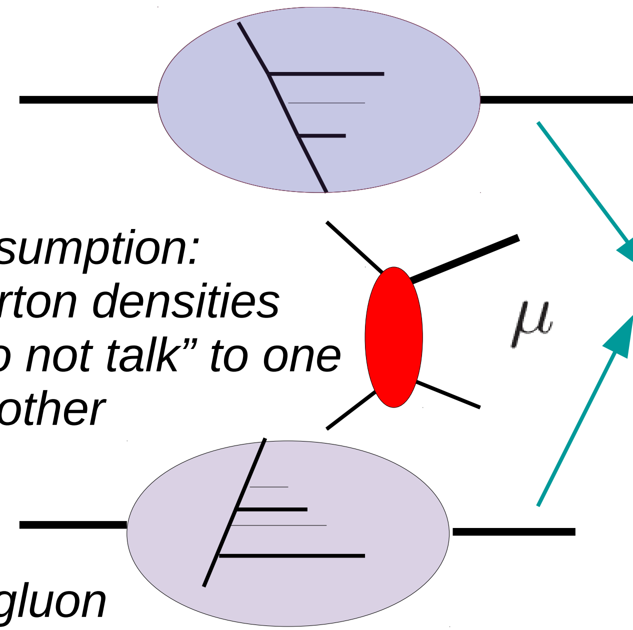
k_T factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{c,d}\int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2s)^2}\overline{|\mathcal{M}_{g^*g^*\to cd}|^2}\,\mathcal{F}_A(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_B(x_2,k_{2t}^2,\mu^2)\frac{1}{1+\delta_{cd}}$$

originally proposed for gg → QQ

developed for low x

rapidities and pt
of produced jets



assumption:
parton densities
“do not talk” to one
another

decreasing longitudinal
momentum fractions
of off-shell partons
BFKL like

decreasing longitudinal
momentum fractions
of off-shell partons
BFKL like

μ — *hard scale e.g. average of p_t of jets*

k_{t1}, k_{t2} — *transversal momentum of incoming gluon*

x — *longitudinal momentum of incoming gluon*
In this framewoμrk of x of comparable values

L.V. Gribov, E.M. Levin, M.G. Ryskin
Phys.Rept. 100 (1983) 1-150

S. Catani, M. Ciafaloni, F. Hautmann
Nucl.Phys. B366 (1991) 135-188

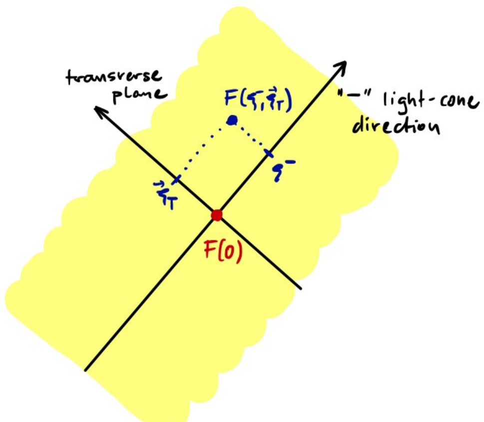
Helicity based method for any tree process
A. van Hameren, P. Kotko, K. Kutak
JHEP 1301 (2013) 078

Definition of TMD – gauge links

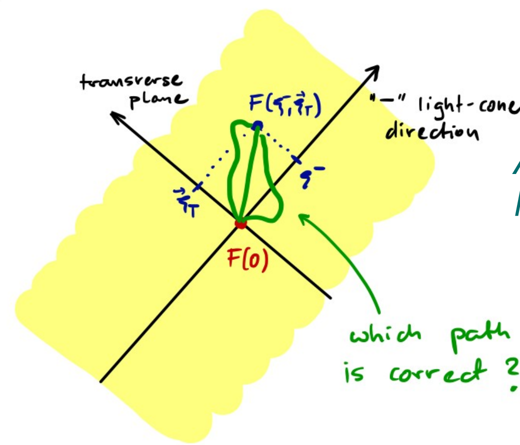
The formula for HEF is strictly valid for large transversal momentum and was obtained in a specific gauge. Ultimately one want to go beyond this.

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\left\langle P\right|\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}\left|P\right\rangle$$

naive definition of gluon distribution



from P. Kotko, Bialasówka 2019



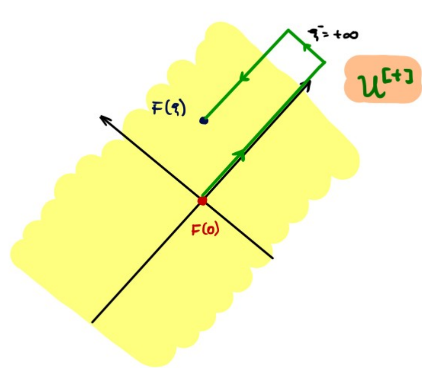
A. Belitsky, X. Ji, F. Yuan Nucl.Phys. B656 (2003) 165-198

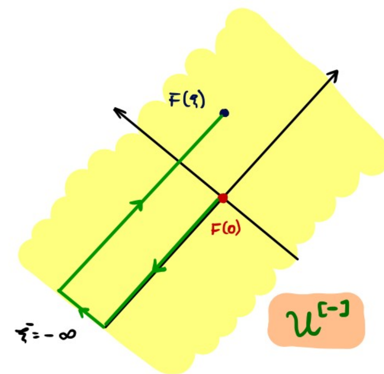
The generalization is achieved via gauge link which accounts for exchange of collinear gluons between the soft and hard parts renders the gluon density gauge invariant….

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{\left(2\pi\right)^3 P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\left\langle P\right|\mathrm{Tr}\left\{\hat{F}^{i+}\left(0\right)\mathcal{U}_{C_1}\hat{F}^{i+}\left(\xi\right)\mathcal{U}_{C_2}\right\}\left|P\right\rangle$$

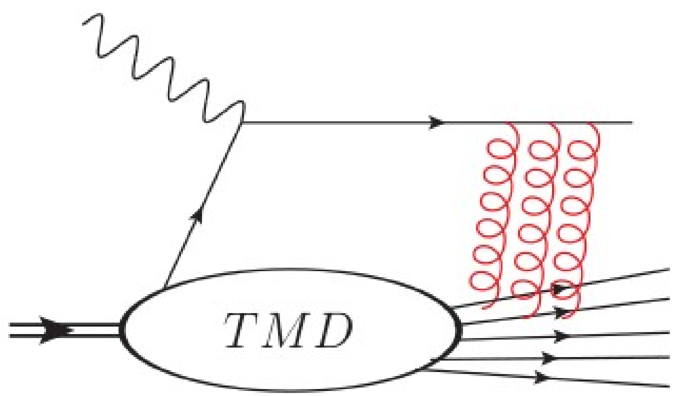
Definition of TMD – gauge links

Two basic structures arise:



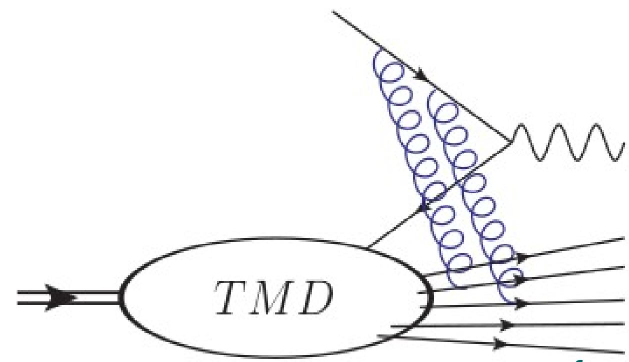


Semi Inclusive DIS



final state interactions

Drell-Yan



initial state interactions

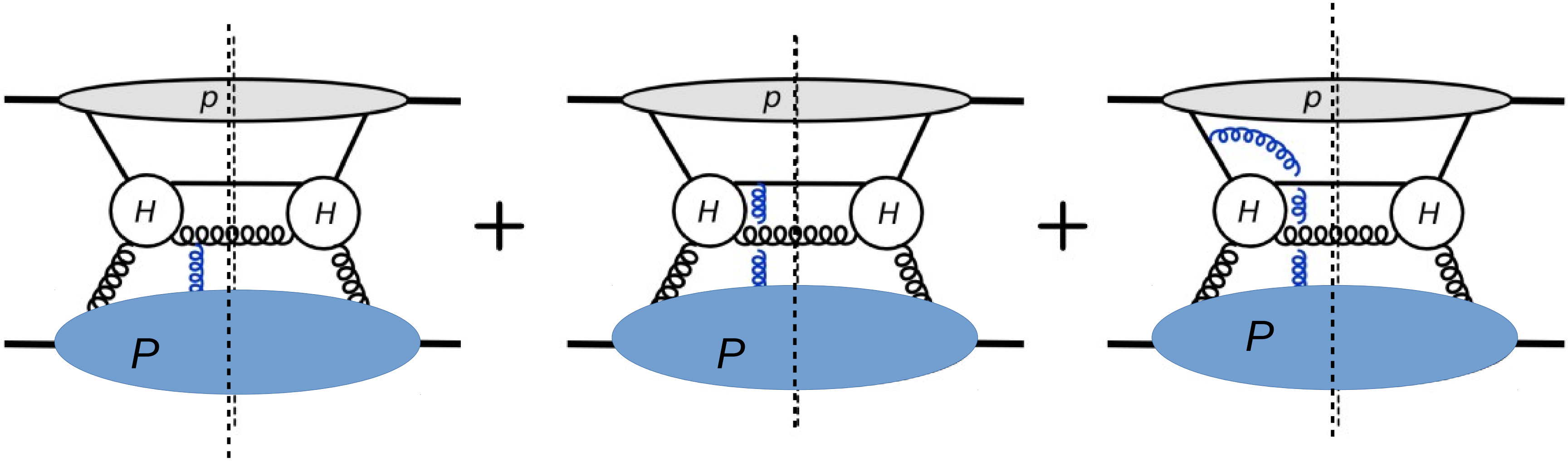
from R. Boussarie Initial Stages 2019

$$\Phi_q^{[+]}(x,p_T) = \int \frac{\mathrm{d}(\xi\cdot P)\,\mathrm{d}^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle H|\,\overline{\psi}(0)\,\mathcal{U}^{[+]}\,\psi(\xi)\,|H\rangle$$

C.J. Bomhof, P.J. Mulders, F. Pijlman Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{U}^{[\pm]} = U^n_{[(0^-,\mathbf{0}_T);(\pm\infty^-,\mathbf{0}_T)]}\,U^T_{[(\pm\infty^-,\mathbf{0}_T);(\pm\infty^-,\boldsymbol{\infty}_T)]}\,U^T_{[(\pm\infty^-,\boldsymbol{\infty}_T);(\pm\infty^-,\boldsymbol{\xi}_T)]}\,U^n_{[(\pm\infty^-,\boldsymbol{\xi}_T);(\xi^-,\boldsymbol{\xi}_T)]}$$

Gauge links and dijets



+ *similar diagrams with 2,3,….gluon exchanges.*

All this need to be resummed

C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

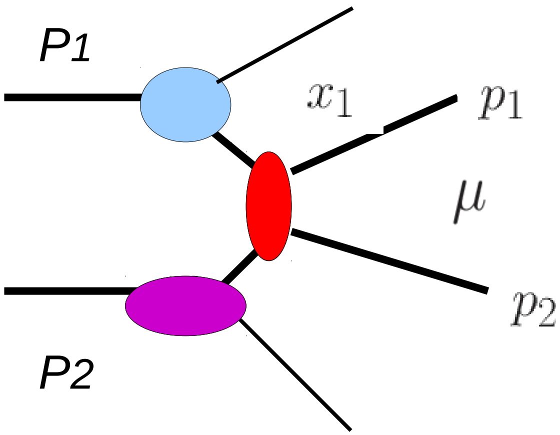
$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle P| \operatorname{Tr}\left\{ \hat{F}^{i+}(0)\,\mathcal{U}_{C_1} \hat{F}^{i+}(\xi)\,\mathcal{U}_{C_2} \right\} |P\rangle$$

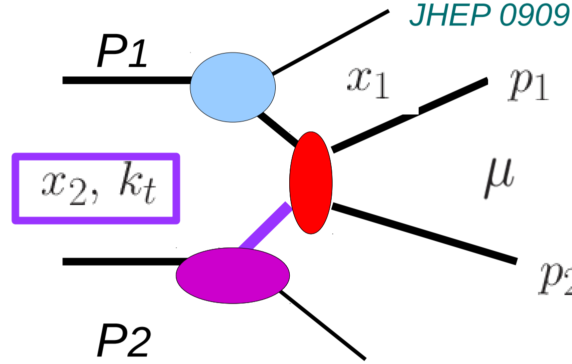
Hard part defines the path of the gauge link

$$\mathcal{U}^{[C]}(\eta;\xi) = \mathcal{P}\exp\left[-ig \int_C \mathrm{d}z \cdot A(z) \right]$$

Improved Transversal Momentum Dependent Factorization

$$\frac{d\sigma^{P_1P_2\to \text{dijets}+X}_{\text{SPS}}}{dy_1dy_2dp_{1t}dp_{2t}d\Delta\phi} = \frac{p_{1t}p_{2t}}{8\pi^2(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/P_1}(x_1,\mu^2)\,|\overline{\mathcal{M}_{ag^*\to cd}}|^2 \quad \mathcal{F}_{g/P_2}(x_2,k_t^2)\frac{1}{1+\delta_{cd}}$$





A, Dumitru, A. Hayashigaki J. Jalilian-Marian Nucl.Phys. A765 (2006) 464-482M. Deak, F. Hautmann, H. Jung, K. Kutak JHEP 0909 (2009) 121

Improved because it accounts for saturation and for k_t in ME

Generalization of hybrid formula but no kt in ME

Fabio Dominguez, Bo-Wen Xiao, Feng Yuan Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan Phys.Rev. D83 (2011) 105005

Appropriate in back-to-back configuration

Using HEF motivated sum over polarization for low x gluons we included kt in ME

Conjecture P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

Appropriate in any configuration

Can be obtained from CGC

T. Altinoluk, R. Boussarie, Piotr Kotko JHEP 1905 (2019) 156

gauge invariant amplitudes with kt and TMDs

Example for g* g → g g

$$\frac{d\sigma^{pA\to ggX}}{d^2P_td^2k_tdy_1dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\; x_1f_{g/p}(x_1,\mu^2)\sum_{i=1}^{6}\mathcal{F}_{gg}^{(i)}H_{gg\to gg}^{(i)}$$

Improved Transversal Momentum Dependent Factorization

$$\frac{d\sigma_{\rm SPS}^{P_1P_2\to {\rm dijets}+X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t}p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \quad \mathcal{F}_{g/P_2}(x_2,k_t^2)\frac{1}{1+\delta_{cd}}$$

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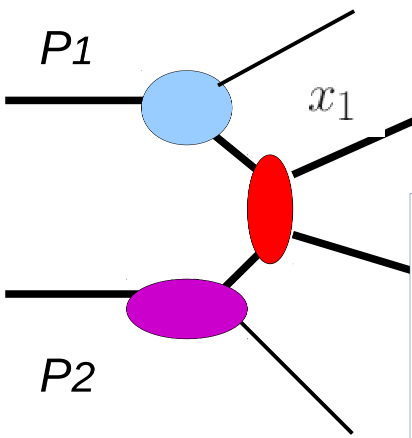

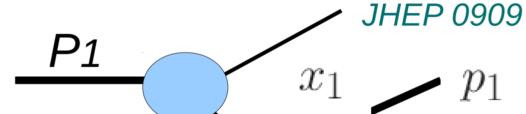

Improved
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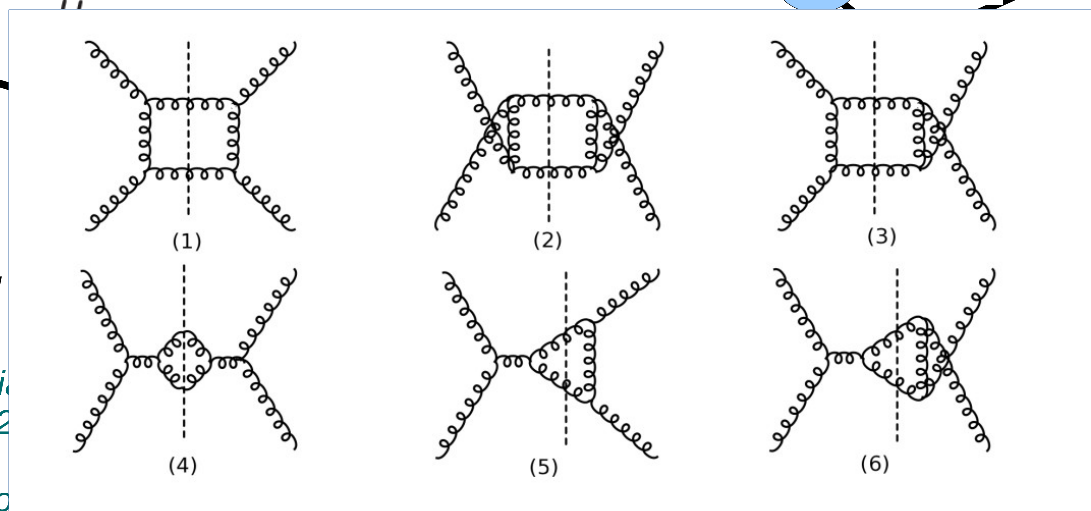

Generalization of hybrid
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Phys.Rev.Lett. 106 (2011) 022

F. Dominguez, C. Marquet, Bo
Phys.Rev. D83 (2011) 105005
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Marquet , E. Petreska , S. Sapeta,
A. van Hameren, JHEP 1509 (2015) 106
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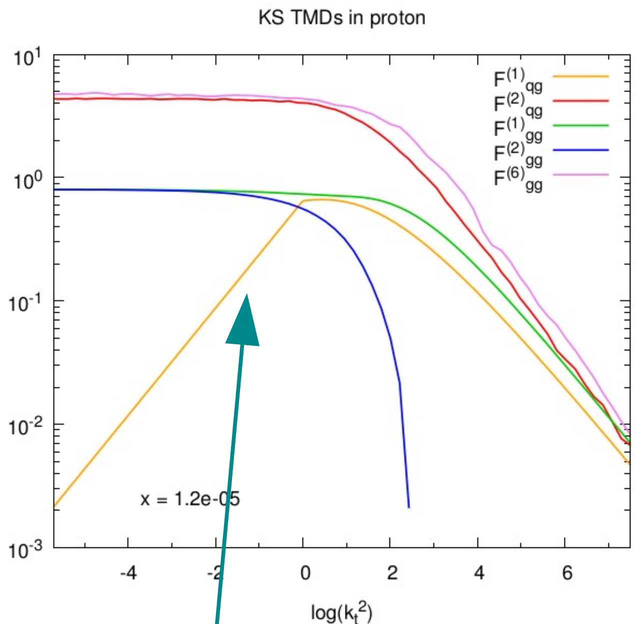
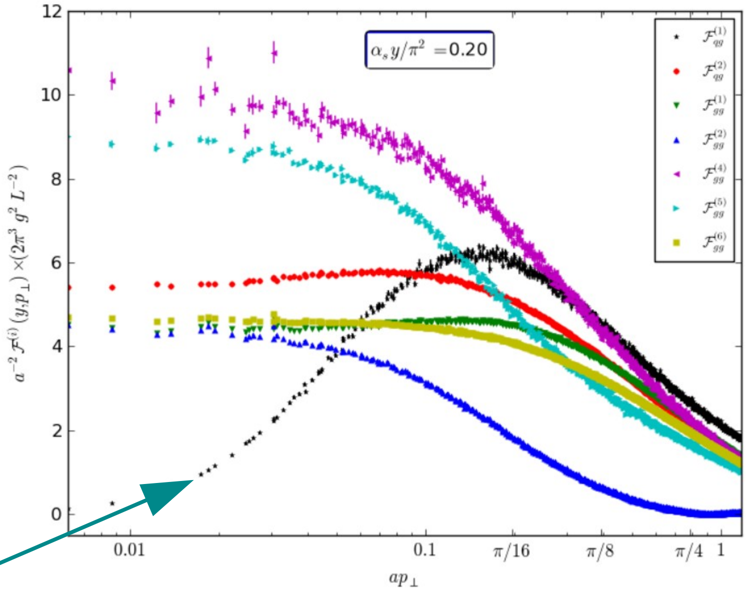
Can be obtained from CGC
T. Altinoluk, R. Boussarie, Piotr Kotko JHEP 1905 (2019) 156

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\; x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

Plots of ITMD gluons





Calculation – in large Nc approximation with analitic model for dipole gluon density – all gluons can be calculated from the dipole one. KS gluon used.

Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren
JHEP 1612 (2016) 034

Standard HEF gluon density

Obtained from solutions of evolution equation which accounts for finite Nc. JIMWLK equation used to obtain Evolved gluon densities.

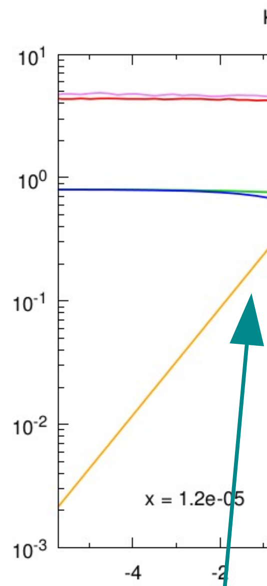
The JIMWLK equation is a renormalization group equation for the Wilson lines, obtained by integrating out the quantum fluctuations at smaller and smaller Bjorken-x.
C. Marquet, E. Petreska, C. Roiesnel
JHEP 1610 (2016) 065

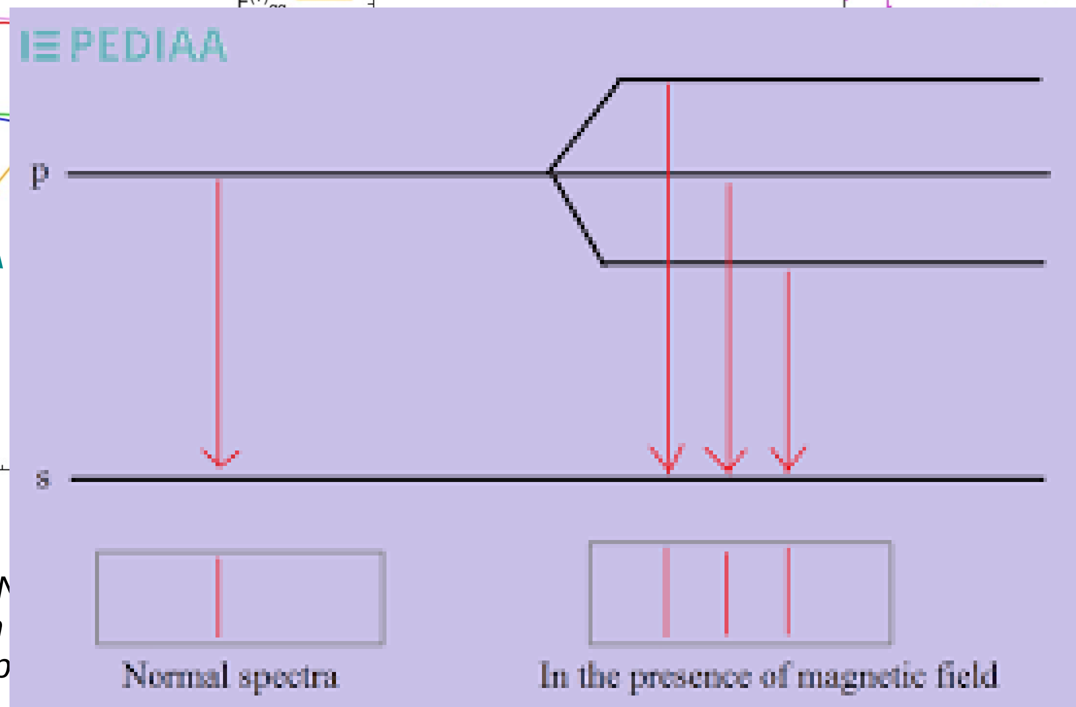
The other densities are flat at low k_t → less saturation

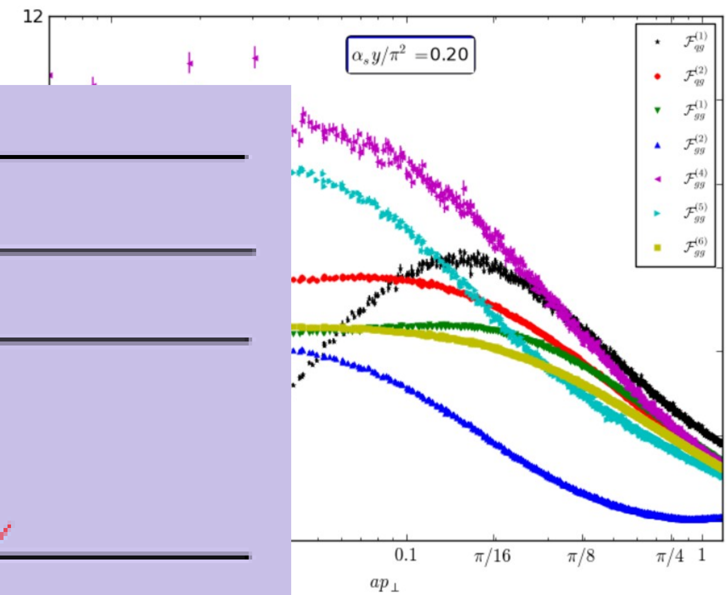
Not negligible differences at large k_t → differences at small angles

Plots of ITMD gluons

rough analogy to splitting of spectral lines
in presence of a new scale







Calculation – in large N
model for dipole gluon
calculated from the dip

Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren
JHEP 1612 (2016) 034

Standard HEF gluon density

of evolution equation which
MWLK equation used to obtain
Evolved gluon densities.

The JIMWLK equation is a renormalization group equation for the Wilson lines, obtained by integrating out the quantum fluctuations at smaller and smaller Bjorken-x.
C. Marquet, E. Petreska, C. Roiesnel
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The other densities are flat at low k_t → less saturation

Not negligible differences at large k_t → differences at small angles

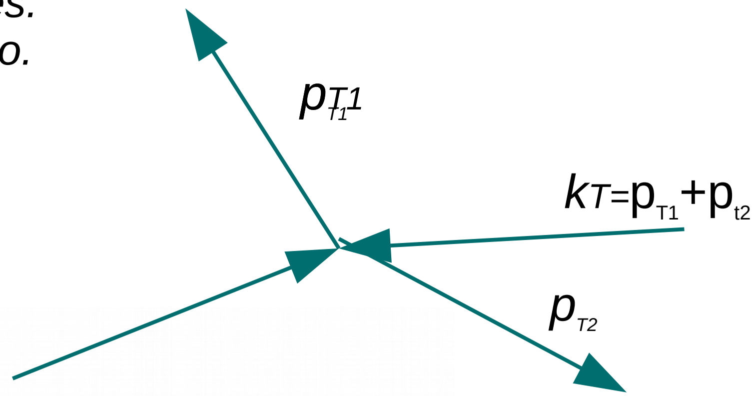
Forward physics and Sudakov form factor

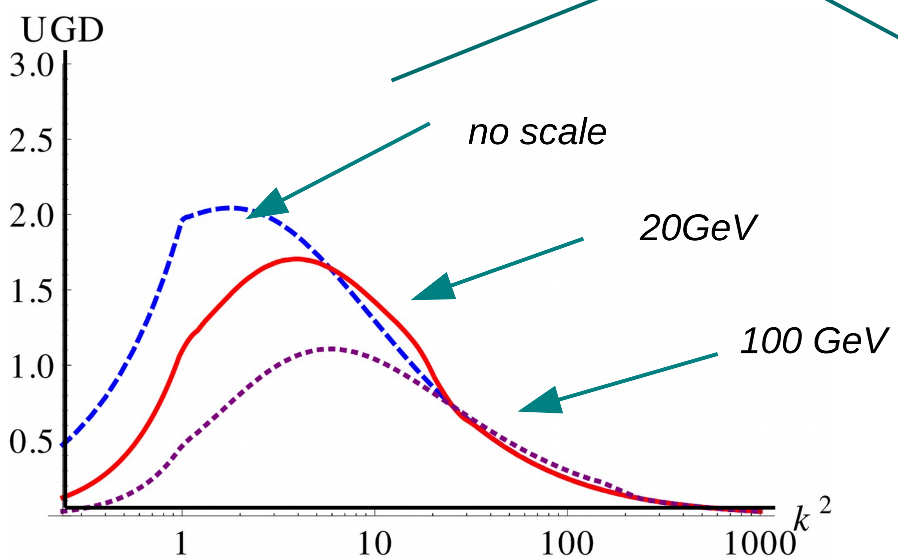
Sudakov - no emision probability between two hard scales. Standard thing in Monte Carlo.

Applies when

$|p_{T1}+p_{t2}|^2 << ½ \, |p_{T1}-p_{t2}|^2$

Low kt gluons are suppressed. The conservation of probability leads to change of shape of gluon density which depends on the hard scale





$$T_s(\mu^2, k^2) = \exp\left(- \int_{k^2}^{\mu^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} dz' P_{a'a}(z') \right)$$

A. H. Mueller, Bo-Wen Xiao, Feng Yuan
Phys.Rev.Lett. 110 (2013) no.8, 082301

Phys. Rev. D 88, 114010 (2013)
A. H. Mueller, Bo-Wen Xiao, Feng Yuan

Phys.Lett. B737 (2014) 335-340
A. van Hameren, P. Kotko, K. Kutak, S. Sapeta

K. Kutak
Phys.Rev. D91 (2015) no.3, 034021

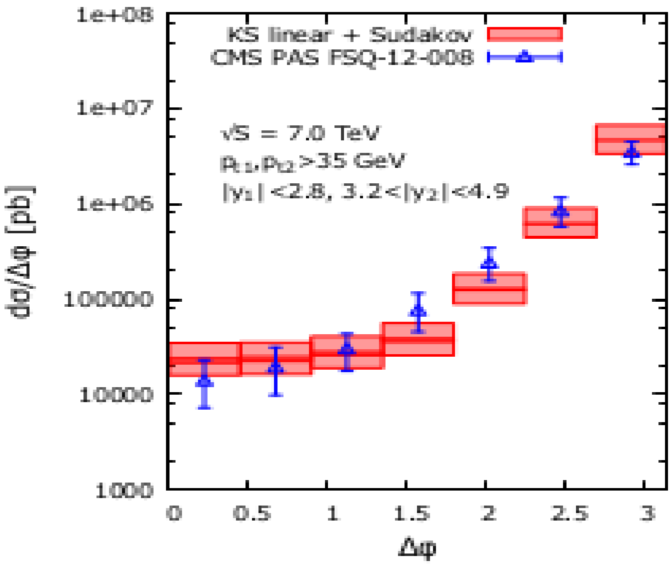
I. Balitsky, A. Tarasov
JHEP 1510 (2015) 017

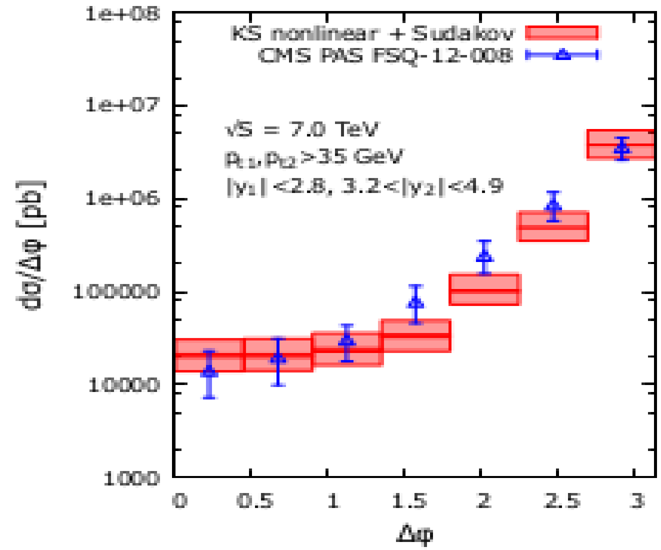
A.H. Mueller, Lech Szymanowski, Samuel Wallon, Bo-Wen Xiao, Feng Yuan
JHEP 1603 (2016) 096

Nucl.Phys. B921 (2017) 104-126
B. Xiao, F. Yuan, J. Zhou.

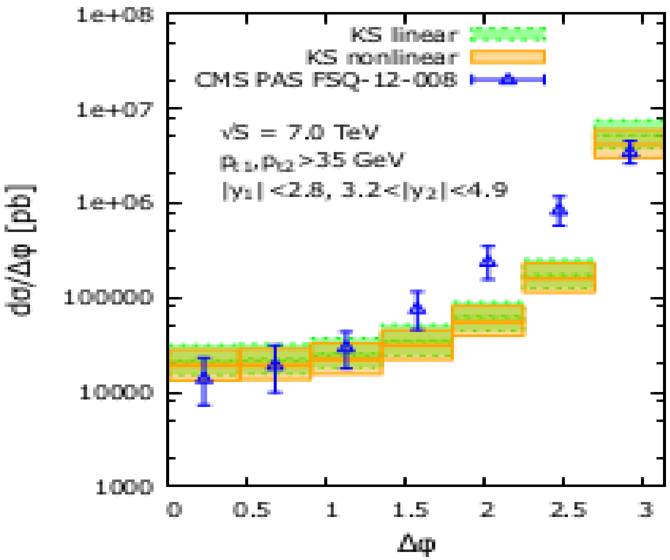
Motivated by Catani, Ciafaloni, Fiorani Marchesini and Kwiecinski, Kimber, Martin, Stasto.

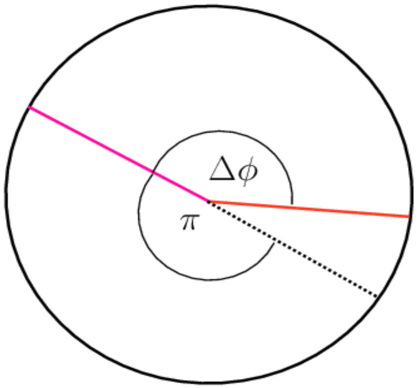
Decorelations inclusive scenario-central forward





No saturation…
visibleSudakov effects





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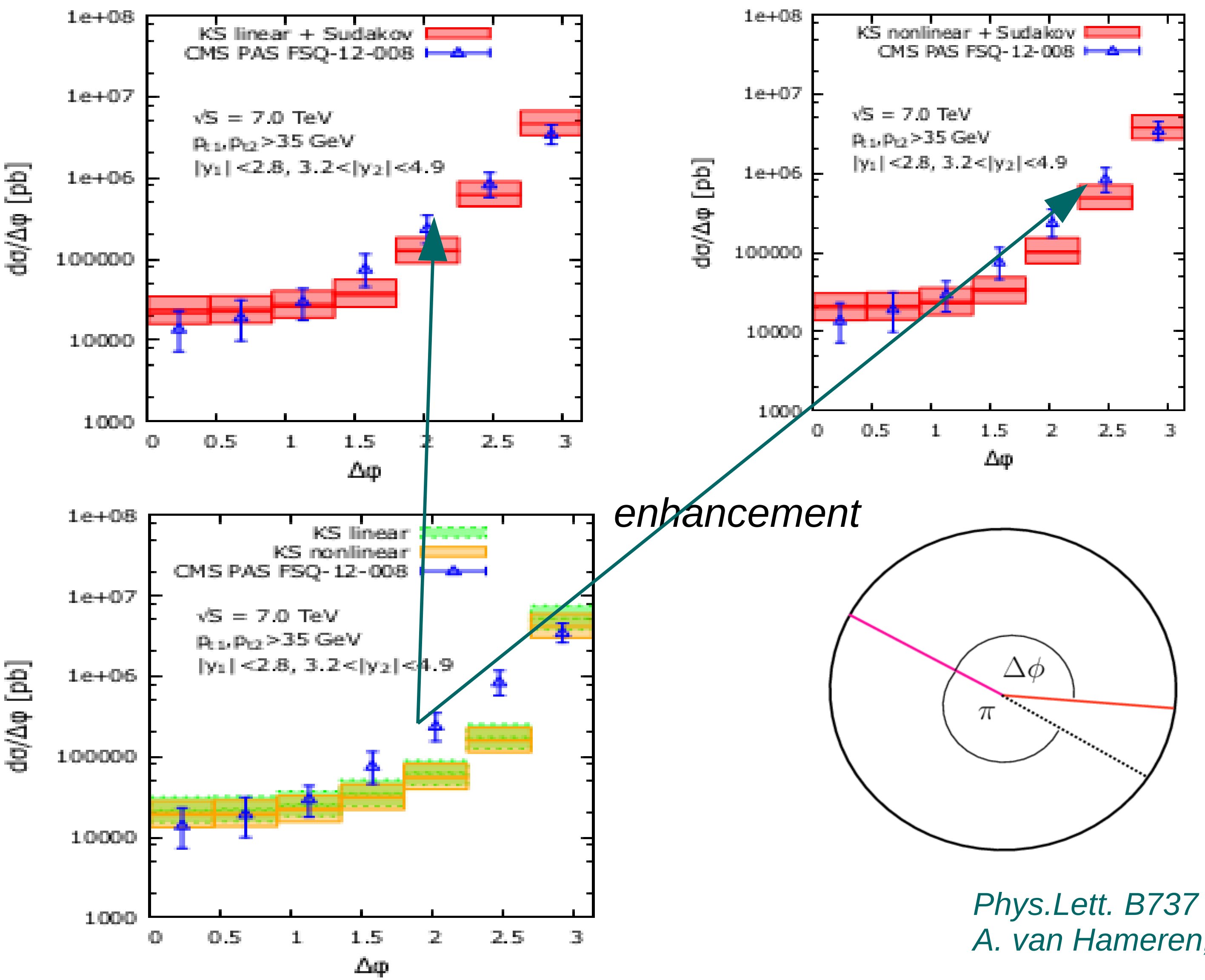
Decorelations inclusive scenario-central forward



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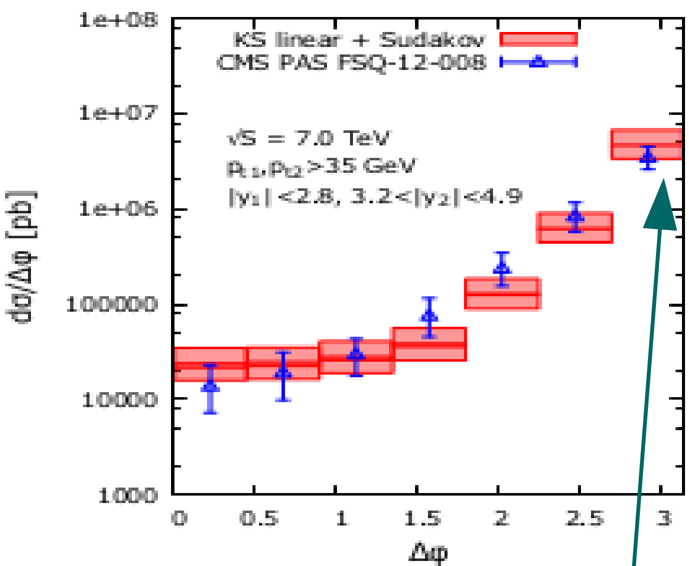
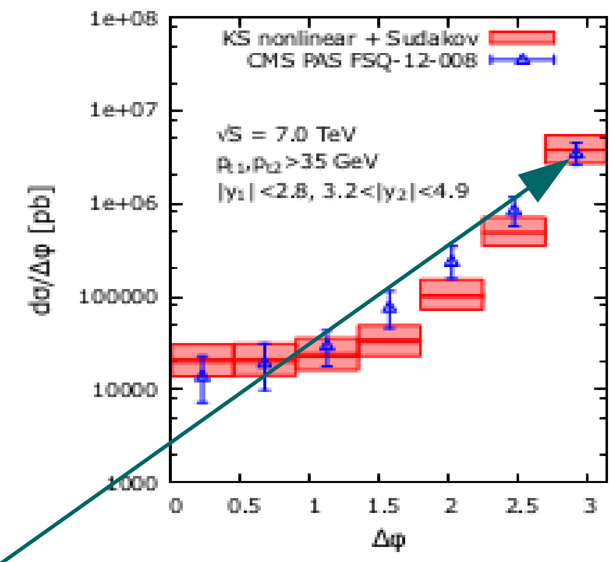
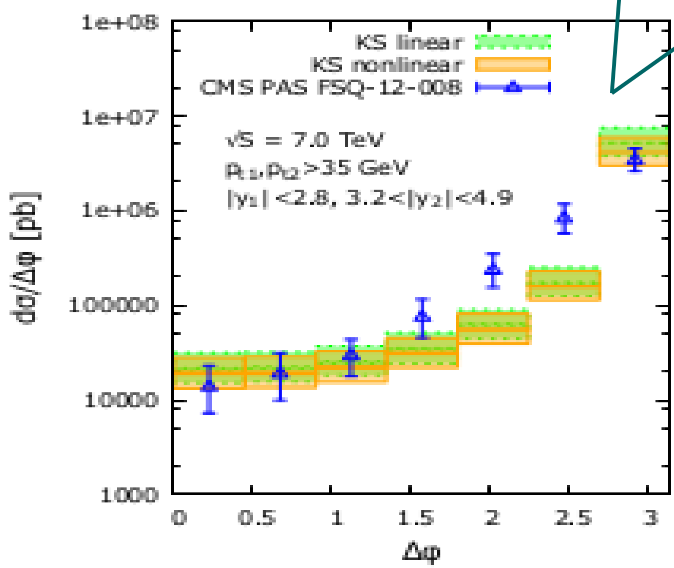
Decorelations inclusive scenario-central forward

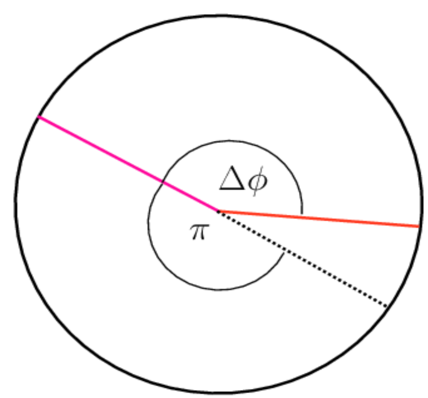




No saturation…
visibleSudakov effects

suppression





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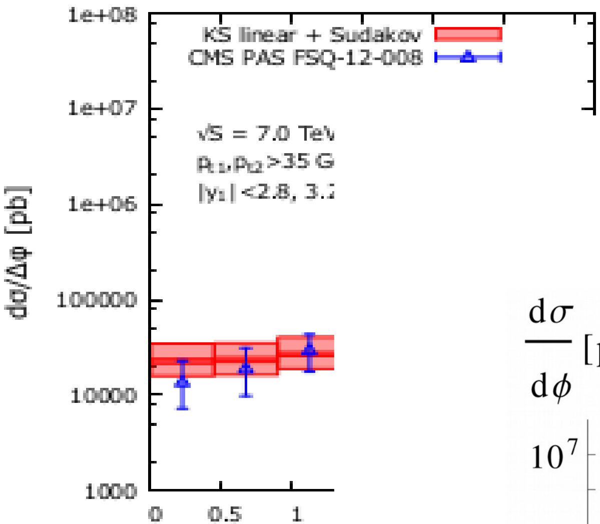
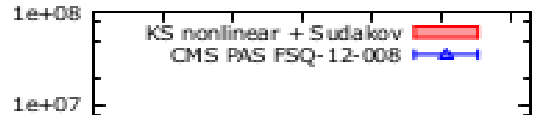
Decorelations inclusive scenario





No saturation…
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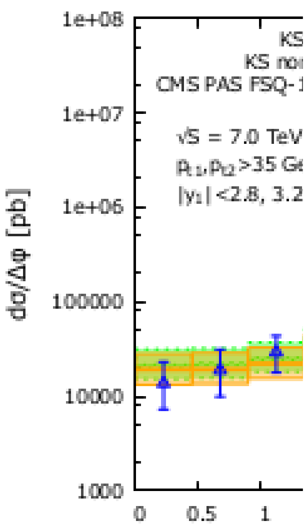
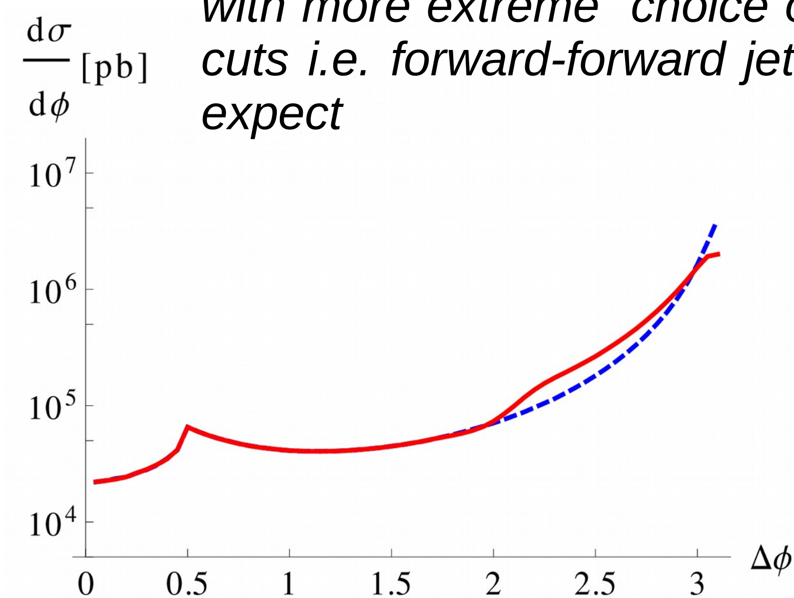


with more extreme choice of rapidity cuts i.e. forward-forward jets we can expect



K. Kutak
Phys.Rev. D91 (2015) no.3, 034021

Forward-forward dijets- elements going into our prediction

The ITMD gluon's were obtained using:
Proton's KS gluon density – fitted to F_2 proton HERA data Balitsky-Kovchegov equation + kinematical constraint + subleading in low x, low z parts of splitting function.
Lead's KS gluon density – normalized to number of nucleons. Modified radius as compared to proton's radius

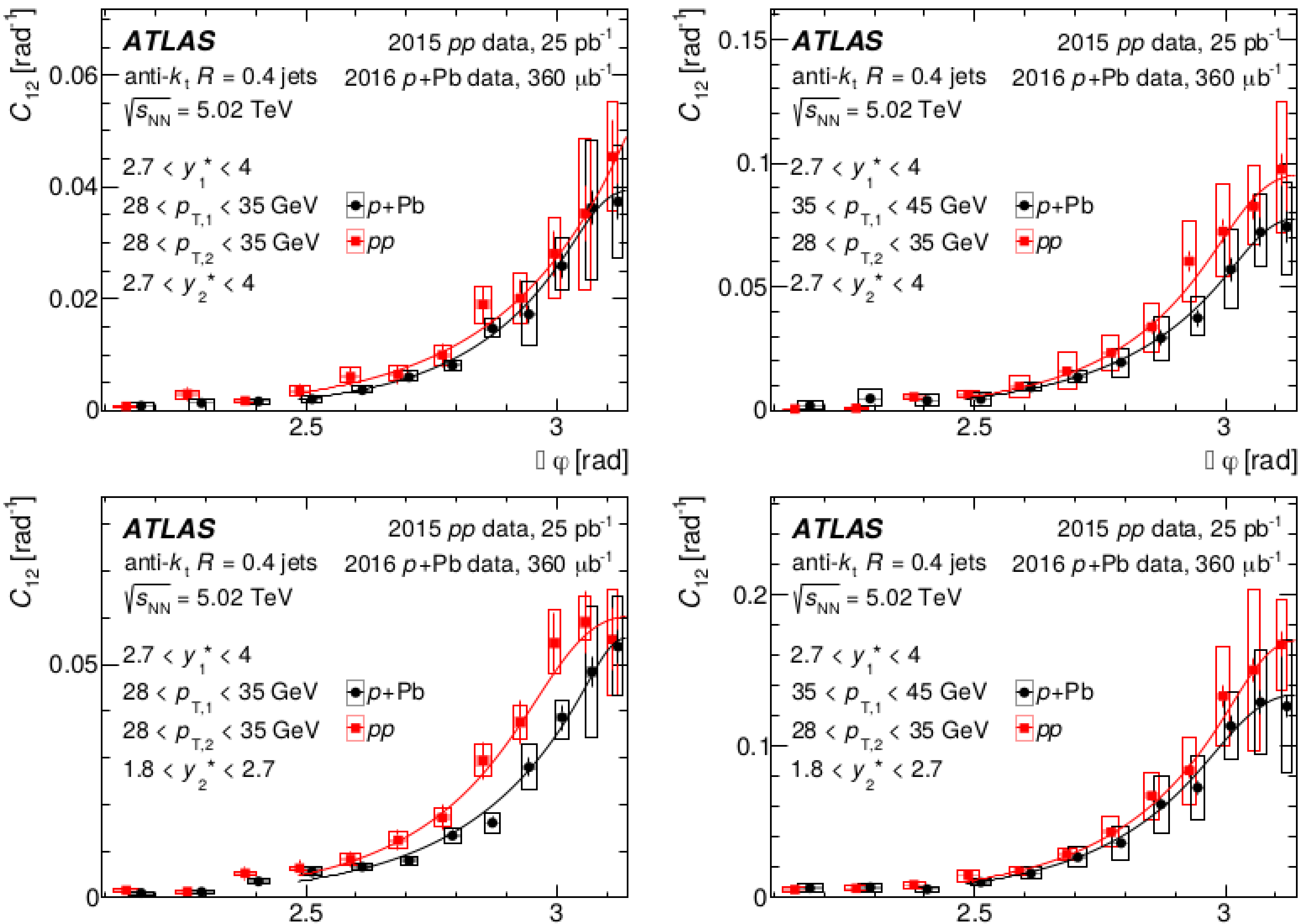
The Sudakov:
It has been was obtained from exponentiation of DGLAP splitting function
Total cross section is unchanged. Cross section at large angles is suppressed. Events with moderate angles are enhanced.

The cross section:
was calculated using:

- *KaTie Monte Carlo - MC for p-p, p-A, soon DIS and A-A, calculates matrix elements in kt factorization and ITMD, matrix elements agree with the once obtained from Lipatov effectve action. Via merging with CASCADE accounts for ISR and FSR*

- *cross-checked with LxJet Monte Carlo – dedicated MC for jets in kt factorization and MC*

Data

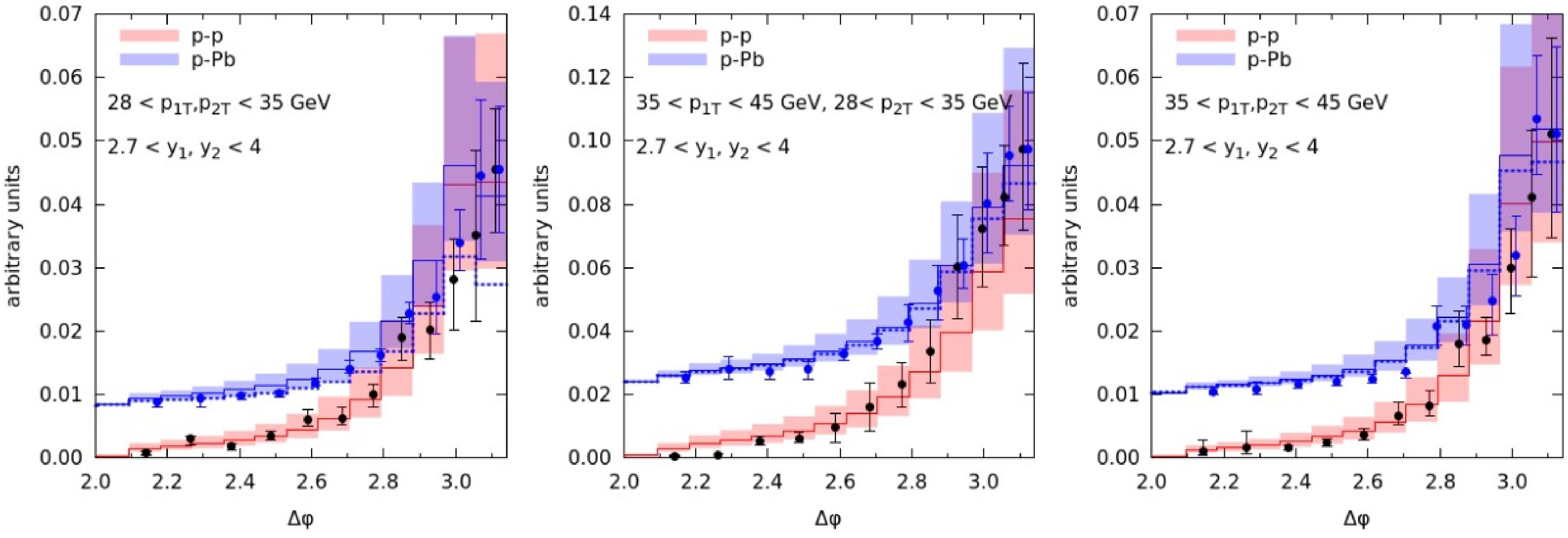


ATLAS
2015 pp data, 25 pb-1
anti-kt R = 0.4 jets
2016 p+Pb data, 360 μb-1
√sNN = 5.02 TeV
2.7 < y1* < 4
28 < pT,1 < 35 GeV
28 < pT,2 < 35 GeV
2.7 < y2* < 4
35 < pT,1 < 45 GeV
1.8 < y2* < 2.7
p+Pb
pp
C12 [rad-1]
φ [rad]

Signature of broadening in forward-forward dijets

ATLAS Phys.Rev. C100 (2019) no.3, 034903

A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

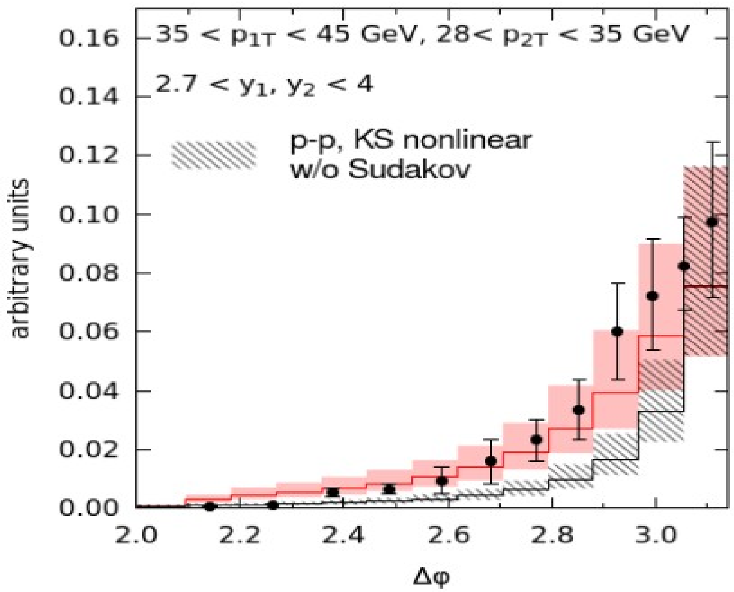


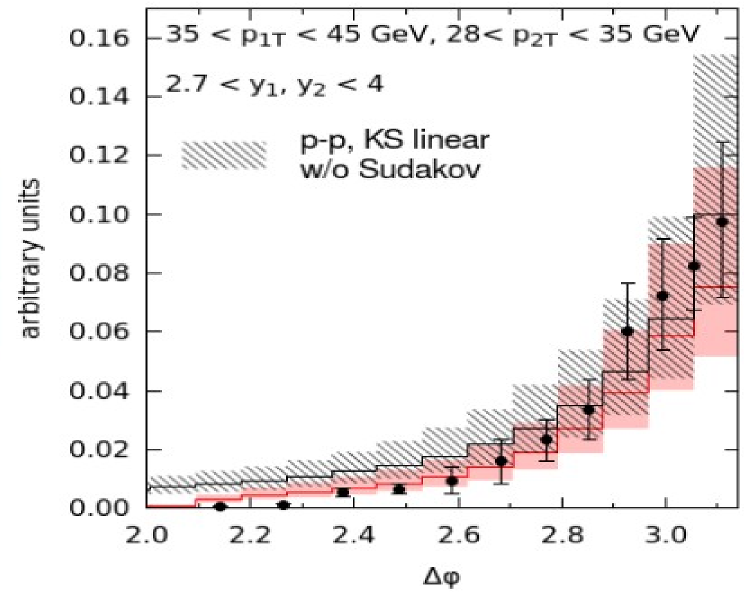
Data: number of dijets normalized to number of single inclusive jets. We can not calculate that. We can compare shapes.
Procedure: fit normalization to p-p data.
Use that both for p-p and p-Pb. Shift p-Pb data
The procedure allows for visualization of broadening

Other approaches

nonlinearity
no Sudakov

too narrow distribution



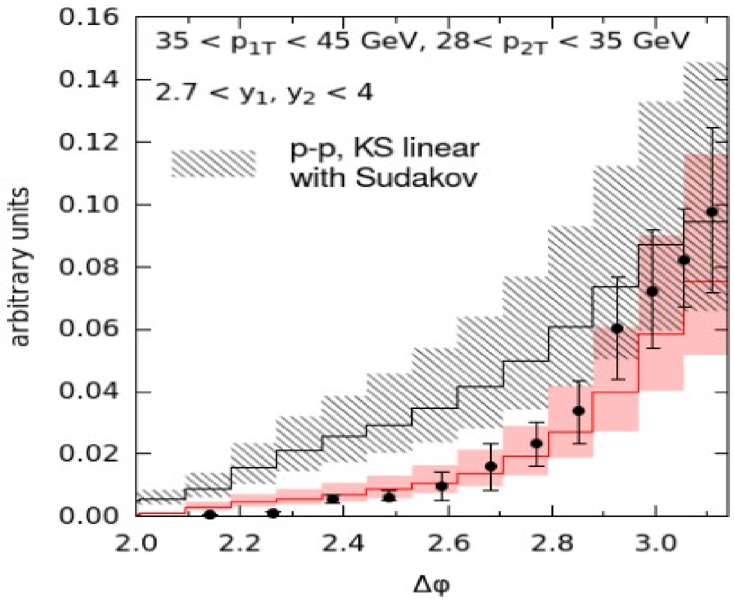


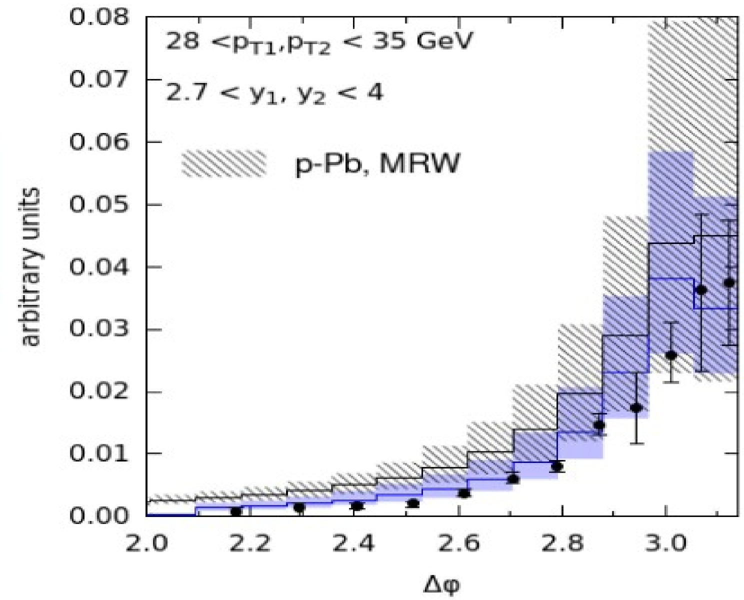
linear
no Sudakov

not too bad but different shape

linear

too wide Sudakov acts too much





Linear +
Sudakov

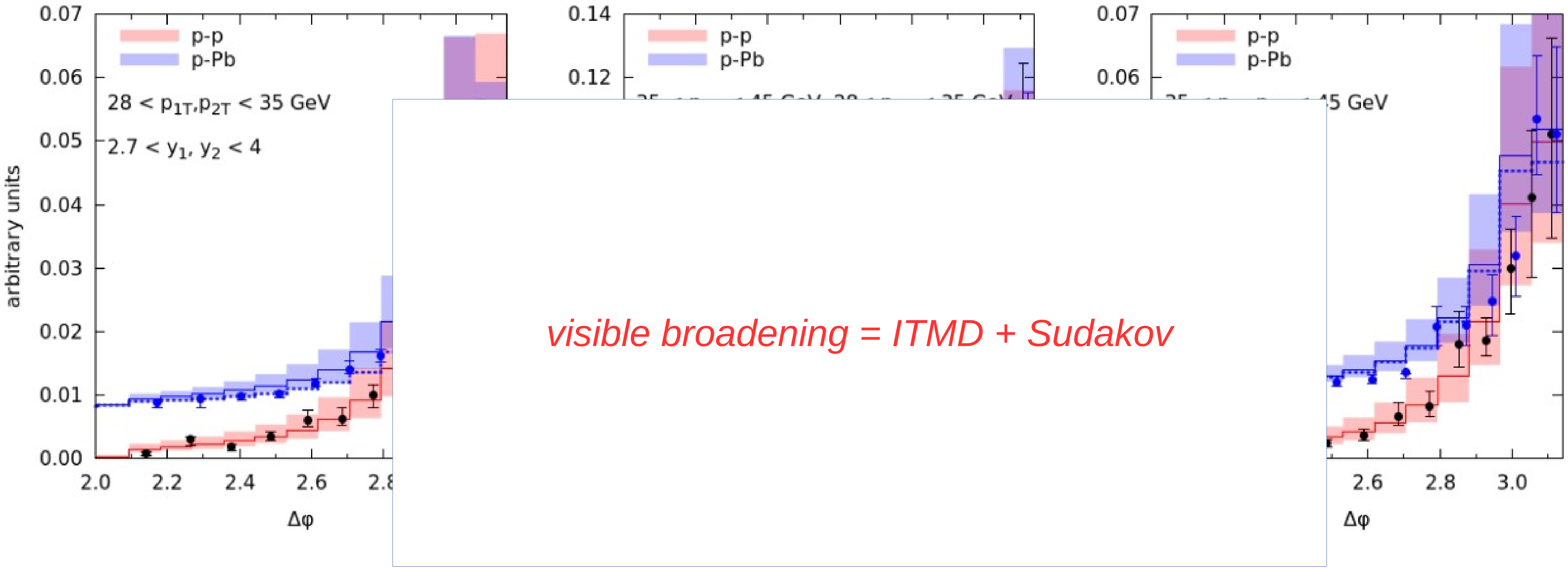
Ordering in k_t

not too bad but different shape

Signature of saturation in forward-forward dijets

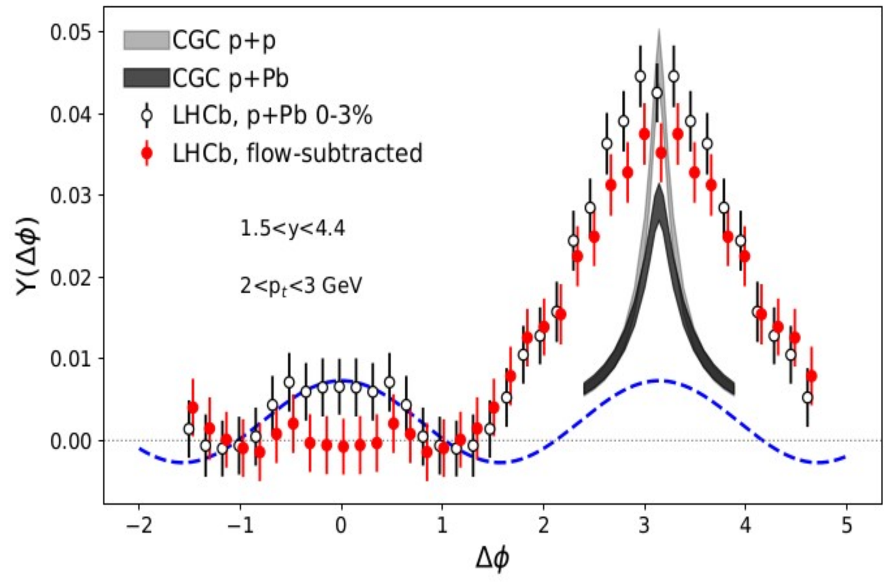
A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

ATLAS 1901.10440



Data: number of dijets normalized to number of single inclusive jets. We can not calculate that.
We can compare shapes.
Procedure: fit normalization to p-p data.
Use that both for p-p and p-Pb. Shift p-Pb data
The procedure allows for visualization of broadening

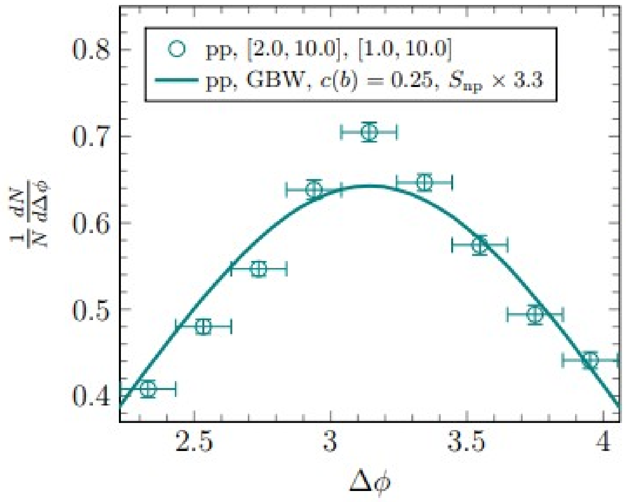
Other calculations which support our result - di-hadrons production

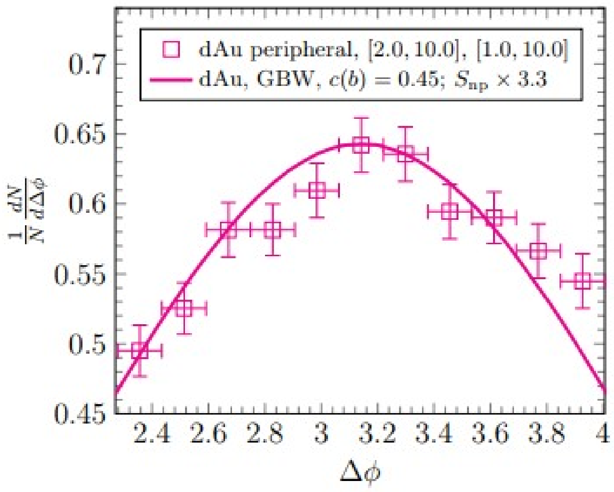


ITMD, no Sudakov

Expectation:
Sudakov
will broaden the distribution

G. Giacalone, C. Marquet, M. Matas
Phys.Rev. D99 (2019) no.1, 014002



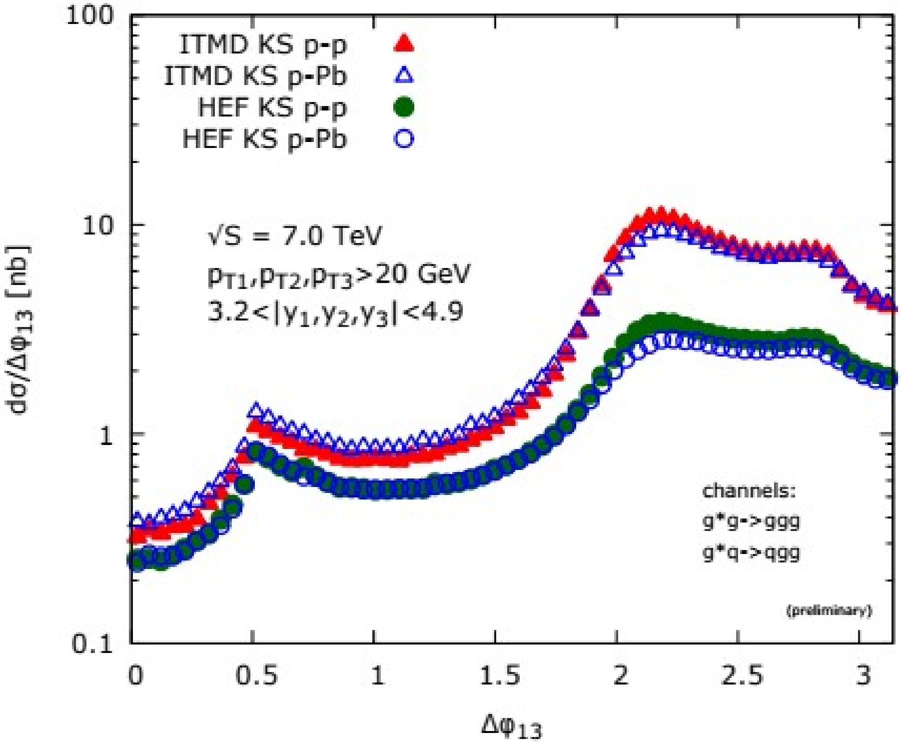


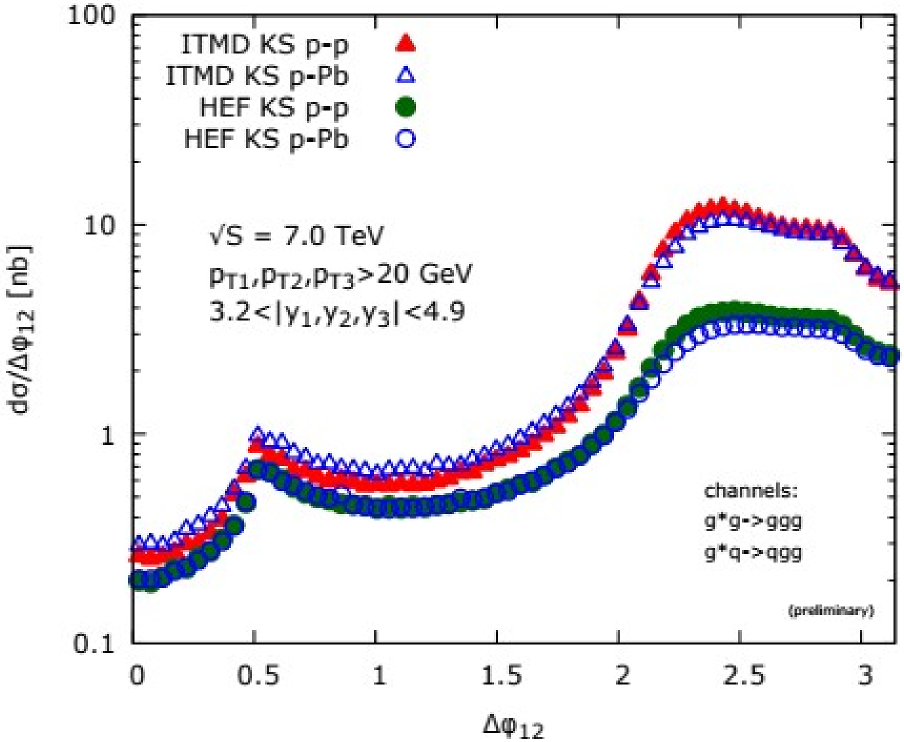
Correlation limit of CGC + Sudakov no k_t in ME

A.Stasto, S. Wei, B. Xiao, F. Yuan
Phys.Lett. B784 (2018) 301-306

Preliminary - ITMD vs. HEF - Tri-jet case

Operator structures of TMDs obtained in *Bury, Kutak, Kotko Eur.Phys.J. C79 (2019) no.2, 152*





Angle between leading jet and softest jet

Angle between leading jets

Main difference comes from change from HEF to ITMD

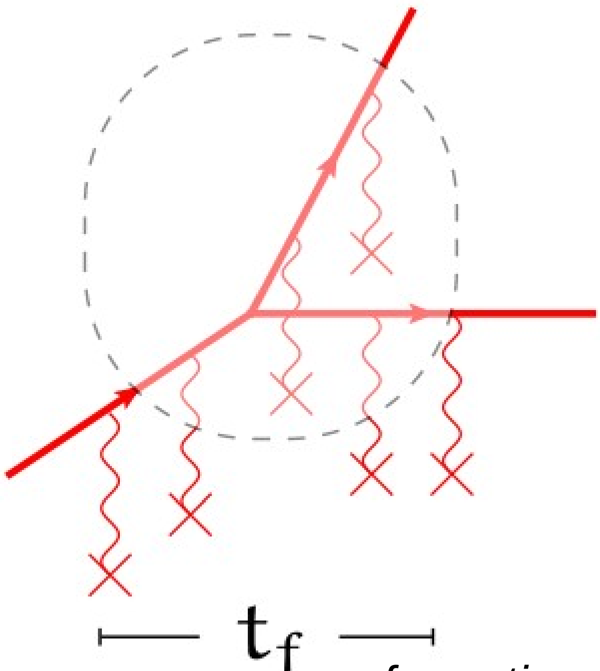
Saturation effects are hardly visible for this observable

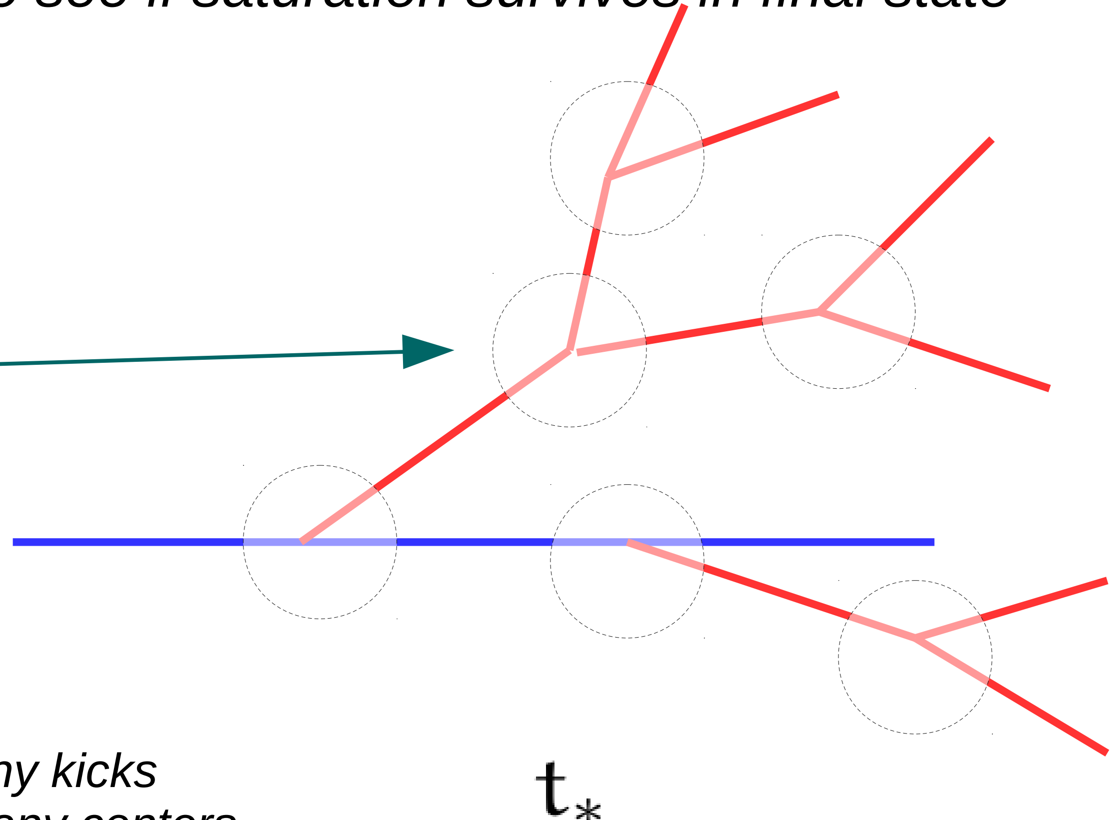
see also T. Altinoluk, R. Boussarie, C. Marquet, P. Taels '18 for 3 jets in correlation limit in $\gamma + A \rightarrow 3\,j$

Pb-Pb

Jet plasma interaction

Jet interacts multiply with plasma. It fragments and broadens. Ultimately it is interesting to combine final state effect i.e. jet quenching with initial state effect i.e. saturation to see if saturation survives in final state observables in A A

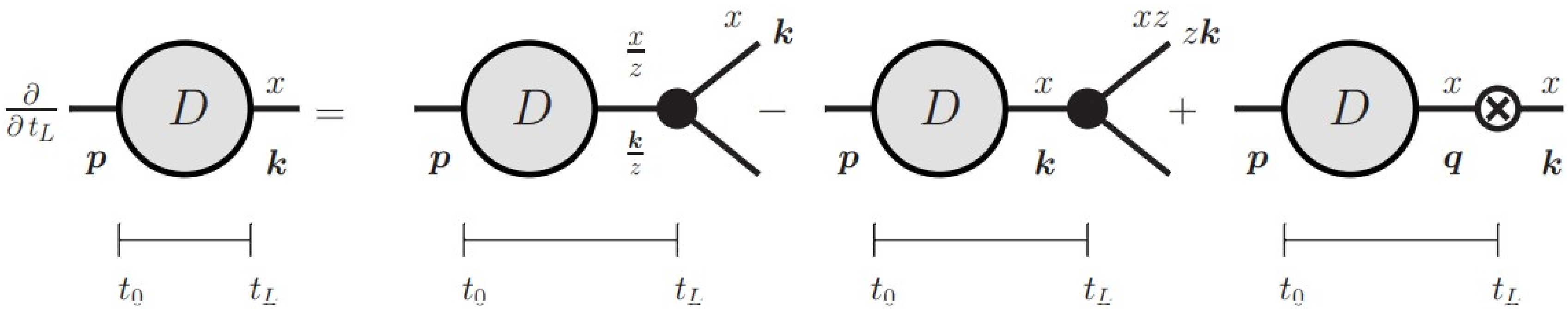




formation time; many kicks before radiation; many centers act as a single source

stopping time → time at which energy has been emitted in form of soft gluons

The BDIM equation



Blaizot, Dominguez, Iancu, Mehtar-Tani '12

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t)=\frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x)-\frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$

$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t)$$

Inclusive gluon distribution as produced by hard jet

$$\frac{1}{t^*}=\frac{\bar{\alpha}}{\tau_{\rm br}(E)}=\bar{\alpha}\sqrt{\frac{\hat{q}}{E}}$$

$$C(\mathbf{q})=w(\mathbf{q})-\delta(\mathbf{q})\int d^2\mathbf{q}'\,w(\mathbf{q}')$$

$$\mathcal{K}(z)=\frac{[f(z)]^{5/2}}{[z(1-z)]^{3/2}}\qquad f(z)=1-z+z^2$$

Equation describes interplay of rescatterings and branching. This particular equation has k_t independent kernel. This is an approximation. The whole broadening comes from rescattering. Energy of emitted gluon is much larger than its transverse momentum

Rearrangement of the equation for gluon density

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$
$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t),$$

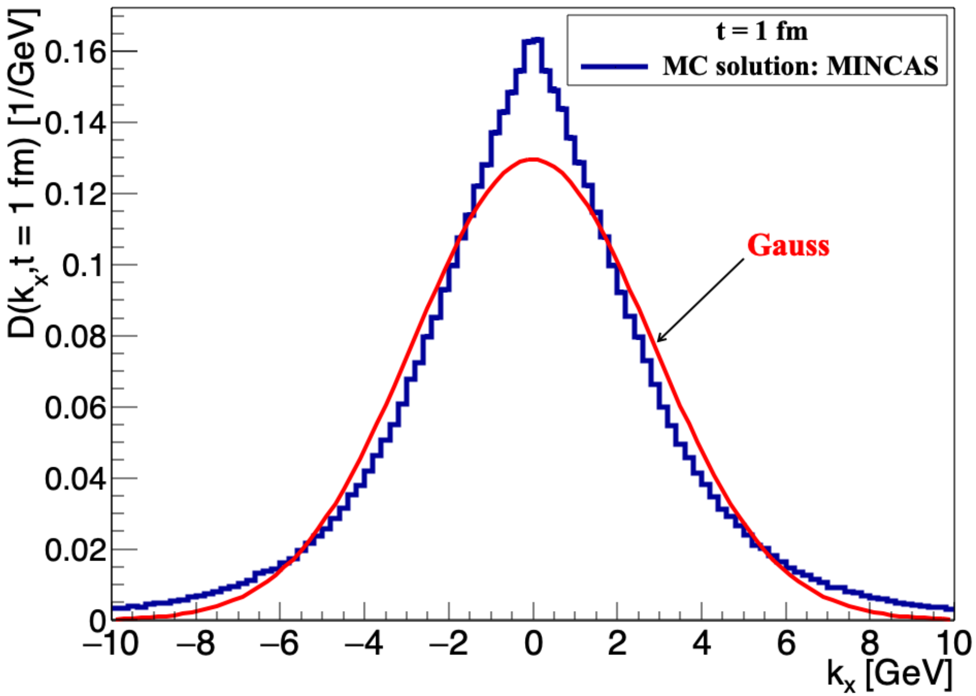
Kutak, Płaczek, Straka EPJC '19

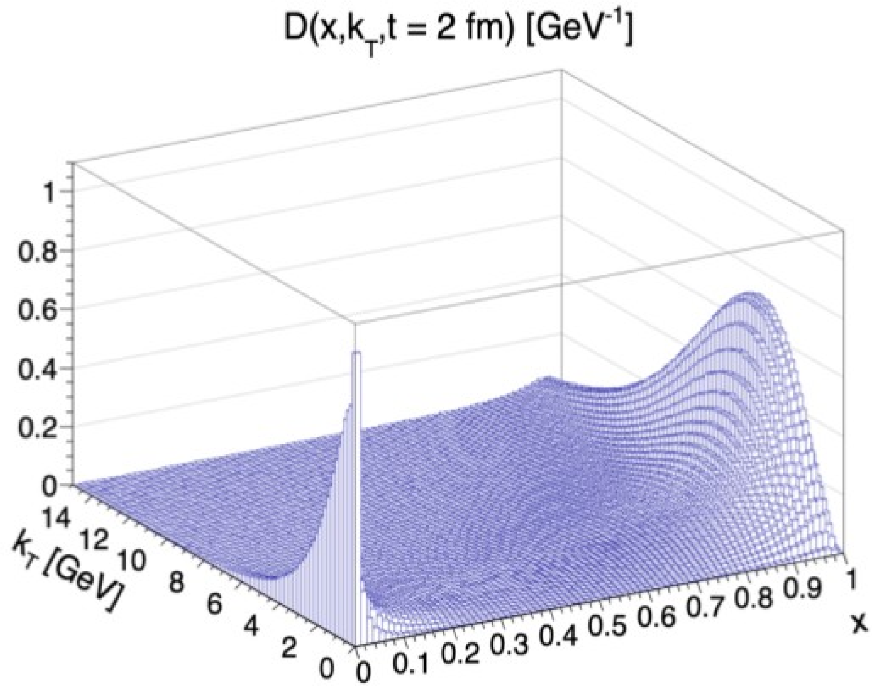
$\downarrow$

Reformulation: virtual contribution and broadening can be exponentiated to Sudakov form factor equation can be solved by MC method

$$D(x,\mathbf{k},\tau) = e^{-\Psi(x)(\tau-\tau_0)}D(x,\mathbf{k},\tau_0)$$
$$+\int_{\tau_0}^{\tau}d\tau'\int_0^1 dz\int_0^1 dy\int d^2\mathbf{k}'\int d^2\mathbf{q}\;\mathcal{G}(z,\mathbf{q})$$
$$\times\,\delta(x-zy)\,\delta(\mathbf{k}-\mathbf{q}-z\mathbf{k}')\,e^{-\Psi(x)(\tau-\tau')}D(y,\mathbf{k}',\tau')$$

Non gaussianity



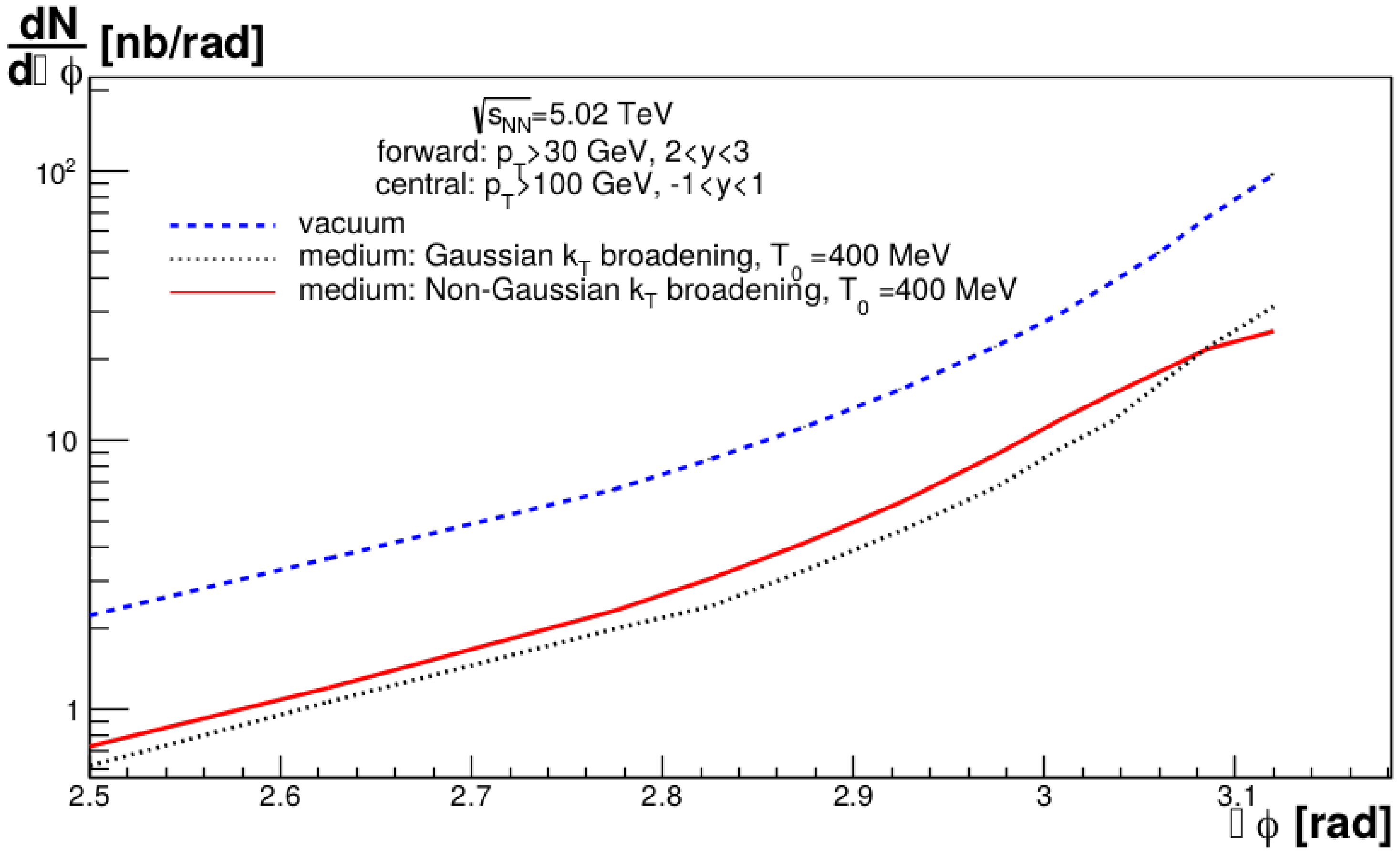


Sum of many gaussians with different width.

This is a result of the exact treatment of the gluon transverse-momentum broadening due to an arbitrary number of the collisions with the medium together with its shrinking due an arbitrary number of the emission branching.

Non gaussianity – observable level

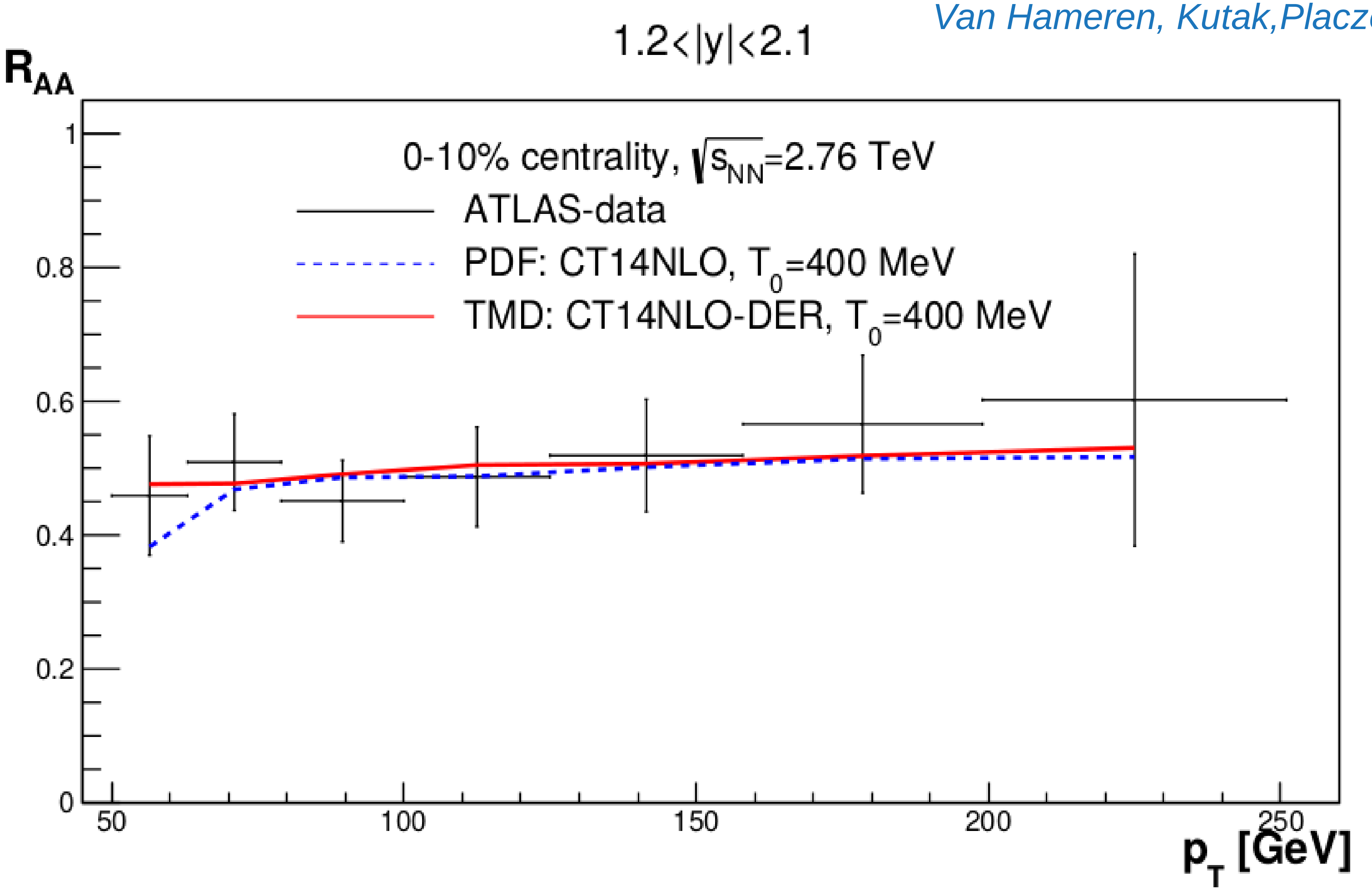
1911.05463
Van Hameren, Kutak,Placzek, Rohrmoser



Non-gaussian broadening leads too suppression at large angles and enhancement at moderate angles

R_{AA} nuclear modificatio ratio

1911.05463
Van Hameren, Kutak,Placzek, Rohrmoser



Summary and outlook

New factorization formula for dilute-dense collision has been obtained

- *accounts for nonlinear evolution of low x gluon density*
- *accounts for correct gauge structure of the theory*
- *can be obtained from Color Glass Condensate in appropriate limit*

Evidence for need for Sudakov and saturation in forward jets has been found – visible broadening

To check model dependence use plan to use the other formalisms combining Sudakov and ITMD gluons.

Update dipole gluon density – new fits etc.

Longer time perspective: NLO (talk by M. Nefedov)

Tri-jets, four-jets → the ITMD formula is already there Bury, Kutak, Kotko Eur.Phys.J. C79 (2019) no.2, 152

Effects of showers, maching talks by (V. Saleev, M. Bermudez-Martinez)

BACKUP

observable

total cross-section

$$\mathcal{O} = \frac{\sigma}{W} \sum_i w_i F_i^{\mathcal{O}} \left(X_i\right)$$

function defining observable: cuts etc.

$$\mathcal{O} = \frac{\sigma}{W} \left[\sum_i w_i F_i^{\mathcal{O}} \left(X_i\right) \Theta\left(\mu_i > k_{Ti}\right) + \sum_j w_j F_j^{\mathcal{O}} \left(X_j\right) \Theta\left(k_{Tj} > \mu_j\right) \right]$$

$W = \sum_i w_i$ *total weight*

of order 1

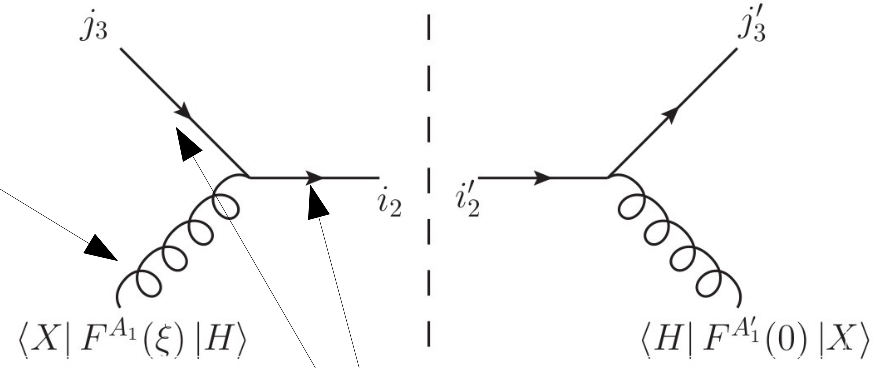
$$\overline{\mathcal{O}} = \frac{\sigma}{\overline{W}} \left[\sum_i w_i \Delta\left(\mu_i, k_{Ti}\right) F_i^{\mathcal{O}} \left(X_i\right) \Theta\left(\mu_i > k_{Ti}\right) + \frac{\widetilde{W}}{W} \sum_j w_j F_j^{\mathcal{O}} \left(X_j\right) \Theta\left(k_{Tj} > \mu_j\right) \right]$$

$$\overline{W} = \sum_i w_i \Delta\left(\mu_i, k_{Ti}\right) \Theta\left(\mu_i > k_{Ti}\right) + \frac{\widetilde{W}}{W} \sum_j w_j \Theta\left(k_{Tj} > \mu_j\right)$$

modified weight

Example $qg \to q$

We want to get TMD distribution of



Resummation

replacement of deltas with operators

We need to resum all collinear emissions from

$$\mathcal{M} = \left(t^{A_1}\right)^{i_2}_{j_3} \mathcal{A}(2,1,3)$$

$$\mathcal{M}^* \mathcal{M}\, \delta^{i_2 i'_2}\, \delta_{j_3 j'_3} = \left(t^{A_1}\right)^{i_2}_{j_3} \left(t^{A'_1}\right)^{j'_3}_{i'_2} \delta^{A_1 A'_1} \delta^{i_2}_{i'_2}\, \delta^{j'_3}_{j_3}\, \mathcal{A}^*(2,1,3)\mathcal{A}(2,1,3)$$

$$\left(t^{A_1}\right)^{i_2}_{j_3} \left(t^{A'_1}\right)^{j'_3}_{i'_2} \left(\mathcal{U}^{[+]}\right)^{i_2}_{i'_2} \left(\mathcal{U}^{[-]\dagger}\right)^{j'_3}_{j_3} F^{A'_1}(0) F^{A_1}(\xi) =$$

$$= \left(F(\xi)\right)^{i_2}_{j_3} \left(\mathcal{U}^{[-]\dagger}\right)^{j'_3}_{j_3} \left(F(0)\right)^{j'_3}_{i'_2} \left(\mathcal{U}^{[+]}\right)^{i_2}_{i'_2} = \mathrm{Tr}\left[F(\xi)\mathcal{U}^{[-]\dagger} F(0) \mathcal{U}^{[+]}\right]$$

$$\mathcal{F}^{(1)}_{qg}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\, \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0)\, \mathcal{U}^{[+]}\right]\right\rangle$$