Quasi-TMDPDFs and the Collins-Soper Kernel

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Workshop on Resummation, Evolution, Factorization
University of Pavia
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Based On: M. Ebert, IS, Y. Zhao, Phys. Rev. D99, 034505 (2019) [1811.00026]
M. Ebert, IS, Y. Zhao, JHEP 1909, 037 (2019) [1901.03685]
M. Ebert, IS, Y. Zhao, 1910.08569
P. Shanahan, M. Wagman, Y. Zhao, 1911.00800, and work in progress

Massachusetts Institute of Technology

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TMD Collaboration

Outline

- **TMD Factorization & Evolution**

  Goal: determine CS Kernel (rapidity anomalous dimension)

  $\gamma_\zeta^q(\mu, b_T)$ **nonperturbatively** $[\gamma_\zeta^q = K = \mathcal{D}]$

- **TMDPDF = Hadronic and Vacuum matrix elements**

- **Quasi-TMDPDFs for Lattice QCD calculations**

- **Nonperturbative proposal to obtain** γ_ζ^q **(& prelim. results)**

TMD Factorization

- rigorous QFT derivation of cross sections

eg. Drell-Yan $q_T \ll Q$

$$\sigma(q_T, Q, Y) = H(Q,\mu) \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b) \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

$\overline{\rm MS}$ **Hard function (virtual corrections)**

ζ = Collins-Soper parameter

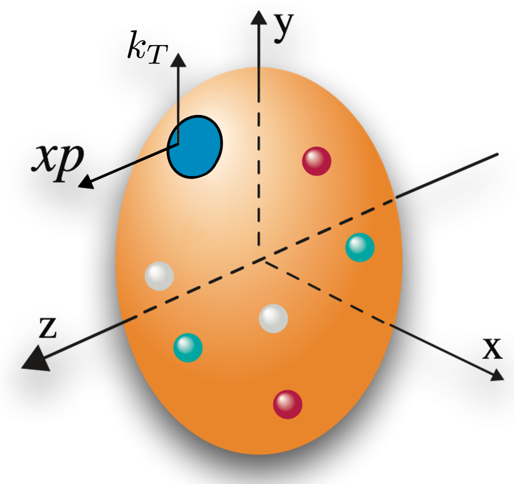
$\zeta_a \zeta_b = Q^4$ $\qquad \zeta_a = (x_a P_a^-)^2 = (2x_a P^z)^2$

here: $\zeta_{a,b} \sim Q^2$

- **nonperturbative**

  $k_T \sim b_T^{-1} \sim \Lambda_{\rm QCD}$

$f_q(x, \vec{k}_T, \mu, \zeta)$



- **perturbative**

  $k_T \sim b_T^{-1} \gg \Lambda_{\rm QCD}$

$$f_q(x, \vec{k}_T, \mu, \zeta) = \sum_i \int \frac{dy}{y}\, C_{qi}\left(\frac{x}{y}, \vec{k}_T, \mu, \zeta\right) f_i(y, \mu)$$

perturbative PDF

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

$$\mu \frac{d}{d\mu} \gamma_\zeta^q(\mu, b_T) = 2\zeta \frac{d}{d\zeta} \gamma_\mu^q(\mu, \zeta) = -2\Gamma_{\rm cusp}^q[\alpha_s(\mu)]$$

path independence

All Orders form:

$$\gamma_\mu^q(\mu, \zeta) = \Gamma_{\rm cusp}^q[\alpha_s(\mu)] \ln \frac{\mu^2}{\zeta} + \gamma_\mu^q[\alpha_s(\mu)]$$

$$\gamma_\zeta^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^q[\alpha_s(\mu')] + \gamma_\zeta^q[\alpha_s(1/b_T)]$$

- **Perturbative at short distance** $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$

3-loop result: Li, Zhu 2016; Vladimirov 2016

$$\gamma_\zeta^q[\alpha_s] = \alpha_s\, \gamma_\zeta^{q(1)} + \alpha_s^2\, \gamma_\zeta^{q(2)} + \alpha_s^3\, \gamma_\zeta^{q(3)} + \ldots$$

→ **LL, NLL, NNLL, N3LL, … results**

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

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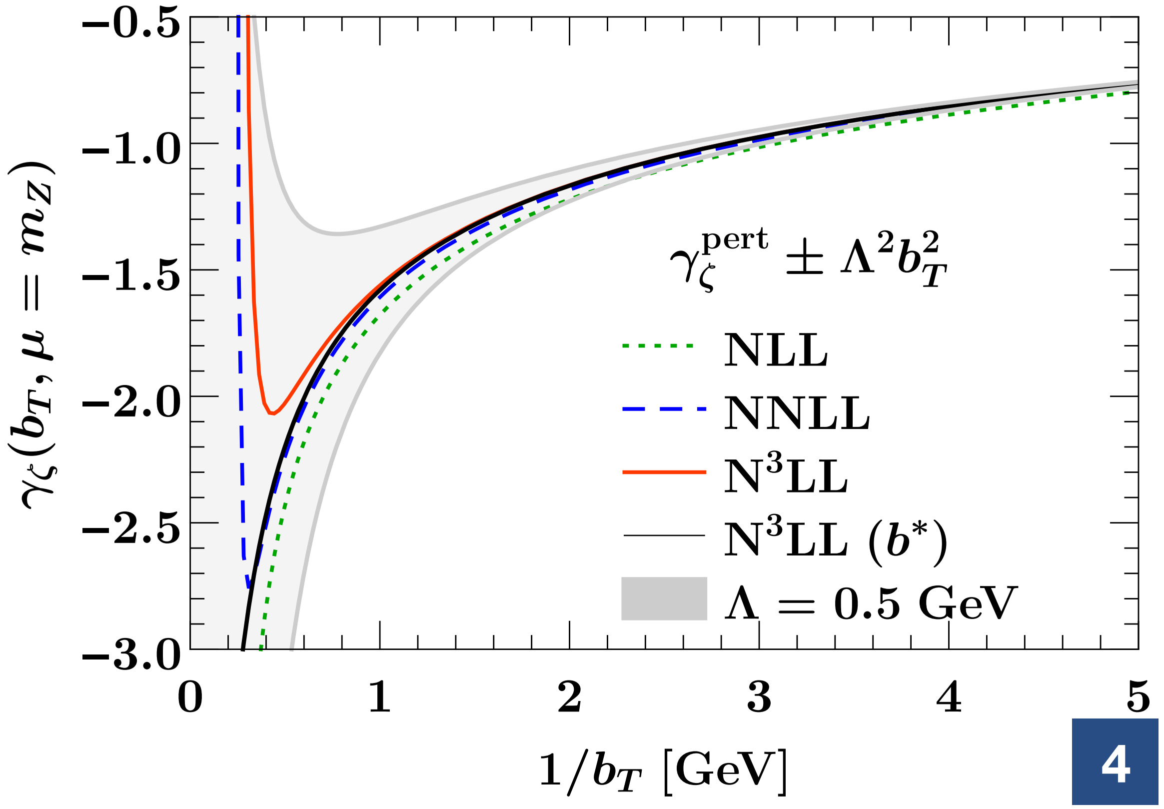
path independent

All Orders form:

$$\gamma_\mu^q(\mu, \zeta) = \Gamma_{\rm cusp}^q[\alpha_s(\mu)] \ln \frac{\mu^2}{\zeta} + \gamma_\mu^q[\alpha_s(\mu)]$$

$$\gamma_\zeta^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^q[\alpha_s(\mu')] + \gamma_\zeta^q[\alpha_s(1/b_T)]$$

- **For** $b_T^{-1} \sim \Lambda_{\rm QCD}$ **the CS kernel** $\gamma_\zeta^q(\mu, b_T)$ **becomes nonperturbative**



TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

Solution:

$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\Big] \exp\Big[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\Big] \times f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

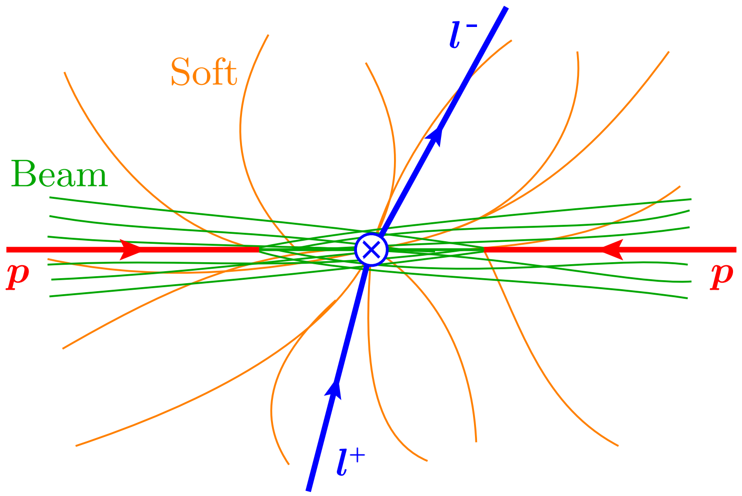
Use to: Connect Lattice calculation (or model) with $\mu \sim P^z \sim \text{few GeV}$

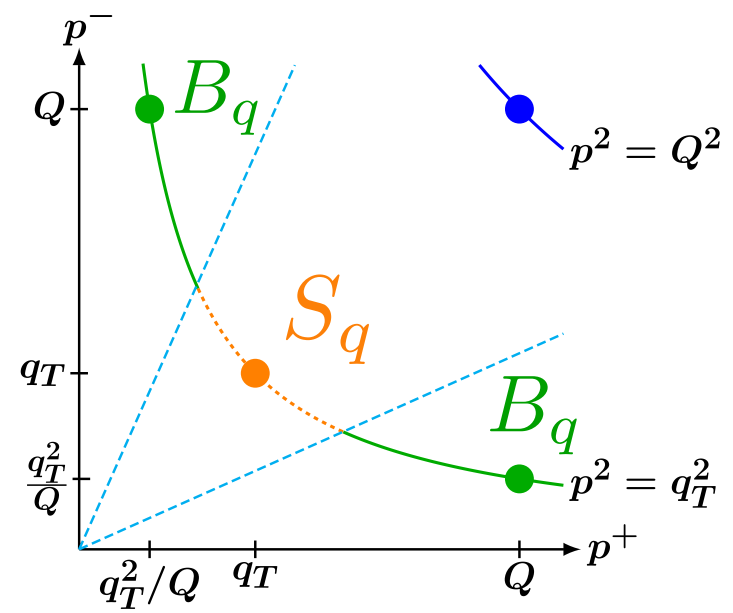
to scales needed in factorization theorem: $\mu \sim Q\ ,\ \ P^z \sim Q/x$

TMD Definitions

Beam Function **Soft factor**

$$f_q(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to 0,\tau\to 0} Z_{\rm uv}(\epsilon,\mu,\zeta) B_q(x,\vec{b}_T,\epsilon,\tau,\zeta)\ \Delta_q(b_T,\epsilon,\tau)$$



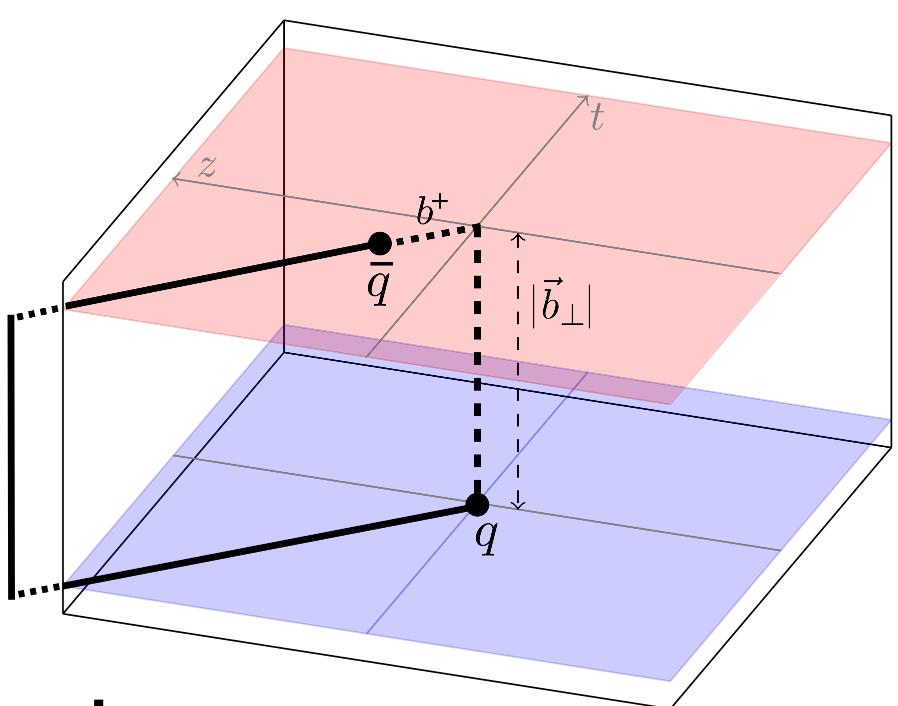


contains S_q & subtractions

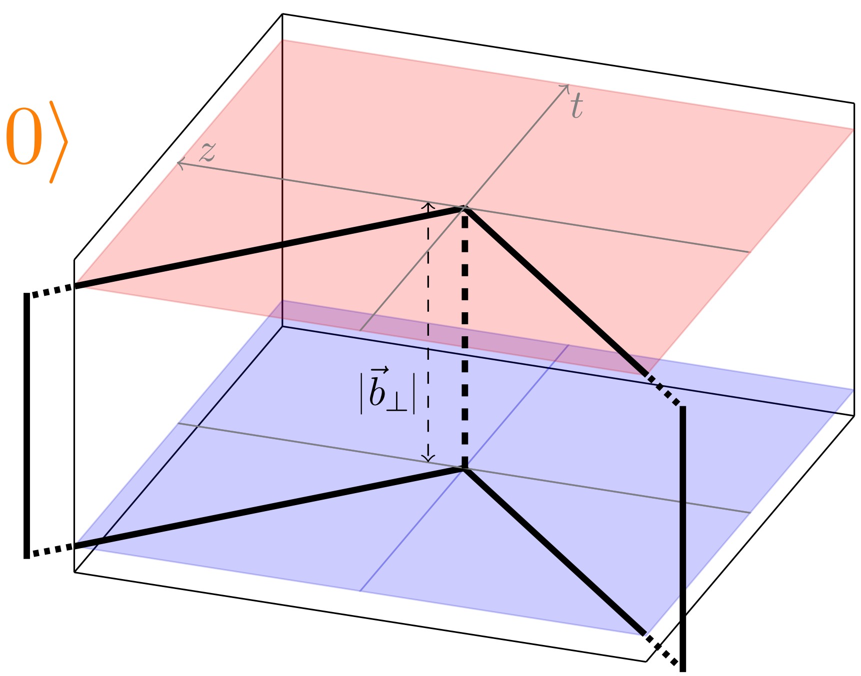
$$B_q = {\rm FT}_{b^+} \langle p|O_B|p\rangle$$

$$S_q = \langle 0|O_S|0\rangle$$

O_B :



O_S :



staple shaped Wilson lines

two light-cone directions

depends on color rep. (q or g)

TMD Definitions

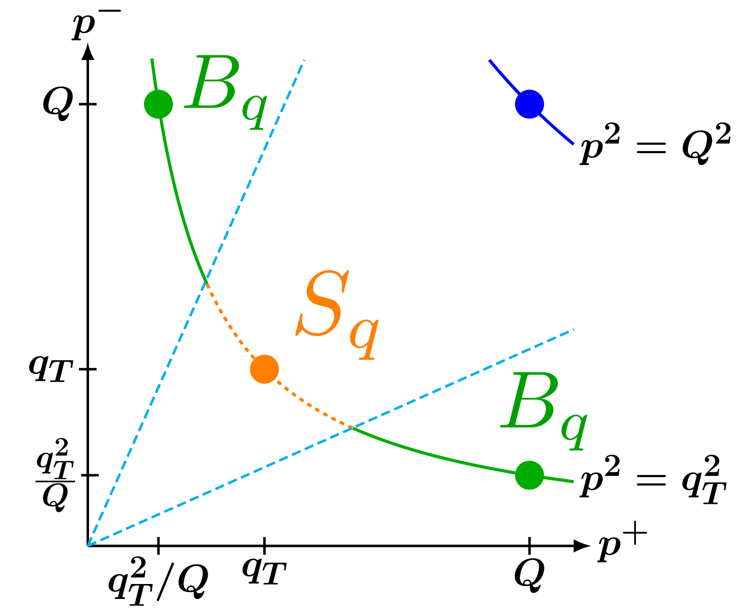
"Beam Function" "Soft Factor"

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0, \tau \to 0} Z_{\rm uv}(\epsilon, \mu, \zeta) B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)\ \Delta_q(b_T, \epsilon, \tau)$$

TMDPDF

ϵ : regulates UV divergences

τ : regulates rapidity divergences



Schemes: Same f_q , ie. universal (across most schemes)
Different B_q & Δ_q

- Wilson lines off the light cone (Modern Collins '11) $\Delta_q = 1/\sqrt{S_q}$
- Delta regulator $(k^\pm + i\delta^\pm)$ (Echevarria,Idilbi,Scimemi '11) $\Delta_q = 1/\sqrt{S_q}$
- η regulator $|k^z/\nu|^{-\eta}$ (Chiu,Jain,Neill,Rothstein '12) $\Delta_q = \sqrt{S_q}$
- Exponential regulator $e^{-k^0\tau}$ (Li,Neill,Zhu '16) $\Delta_q = 1/\sqrt{S_q}$

TMD Definitions

"Beam Function" **"Soft Factor"**

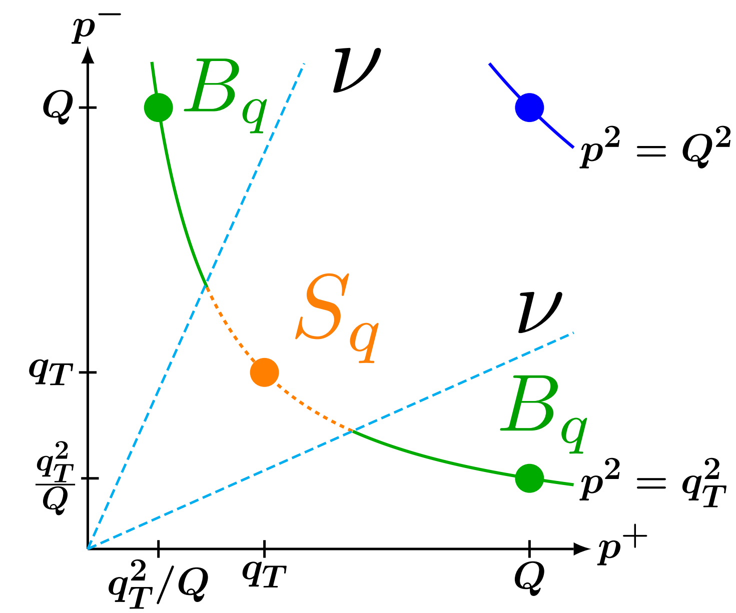
$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\mathrm{uv}}(\epsilon, \mu, \zeta) B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)\ \Delta_q(b_T, \epsilon, \tau)$$

TMDPDF

ϵ : regulates UV divergences

τ : regulates rapidity divergences

$\nu =$ cutoff after renormalization



Schemes: **Same f_q, ie. universal (across most schemes)**
Different B_q & Δ_q

$$f_q(x, \vec{b}_T, \mu, \zeta) = B_q^{\mathrm{ren}}(x, \vec{b}_T, \mu, \nu^2/\zeta)\ \Delta_q^{\mathrm{ren}}(b_T, \mu, \nu)$$

$$\gamma_\zeta(b_T, \mu) = 2\zeta \frac{d}{d\zeta} \ln f_q = -\nu \frac{d}{d\nu} \ln B_q^{\mathrm{ren}} = \frac{1}{2}\nu \frac{d}{d\nu} \ln \Delta_q^{\mathrm{ren}}$$

CS kernel
= rapidity anom.dim.

Vacuum matrix element, so $\gamma_\zeta(b_T, \mu)$ is <u>independent of hadronic state</u>.

Lattice QCD and quasi-distributions

$$\int D\psi D\bar{\psi} D\mathcal{A}\, \mathcal{O}\, e^{-S_E}$$

Euclidean path integral

- **PDF and TMDPDF operators are intrinsically Minkowski**

$$n_E^2 = 0 \;\Leftrightarrow\; n_E^\mu = 0$$

Quasi-PDF (warmup)

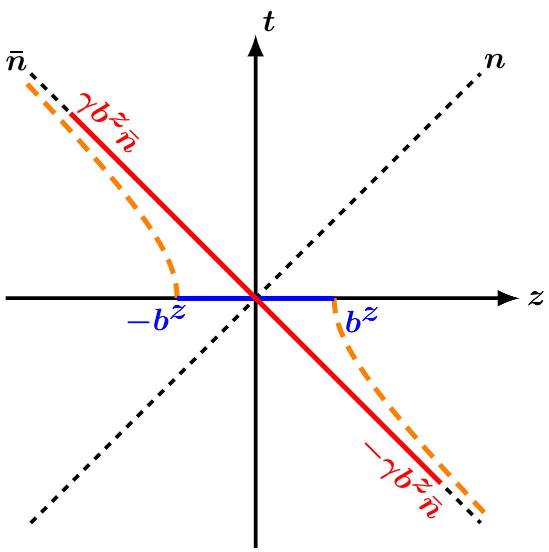
(Ji 2013)

$$\tilde{f}_q(x, P^z, \epsilon) = \int \frac{db^z}{4\pi} e^{ib^z x P^z} \langle p(P) | \bar{q}(b^z) W_z(b^z, 0) \gamma^0 q(0) | p(P) \rangle$$

- **Same IR as PDF, related to PDF by a boost**

boost to $\mathcal{O}$ $\Leftrightarrow$ boost to proton state

take $\Lambda_{\mathrm{QCD}} \ll P^z$ **"LaMET"**



- **Perturbative matching**

$$\tilde{f}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right) \boxed{f_j(y, \mu)} + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

simulation & renormalization on lattice

Perturbative matching coefficient

PDF

Power corrections

Proven factorization theorem [Xiong, Ji, Zhang, Zhao '13; Ma, Qiu '14 '17; Izubuchi, Ji, Jin, Stewart, Zhao '18]

Lattice QCD and quasi-distributions

$$\int D\psi D\bar{\psi} D\mathcal{A}\ \mathcal{O}\ e^{-S_E}$$

Euclidean path integral

- **PDF and TMDPDF operators are intrinsically Minkowski**

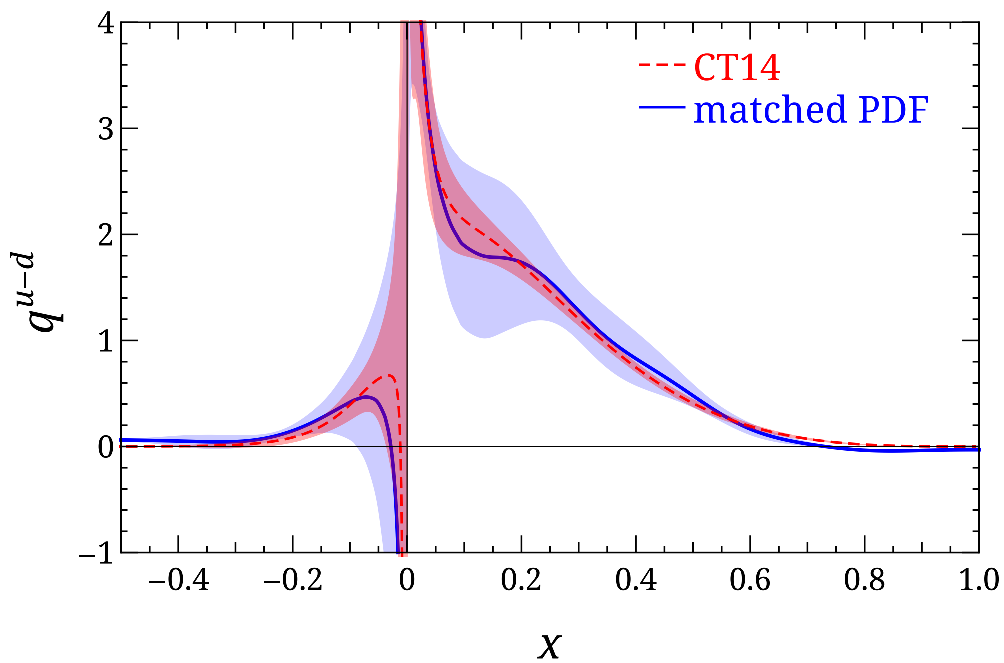
$$n_E^2 = 0 \ \Leftrightarrow \ n_E^\mu = 0$$

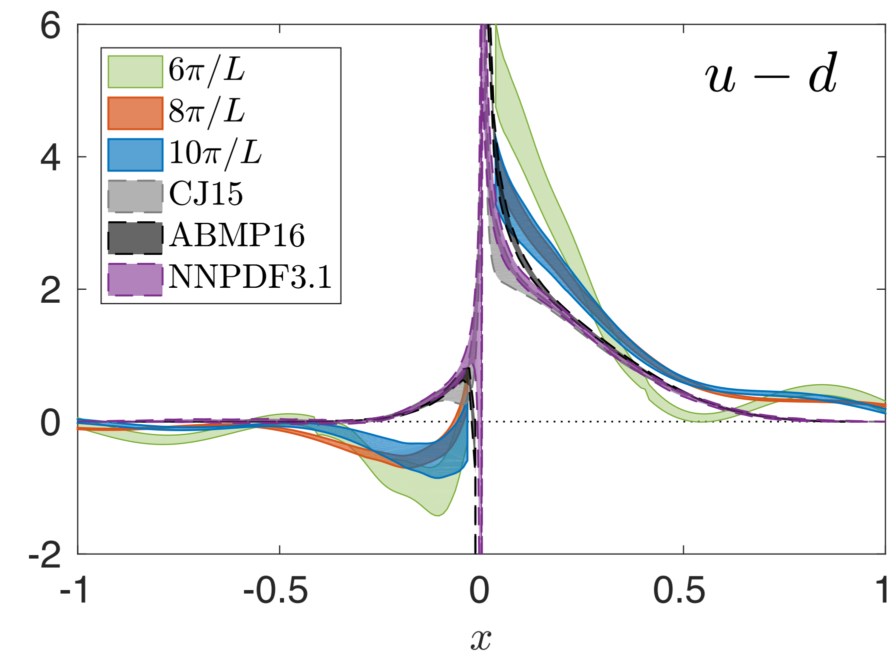
Quasi-PDF (warmup)

$$\tilde{f}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\ C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right)\ f_j(y,\mu)\ +\ \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

State-of-the art calculations at physical pion mass:

- [Lin et al (LP3)]: $P^z = \{2.2,\ 2.6,\ 3.0\}$ GeV
- [Alexandrou et al (ETMC)]: $P^z = \{0.83, 1.11, 1.38\}$ GeV





Quasi-TMDPDFs

UV renormalization

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0 \\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a)$$

$$\times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

quasi-Beam function quasi-soft factor

a = lattice spacing (UV regulator)

- **needs to be computable with Lattice QCD**
- **must have same IR physics as TMDPDF**

(including $b_T \sim \Lambda_{\rm QCD}^{-1}$ dependence)

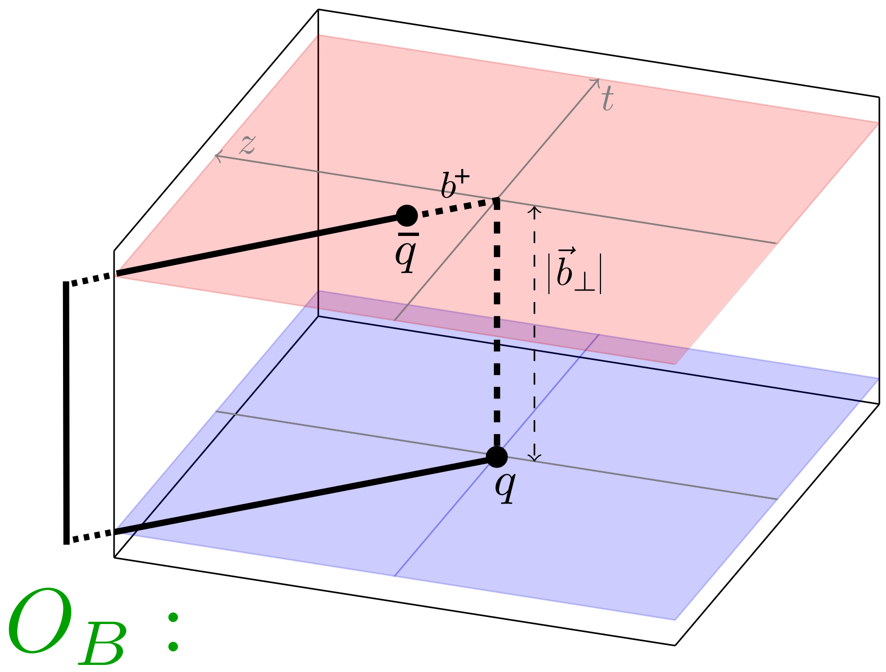
(isovector quark operators u-d, from here on)

Quasi-Beam Functions

$$\tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$$

Beam Function

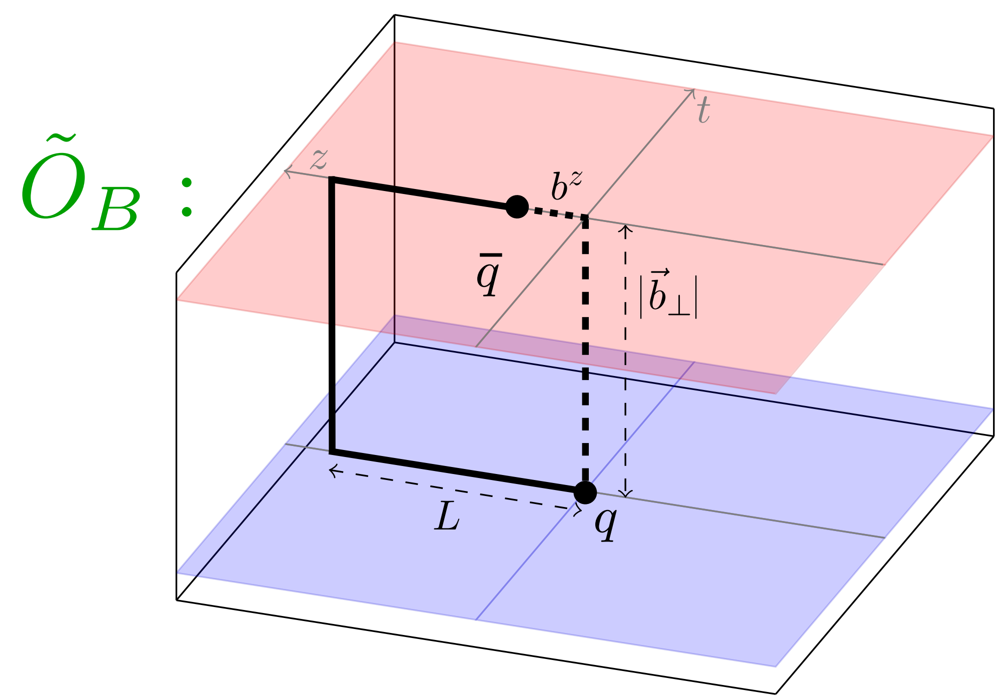
$$B_q = \langle p | O_B | p \rangle$$



Connected by boost (for bare operators)

Natural Quasi-Beam Function

$$\tilde{B}_q = \langle p | \tilde{O}_B | p \rangle$$



- **Finite length L for Wilson lines, regulates rapidity divergences** $\dfrac{1 - e^{-\mathrm{i}k^z L}}{k^z}$

  $$\frac{1}{P^z} \ll b_T \ll L$$

- **Spatial lines, so have power law UV divergence** $\propto \text{length} = 2L + b_T - b^z$

Quasi-Soft Function

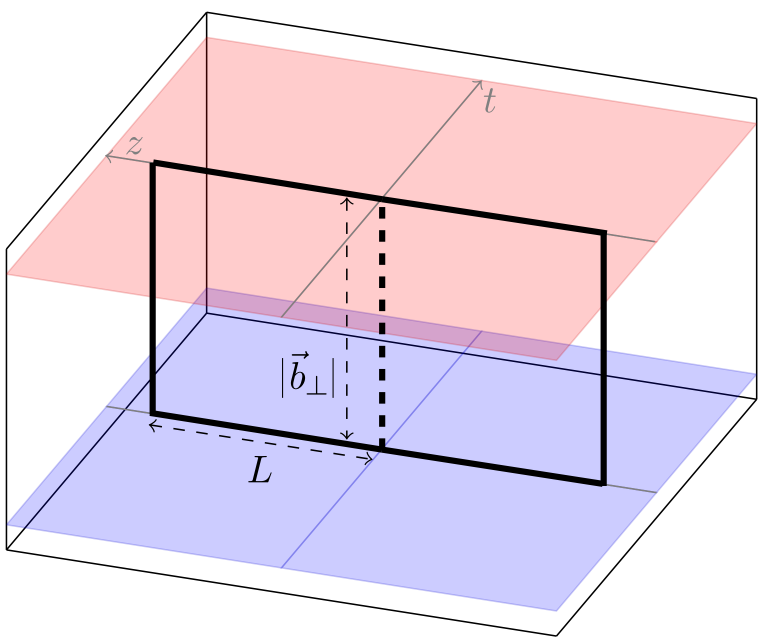
$\tilde{\Delta}_S^q = 1/\sqrt{\tilde{S}_q}$ $\tilde{S}_q = \langle 0|\tilde{O}_S|0\rangle$

- **Cancel power law dependence on L, length =** $2(2L + b_T)$
- **Needed to reproduce infrared structure.**
- **Free to invent a** $\tilde{O}_S$ **to achieve this.**

[Ebert, IS, Zhao '18]
[Ji, Sun, Xiong, Yuan '14]

(1) "naive" quasi-soft

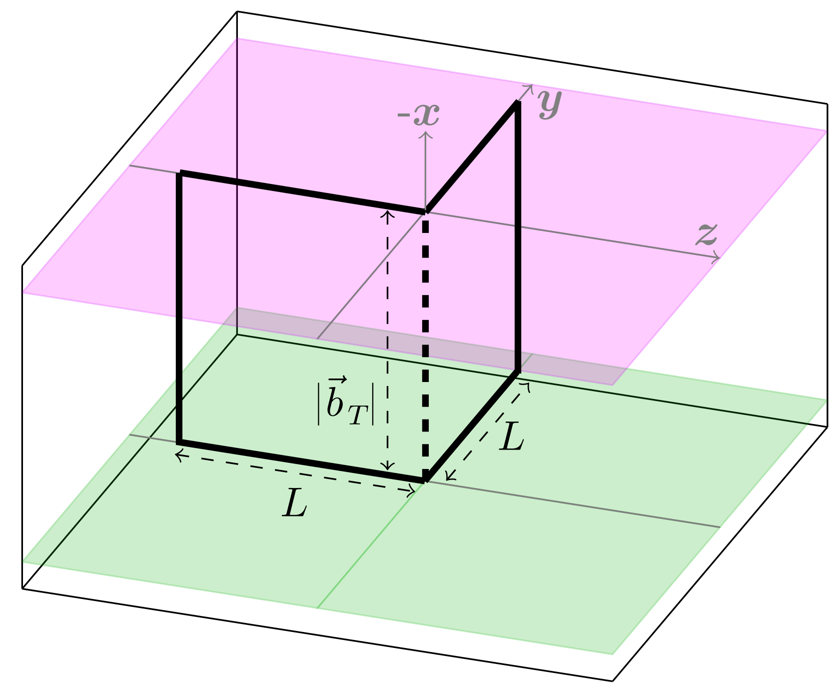
Invalid: IR differs @ 1-loop



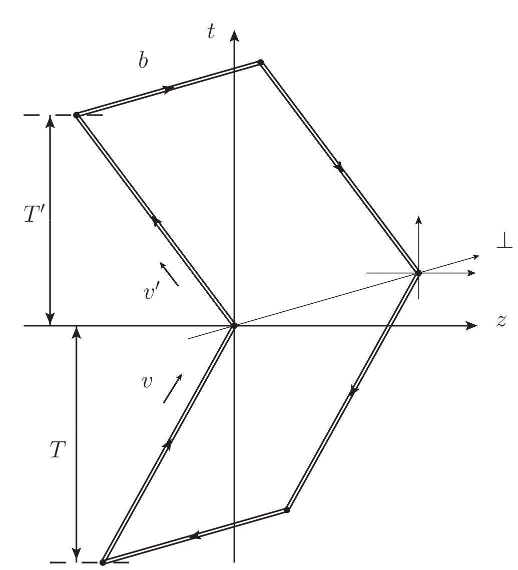
(2) "bent" quasi-soft

IR agrees @ 1-loop,

beyond?



(3) quasi-soft from boosted HQET QQ



[Ji, Liu, Liu 1910.11415]

Quasi-TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0\\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a) \times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

- **linear divergences in L cancel**
- $\tilde{Z}^q_{\rm uv}$ **multiplicative, and removes linear** b^z/a **divergence**
- $\tilde{Z}'_q$ **converts lattice friendly scheme (**$\tilde{\mu}$**) to** $\overline{\rm MS}$ **(**μ**)**

Relation between Quasi-TMDPDF & TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\mathrm{TMD}}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

nonperturbative quasi-TMDPDF

perturbative kernel

nonperturbative factor that changes based on choice for $\tilde{\Delta}_S^q$

nonperturbative CS kernel

nonperturbative TMDPDF

Confirmed explicitly at one-loop (so far)

(Note: no convolution in x)

Determination of $\gamma_\zeta^q(\mu, b_T)$

- **independent of $g_q^S(b_T, \mu)$**

(full matching would require $g_q^S(b_T, \mu) = 1$)

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\mathrm{TMD}}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMDPDF

Hold ζ fixed, take ratio with two different P^zs:

$$\frac{\tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{\tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)} = \frac{C^{\mathrm{TMD}}(\mu, xP_1^z)\, \cancel{g_q^S(b_T, \mu)} \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP_1^z)^2}{\zeta}\right] \cancel{f_q(x, \vec{b}_T, \mu, \zeta)}}{C^{\mathrm{TMD}}(\mu, xP_2^z)\, \cancel{g_q^S(b_T, \mu)} \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP_2^z)^2}{\zeta}\right] \cancel{f_q(x, \vec{b}_T, \mu, \zeta)}}$$

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMDPDF

Hold ζ fixed, take ratio with two different P^zs:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

One-loop illustration:

$$\frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)} = 1 + \frac{\alpha_s C_F}{\pi} \ln\frac{P_1^z}{P_2^z}\left(\ln\frac{4x^2 P_1^z P_2^z}{\mu^2} - 1\right) + \mathcal{O}(\alpha_s^2)$$

$$\frac{\tilde{f}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{\tilde{f}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)} = 1 + \frac{\alpha_s C_F}{\pi} \ln\frac{P_1^z}{P_2^z}\left(-\ln\frac{x^2 P_1^z P_2^z b_T^2}{e^{-2\gamma_E}} + 1\right) + \mathcal{O}(\alpha_s^2)$$

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln\frac{P_1^z}{P_2^z}} \ln\left[1 - \frac{\alpha_s C_F}{\pi} \ln\frac{P_1^z}{P_2^z} \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}} + \mathcal{O}(\alpha_s^2)\right] = -\frac{\alpha_s C_F}{\pi} \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}} + \mathcal{O}(\alpha_s^2)$$

✔

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu)\, \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMDPDF

Hold ζ fixed, take ratio with two different P^zs:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}\ln\frac{C^{\rm TMD}(\mu, xP_2^z)\,\tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\,\tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

Recall:

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int\frac{db^z}{2\pi}e^{ib^z(xP^z)}\lim_{\substack{a\to 0\\ L\to\infty}}\tilde{Z}'_q(b^z, \mu, \tilde{\mu})\tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a) \times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)\tilde{\Delta}_S^q(b_T, a, L)$$

- $\tilde{\Delta}_S^q$ **factors cancel out in the ratio**

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMDPDF

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z) \int db^z e^{ib^z xP_1^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z) \int db^z e^{ib^z xP_2^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_2^z)}$$

quasi-Beam fns.

- **needs $\tilde{B}_q$, $\tilde{Z}^q_{\rm uv}$, $\tilde{Z}'_q$, $C^{\rm TMD}$ (does not require $\tilde{\Delta}_S^q$)**
- **LHS independent of P_1^z, P_2^z, x, hadron state, spin**
- **can setup $\tilde{Z}^q_{\rm uv}$ to remove power law divergences**

Important universal QCD function obtainable from Lattice QCD

Ratios of proton $\tilde{B}_q$ s also studied by [Musch et al ’10’12; Engelhardt et al ’15; Yoon et al ’17]

$C^{\mathrm{TMD}}(\mu, xP^z)$ in $\overline{\mathrm{MS}}$ at 1-loop

[Ji, Jin, Yuan, Zhang, Zhao ’18]

[Ebert, IS, Zhao ’18]

$$C^{\mathrm{TMD}}(\mu, xP^z) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{(2xP^z)^2}{\mu^2} + 2\ln\frac{(2xP^z)^2}{\mu^2} - 4 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

$\tilde{Z}'_q$

conversion factor between RI/MOM scheme and $\overline{\mathbf{MS}}$ at 1-loop

M. Ebert, IS, Y. Zhao, 1910.08569

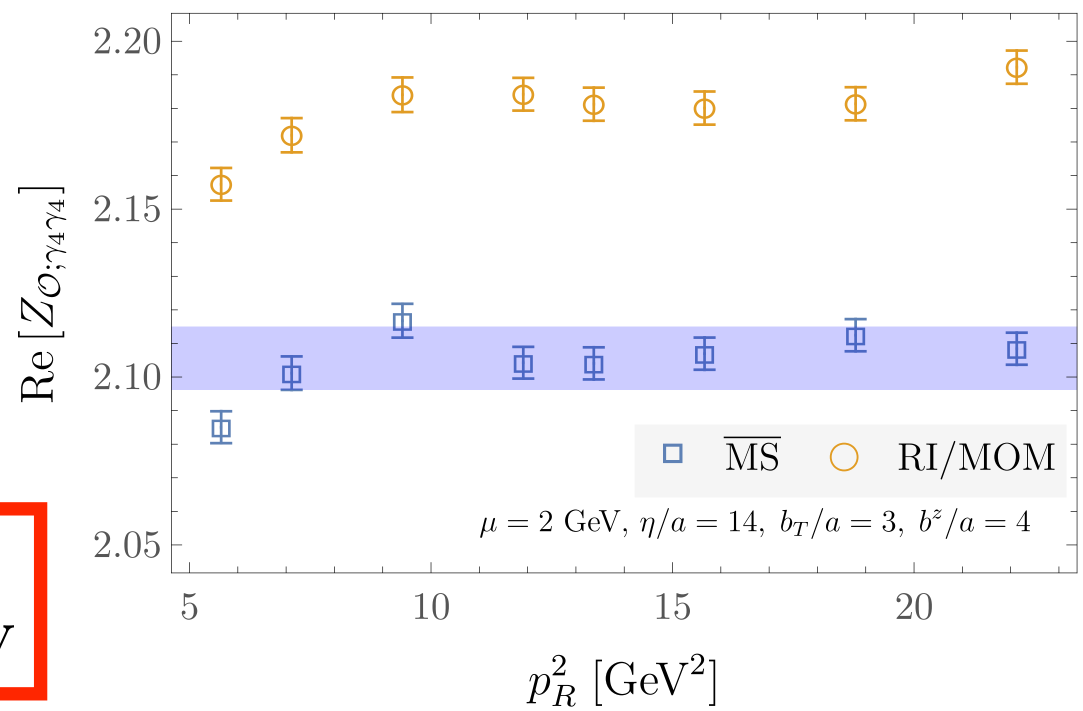
$\tilde{Z}^q_{\rm uv}$

P. Shanahan, M. Wagman, Y. Zhao, 1911.00800

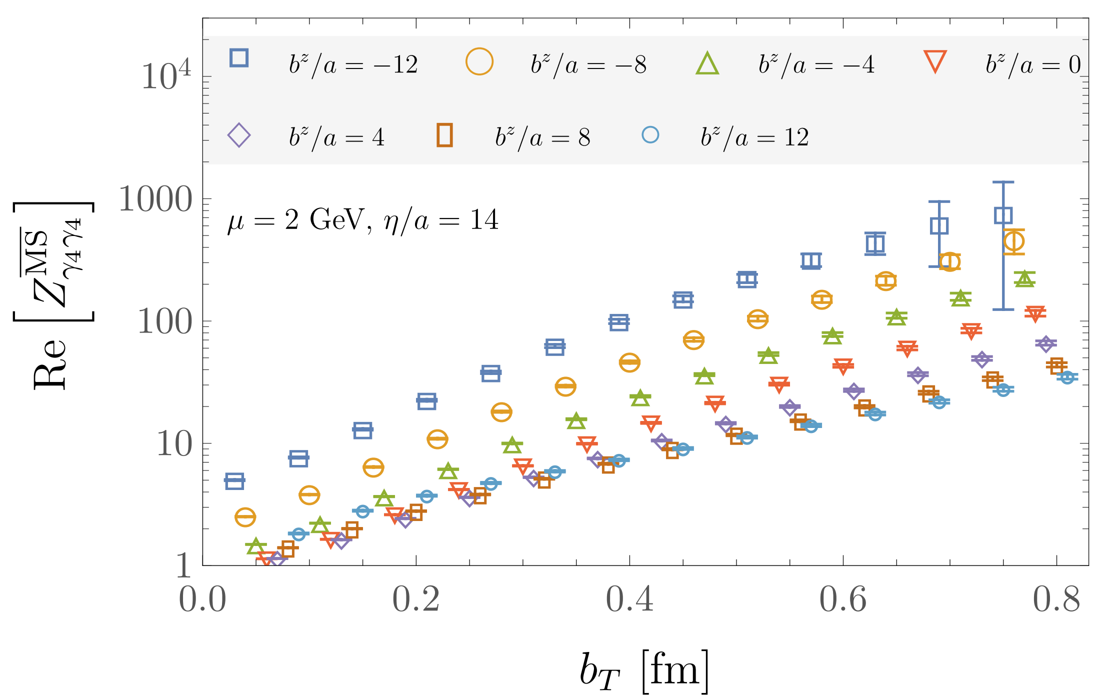
Calculate Nonperturbatively on Lattice in an RI/MOM scheme

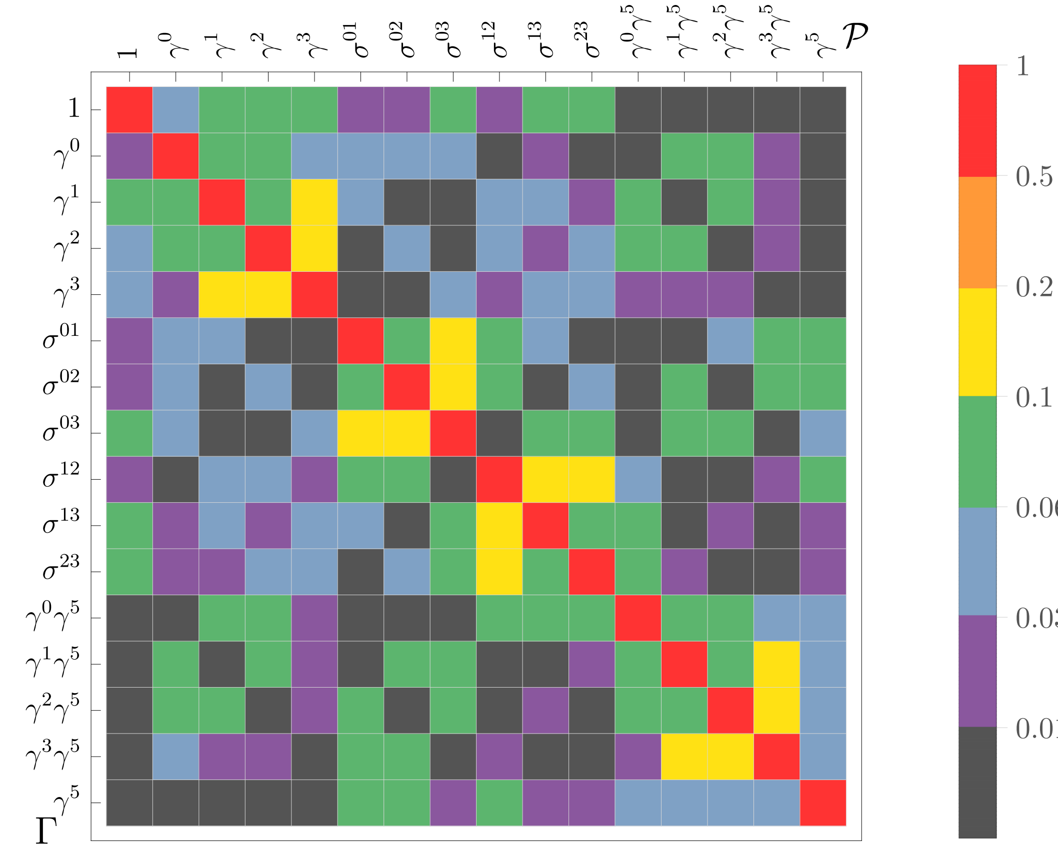
$$Z_q^{-1}(p_R) Z^{\rm RI'/MOM}_{\mathcal{O}_{\Gamma\Gamma'}}(p_R) \Lambda^{\mathcal{O}_{\Gamma'}}_{\alpha\beta}(p)\big|_{p^\mu = p^\mu_R} = \Lambda^{\mathcal{O}_\Gamma;\rm tree}_{\alpha\beta}(p)$$

quenched (nf=0)
improved Wilson fermions
smearing (Wilson flow) on gauge links
a=0.04, 0.06, 0.08 fm
volume ~ 2 fm
$m_\pi \sim 1.2\,\mathrm{GeV}, 340\,\mathrm{MeV}$
various L, bT, bz, pR
full 16x16 mixing matrix



$\tilde{Z}'_q \; \tilde{Z}^q_{\rm uv}$





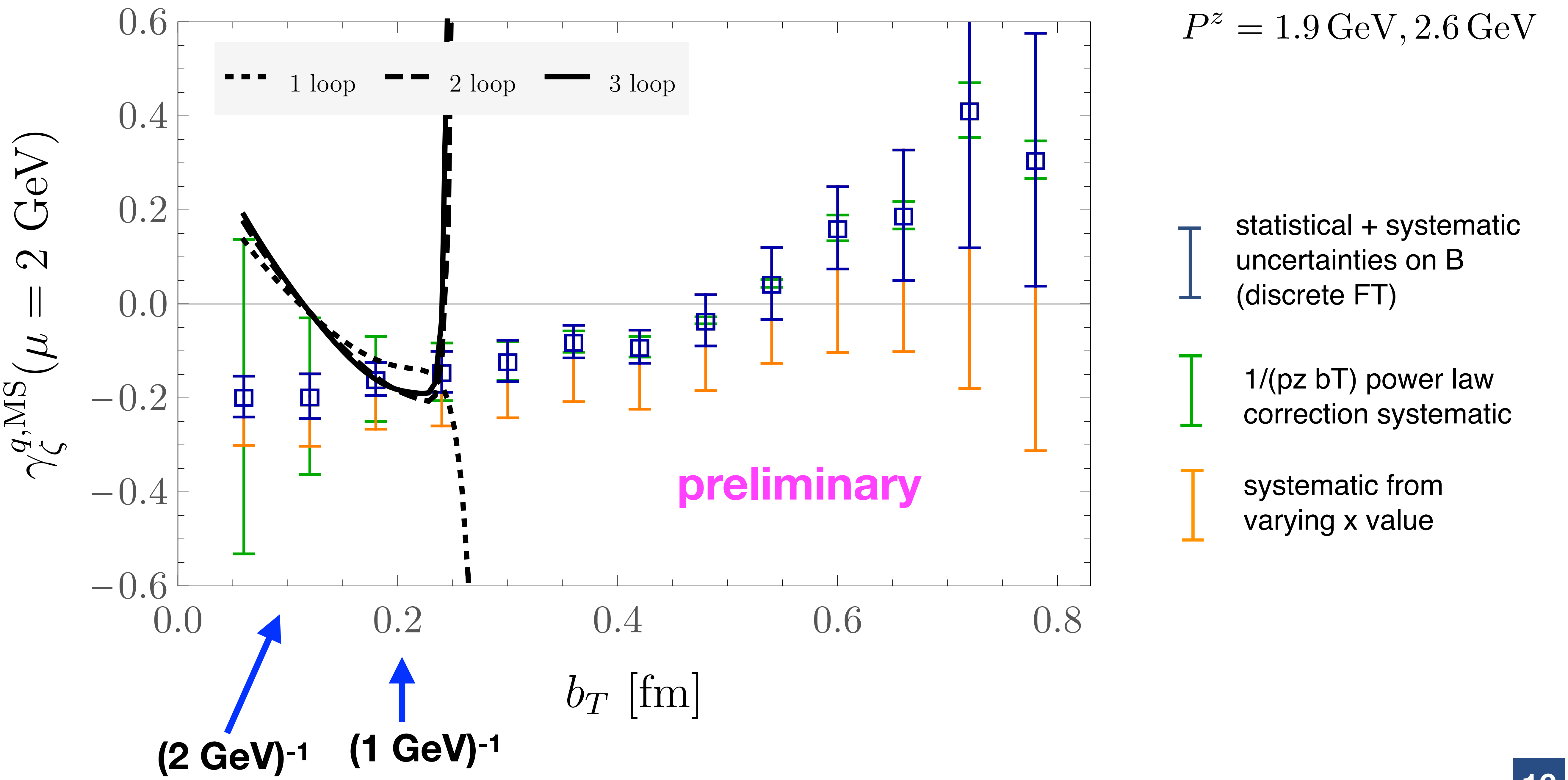
Lattice Results for Rapidity Anomalous Dimension

Ongoing work by P. Shanahan, M. Wagman, Y. Zhao

Exploratory quenched ($n_f = 0$) simulation

Exploits universality: uses 1.2 GeV pseudoscalar meson

Includes renormalization



Summary

- TMDs are rich field theory objects: Wilson line paths, rapidity divergences, hadronic and vacuum matrix elements
- Proposed a method to determine CS kernel with Lattice QCD

Future

- Proof for TMDPDF matching, Lattice for quasi-soft function
- Study form of power corrections
- Further Lattice Simulations for γ_ζ^q

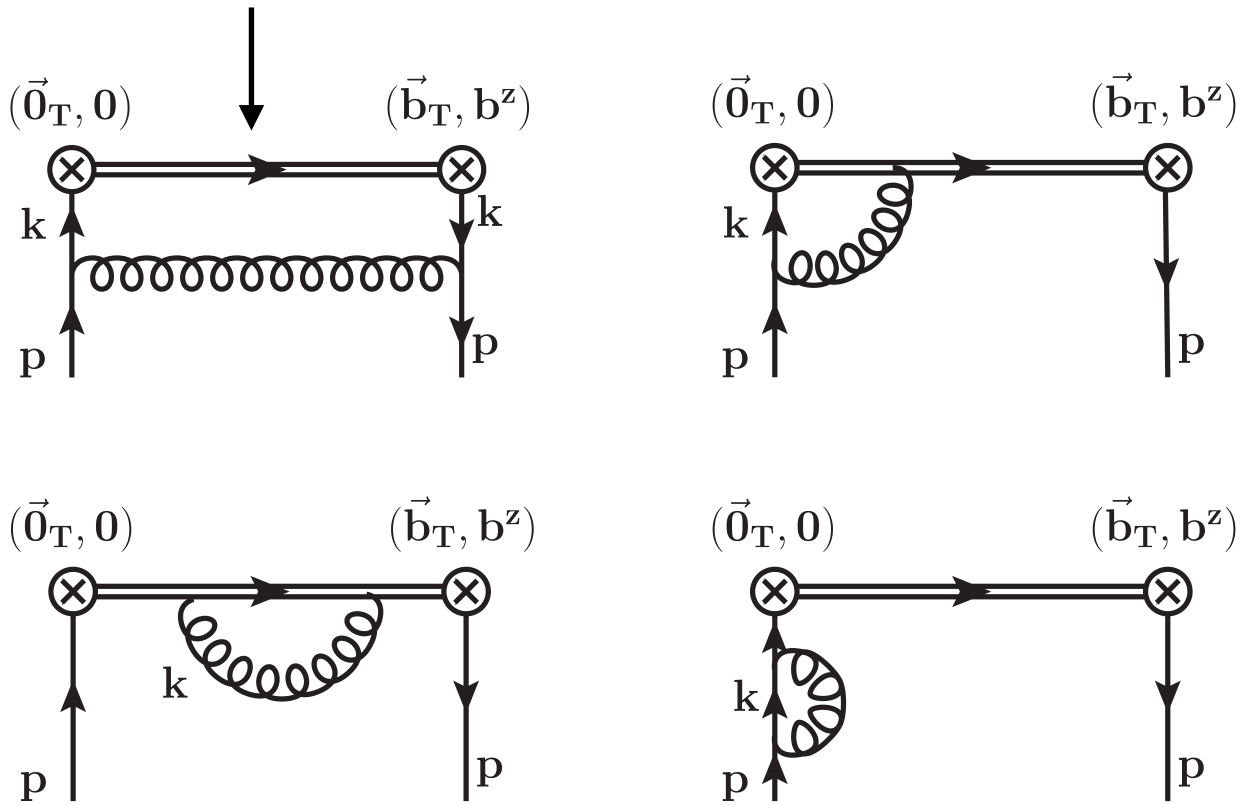
→ Improved precision for DY, SIDIS, … at small qT

Backup

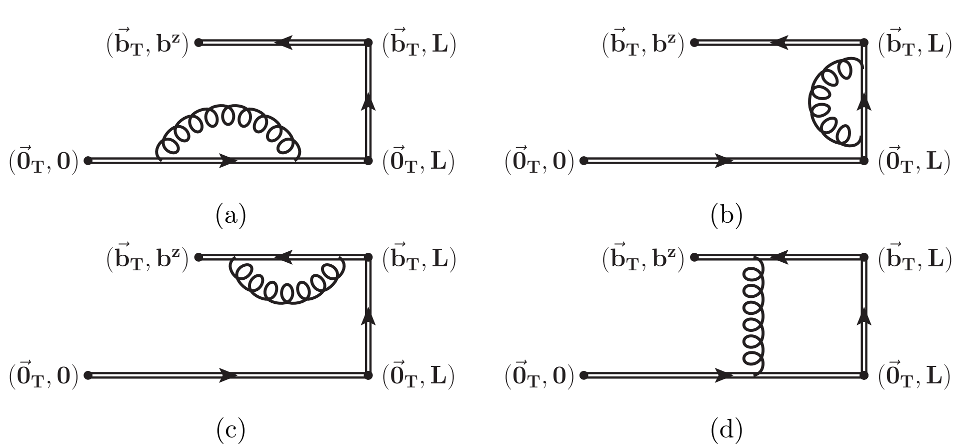
One-Loop Diagrams

Quasi-Beam Function

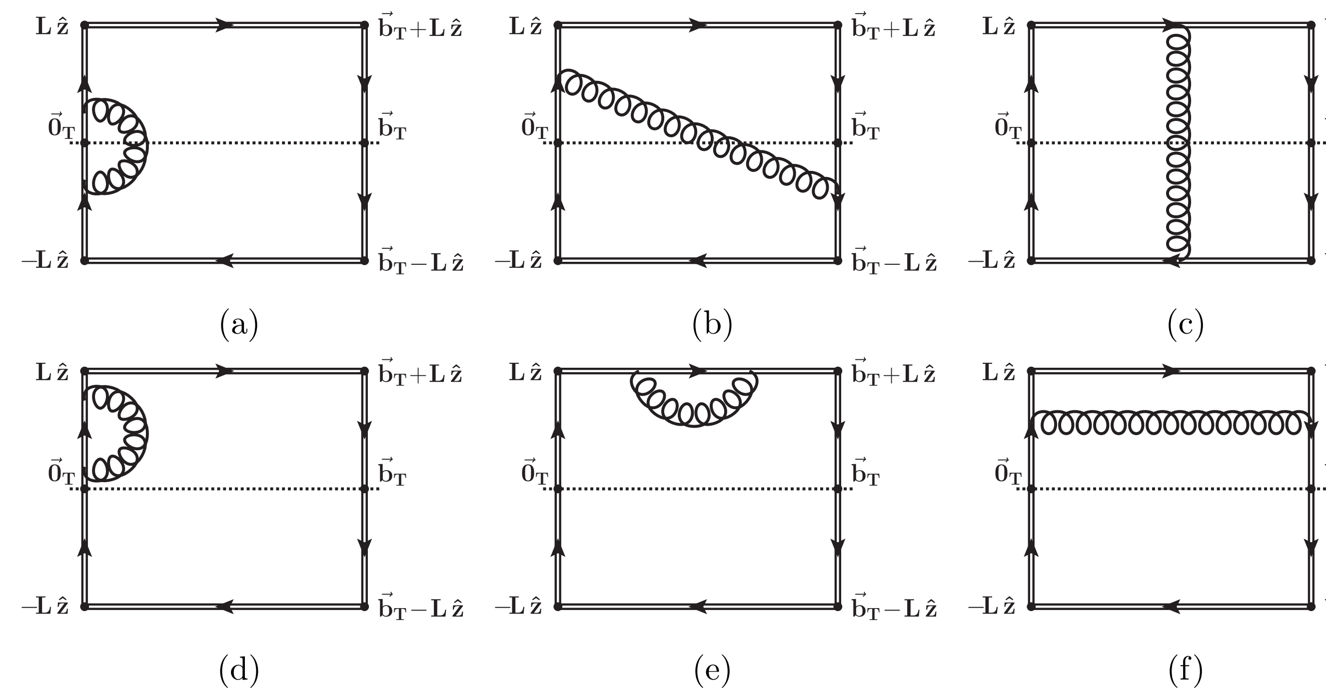
staple!



eg. this
is equal to



Quasi-Soft Function (eg. of naive case)



One-Loop Matching

Legend: match the colors!

$$\tilde{f}_q(x,\vec{b}_T,\mu,P^z) = C^{\mathrm{TMD}}(\mu,xP^z)\, \underline{g_q^S(b_T,\mu)}\, \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu,b_T)\ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x,\vec{b}_T,\mu,\zeta)$$

$$L_b = \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}$$

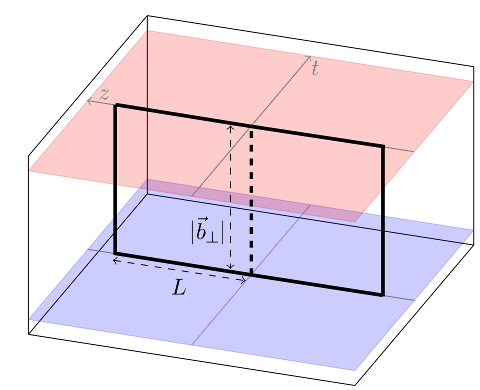
$$L_{P_z} = \ln\frac{\mu^2}{(2xP^z)^2}$$

TMDPDF:

$$f_q^{(1)}(x,\vec{b}_T,\mu,\zeta) = \frac{C_F\alpha_s}{2\pi}\Big[-\Big(\frac{1}{\epsilon_{\mathrm{IR}}}+L_b\Big)P_{qq}(x) + (1-x)_+ + \delta(1-x)\Big(-\frac{1}{2}L_b^2+\frac{3}{2}L_b + L_b\ln\frac{\mu^2}{\zeta} + \frac{1}{2} - \frac{\pi^2}{12}\Big)\Big]$$

quasi-TMDPDF:

(1) "naive" $\tilde{S}_q$:



$$\tilde{f}_q^{(1)}(x,\vec{b}_T,\mu,P^z) = \frac{C_F\alpha_s}{2\pi}\Big[-\Big(\frac{1}{\epsilon_{\mathrm{IR}}}+L_b\Big)P_{qq}(x) + (1-x)_+ + \delta(1-x)\Big(-\frac{1}{2}L_b^2+\frac{3}{2}L_b + L_b + L_b\ln\frac{\mu^2}{(2xP^z)^2} - \frac{1}{2}L_{P_z}^2 - L_{P_z} - \frac{3}{2}\Big)\Big]$$

extra IR log (matching fails!) (ok for γ_ζ^q)

agree: **[Ji, Jin, Yuan, Zhang, Zhao '18]** **[Ebert, IS, Zhao '18]**

One-Loop Matching

Legend: match the colors!

$$\tilde{f}_q(x,\vec{b}_T,\mu,P^z) = C^{\rm TMD}(\mu,xP^z)\, g_q^S(b_T,\mu)\, \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu,b_T)\ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x,\vec{b}_T,\mu,\zeta)$$

$$L_b = \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}$$

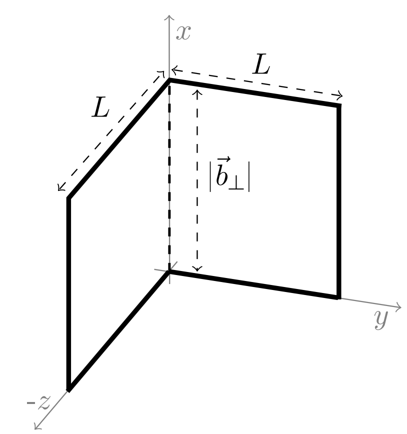
$$L_{P_z} = \ln\frac{\mu^2}{(2xP^z)^2}$$

TMDPDF:

$$f_q^{(1)}(x,\vec{b}_T,\mu,\zeta) = \frac{C_F\alpha_s}{2\pi}\left[-\left(\frac{1}{\epsilon_{\rm IR}}+L_b\right)P_{qq}(x)+(1-x)_+ + \delta(1-x)\left(-\frac{1}{2}L_b^2+\frac{3}{2}L_b+L_b\ln\frac{\mu^2}{\zeta}+\frac{1}{2}-\frac{\pi^2}{12}\right)\right]$$

quasi-TMDPDF:

(2) "bent" $\tilde{S}_q$:



$$\tilde{f}_q^{(1)}(x,\vec{b}_T,\mu,P^z) = \frac{C_F\alpha_s}{2\pi}\left[-\left(\frac{1}{\epsilon_{\rm IR}}+L_b\right)P_{qq}(x)+(1-x)_+ + \delta(1-x)\left(-\frac{1}{2}L_b^2+\frac{3}{2}L_b+L_b\ln\frac{\mu^2}{(2xP^z)^2}-\frac{1}{2}L_{P_z}^2-L_{P_z}-\frac{3}{2}\right)\right]$$

matching works! (and fine for γ_ζ^q)

$$C^{\rm TMD}(\mu,xP^z)$$

[Ebert, IS, Zhao '19]

Details on IR Logs

$$L_b = \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}}$$

Regulator	Beam function B_q	Soft factor Δ_S^q	TMDPDF $f_q^{\rm TMD} = B_q \Delta_S^q$
Collins	$-\frac{1}{2}L_b^2\,,\ \frac{5}{2}L_b$	$-L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
δ regulator	$\frac{3}{2}L_b$	$-\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
η regulator	$\frac{3}{2}L_b$	$-\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
Exp. regulator	$-L_b^2\,,\ \frac{3}{2}L_b$	$\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
	quasi $\tilde{B}_q$	quasi $\tilde{\Delta}_S^q$	quasi $\tilde{f}_q^{\rm TMD} = \tilde{B}_q \tilde{\Delta}_S^q$
Finite L, naive $\tilde{\Delta}_S^q$	$-\frac{1}{2}L_b^2\,,\ \frac{9}{2}L_b$	$-2L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{5}{2}L_b$
Finite L, bent $\tilde{\Delta}_S^q$	$-\frac{1}{2}L_b^2\,,\ \frac{9}{2}L_b$	$-3L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$

- **Matching for $\tilde{B}_q \leftrightarrow B_q$ fails in all schemes. So boost argument fails for regulated beam functions.**
- **Matching for $\tilde{\Delta}_S^q \leftrightarrow \Delta_S^q$ alone also fails.**
- **Matching works for $\tilde{f}_q \leftrightarrow f_q$ at 1-loop with bent quasi-soft factor.**