Double parton distribution of valence quarks in the pion in chiral quark models

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- **"Double parton distribution of valence quarks in the pion in chiral quark models"**
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- **"Valence double parton distributions of the nucleon in a simple model"**
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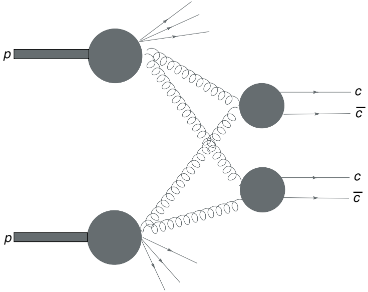
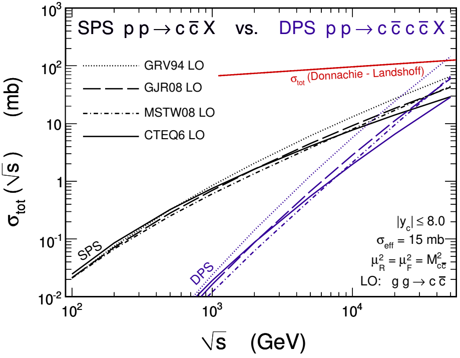
Motivation for multi-parton distributions

- Old story (Fermilab), renewed interest (e.g., ATLAS measurement for pp→ W+2 jets 2013) [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schäfer 2012, . . . , reviews: Bartalani et al. 2011, Snigirev 2011, Rinaldi,Ceccopieri 2018]
- Model exploration [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB+ERA 2013], constituent quarks: Rinaldi, Scopetta, Vento 2013, Rinaldi, Scopetta, Traini, Vento 2018, **Pion** Courtoy, Noguera,Scoppeta-2019]
- Questions: factorization, interference of single- and double-parton scattering [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013], transverse momentum dependence [Casemets, Gaunt 2019]
- Gaunt-Stirling sum rules [Gaunt, Stirling 2010, WB+ERA 2013, Diehl, Plöß, Schäfer 2019]

Double parton scattering

[example from Łuszczak, Maciuła, Szczurek 2011]





DPS can be comparable to SPS at the LHC

Assumption: $D_{gg}(x_1, x_2, \boldsymbol{b}) = g(x_1) g(x_2) F(\boldsymbol{b})$

– no correlations, transverse-longitudinal factorization

Definition

Intuitive probabilistic definition:

Multi-parton distribution = probability distribution that struck partons have LC momentum fractions x_i

Field-theoretic definition of (spin-averaged) sPDF and dPDF [Diehl, Ostermeier, Schaeffer 2012] of a hadron with momentum p:

$$D_j(x) = \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i x z^- p^+} \langle p | \mathcal{O}_j(0,z) | p \rangle \Big|_{z^+=0\,,\boldsymbol{z}=\boldsymbol{0}}$$

$$F_{j_1 j_2}(x_1, x_2, \boldsymbol{y}) = 2p^+ \int \mathrm{d}y^-\, \frac{\mathrm{d}z_1^-}{2\pi}\, \frac{\mathrm{d}z_2^-}{2\pi}\, e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle p | \mathcal{O}_{j_1}(y, z_1)\, \mathcal{O}_{j_2}(0, z_2) | p \rangle \Big|_{z_1^+=z_2^+=y^+=0\,,\boldsymbol{z}_1=\boldsymbol{z}_2=\boldsymbol{0}}$$

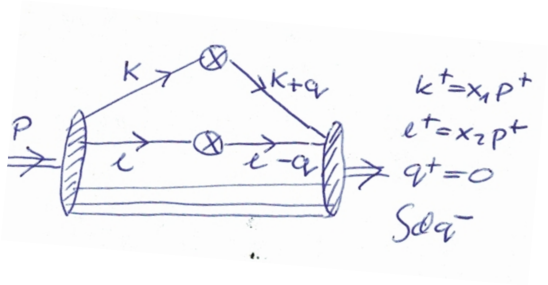
$\mathcal{O}_q(y,z) = \frac{1}{2}\, \bar{q}(y - \frac{z}{2})\gamma^+ q(y + \frac{z}{2}),\ \ldots$ $\qquad v^\pm = (v^0 \pm v^3)/\sqrt{2}$

$\boldsymbol{y}$ plays the role of the transverse distance between the two quarks

dPDF in momentum space

Fourier transform in $\boldsymbol{y}$

$$F_{j_1 j_2}(x_1, x_2, \boldsymbol{y}) \to \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q})$$



Special case of $\boldsymbol{q} = \boldsymbol{0}$:

$$D_{j_1 j_2}(x_1, x_2) = \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q} = \boldsymbol{0})$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$|P\rangle = \sum_N \int d[x,\boldsymbol{k}]_N \Phi(\{x_i,\boldsymbol{k}_i\})|\{x_i,\boldsymbol{k}_i\}\rangle_N$$

$$d[x,\boldsymbol{k}]_N = \prod_{i=1}^{N}\left[\frac{dx_i d^2 k_i}{\sqrt{2(2\pi)^3 x_i}}\right]\delta\left(1-\sum_{i=1}^{N} x_i\right)\delta^{(2)}\left(1-\sum_{i=1}^{N}\boldsymbol{k}_i\right)$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]
Fock-space decomposition on LC + conservation laws →

$$\sum_i \int_0^{1-x_2} dx_1\, x_1 D_{ij}(x_1,x_2) = (1-x_2) D_j(x_2) \quad \text{(momentum)}$$

$$\int_0^{1-x_2} dx_1\, D_{i_{\rm val} j}(x_1,x_2) = (N_{i_{\rm val}} - \delta_{ij} + \delta_{\bar{i}j}) D_j(x_2) \quad \text{(quark number)}$$

$(A_{i_{\rm val}} \equiv A_i - A_{\bar{i}})$

$$N_{i_{\rm val}} = \int_0^1 dx\, D_{i_{\rm val}}(x)$$

- Preserved by DGLAP evolution
- Non-trivial to satisfy with the (guessed) function
- Checked in light-front perturbation theory and in lowest-order covariant calculations in [Diehl, Plößl, Schäfer 2019]

Important and fundamental constraints!

Simple example (valon model)

$|\Lambda\rangle = |uds\rangle$ (to avoid the complications of indistinguishable partons)

$$
\begin{aligned}
D_{uds}(x_1,x_2,x_3) &= f(x_1,x_2,x_3)\delta(1-x_1-x_2-x_3) \\
D_{ud}(x_1,x_2) &= \int dx_3\, D_{uds}(x_1,x_2,x_3) = f(x_1,x_2,1-x_1-x_2) \\
D_{us}(x_1,x_3) &= \dots \\
D_u(x_1) &= \int_0^{1-x_1} dx_2\, D_{ud}(x_1,x_2) = \int_0^{1-x_1} dx_3\, D_{us}(x_1,x_3)
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^{1-x_1} dx_2\, x_2 D_{ud}(x_1,x_2) + \int_0^{1-x_1} dx_3\, x_3 D_{us}(x_1,x_3) \\
&= \int dx_2 dx_3 (x_2+x_3) D_{uds}(x_1,x_2,x_3) = \int dx_2 dx_3 (1-x_1) D_{uds}(x_1,x_2,x_3) \\
&= (1-x_1) D_u(x_1)
\end{aligned}
$$

Attempts of bottom-up construction

- Gaunt, Stirling (2011)

$$D_{ij}(x_1, x_2) = D_i(x_1)\, D_j(x_2)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}$$

(do not satisfy the GS sum rules)

- Lewandowska, Golec-Biernat 2014

$$D_{ij}(x_1, x_2) = \frac{1}{1 - x_2} D_i\!\left(\frac{x_1}{1 - x_2}\right) D_j(x_2)$$

$\ldots$

(no parton exchange symmetry, negative D_{qq} at large x)

- Can never be unique: marginal projections do not determine the two-particle distribution

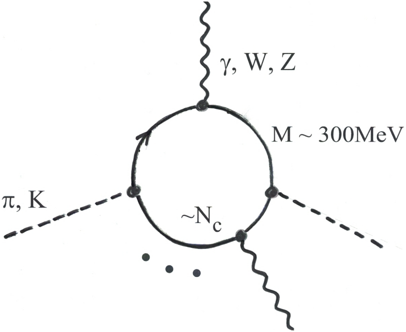
Problems!

Message

- Construct the multiparticle distribution (model, data?) and go down with marginal projections

[cf. a similar in spirit “top-down” study by M. Rinaldi et al. 2018 with the Brodsky - de Teramond AdS/CFT soft wall pion wave function]

Chiral quark models

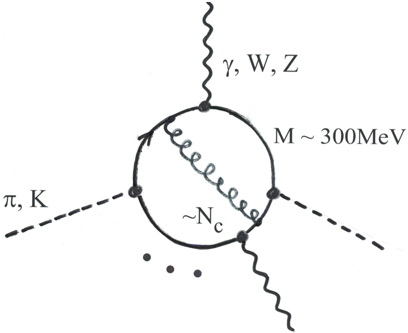


- χSB breaking $\to$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, W, Z)
- Large-N_c $\to$ one-quark loop
- Regularization

pion – Goldstone boson of χSB, fully relativistic $q\bar{q}$ bound state of the Bethe-Salpeter equation

Quantities evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

Chiral quark models



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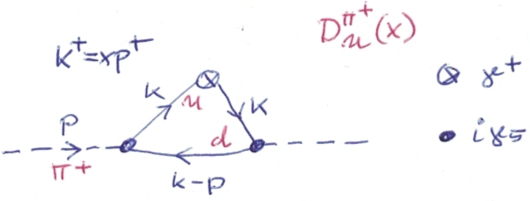
Need for evolution
Gluon dressing, renorm-group improved

sPDF in NJL

[Davidson, Arriola, 1995]



$$q_{\rm val}(x;Q_0) = 1 \times \theta[x(1-x)]$$

(proper treatment of symmetries with regularization)

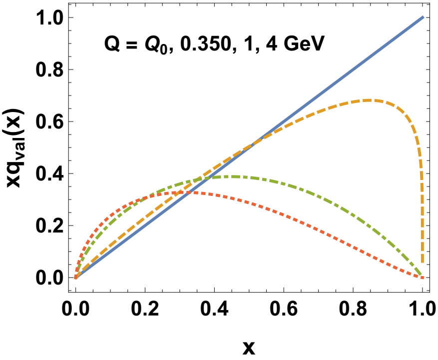
Quarks are the only degrees of freedom, hence saturate the sPDF sum rules:
$\int_0^1 dx\, q_{\rm val}(x;Q_0) = 1$ (valence), $2\int_0^1 dx\, x q_{\rm val}(x;Q_0) = 1$ (momentum)

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$F(x, Q_0)|_{\text{model}} = F(x, Q_0)|_{\text{QCD}}\,, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100% of momentum at Q_0, adjusted such that when evolved to $Q = 2$ GeV, they carry the experimental value of 47% (radiative generation of gluons and sea quarks)



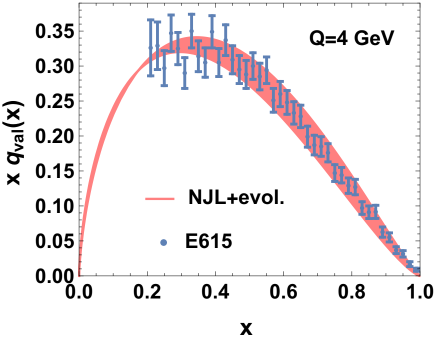
LO DGLAP evolution

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

NLO close to LO

$$\sim (1-x)^{p + \frac{4C_F}{\beta_0} \log \frac{\alpha(Q_0)}{\alpha(Q)}}$$

Pion valence quark PDF, NJL vs E615

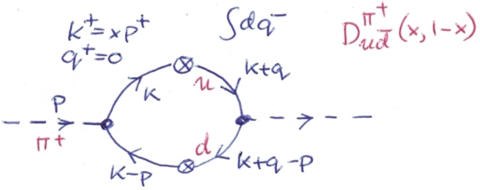


points: Fermilab E615
Drell-Yan, $\pi^\pm W \to \mu^+\mu^- X$

band: QM + LO DGLAP from
$Q_0 = 313^{+20}_{-10}$ MeV to
$Q = 4$ GeV

Many predictions for related quantities: DA, GPD, TDA, TMD, quasi/pseudo DA/PDF...

dPDF of the pion in NJL model



$$D_{u\bar{d}}(x_1, x_2) = 1 \times \delta(1 - x_1 - x_2)\theta(x_1)\theta(x_2)$$

- Special case of the valon model
- GS sum rules satisfied
- ... at the quark-model scale → need for evolution

dDGLAP evolution in the Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving dDGLAP based on the Mellin moments, similarly to sPDF

$$M_j^n = \int_0^1 dx\, x^n D_j(x), \quad M_{j_1 j_2}^{n_1 n_2} = \int_0^1 dx_1 \int_0^1 dx_2 \theta(1-x_1-x_2) x_1^{n_1} x_2^{n_2} D_{j_1 j_2}(x_1, x_2)$$

$$\frac{d}{dt} M_{j_1 j_2}^{n_1 n_2} = \sum_i P_{i\to j_1}^{n_1} M_{i j_2}^{n_1 n_2} + \sum_i P_{i\to j_2}^{n_2} M_{j_1 i}^{n_1 n_2} + \sum_i \left(P_{i\to j_1 j_2}^{n_1 n_2} + \tilde{P}_{i\to j_1 j_2}^{n_1 n_2} \right) M_i^{n_1+n_2}$$

$t = \frac{1}{2\pi\beta} \log\left[1 + \alpha_s(\mu)\beta \log(\Lambda_{\rm QCD}/\mu)\right]$ (single scale for simplicity), $\beta = \frac{11N_c - 2N_f}{12\pi}$
(inhomogeneous term from coupling to sPDF's)

For valence-valence distributions there are no partons i decaying into a pair of valence quarks ($P_{i\to j_1 j_2} = 0$) $\to$ inhomogeneous term vanishes

$$\text{dPDF}: \quad \frac{d}{dt} M_{j_1 j_2}^{n_1 n_2}(t) = \left(P_{j_1 \to j_1}^{n_1} + P_{j_2\to j_2}^{n_2} \right) M_{j_1, j_2}^{n_1 n_2}(t)$$

$$\text{sPDF}: \quad \frac{d}{dt} M_j^n(t) = P_{j\to j}^n M_j^n(t)$$

Solution

[10 lines in Mathematica (!)]

$$M_j^n(t) = e^{P_{j\to j}^n (t-t_0)} M_j^n(t_0)$$
$$M_{j_1 j_2}^{n_1 n_2}(t) = e^{\left[P_{j_1\to j_1}^{n_1} + P_{j_2\to j_2}^{n_2}\right](t-t_0)} M_{j_1 j_2}^{n_1 n_2}(t_0)$$

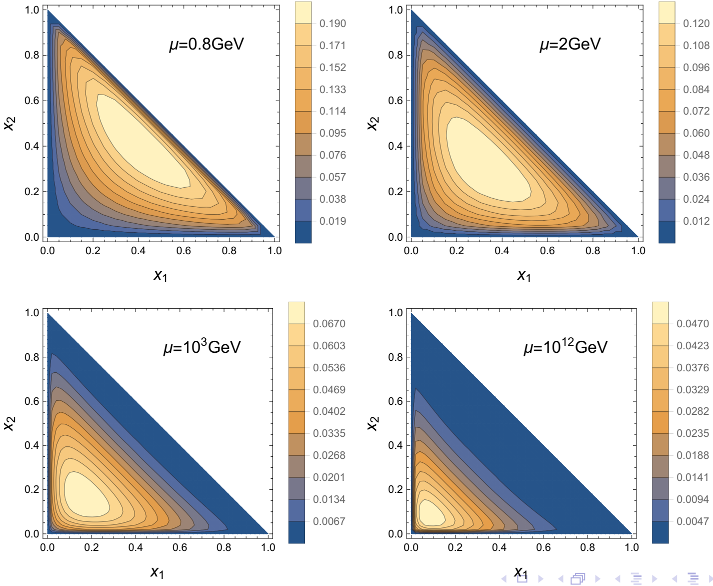
inverse Mellin transform:

$$D_j(x;t) = \int_C \frac{dn}{2\pi i} x^{-n-1} M_j^n(t)$$
$$D_{j_1 j_2}(x_1, x_2; t) = \int_C \frac{dn_1}{2\pi i} x_1^{-n_1-1} \int_{C'} \frac{dn_2}{2\pi i} x_2^{-n_2-1} M_{j_1, j_2}^{n_1, n_2}(t)$$

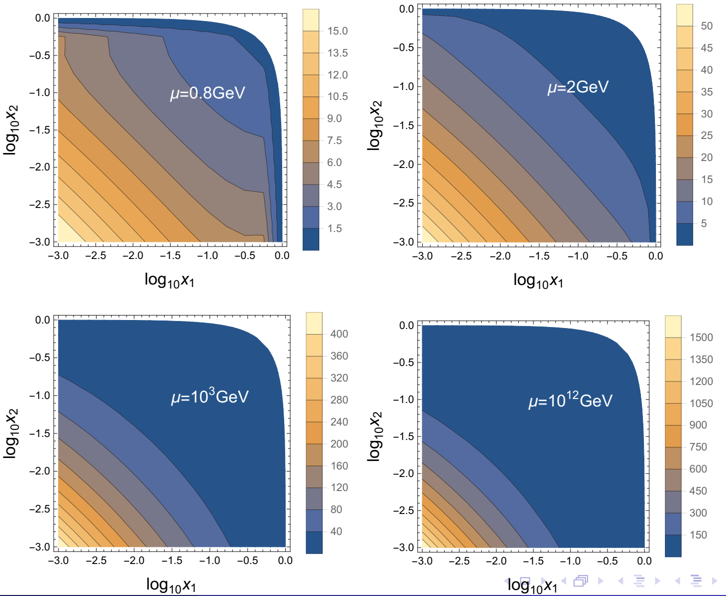
n and n' are complex variables and the contours C and C' lie right to singularities of M

- correlations $\to M_{j_1 j_2}^{n_1 n_2}(t) \neq M_{j_1}^{n_1}(t) M_{j_2}^{n_2}(t)$ – no separability
- valence-valence: $M_{j_1 j_2}^{n_1 n_2}(t)/[M_{j_1}^{n_1}(t) M_{j_2}^{n_2}(t)]$ independent of t

$x_1 x_2 D^{\pi^+}_{u\bar{d}}(x_1, x_2)$

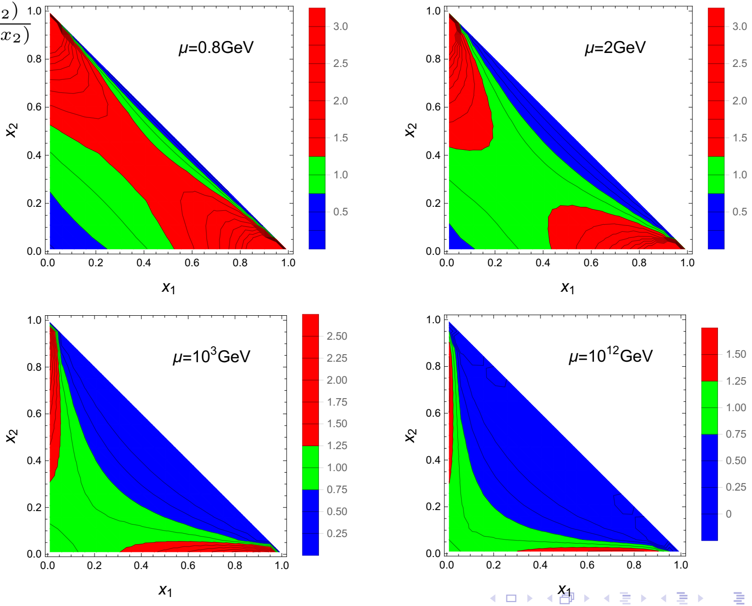


$D^{\pi^+}_{u\bar{d}}(x_1, x_2)$ – log scale



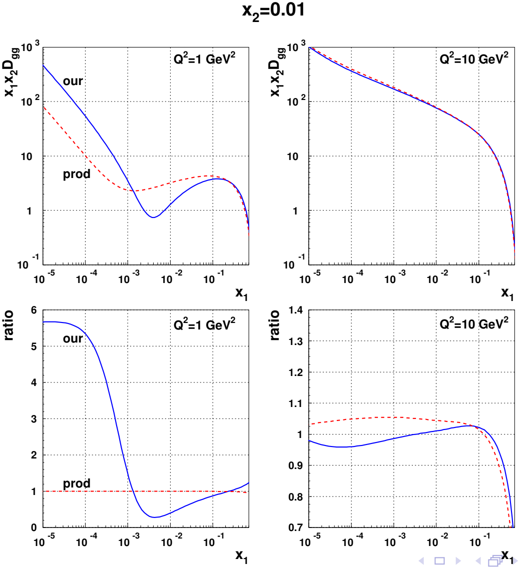
Correlation

$$\frac{D^{\pi^+}_{u\bar{d}}(x_1,x_2)}{D_u(x_1)D_{\bar{d}}(x_2)}$$



Gluon correlation

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto 2015]



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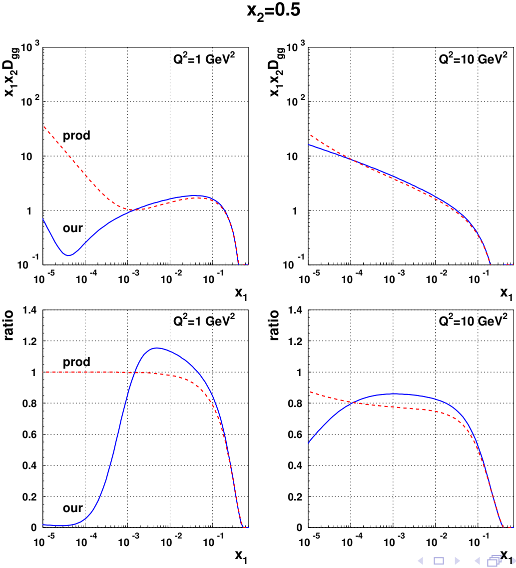
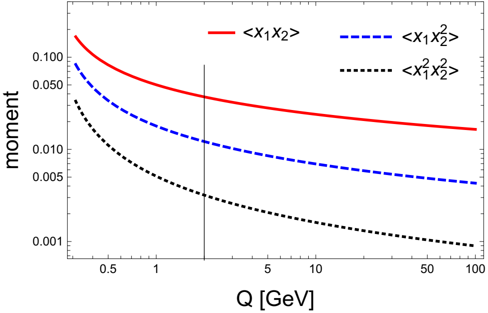


Valence moments in NJL

$$\frac{\langle x_1^n x_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \quad \text{(NJL, any scale)}$$

(independent of the evolution scale)

1	1	1	1	1	1
1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$
1	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{3}{28}$
1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{4}{35}$	$\frac{1}{14}$	$\frac{1}{21}$
1	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{5}{126}$	$\frac{1}{42}$
1	$\frac{2}{7}$	$\frac{3}{28}$	$\frac{1}{21}$	$\frac{1}{42}$	$\frac{1}{77}$



Double moments reduced compared to product of single moments
[lattice results coming shortly, Zimmermann et al.]

Transverse structure: Regularization vs positivity

- Formally

$$F(\boldsymbol{q}) = \frac{N_c M^2}{(2\pi)^3 f^2} \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2 \delta(\boldsymbol{k_1} - \boldsymbol{k_2} + \boldsymbol{q}) \times \left[\Psi(\boldsymbol{k_1})\boldsymbol{k_1} \cdot \Psi(\boldsymbol{k_2})\boldsymbol{k_2} + M^2 \Psi(\boldsymbol{k_1})\Psi(\boldsymbol{k_2})\right]$$

- Momentum vs coordinate

$$\Psi(\boldsymbol{k}) = \frac{1}{\sqrt{\boldsymbol{k}^2 + M^2}} \qquad \Phi(\boldsymbol{b}) = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \Psi(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{b}} = \frac{e^{-bM}}{2\pi b}, \tag{1}$$

- Positive but divergent !

$$F(\boldsymbol{q}) = \frac{N_c M^2}{2\pi f^2} \int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\boldsymbol{q}} \left[\nabla\Phi(\boldsymbol{b})^2 + M^2 \Phi(\boldsymbol{b})^2\right], \tag{2}$$

- Example $\langle 0|\phi^2|0\rangle$ positive but divergent : $\phi^2 := \phi^2 - \langle 0|\phi^2|0\rangle$ convergent but not positive.

Transverse structure I

Regularization dependence, Gauge invariance

- NJL model (Pauli-Villars regularization)

$$
\begin{aligned}
A(M)|_{\mathrm{NJL}} &= \sum_i c_i A\left(\sqrt{\Lambda_i^2 + M^2}\right) \\
f_\pi^2 &= -\frac{4N_c^2 M^2}{(4\pi)^2} \sum_i c_i \log(\Lambda_i^2 + M^2)
\end{aligned}
$$

- Spectral Quark Model (Vector meson dominance)

$$
\begin{aligned}
A(M)|_{\mathrm{SQM}} &= \int_C dw \rho(w) A(w) \\
\rho(w) &: \quad F_{\mathrm{em}}(q^2) = \frac{m_\rho^2}{m_\rho^2 + q^2}, \quad f_\pi^2 = \frac{N_c m_\rho^2}{24\pi^2}
\end{aligned}
$$

Transverse structure II

- Form factor

$$F(\boldsymbol{q}) = \frac{m_\rho^4 - \boldsymbol{q}^2 m_\rho^2}{\left(m_\rho^2 + \boldsymbol{q}^2\right)^2},$$

- Impact parameter

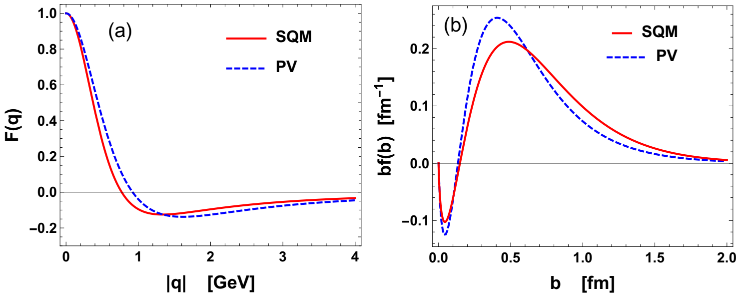
$$f(b) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}} F(\boldsymbol{q}) = \frac{m_\rho^2}{2\pi} \left[b m_\rho K_1\left(b m_\rho\right) - K_0\left(b m_\rho\right)\right]$$

- Effective cross section (coincides with geometrical)

$$\sigma_{\rm eff} = \frac{1}{\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} F(\boldsymbol{q}) F(-\boldsymbol{q})} = \pi \frac{12}{m_\rho^2} = \pi \langle b^2 \rangle_{\rm dPDF} = 23 \text{ mb}$$

(Rinaldi and Ceccopieri bounds $\pi\langle b^2\rangle \leq \sigma_{\rm eff} \leq 3\pi\langle b^2\rangle$)

Transverse structure III



- Is $F(q)$ positive definite ? (A convolution of a function with a node may become negative)
- Negative values for large q or small b could be checked on lattice ($\sim a = 0.1$)
- Similar results as Courtoy, Noguera,Scoppeta-2019

Summary

- Top-down strategy of constructing multi-parton distributions $\rightarrow$ formal features guaranteed, in particular GS sum rules
- Phenomenological sPDF's as constraints
- NJL or SQM $\rightarrow$ valon initial condition, $\mathrm{const} \times \delta(1 - x_1 - x_2)$, dDGLAP
- Correlations decrease with increasing evolution scale and are probably not very important ($\pm 25\%$) in the range probed by experiments, justifying the product ansatz in that limit
- Moments measure the $x_1 - x_2$ factorization breaking; can be verified in forthcoming lattice calculations
- Transverse distributions may become negative
- Effective cross section is geometrical