Thank you.

Different forms of the kinematical constraint in BFKL

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Motivation (What and Why)

- **Kinematical constraint (k.c.) captures some higher order corrections to the BFKL kernel**
  - What NLL and NNLL corrections does it provide?

- **k.c. the same in QCD and N=4 sYM**
  - Can we cross check the NLL and NNLL corrections generated by k.c. against the N=4 sYM result?

- **How do different forms of k.c. compare against each other?**
  - How do they compare in Mellin/momentum space?

The BFKL equation

In momentum space:

$$\mathcal{F}(x, k_T^2) = \mathcal{F}^{(0)}(x, k_T^2) + \bar{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}_T}{\pi q_T^2} \left[\mathcal{F}\left(\frac{x}{z}, |\mathbf{k}_T + \mathbf{q}_T|^2\right) - \Theta(k_T^2 - q_T^2)\mathcal{F}\left(\frac{x}{z}, k_T^2\right) \right]$$

In Mellin space:

$$\overline{\mathcal{F}}(\omega, k_T^2) = \int_0^1 \frac{dz}{z} z^{\omega} \mathcal{F}(z, k_T^2)$$

$$\tilde{\mathcal{F}}(\omega, \gamma) = \int_0^{\infty} dk_T^2 \left(k_T'^2\right)^{-\gamma} \mathcal{F}(\omega, k_T^2)$$

$$\tilde{\mathcal{F}}(\omega, \gamma) = \tilde{\mathcal{F}}^{(0)}(\omega, \gamma) + \frac{\bar{\alpha}_s}{\omega} \chi(\gamma, \omega) \tilde{\mathcal{F}}(\omega, \gamma)$$

χ is the BFKL kernel

The BFKL kernel in Mellin space

BFKL equation in Mellin space

- Solving the equation for eigenvalues of the BFKL kernel Mellin transform

$$\omega=\chi(\omega,\gamma)$$

$$\omega=\chi_{\mathrm{eff}}(\bar{\alpha}_S,\gamma)$$

- BFKL kernel without the kinematical constraint

$$\chi_{\mathrm{eff}}(\bar{\alpha}_S,\gamma)=\bar{\alpha}_S(\psi(1)-\psi(\gamma)-\psi(1-\gamma))$$

  – $1/\gamma$ pole

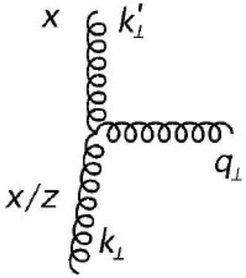
Kinematical constraint

Follows from consistency requirements for the virtuality of the t-channel gluon ➡ $k^2 < 0$

$$k^2 = -z\,\bar{z}\,\hat{s} - k_T^2$$

$$\big\downarrow \; k^2<0$$

$$k_T^2 > z\,\bar{z}\,\hat{s}$$



$$\mathbf{k}_\perp'^2 = \mathbf{k}_\perp^2 - 2\cos(\phi)\,\mathbf{k}_\perp \mathbf{q}_\perp + \mathbf{q}_\perp^2$$

- Inserting the on-shell condition for emitted parton

$$q^2 = \bar{z}(1-z)\hat{s} - {q_T}^2 = 0 \quad \rightarrow \quad \boxed{k_T^2 > \frac{z\,{q_T}^2}{1-z}}$$

The BFKL equation

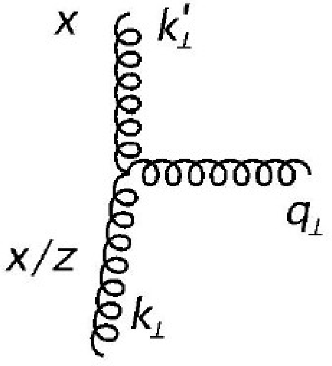
- Follows from consistency requirement for the virtuality of the *t*-channel gluon → $k^2<0$
- Effectively implements energy conservation

$$\mathbf{k}_\perp^2 > \frac{\mathbf{z}}{\mathbf{1-z}}\,\mathbf{q}_\perp^2$$

- Complicated expression → approximations possibly useful (used in literature):

small z $\qquad$ $\mathbf{k}_\perp^2 > \mathbf{z}\ \mathbf{q}_\perp^2$

small z and $k'_T \approx q_T$ $\qquad$ $\mathbf{k}_\perp^2 > \mathbf{z}\ \mathbf{k}_\perp'^2$



$$\mathbf{k}_\perp'^2 = \mathbf{k}_\perp^2 - 2\cos(\phi)\,\mathbf{k}_\perp \mathbf{q}_\perp + \mathbf{q}_\perp^2$$

J. Kwiecinski, A. D. Martin and P. J. Sutton

Scale choice in the BFKL equation

Mueller-Navalet jet cross section:

$$\sigma = \int \frac{d\omega}{2\pi i} \int \frac{d^2\mathbf{k}_T}{k_T^2} \frac{d^2\mathbf{k}_{T0}}{k_{T0}^2} \left(\frac{s}{QQ_0}\right)^{\omega} h^A(Q,\mathbf{k}_T)\, G_\omega(\mathbf{k}_T,\mathbf{k}_{T0})\, h^B(Q_0,\mathbf{k}_{T0})$$

uPDF:

$$\bar{\mathcal{F}}(\omega,\mathbf{k}_T) \;=\; \int \frac{d^2\mathbf{k}_{T0}}{k_{T0}^2} G_\omega(\mathbf{k}_T,\mathbf{k}_{T0})\, h(Q_0,\mathbf{k}_{T0})$$

Different scale choices:

$$G(Y;\mathbf{k}_T,\mathbf{k}_{T0}) = \int \frac{d\omega}{2\pi i}\, e^{\omega Y}\, G_\omega(\mathbf{k}_T,\mathbf{k}_{T0})$$

$$Y = \ln(\boxed{\nu/k_T k_{T0}})$$

ν - energy scale

$$\nu/k_{T0}^2 \qquad \nu/k_T^2 \qquad \nu/k_T k_{T0}$$

$$\int \frac{d\omega}{2\pi i} \left(\frac{\nu}{k_T k_{T0}}\right)^{\omega} G_\omega(\mathbf{k}_T,\mathbf{k}_{T0}) = \int \frac{d\omega}{2\pi i} \left(\frac{\nu}{k_T^2}\right)^{\omega} \left(\frac{k_T}{k_{T0}}\right)^{\omega} G_\omega(\mathbf{k}_T,\mathbf{k}_{T0})$$

Symmetric vs. Assymetric scale

The same change in the kernel:

$$K_\omega^{s_0=k_T^2}(\mathbf{k}_T,\mathbf{k}_{T0}) = K_\omega^{s_0=k_T k_{T0}}(\mathbf{k}_T,\mathbf{k}_{T0})\left(\frac{k_T}{k_{T0}}\right)^\omega$$

$$K_\omega^{s_0=k_{T0}^2}(\mathbf{k}_T,\mathbf{k}_{T0}) = K_\omega^{s_0=k_T k_{T0}}(\mathbf{k}_T,\mathbf{k}_{T0})\left(\frac{k_{T0}}{k_T}\right)^\omega$$

depending on the scale choice

Shift in poles of γ by $\omega/2$:

$$\chi^S(\gamma') = \chi^S\left(\gamma - \frac{\omega}{2}\right) = \chi^A(\gamma)$$

M. Ciafaloni and G. Camici, 1998

Scale changing transformation at NLL

Shift in γ: $$\chi_0\left(\gamma-\frac{\omega}{2}\right)+\bar{\alpha}_s\chi_1\left(\gamma-\frac{\omega}{2}\right)$$ S $\to$ A

Expansion in ω up to $\bar{\alpha}_s$: $$\chi_0(\gamma)-\frac{\omega}{2}\frac{\partial\chi_0}{\partial\gamma}+\bar{\alpha}_s\chi_1(\gamma)$$

$$\Big\Downarrow \quad \omega=\bar{\alpha}_s\chi_0(\gamma)$$

$$-\frac{1}{2}\bar{\alpha}_s\chi_0(\gamma)\frac{\partial\chi_0}{\partial\gamma}$$

Leading poles change:

$$-\frac{1}{2}\bar{\alpha}_s\chi_0(\gamma)\frac{\partial\chi_0}{\partial\gamma}\sim\frac{\bar{\alpha}_s}{2\gamma^3}-\frac{\bar{\alpha}_s}{2(1-\gamma)^3}$$

$$\chi_1^S\sim-\frac{1}{2\gamma^3}-\frac{1}{2(1-\gamma)^3}$$

$$\chi_1^A\sim\frac{0}{2\gamma^3}-\frac{1}{(1-\gamma)^3}\,.$$

Comparison with N=4 sYM

N=4 sYM result available for a symmetric scale choice

$\boxed{k_T'^2 < \frac{k_T^2}{z}}$ case

$$\chi^{\omega,A}(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma+\omega)$$

$\Updownarrow$ scale changing transformation

$$\chi^{\omega,S}(\gamma) = 2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1-\gamma+\frac{\omega}{2}\right)$$

NLL contributions to the BFKL kernel

Expansion in ω: $$\chi_0 + \chi^{(1)}\frac{\omega}{2} + \frac{1}{2!}\chi^{(2)}\left(\frac{\omega}{2}\right)^2 + \dots$$

$$\omega = \bar{\alpha}_s \chi_0(\gamma)$$

Keep terms in up to $\bar{\alpha}_s$:

$$\chi(\gamma) = \chi_0 + \frac{1}{2}\bar{\alpha}_s \chi^{(1)}\chi_0$$

The result:

$$\chi_1(\gamma) = \frac{1}{2}\chi^{(1)}\chi_0 = \frac{1}{2}\left[\psi^{(1)}(\gamma) + \psi^{(1)}(1-\gamma)\right]\left[2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\right]$$

Leading poles at NLL and NNLL

Absence of subleading poles $1/\gamma^2$ (NLL) and $1/\gamma^4$ (NNLL)

$$\chi_1(\gamma) = \boxed{-\frac{1}{2\gamma^3}} - \frac{\zeta(2)}{\gamma} + O(1)$$

$$\chi_2 = \boxed{\frac{1}{2\gamma^5}} + \frac{\zeta(2)}{\gamma^3} + \frac{2\zeta(3)}{\gamma^2} + O\left(\frac{1}{\gamma}\right)$$

QCD

$$\chi_1^{sYM} = \boxed{-\frac{1}{2\gamma^3}} - 1.79 + \mathcal{O}(\gamma)\ ,$$

$$\chi_2^{sYM} = \boxed{\frac{1}{2\gamma^5}} - \frac{\zeta(2)}{\gamma^3} - \frac{9\zeta(3)}{4\gamma^2} - \frac{29\zeta(4)}{8\gamma} + \mathcal{O}(1)$$

sYM

In case of $q_T^2 < \frac{1-z}{z} k_T^2$ collinear pole in γ absent

N. Gromov, F. Levkovich-Maslyuk, and G. Sizov, 2015; V. N. Velizhanin, 2015; S. Caron-Huot and M. Herranen, 2018.

Leading and next-to-leading pole structure to all orders

Observation: χ_k

$$\frac{1}{\gamma^{2k+1}}$$ agreement QCD and sYM (NLL and NNLL)

$$\frac{1}{\gamma^{2k}}$$ vanish

Proven by mathematical induction to all orders!

thanks to
Wanchen Li

The BFKL kernel in Mellin space

$$k'^2_T < \frac{k_T^2}{z}$$

$$\chi(\gamma,\omega) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma+\omega)$$

$$q_T^2 < \frac{k_T^2}{z}$$

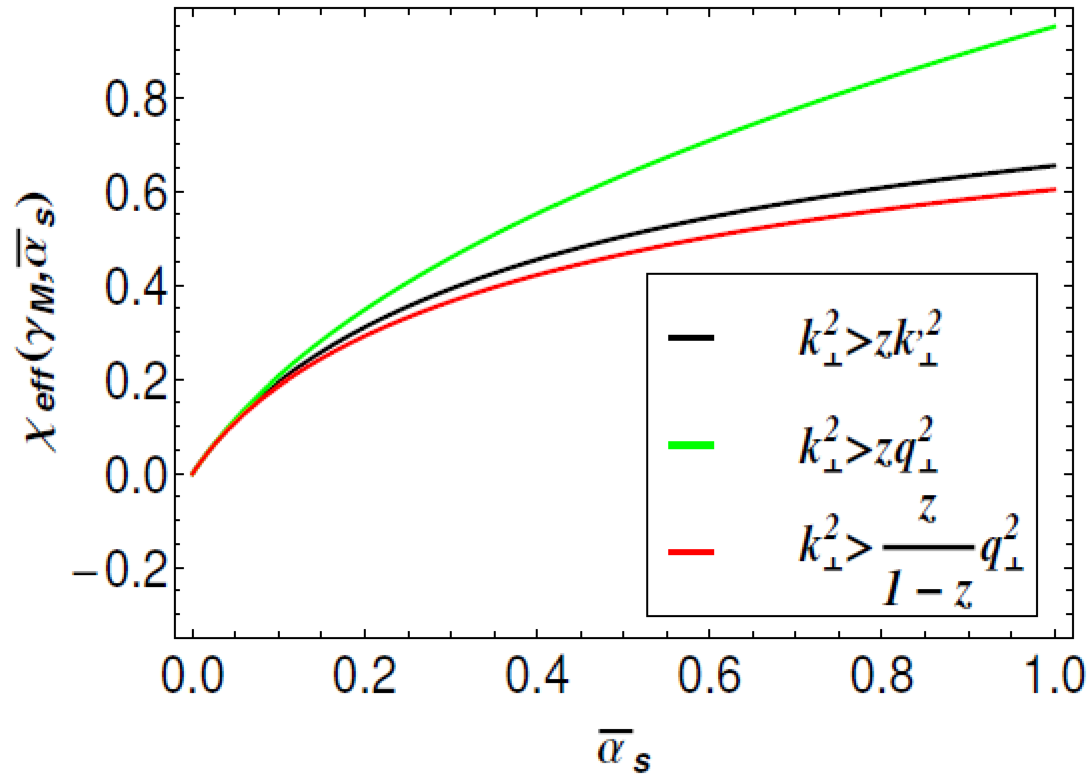
$$\chi(\omega,\gamma) = \int_0^1 \frac{du}{u}\left[\left(1+u^{\omega+1-\gamma}\right) {}_2F_1(1-\gamma,1-\gamma;1;u) - 1\right]$$

$$q_T^2 < \frac{1-z}{z}k_T^2$$

$$\chi(\omega,\gamma) = \int_0^1 \frac{du}{u}\left[(1+u)^{-\omega}\left(1+u^{\omega+1-\gamma}\right) {}_2F_1(1-\gamma,1-\gamma;1;u) - 1\right]$$

Leading behavior for x→0

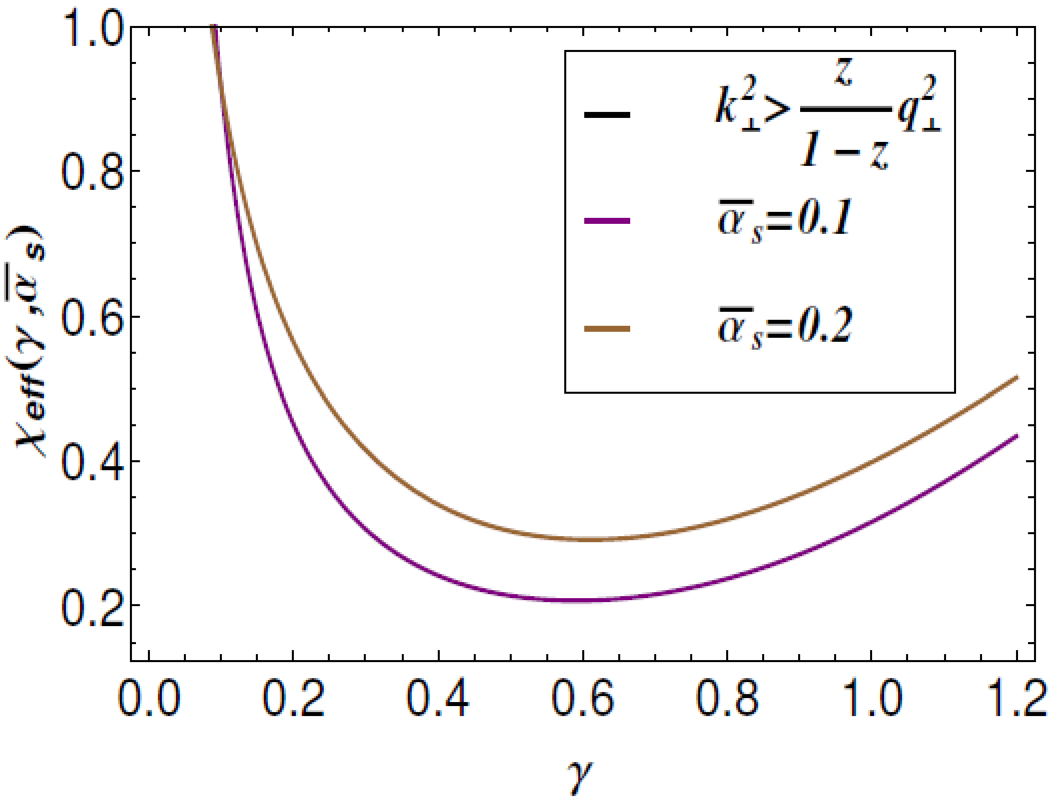
Value at the saddle point - intercept



$$\frac{d\chi_{\rm eff}(\gamma,\bar{\alpha}_s)}{d\gamma}\Big|_{\gamma=\gamma_M}=0\,,\qquad \omega_{I\!P}=\chi_{\rm eff}(\gamma_M,\bar{\alpha}_s)$$

Numerical solution in Mellins space

Different values of fixed $\bar{\alpha}_s$:



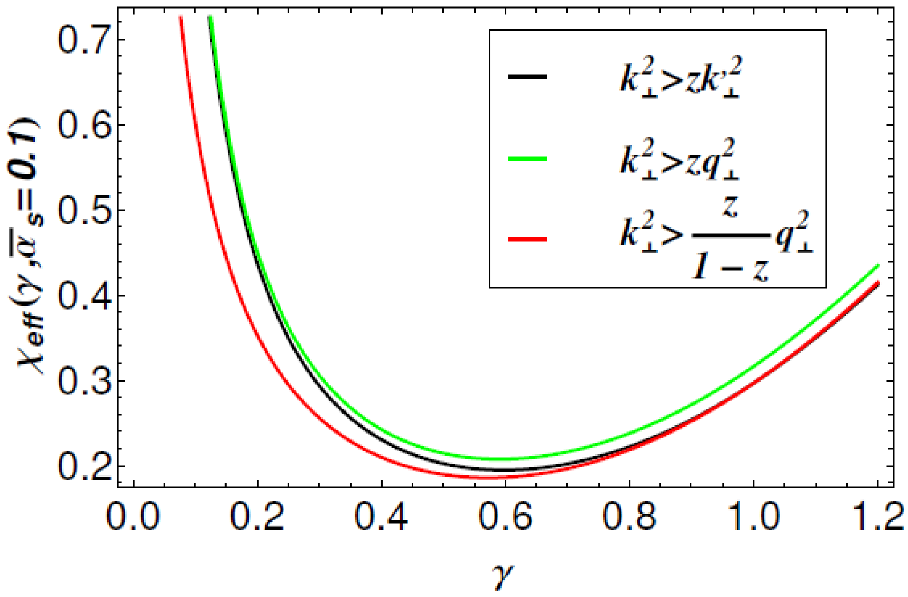
Numerical solution in Mellins space

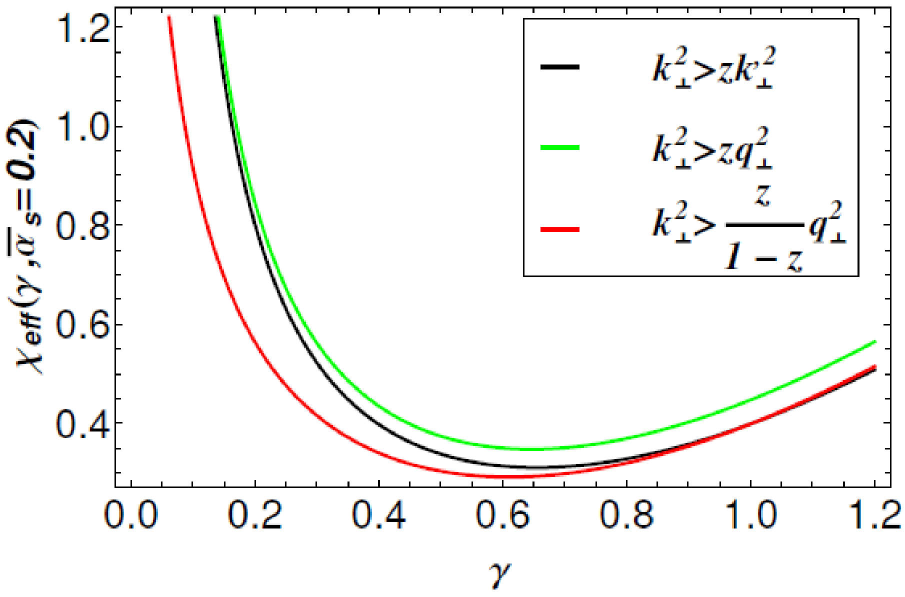
Comparison of different k.c.

$$\omega = \bar{\alpha}_s \chi(\omega, \gamma)$$

$$\omega = \chi_{\text{eff}}(\gamma, \bar{\alpha}_s)$$

Exact agreement at γ=1 of k.c. and k.c.





Differential form of the BFKL with k.c.

kinematical constraint in the form: $\theta\left(k_C\left(\mathbf{q}_T, \mathbf{k}_T\right) - z\right)$

shift of the x variable: $x \rightarrow x \ \max\left\{1, \frac{1}{k_C\left(\mathbf{q}_T, \mathbf{k}_T\right)}\right\} = x'$

Differential form of the BFKL equation:

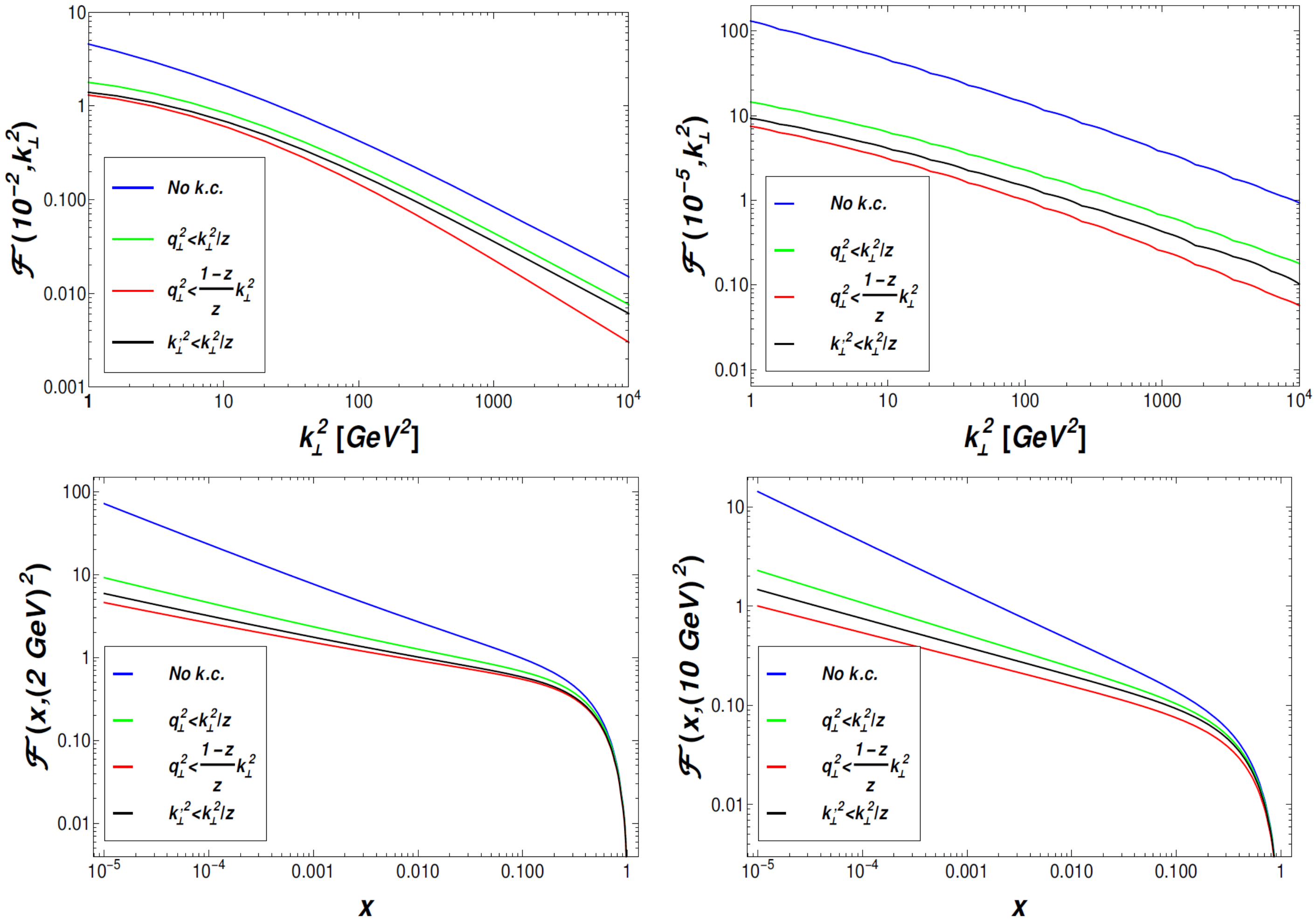
$$\frac{\partial F(x, k_T^2)}{\partial \ln 1/x} = \frac{\partial F_0(x, k_T^2)}{\partial \ln 1/x} + \bar{\alpha}_s \int \frac{d^2\mathbf{q}_T}{\pi {q_T}^2} \left\{ F\left(x', |\mathbf{k}_T + \mathbf{q}_T|^2\right) \Theta\left(k_C\left(\mathbf{q}_T, \mathbf{k}_T\right) - x\right) - \Theta\left(k_T^2 - q_T^2\right) F\left(x, k_T^2\right) \right\}.$$

Numerical results in momentum space

$$\mathcal{F}(x, k_T) = \exp\left[p_0 \log (k_T/\mu)^2 + p_1 \log^2 (k_T/\mu)^2\right] (1-x)^{p_2}$$

$p_0 = p_1 = -0.1$ and $p_2 = 2$ and $\mu = 1\,\mathrm{GeV}$

- expected ordering of magnitudes
- for large $k_\perp$ k.c. tend to differ from k.c. - k.c. modification in the collinear limit
- for small $\boldsymbol{k}_\perp$ k.c. tend to be close to k.c.



Summary

- **We have shown that leading poles at NLL and NNLL originating from the kinematical constraint are identical to the poles contained in the N=4 sYM results**
- **We have compared different forms of kinematical constraints numerically in Mellin and momentum space**