Soft gluon resummation in associated production of top-antitop pair and a H, Z or W boson

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Motivation

- **The main goal:**

  Estimate resummed soft gluon corrections to $t\bar{t}H$, $t\bar{t}Z$, $t\bar{t}W$ hadroproduction at the LHC and improve theoretical precision

- **Reasons:**

  – processes with the heaviest Standard Model particles that probe directly the value of top Yukawa coupling and top quark coupling to electroweak gauge bosons – potentially sensitive to New Physics

  – practical extension of the soft gluon resummation technique to

  more than two final state particles

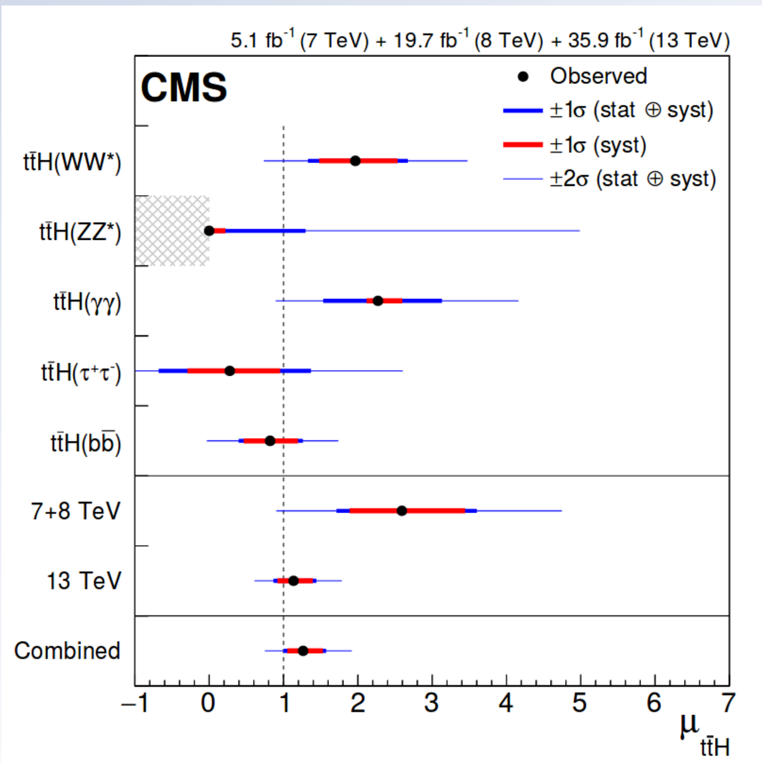
- Report on results of recent and continued work by

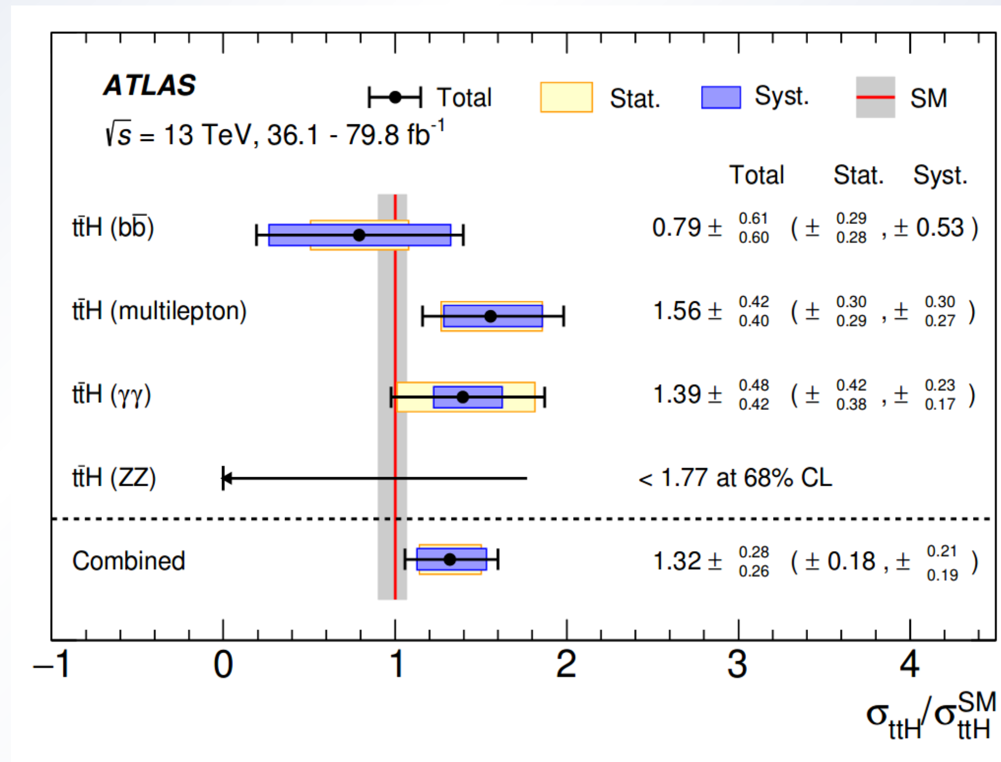
  Anna Kulesza, Tomasz Stebel, Vincent Theeuwes, Daniel Schwartlander, LM

  [JHEP 1603 (2016) 065, Phys.Rev. D97 (2018) 114007,

  Eur.Phys.J. C79 (2019), 249]

Observation of $t\bar{t}$-Higgs at the LHC (2018)





Parallel efforts in soft gluon resummation in $t\bar{t}H$

Currently two active groups:

Direct QCD – Mellin space: A. Kulesza, D. Schwartlander, T. Stebel, V. Theeuwes,LM

Soft Collinear Effective Theory (SCET) A. Broggio, A. Ferroglia, B.D. Pecjak, A. Signer and L.L.Yang, R. Frederix

- *Soft gluon NLL resummation at the absolute threshold* (direct QCD) arXiv:1509.02780
- *Approximate NNLO from soft gluon NNLL resummation formula in Soft Collinear Effective Theory: invariant mass dependent resummation* (SCET) arXiv:1510.01914
- Invariant mass dependent resummation at NLL (direct QCD) arXiv:1609.01619
- Invariant mass dependent resummation at NNLL (SCET) arXiv:1611.00049
- Invariant mass dependent resummation at NNLL (direct QCD) arXiv:1704.03363

- Since then: several new papers of both groups on pp → $t\bar{t}Z$ and $t\bar{t}W$

Soft gluon resummation

- Framework based on proofs of hard factorization by Collins, Soper and Sterman and by Catani and Trentadue (1980-s)

- Logarithmically enhanced soft gluon corrections may factorized and resummed to all order of perturbation theory keeping:

  $(\alpha_s \log^2 \beta)^n$ at LL, $(\alpha_s^n \log^{2n-1} \beta)$ at NLL, $(\alpha_s^n \log^{2n-2} \beta)$ at NNLL accuracy

- We work up to NNLL accuracy: hard scattering at NLO, soft-collinear logs at NNLO + collinear logs (NLO) + soft logs (NLO)

- Collinear logs → incoherent, they redefine massless partons
- Soft wide angle logs → coherent, depend on total color current

- Resummation is performed using Renormalization Group technique

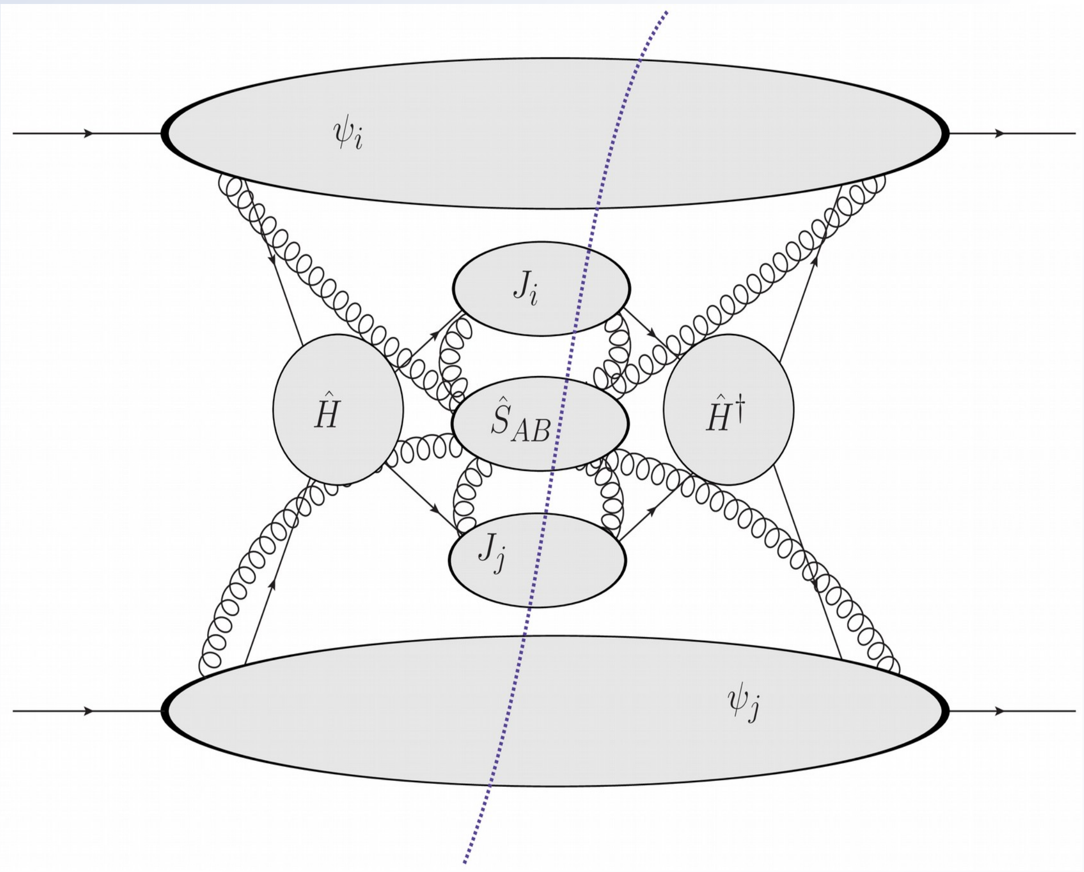
- Those corrections may be factorized in the Mellin moment space:

  particularly simple picture of resummation →

  multiplicative factors at given value of the Mellin moment N

General factorization procedure

$$\hat{\sigma} = \hat{H}^\dagger \otimes \hat{S} \otimes \hat{H} \otimes \psi_i \otimes \psi_j \otimes J_i \otimes J_j$$



S_{AB} — soft gluon matrix

H — hard amplitude matrix

ψ_i — initial state "jet factors"
collinear radiation of incoming partons

J_i — collinear "jet factors"
for final state massless partons

Scale evolution of the soft gluon matrix: renormalization group equations

- The anomalous dimension matrix in color tensor space governs the Soft Matrix evolution:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\hat{S}(\mu,g)=-\hat{\Gamma}_S^{\dagger}\hat{S}(\mu,g)-\hat{S}(\mu,g)\hat{\Gamma}_S$$

$$\hat{\Gamma}_S(g)=-\frac{g}{2}\frac{\partial}{\partial g}\,\mathrm{Res}_{\epsilon\to 0}\hat{Z}_S(g,\epsilon)$$

$$\beta(g)=-g^3\frac{\beta_0}{(4\pi)^2}-g^5\frac{\beta_1}{(4\pi)^4}-\ldots$$

$$\hat{\Gamma}_S=\frac{\hat{\gamma}^{(0)}}{(4\pi)^2}+\frac{\hat{\gamma}^{(1)}}{(4\pi)^4+\ldots}$$

- The evolution between the process scale and the lower cutoff scale on the soft gluon energy: generation of logs of the scale ratio
- Solution in the Mellin space

Compact summary of the resummation formalism

$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{res})}_{ij\to klB}}{dQ^2}(N, Q^2, \{m^2\}, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2) =$$

$$\begin{aligned} &= \mathrm{Tr}\left[\mathbf{H}_{ij\to klB}(Q^2, \{m^2\}, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2)\mathbf{S}_{ij\to klB}(N+1, Q^2, \{m^2\}, \mu_{\mathrm{R}}^2)\right] \\ &\times \Delta^i(N+1, Q^2, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2)\Delta^j(N+1, Q^2, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2), \end{aligned}$$

$$\begin{aligned} \mathbf{S}_{ij\to klB}(N, Q^2, \{m^2\}, \mu_{\mathrm{R}}^2) &= \bar{\mathbf{U}}_{ij\to klB}(N, Q^2, \{m^2\}, \mu_{\mathrm{R}}^2)\, \tilde{\mathbf{S}}_{ij\to klB}(\alpha_{\mathrm{s}}(Q^2/\bar{N}^2)) \\ &\times \mathbf{U}_{ij\to klB}(N, Q^2, \{m^2\}, \mu_{\mathrm{R}}^2), \end{aligned}$$

$$\mathbf{H}_{ij\to klB} = \mathbf{H}^{(0)}_{ij\to klB} + \frac{\alpha_{\mathrm{s}}}{\pi}\mathbf{H}^{(1)}_{ij\to klB} + \ldots \qquad \tilde{\mathbf{S}}_{ij\to klB} = \tilde{\mathbf{S}}^{(0)}_{ij\to klB} + \frac{\alpha_{\mathrm{s}}}{\pi}\tilde{\mathbf{S}}^{(1)}_{ij\to klB} + \ldots$$

$$\mathbf{U}_{ij\to klB}\left(N, Q^2, \{m^2\}, \mu_{\mathrm{R}}^2\right) = \mathrm{P}\exp\left[\int_{\mu_{\mathrm{R}}}^{Q/\bar{N}} \frac{dq}{q}\boldsymbol{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]$$

$$\boldsymbol{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\right) = \left[\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)\boldsymbol{\Gamma}^{(1)}_{ij\to klB} + \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2\boldsymbol{\Gamma}^{(2)}_{ij\to klB} + \ldots\right]$$

Compact summary of the resummation formalism

Ingredients of the NLL calculation:

- LO hard matrix in color tensor basis (computed)
- LO soft matrix at initial scale (trivial)
- One loop soft anomalous dimension matrix (computed)
- NLL collinear factors (known)

NNLL calculation:

- Hard matrix in color tensor basis at one loop (extracted from NLO QCD calculations)
- One-loop soft matrix at one initial scale (computed)
- Two-loop soft anomalous dimension matrix
- NNLL collinear factors (known)

Organizing the perturbative series:

- In Mellin representation $\log\beta$ -s translate into logs of Mellin moments $\rightarrow \log N$
- **LL corresponds to $(\alpha_s \log^2 N)^n$,**

  NLL to $\alpha_s \log N\, (\alpha_s \log^2 N)^{n-1}$

  NNLL to $\alpha_s^2 (\alpha_s \log^2 N)^{n-1}$

- The LL terms come from the LO soft–collinear corrections

- The soft wide angle gluon corrections enter at NLL (they are exponentiated), and the first order corrections at NNLL

- The Mellin space resumming factors organized into functions at given level of accuracy: $g_1(\alpha_s, N)$, $g_2(\alpha_s, N)$ **and** $g_3(\alpha_s, N)$

Matching to NLO QCD and NLO EW

- In order to make full use of available information:
  matching of soft gluon resummation to the existing NLO calculations

- The NLO cross-section implemented in MC codes:
  PowHEG BOX, aMC@NLO and SHERPA

- From the cross-section at the NLL / NNLL accuracy the part beyond the fixed order NLO expansion is taken and combined with the exact NLO result

  → **NLL / NNLL cross-section matched to NLO**

- Also: inclusion of NLO EW corrections into the fixed order part

Beyond NLO, towards NNLL: NLO hard matrix element

- The available NLO calculations give access to the value of NLO correction in the threshold limit

$$\mathbf{H}_{ij\to klB} = \mathbf{H}^{(0)}_{ij\to klB} + \frac{\alpha_s}{\pi}\mathbf{H}^{(1)}_{ij\to klB} + \ldots$$

- Part of this correction coincides with the LL and NLL soft gluon logarithms → taken care by the soft gluon resummation
- The reminder of the NLO correction constant at the threshold limit (as function of the Mellin moment N) → hard matrix at NLO
- Inclusion of NLO correction in the hard matrix element → necessary part of NNLL resummation, but it may be also used as an improvement of the NLL resummation → customary in soft gluon resummation in Higgs boson physics

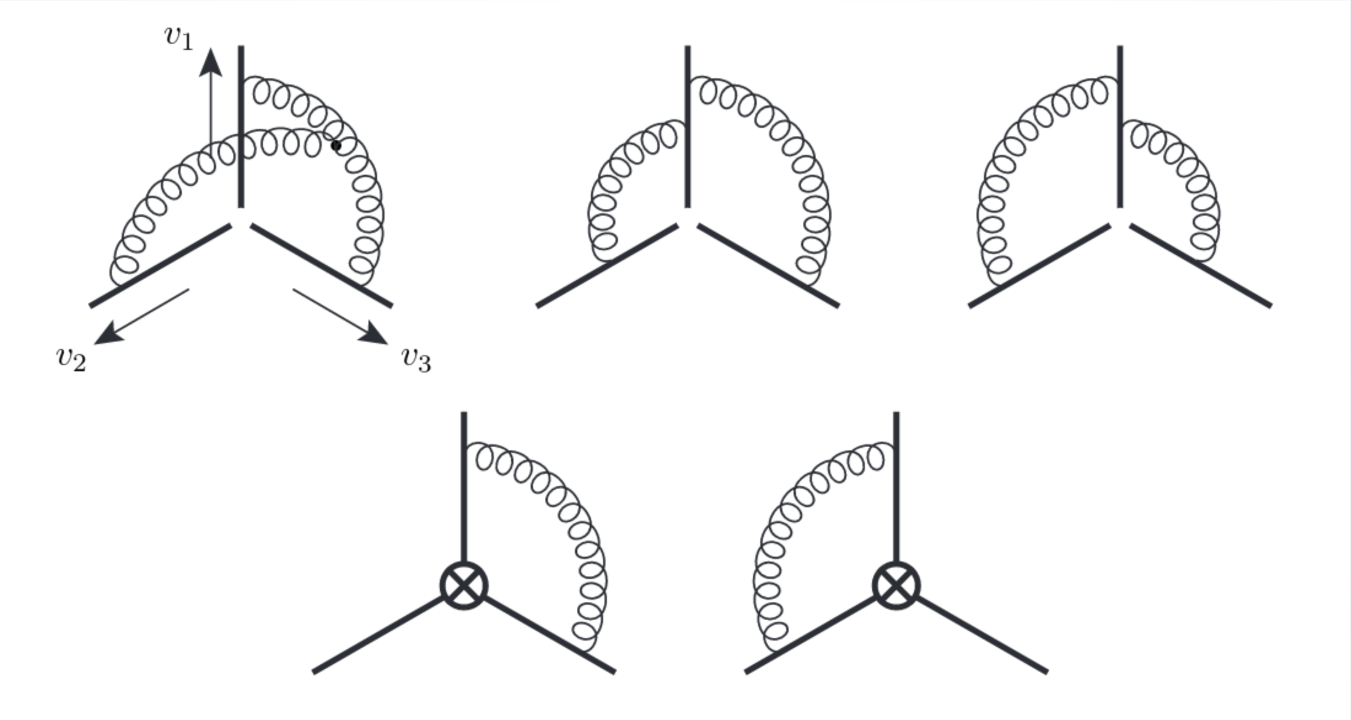
Soft anomalous dimension at NNLL

- Soft anomalous dimension – new topologies with three eikonal lines

- Color structures containing 3 SU(3) generators: $T^a T^b T^c$

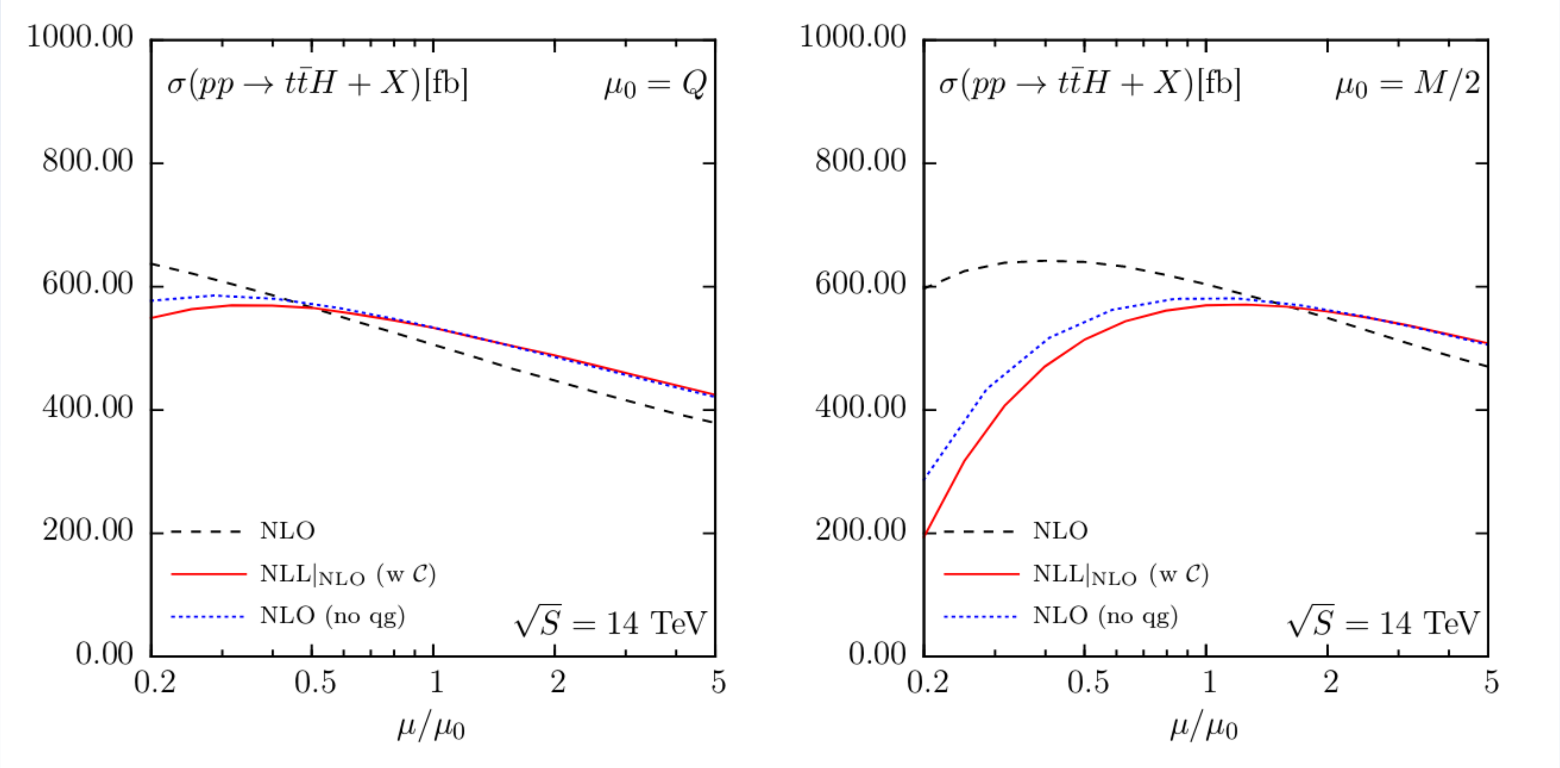
  [A. Ferroglia, M. [Neubert, B. Pecjak, L. L. Yang, Phys.Rev.Lett. 103 (2009) 201601,

  JHEP 0911 (2009) 062]



Results for $t\bar{t}$-Higgs: NLO approximate vs NLO exact

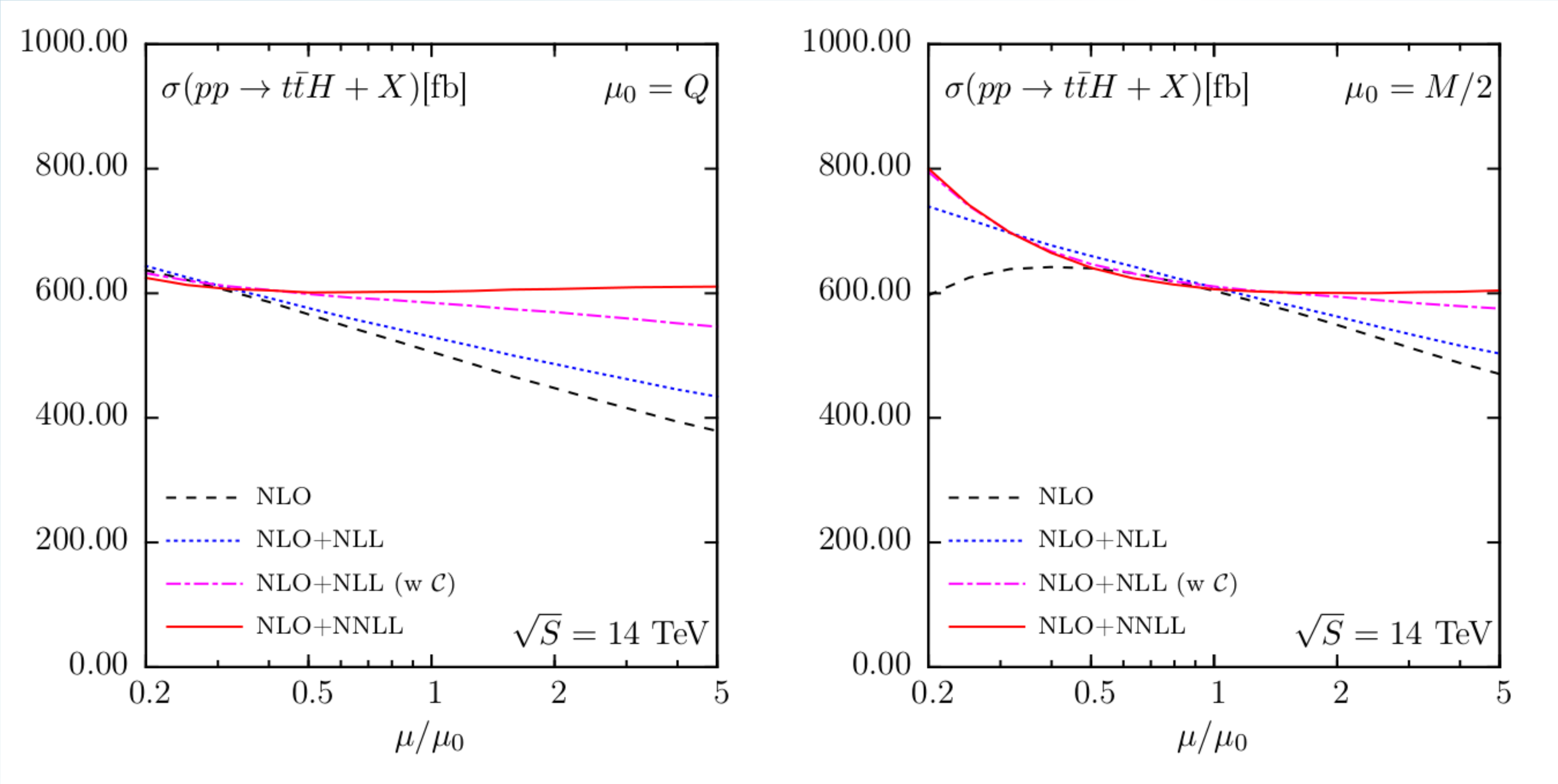
- Quality test of the approach: comparison of **expanded NNLL** result to **exact NLO** result



- In the plot the qg channel that enters at NLO as a subleading piece that is subtracted from the NLO as it is not generated in the soft gluon resummation
- The NLO result is well approximated

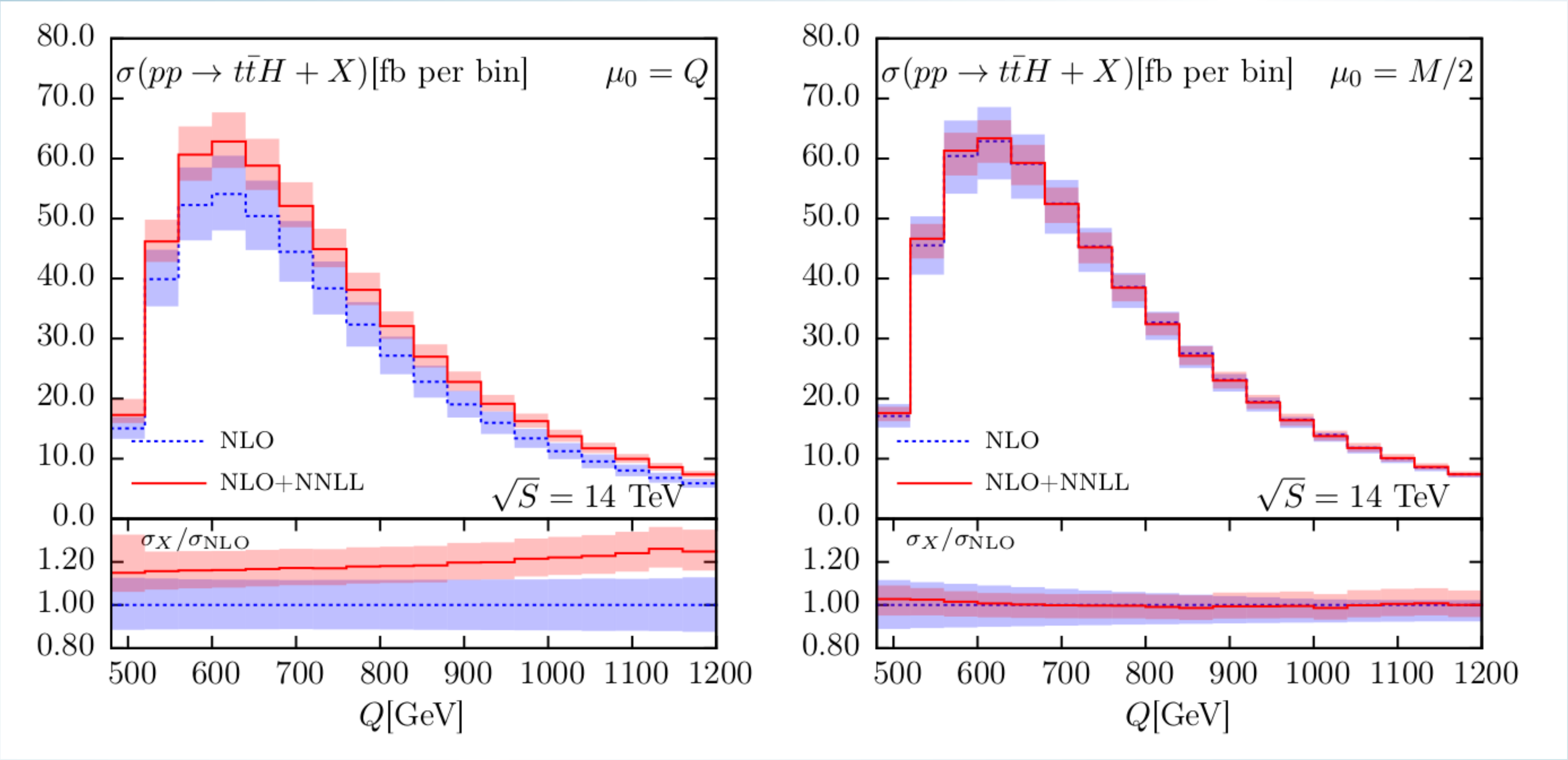
Results for $t\bar{t}H$, Q-dependent threshold resummation at NLO + NNLL: renormalization scale = factorization scale

- Scale dependence of total cross-section



- Sizable improvement of the scale stability of NLO+NLLL w.r.t. the NLO
- Increasing theoretical accuracy: NLO → NLO+NLL → NLO+NLLwC → NLO+NNLL leads to increasing stability of predictions

Results for $t\bar{t}H$: effects of the NNLL resummation on the invariant mass (Q) distribution

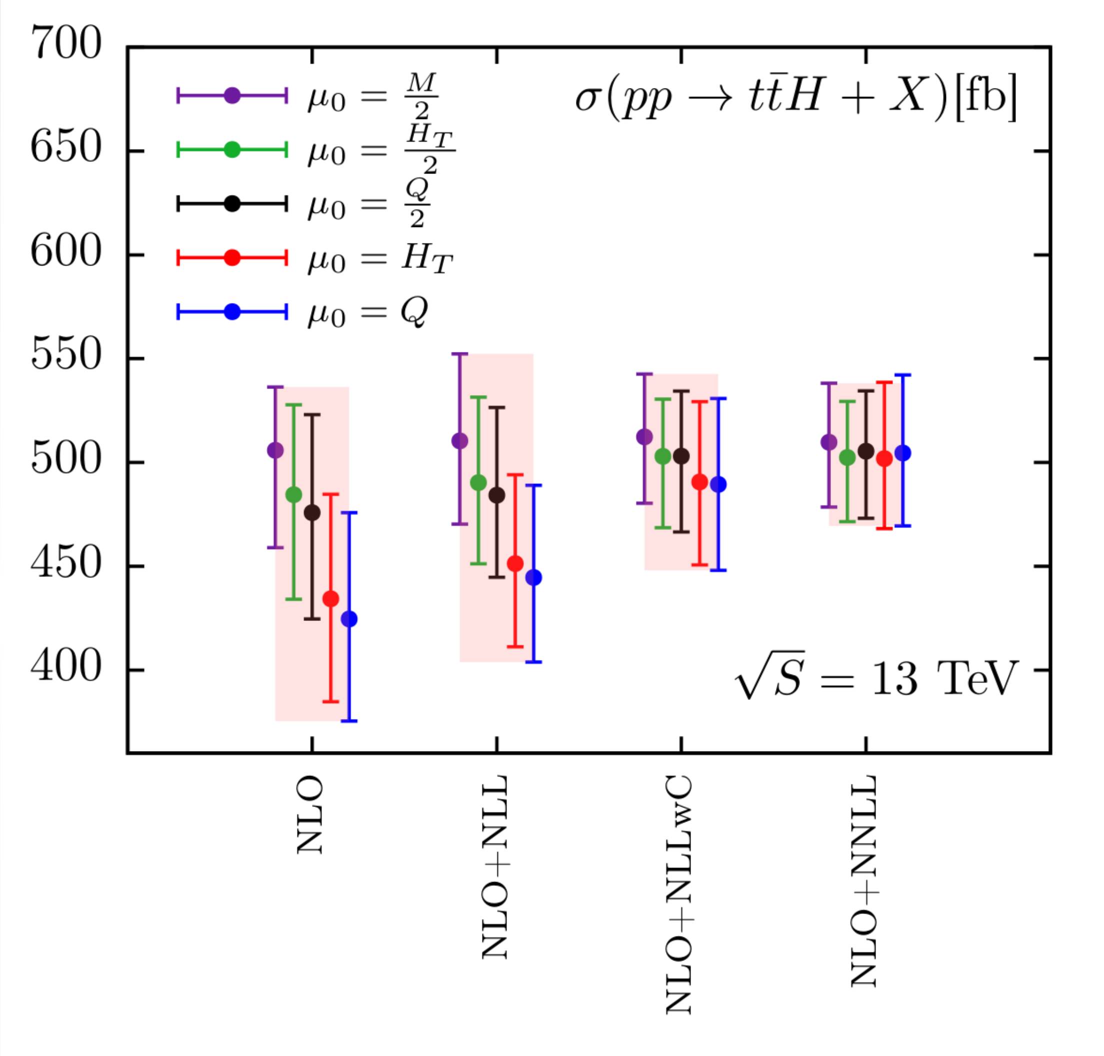


- We use the **7-point method** to estimate of theoretical uncertainty due to independent variations of renormalisation and factorisation scales
- Sizable improvement of theoretical precision of NLO+NNLL w.r.t. the NLO
- Improvement of the results stability w.r.t. the choice of the central scale

Predictions for $t\bar{t}H$: NLO + NNLL invariant mass dependent resummation

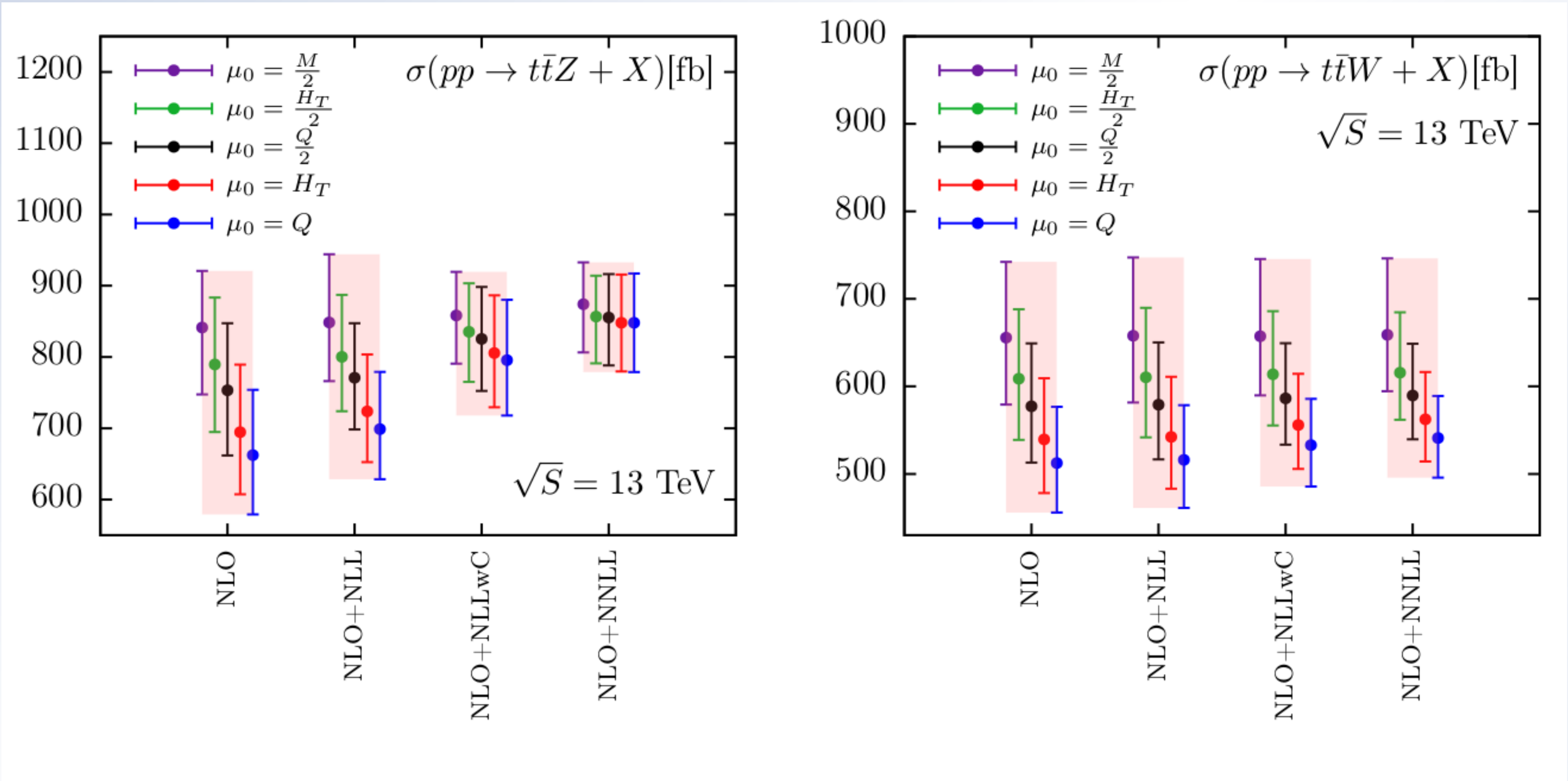
Central scales:

- $M = 2m_t + m_H$
- $Q^2 = (p_t + p_{\bar{t}} + p_H)^2$
- $H_T = \text{sum } M_{Ti}$



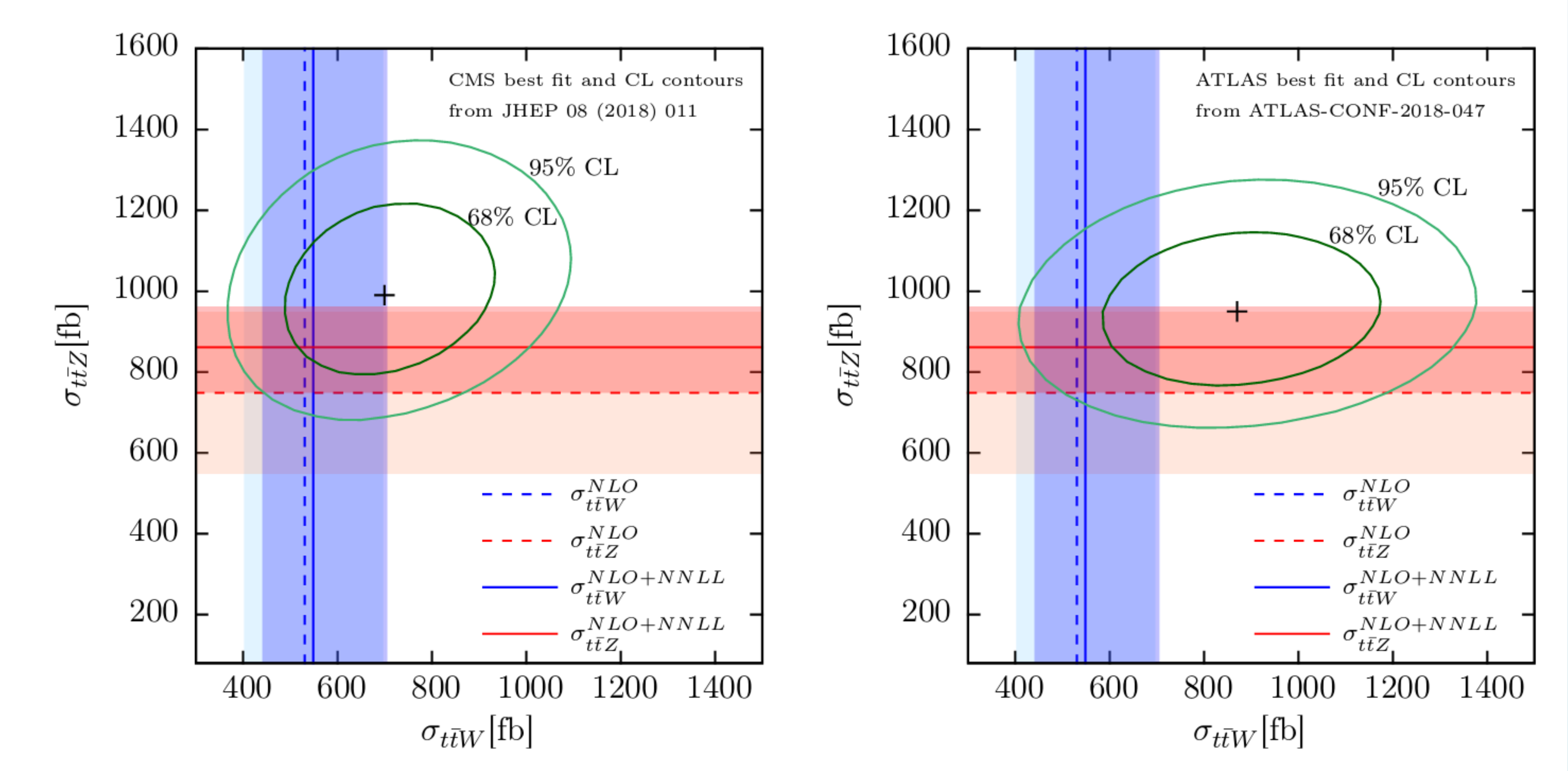
Predictions for $t\bar{t}Z$ and $t\bar{t}W$: NLO + NNLL invariant mass dependent resummation

- The same framework as for associated Higgs boson production



- In the case of Z boson – similar pattern to the Higgs boson
- For the W boson – only small improvement due to resummation → reason: lack of gluon – gluon channel

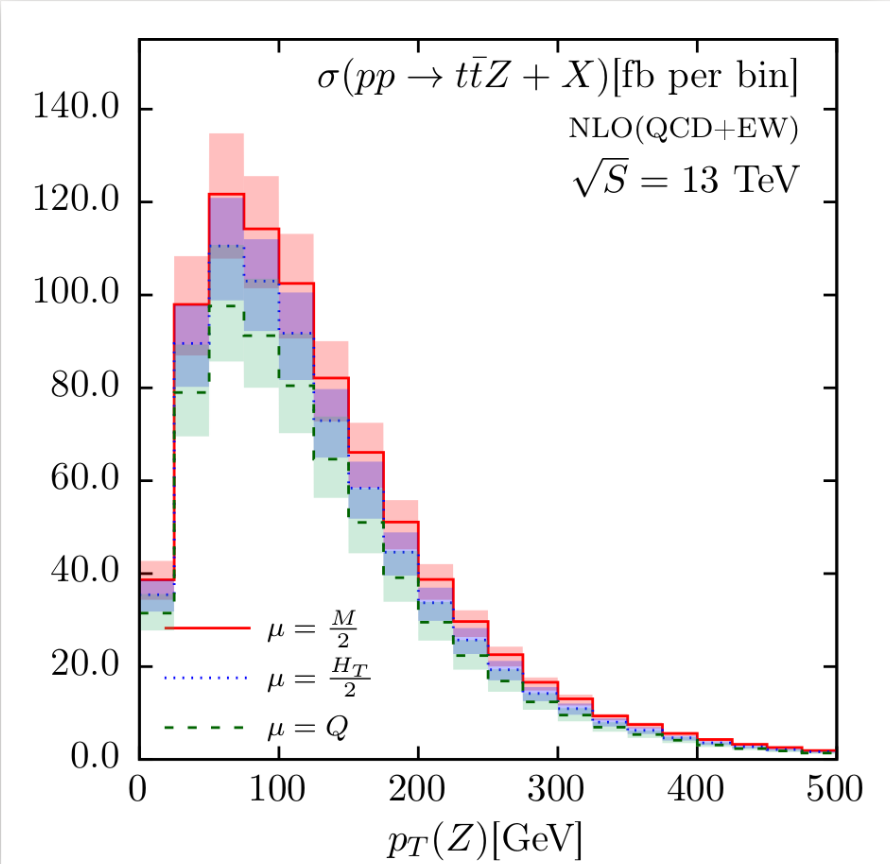
NLO + NNLL invariant mass dependent resummation for $t\bar{t}Z$ and $t\bar{t}W$: comparison to CMS and ATLAS data



- Theory errors reduced, and for Z made lower than experimental ones
- Measurements consistent with the Standard Model predictions

Predictions: NLO+NNLL resummation for Z-boson pT-distribution in hadroproduction of tt̄Z

- Included: NLO+NNLL QCD + NLO EW



- NLO results show quite some sensitivity to the choice of central scale

Predictions: NLO + NNLL resummation for Z-boson pT-distribution in hadroproduction of $t\bar{t}Z$

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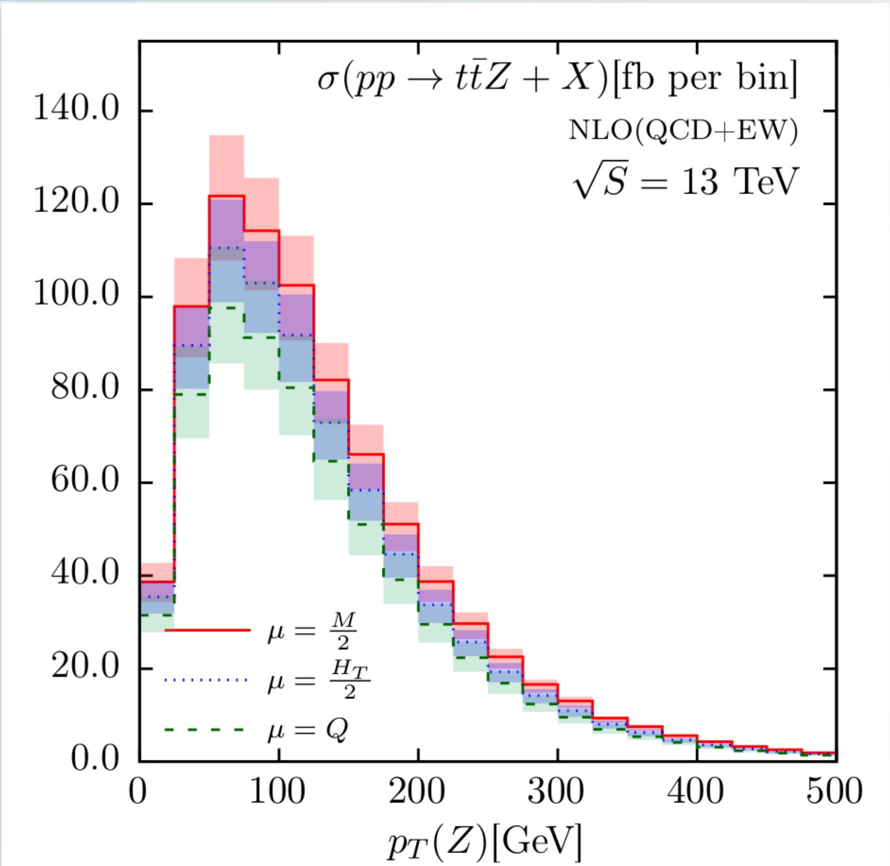
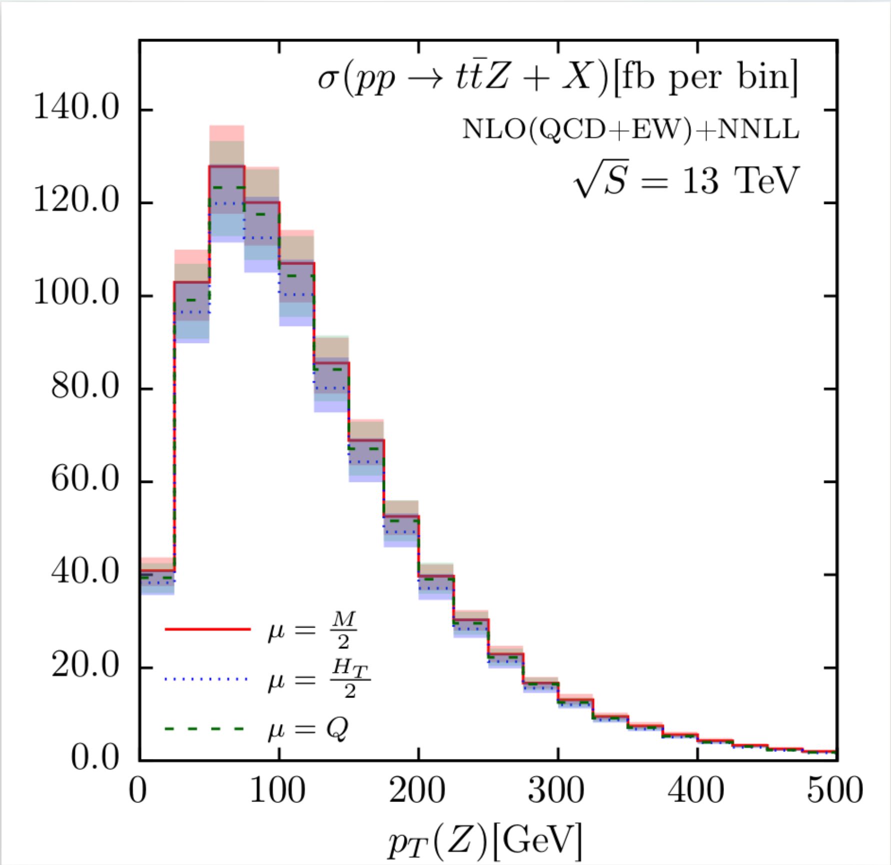




- NLO results show quite some sensitivity to the choice of central scale
- NLO + NNLL results lead to smaller theoretical uncertainty

NLO + NNLL resummation for Z boson pT distribution in t-t̄-Z hadroproduction: comparison to CMS (2019) data

- Accuracy:

  NLO QCD + NLO EW

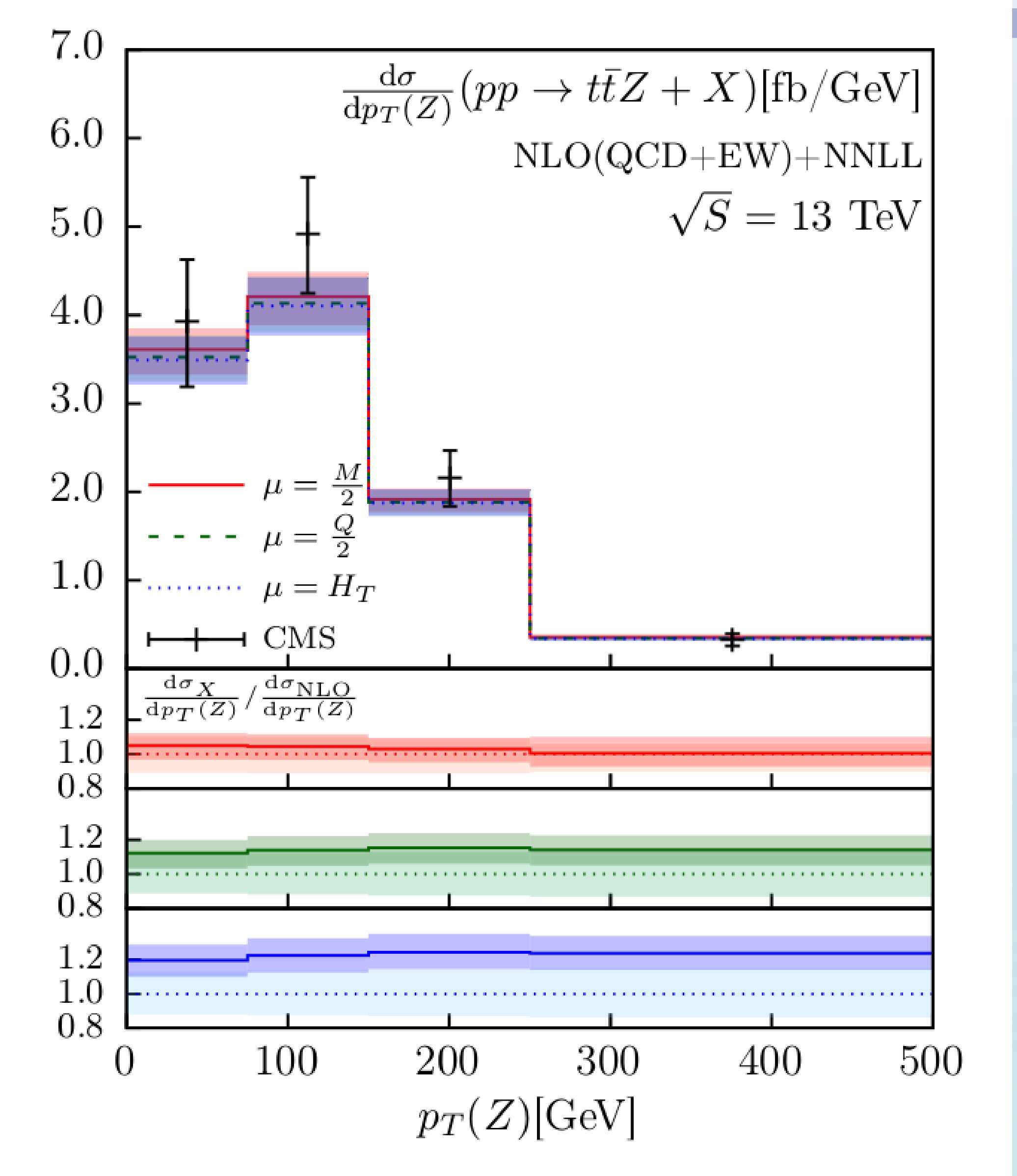
  vs

  NLO + NNLL QCD + NLO EW

- NLO + NNLL results:

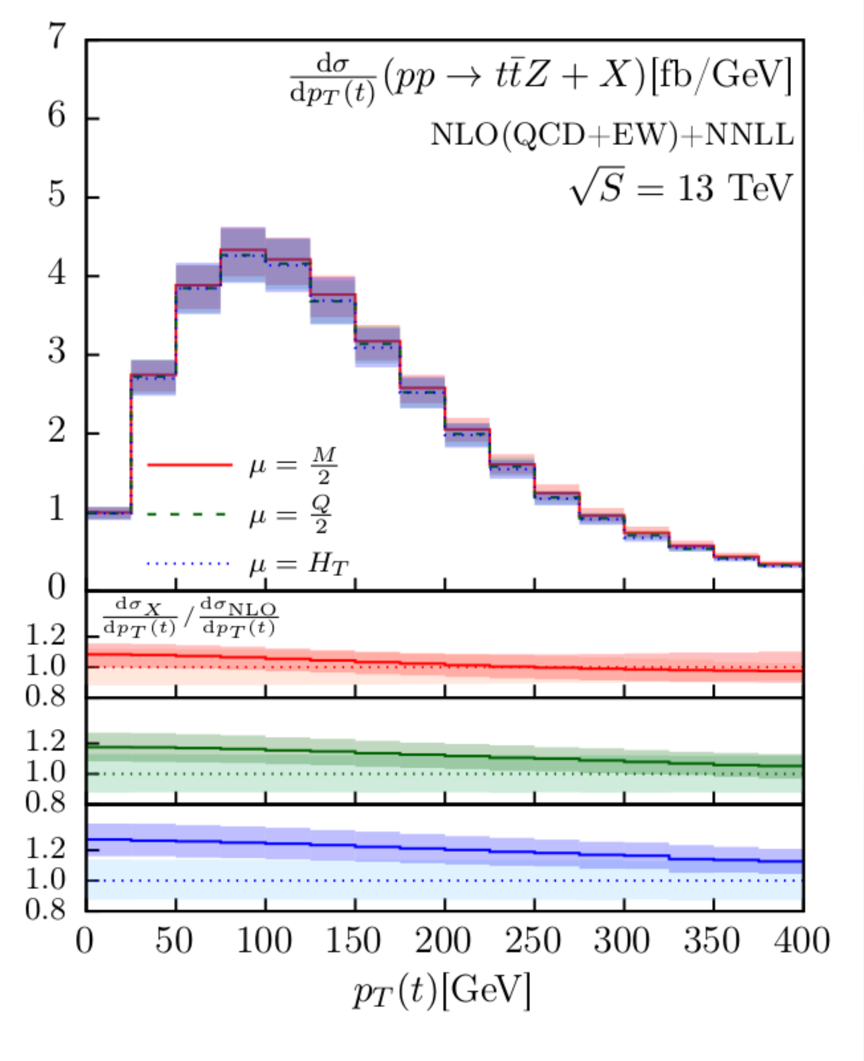
  closer to data,

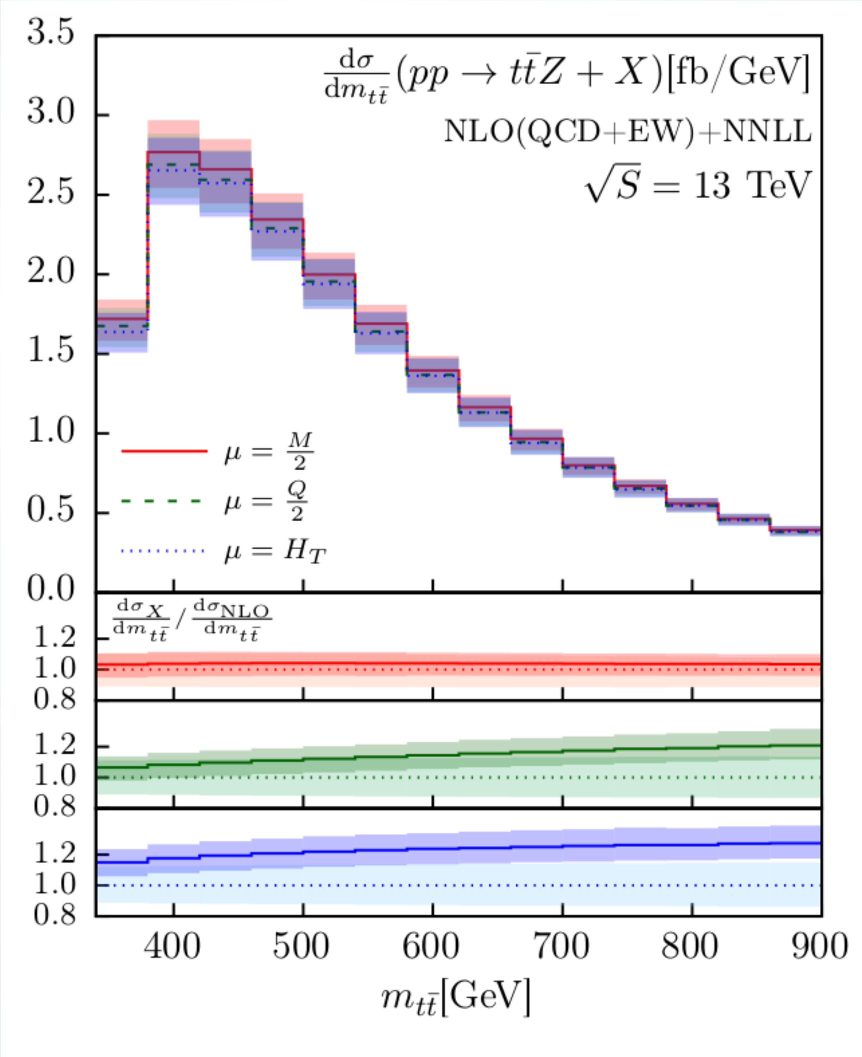
  smaller theoretical uncertainty



Distributions at NLO (QCD+EW)+NNLL for $t\bar{t}Z$ [New!]

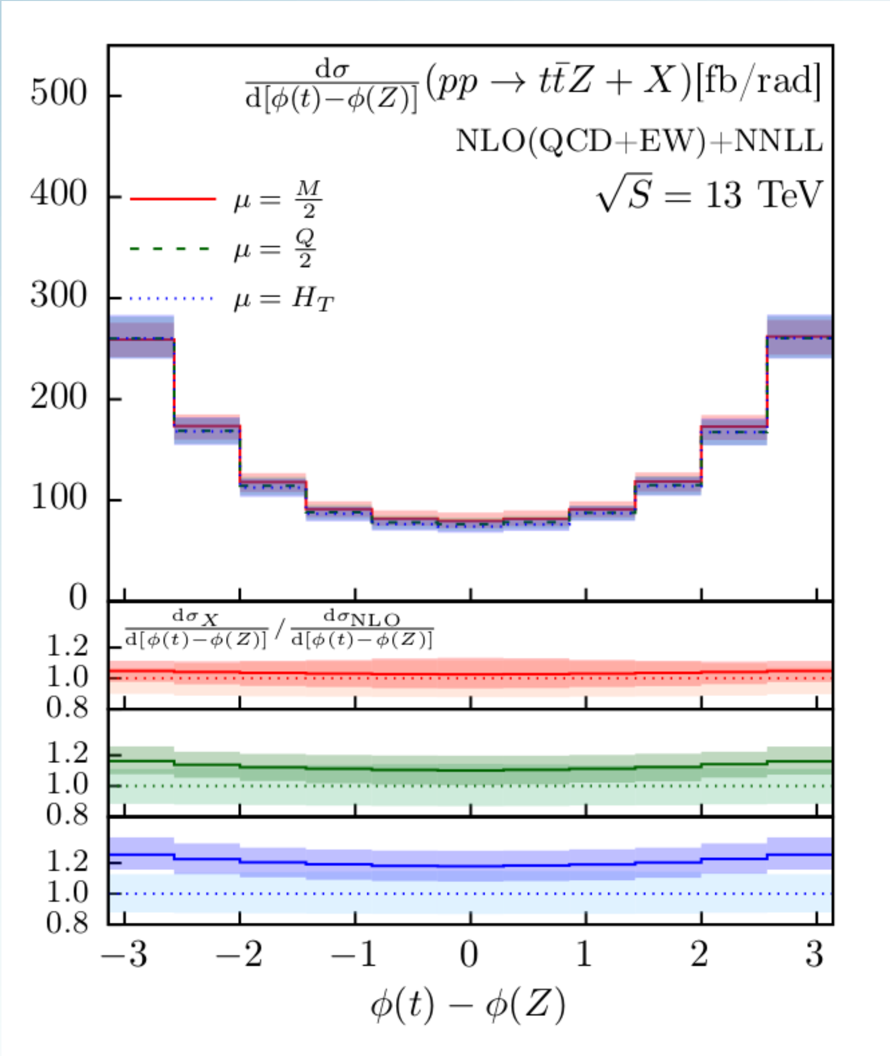
- P_T (top) and M(top-antitop)

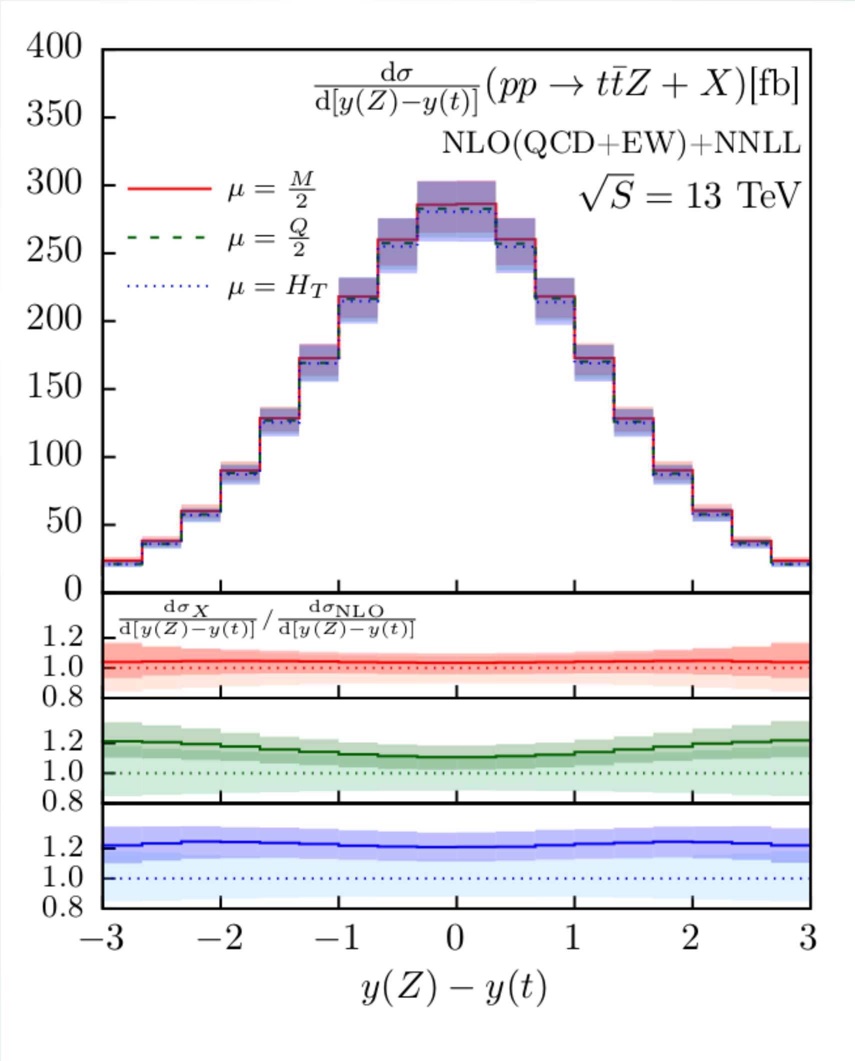




Distributions at NLO (QCD+EW)+NNLL for $t\bar{t}Z$ [New!]

- Relative azimuthal angle of top and Z and the difference of Z – t rapidities





Summary

- Soft gluon resummation in $t\bar{t}H$, $t\bar{t}Z$ and $t\bar{t}W$ production in *pp* collisions have been performed for absolute threshold and inariant mass dependent schemes at NNLL accuracy matched to NLO QCD

- Extension performed of the soft gluon resummation to three body final state with non-trivial colour structure

- Significant improvement of the NLO+NNLL theoretical precision w.r.t. the NLO results

- The currently most precise theoretical estimates obtained of hadroproduction $t\bar{t}H$, $t\bar{t}Z$ and $t\bar{t}W$ cross sections at the LHC including differential distributions

- The obtained results lead to an improved description of data