  

Building maps of kinematical regions in SIDIS

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Mapping the kinematical regimes of SIDIS

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ArXiv:1904.12882, JHEP 1910 (2019) 122

- Sophisticated theoretical frameworks (like perturbation theory and QCD factorization) have long existed for describing specific **underlying physical mechanisms** in terms of **partonic degrees of freedom**.

- However, they always assume specific kinematic limiting cases, e.g. very large or very small transverse momentum, or very large or very small rapidity.

- If all energies and hard scales are extremely large, we are in the asymptotic freedom regime and pictures of partonic interactions rooted in **perturbation theory** can usually be applied confidently and with very high accuracy and precision. However here we are **little sensitive to the inner hadronic structure**.

- Instead, it is in the **moderate-to-low Q range** that we can reasonably expect some sensitivity to **intrinsic properties of hadron structure** and other non-perturbative effects.

Mapping the kinematical regimes of SIDIS

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- The interface between different physical regimes remains unclear in practice, especially when the hard scales involved are not particularly large.

- Estimating the **kinematic boundaries** of any specific QCD approach or approximation beyond very rough orders of magnitude is difficult and subtle. It requires at least some model assumptions, e.g. about the role of parton virtuality and/or the onset of various non-perturbative or hadronic mechanisms generally.

- Need to organize an **interpretation strategy**, applicable with any model of underlying non-perturbative dynamics, independent of assumptions about factorization.

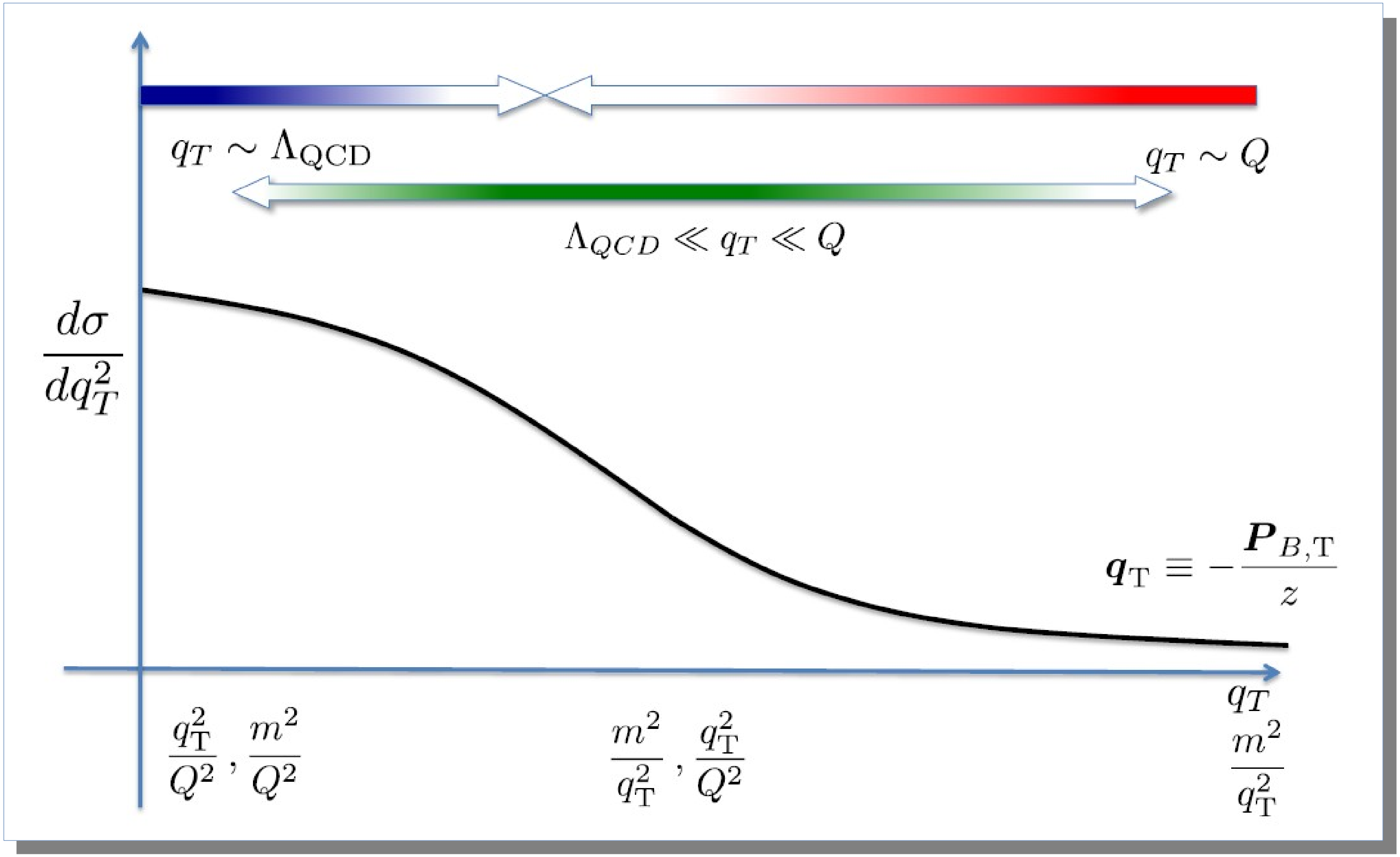
- Probe the proximity of any given kinematic configuration to a conventional **partonic region of SIDIS**, and probe the sensitivity to the various model assumptions needed to make such an assessment.

TMD regions

- For a CSS-like factorization scheme to work, 4 distinct kinematic regions have to be identified
- They should be large enough and well separated

TMD evolution		Matching region (Y factor)	Fixed Order collinear QCD
$q_T \sim \lambda_{QCD}$	$q_T << Q$	$q_T \sim Q$	$q_T \geq Q$
Intrinsic q_T	Soft gluon radiation		Hard gluon emission

Unpolarized cross section vs. transverse momentum

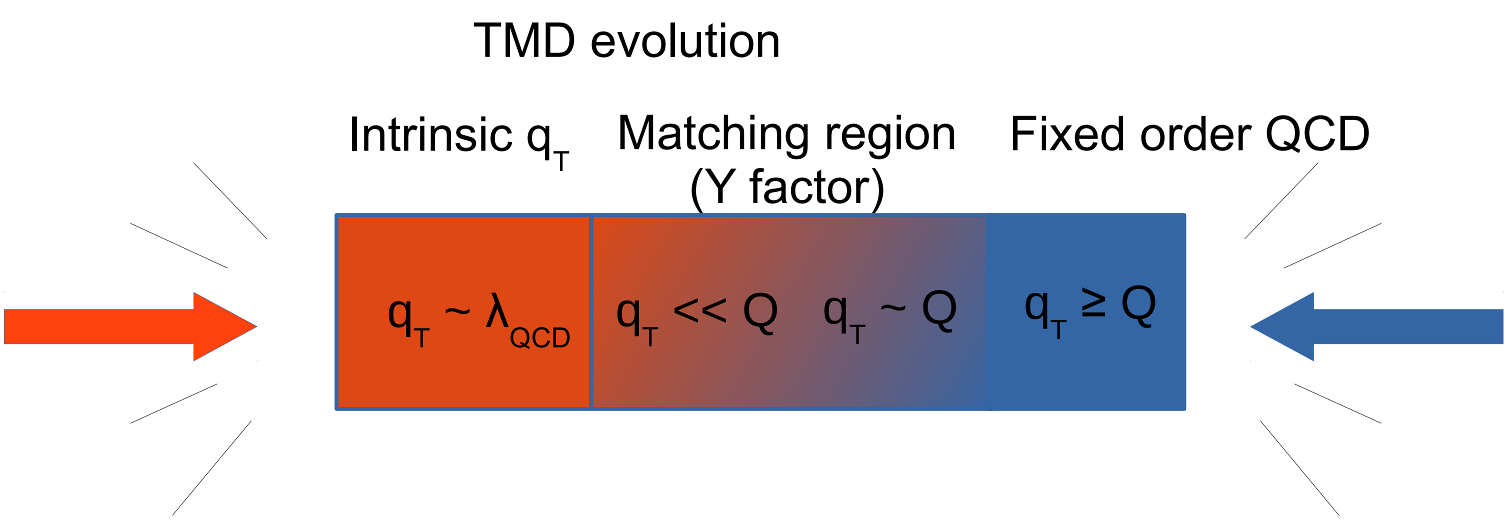


Plot credit: Ted Rogers

TMD regions in SIDIS

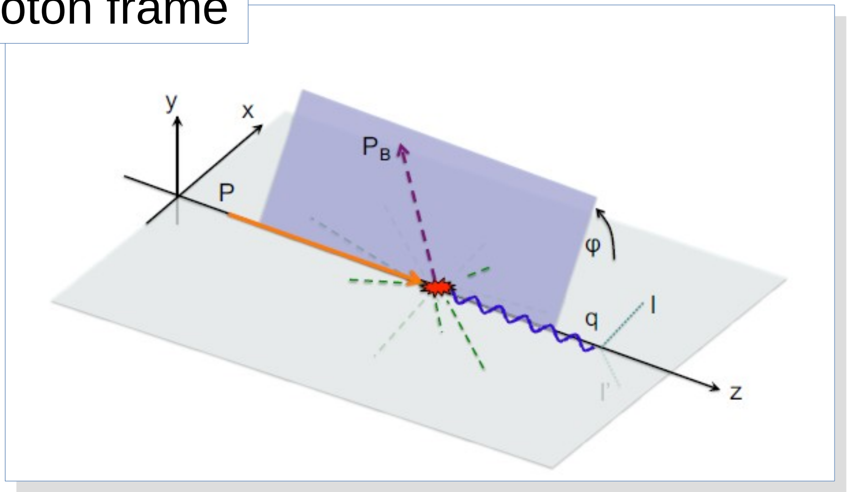
- For this scheme to work, 4 distinct ki… have to be identified
- They should be large… separated

Does not work in SIDIS !



Anatomy of a SIDIS process

Photon frame



$$Q^2 = -q^2 = -(l - l')^2\,, \qquad x_{\rm Bj} = \frac{Q^2}{2P\cdot q}\,,$$

$$\boxed{x_{\rm N} = -\frac{q^+}{P^+} = \frac{2x_{\rm Bj}}{1+\sqrt{1+\frac{4x_{\rm Bj}^2 M^2}{Q^2}}}\,,}$$

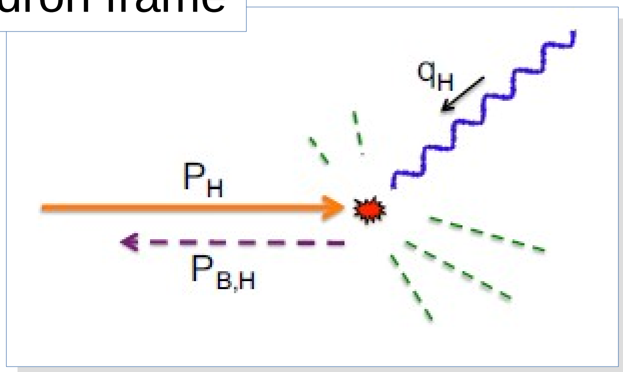
M = mass of the ingoing hadron (target mass)

$$y = \frac{P\cdot q}{P\cdot l}\,, \qquad z_{\rm h} = \frac{P\cdot P_{\rm B}}{P\cdot q} = 2x_{\rm Bj}\frac{P\cdot P_{\rm B}}{Q^2}\,, \qquad \boxed{z_{\rm N} = \frac{P_{\rm B}^-}{q^-}\,,}$$

$$W_{\rm tot}^2 = (q+P)^2\,, \qquad W_{\rm SIDIS}^2 = (q+P-P_{\rm B})^2\,, \qquad s = (l+P)^2$$

Anatomy of a SIDIS process

Hadron frame



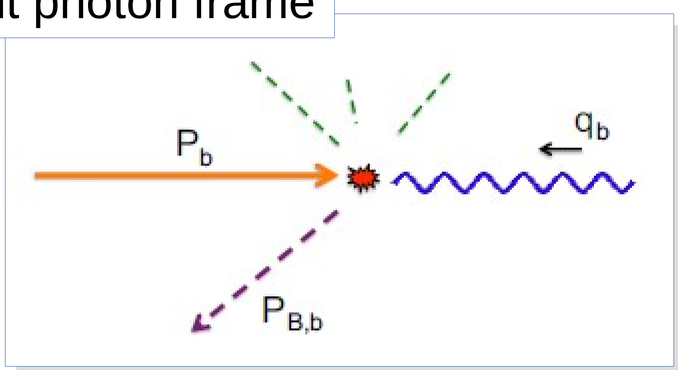
- In the hadron frame, the incoming hadron and final state hadron are exactly back-to-back (zero relative transverse momentum), while the virtual photon has non-zero transverse momentum.
- It is an especially useful frame for setting up factorization.

$$q_{\rm H} = \left(q_{\rm H}^+, q_{\rm H}^-, \mathbf{q}_{\rm H,T}\right) ,\qquad P_{\rm H}^+ = P_\gamma^+ = \frac{Q}{\sqrt{2}x_{\rm N}} ,$$

$$P_{\rm H} = \left(P_{\rm H}^+, \frac{M^2}{2P_{\rm H}^+}, \mathbf{0}_{\rm T}\right) ,\qquad P_{\rm H}^- = P_\gamma^- = \frac{x_{\rm N}M^2}{\sqrt{2}Q} ,$$

$$P_{\rm B,H} = \left(\frac{M_{\rm B}^2}{2P_{\rm B,H}^-}, P_{\rm B,H}^-, \mathbf{0}_{\rm T}\right) \qquad \mathbf{P}_{\rm H,T} = \mathbf{0}_{\rm T} .$$

Breit photon frame

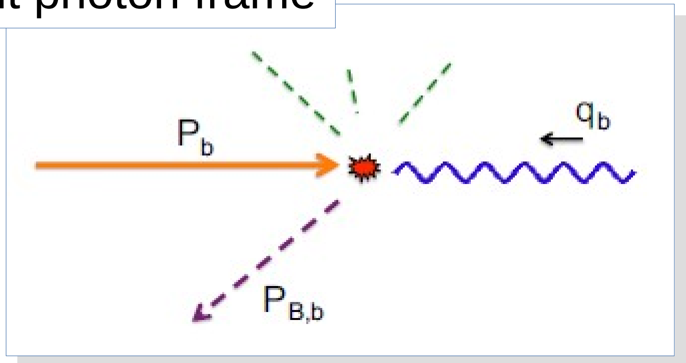


$$q_{\rm b} = \left(-\frac{Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, \mathbf{0}_{\rm T}\right) ,$$

$$P_{\rm b} = \left(\frac{Q}{x_{\rm N}\sqrt{2}}, \frac{x_{\rm N}M^2}{\sqrt{2}Q}, \mathbf{0}_{\rm T}\right) = \left(\frac{M}{\sqrt{2}}\, e^{y_{P,\rm b}}, \frac{M}{\sqrt{2}} e^{-y_{P,\rm b}}, \mathbf{0}_{\rm T}\right)$$

$$P_{\rm B,\gamma}^- = \frac{z_{\rm h}Q^2}{4x_{\rm Bj}P_\gamma^+}\left(1 \pm \sqrt{1 - \frac{4x_{\rm Bj}^2M^2\left(\mathbf{P}_{\rm B,\gamma,T}^2 + M_{\rm B}^2\right)}{z_{\rm h}^2Q^4}}\right) \approx \frac{z_{\rm h}Q^2}{2x_{\rm Bj}P_\gamma^+}$$

Final hadron variables in the Breit photon frame



$$P_{\mathrm{B,b}} = \left(\frac{M_{\mathrm{B}}^2 + z_{\mathrm{N}}^2 \mathbf{q}_{\mathrm{T}}^2}{\sqrt{2} z_{\mathrm{N}} Q}, \frac{z_{\mathrm{N}} Q}{\sqrt{2}}, -z_{\mathrm{N}} \mathbf{q}_{\mathrm{T}} \right)$$

$$\mathbf{q}_{\mathrm{T}} \equiv -\frac{\mathbf{P}_{\mathrm{B,b,T}}}{z_{\mathrm{N}}} = -\frac{q_{\mathrm{b}}^- \mathbf{P}_{\mathrm{B,b,T}}}{P_{\mathrm{B,b}}^-}$$

$$\boxed{z_{\mathrm{N}} = \frac{x_{\mathrm{N}} z_{\mathrm{h}}}{2 x_{\mathrm{Bj}}} \left(1 + \sqrt{1 - \frac{4 M^2 M_{\mathrm{B,T}}^2 x_{\mathrm{Bj}}^2}{Q^4 z_{\mathrm{h}}^2}} \right)} \approx z_{\mathrm{h}}$$

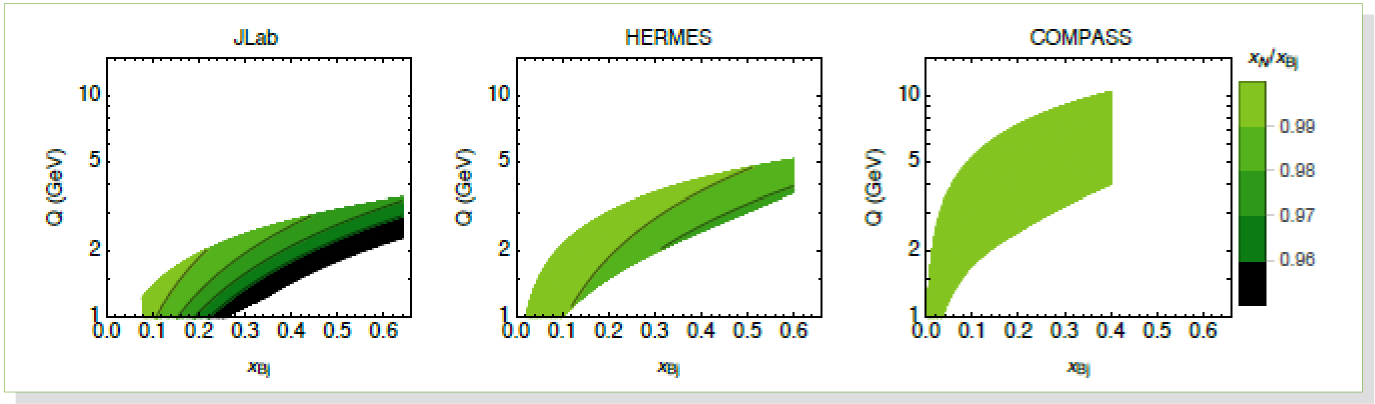
If hadron masses, M and M_B, and $P_{B,b,T}$ are much smaller than the hard scale Q

$$M_{B,T}^2 = P_{B,b,T}^2 + M_B^2$$

$$x_{\mathrm{N}} = x_{\mathrm{Bj}} \left[1 - \frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} + O\left(\frac{x_{\mathrm{Bj}}^4 M^4}{Q^4} \right) \right], \tag{3.1}$$

$$z_{\mathrm{N}} = z_{\mathrm{h}} \left[1 - \frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} \left(1 + \frac{\mathbf{P}_{\mathrm{B,b,T}}^2}{z_{\mathrm{h}}^2 Q^2} \right) + \left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} \right)^2 \left(\frac{\mathbf{P}_{\mathrm{B,b,T}}^2}{z_{\mathrm{h}}^2 Q^2} - \frac{\mathbf{P}_{\mathrm{B,b,T}}^4}{z_{\mathrm{h}}^4 Q^4} + 2 - \frac{M_{\mathrm{B}}^2}{z_{\mathrm{h}}^2 M^2 x_{\mathrm{Bj}}^2} \right) + O\left(\frac{x_{\mathrm{Bj}}^6 M^6}{Q^6} \right) \right]. \tag{3.2}$$

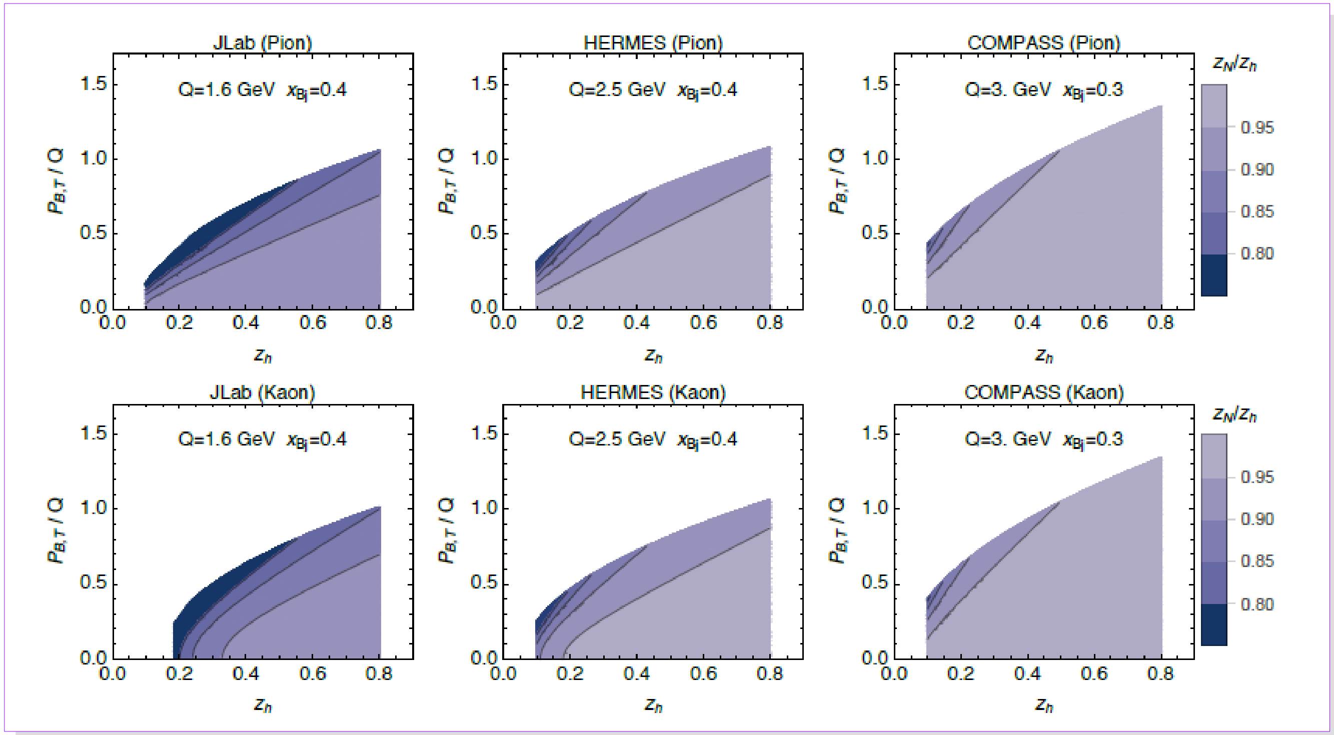
Massless Hadron Approximations (MHA)



The ratio x_N/x_{Bj} is a measure of the quality of the MHA approximation. It must not deviate too much from 1 if the standard massless approximations are to be considered valid

- Kinematics regions of Q and x_{Bj} covered by JLab 12, HERMES and COMPASS.
- Shaded areas are obtained by applying appropriate experimental cuts.
- Q and x_{Bj} are strongly correlated: large values of x_{Bj} can only be accessed when Q is suffciently large. When Q is relatively small, only limited values of x_{Bj} can be reached.
- The values of x_N / x_{Bj} are colour-coded: the lightest shade corresponds to values very close to one, while darker shades correspond to regions where the ratio x_N=x_{Bj} increasingly deviates from 1 and the quality of the MHA deteriorates.
- While mass corrections are more important for JLab 12 kinematics, for all three experiments the value x_N=x_{Bj} remains very close to 1 to a very good approximation.

Massless Hadron Approximations (MHA)

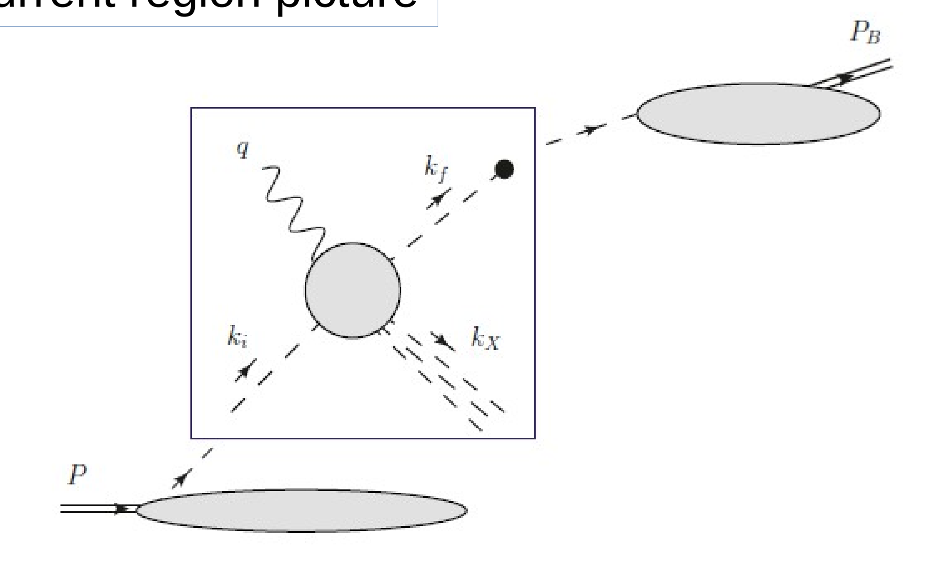


The ratio z_N/z_h is a measure of the quality of the MHA approximation. It must not deviate too much from 1 if the standard massless approximations are to be considered valid

- The ratio z_N/z_h is represented over the kinematic coverage in (z_h; P_{BT};Q) for JLab 12, HERMES and COMPASS, at some fixed values of x_{Bj} and Q.
- The values of z_N/z_h , for pion and kaon production are colour-coded: the lightest shade corresponds to values very close to one, while darker shades correspond to regions where the ratio increasingly deviates from 1 and the quality of the MHA deteriorates.
- Deviations from 1 are more sizeable as compared to those of x_N/x_{Bj} , particularly in the JLab case.

Mapping SIDIS kinematics and partonic sub-processes

Current region picture



Assuming that the configuration of initial and final hadrons is the result of scattering and fragmentation of small-mass constituents (partons), **what are the possible kinematics configurations of those constituents,** given a set of assumptions about their intrinsic properties?

- Lower blob ↔ incoming hadron.
- Diagram in central square ↔ partonic subprocess of interaction partons - virtual photon
- Dashed lines ↔ flow of momenta
- Upper blob ↔ fragmentation of the outgoing parton into the observed hadron B
- k_X labels the total momentum of all other unobserved partons combined

Parton kinematics

$$k_{\rm i}^{\rm b} = \left(\frac{Q}{\hat{x}_{\rm N}\sqrt{2}}, \frac{\hat{x}_{\rm N}(k_{\rm i}^2 + \mathbf{k}_{\rm i,b,T}^2)}{\sqrt{2}Q}, \mathbf{k}_{\rm i,b,T} \right) , \qquad k_{\rm f}^{\rm b} = \left(\frac{\mathbf{k}_{\rm f,b,T}^2 + k_{\rm f}^2}{\sqrt{2}\hat{z}_{\rm N}Q}, \frac{\hat{z}_{\rm N}Q}{\sqrt{2}}, \mathbf{k}_{\rm f,b,T} \right)$$

$$k_{\rm i}^+ \equiv \xi P_{\rm b}^+ , \qquad P_{\rm B,b}^- \equiv \zeta k_{\rm f}^- , \qquad \hat{x}_{\rm N} \equiv -\frac{q_{\rm b}^+}{k_{\rm i,b}^+} = \frac{x_{\rm N}}{\xi} , \qquad \hat{z}_{\rm N} \equiv \frac{k_{\rm f,b}^-}{q_{\rm b}^-} = \frac{z_{\rm N}}{\zeta}$$

$$\boxed{\mathbf{k}_{f,b,T} = -z_N \mathbf{q_T} + \delta \mathbf{k_T}}$$

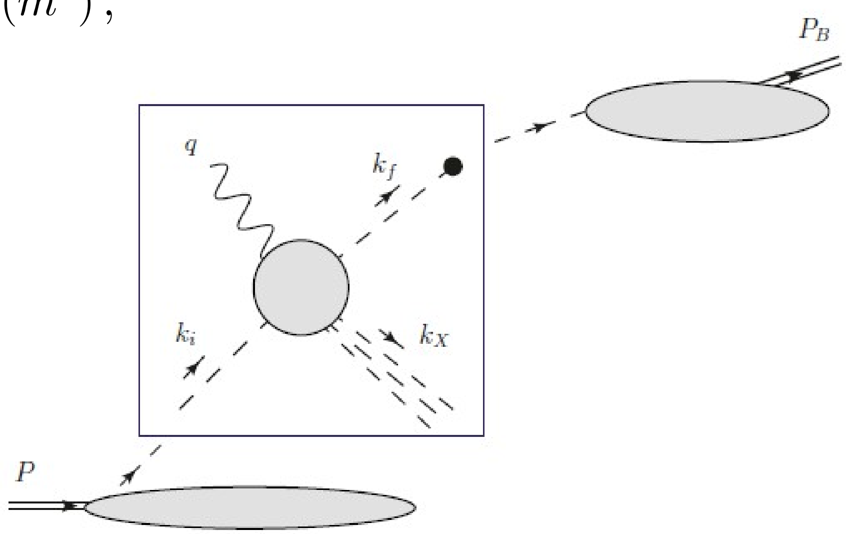
- δk_T plays the role of “intrinsic” transverse momentum
- It survives when $q_T \to 0$

For nearly on-shell partons

$$\left\{ \begin{array}{l} |k_i^2| = \mathcal{O}(m^2) , \quad |k_f^2| = \mathcal{O}(m^2) , \\ \delta k_T^2 = \mathcal{O}(m^2) \end{array} \right.$$

From momentum conservation:

$$k_X^2 = (k_i + q - k_f)^2$$



Parton kinematics

$$\boxed{\mathbf{k}_{f,b,T} = -z_N \mathbf{q_T} + \delta \mathbf{k_T}}$$

- δk_T plays the role of "intrinsic" transverse momentum
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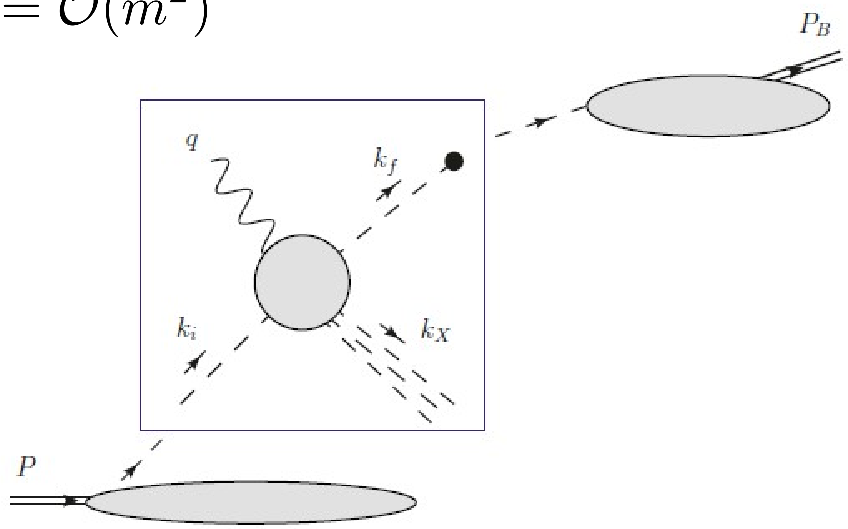
For nearly on-shell partons
$$\begin{cases} |k_i^2| = \mathcal{O}(m^2)\,, \quad |k_f^2| = \mathcal{O}(m^2) \\ \delta k_T^2 = \mathcal{O}(m^2) \end{cases}$$



From momentum conservation

$$k_X^2 = (k_i + q - k_f)^2$$

Let's also define an auxiliary momentum

$$k \equiv k_f - q$$

When $k_i^2 = k_f^2 = k_{i,b,T}^2 = \delta k_T^2 = 0$ then:

$$\begin{cases} k_b^+ = \dfrac{Q}{\sqrt{2}}(1 + \dfrac{q_T^2}{Q^2} + \ldots)\,, \qquad k_b^- = -\dfrac{\hat{x}_N Q}{(1-\hat{x}_N\sqrt{2}})(\dfrac{q_T^2}{Q^2} + \dfrac{k_X^2}{Q^2} + \ldots) \\ k_T = -q_T\Big(1 - \dfrac{\hat{x}_N}{1-\hat{x}_N}(\dfrac{q_T^2}{Q^2} + \dfrac{k_X^2}{Q^2}) + \ldots\Big) \end{cases}$$

Mapping SIDIS kinematics and partonic sub-processes

- We are interested in the kinematics of the $k_i + q \rightarrow k_f + k_X$ subprocess and how closely it matches the overall $P + q \rightarrow P_B + X$ process under very general assumptions

- Basic partonic approximation 1: masses and off-shellness of partons are small relative to Q:

$$\frac{k_i^2}{Q^2} \rightarrow 0\,, \quad \frac{k_f^2}{Q^2} \rightarrow 0$$

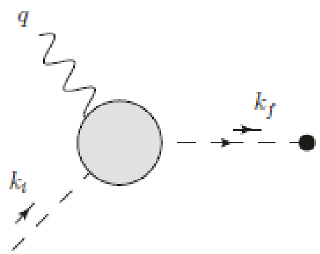
- Basic partonic approximation 2: in the current region, k_f is aligned with the final state hadron

$$k_f \cdot P_B \rightarrow 0$$

Mapping SIDIS kinematics and partonic sub-processes

- Further approximations apply to different specic partonic subprocesses

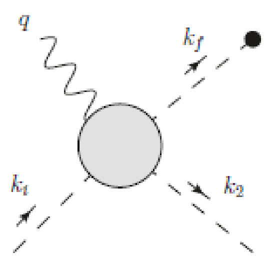
$2 \to 1$



$$\frac{q_T^2}{Q^2} \to 0\,, \ \frac{k_X^2}{Q^2} \to 0\,, \ k_i \to k$$

The kinematics of the struck parton approaches the kinematic configuration of TMD factorization, in the current region

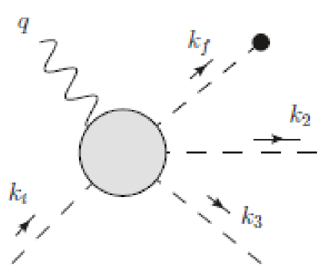
$2 \to 2$



$$|k^2| \to Q^2\,, \ \frac{k_X^2}{Q^2} \to 0$$

The kinematics of the subprocess is that of a hard 2 to 2 partonic process

$2 \to 3$



$$|k^2| \to Q^2\,, \ |k_X^2| \to Q^2$$

Here at least three partons are ejected at wide angles from the hard collision

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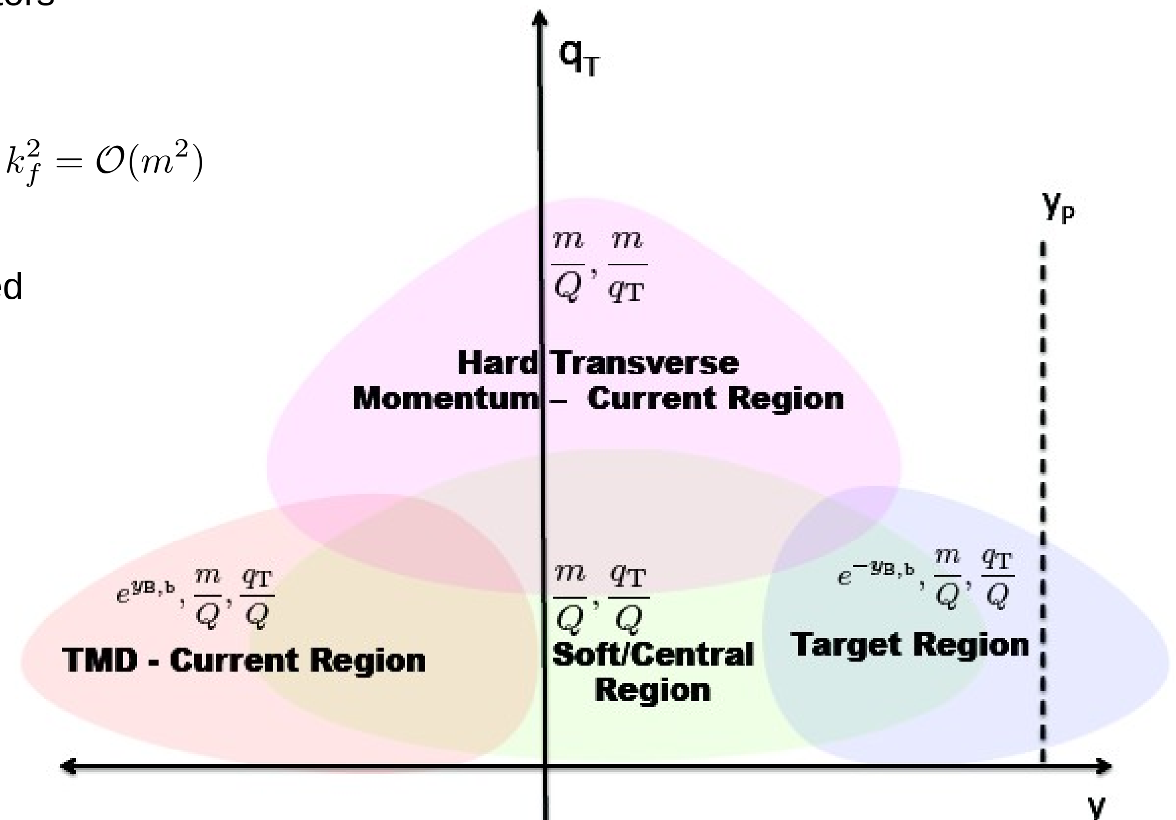
- Sketch of kinematical regions of SIDIS in terms of the produced hadron's Breit frame rapidity and transverse momentum.
- In each region, the type of suppression factors that give factorization are shown.
- In the Breit frame, partons in the handbag configuration are centered on y=0 if $-k_i^2 \sim k_f^2 = \mathcal{O}(m^2)$
- The shaded regions in the sketch are shifted somewhat toward the target rapidity $y_{P;b}$ (the vertical dashed line) to account for the behaviour of the rapidities when z_N and x_N are small.



Plot credit: Ted Rogers

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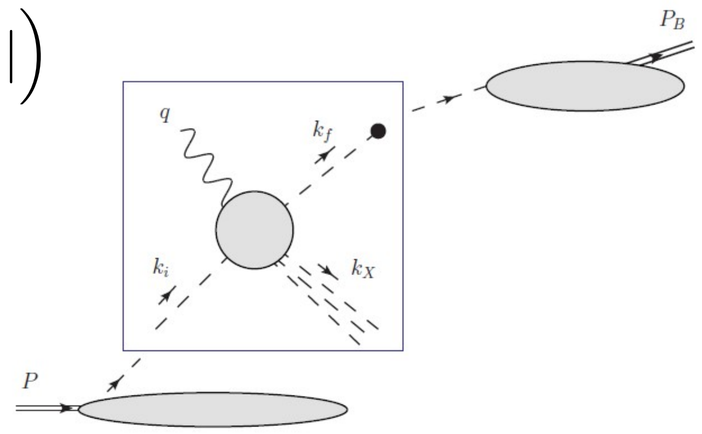
$$\text{General Hardness Ratio :}\quad R_0 \equiv \max\left(\left|\frac{k_i^2}{Q^2}\right|, \left|\frac{k_f^2}{Q^2}\right|, \left|\frac{\delta k_f^2}{Q^2}\right|\right)$$

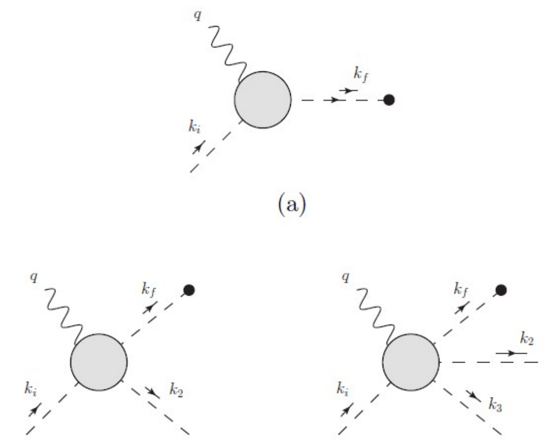
$$\text{Collinearity :}\quad R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

$$\text{Transverse Hardness Ratio :}\quad R_2 \equiv \frac{|k^2|}{Q^2}$$

$$\text{Spectator Virtuality Ratio :}\quad R_3 \equiv \frac{|k_X^2|}{Q^2}$$



	R_0	R_1	R_2	R_3	R_1'
TMD Current region	small	small	small	X	large
Hard region	small	small	large	small (low order pQCD)	large
	small	small	large	large (high order pQCD)	large
Target region	small	large	X	X	small
Soft region	small	large	small	X	large



(a)

Non-perturbative parameters

- Typical estimate of non-perturbative mass scales ~ 300 MeV, so we set

$$k_i = k_f = k_T = 300 \text{ MeV}$$

- Also, to start with we assume that

$$\mathbf{q}_T \cdot \delta\mathbf{k}_T = q_T \cdot \delta k_T$$

Azimuthal effects may be added later

- Finally we set the following representative values:

$$\xi = 0.3\,, \quad \zeta = 0.3\,, \quad z_h = 0.3\,, \quad q_T = 0.3 \text{ GeV}$$

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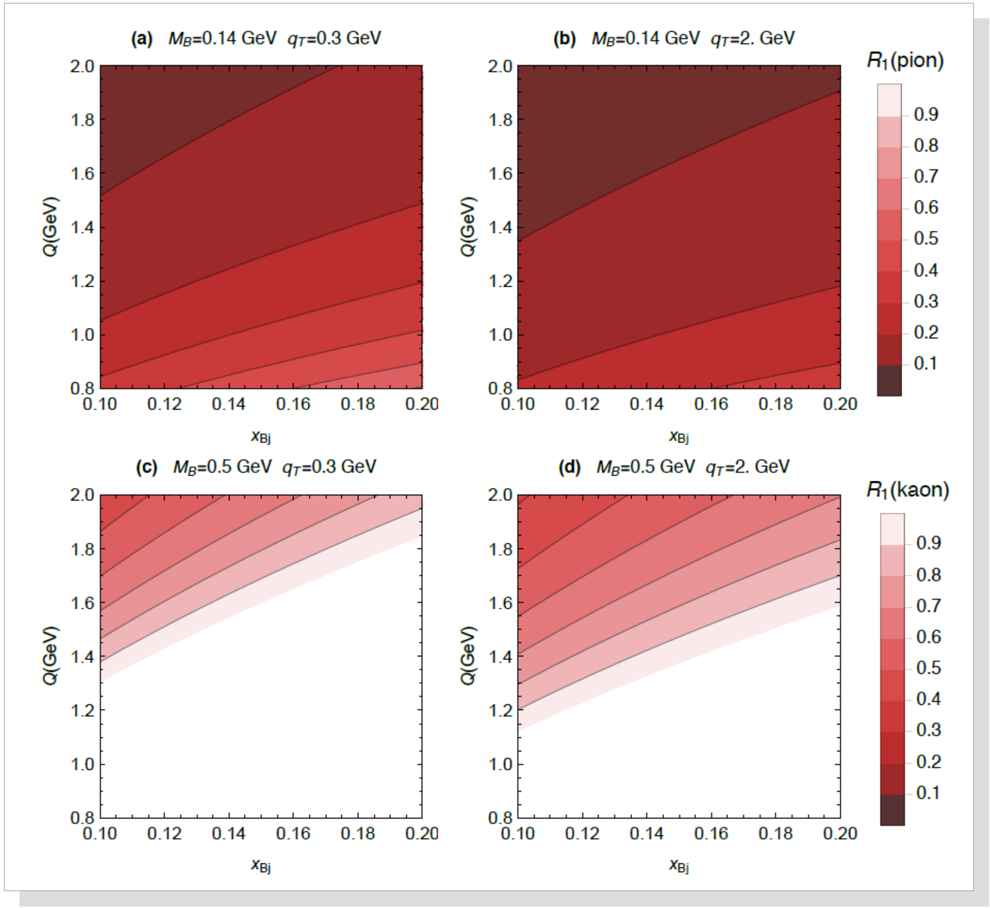
Collinearity R_1

- At large Q and not-too-large x_{Bj}, R_1 remains small for all transverse momenta, while corrections might be necessary at smaller Q and larger x_{Bj}.
- Notice the strong flavour dependence.



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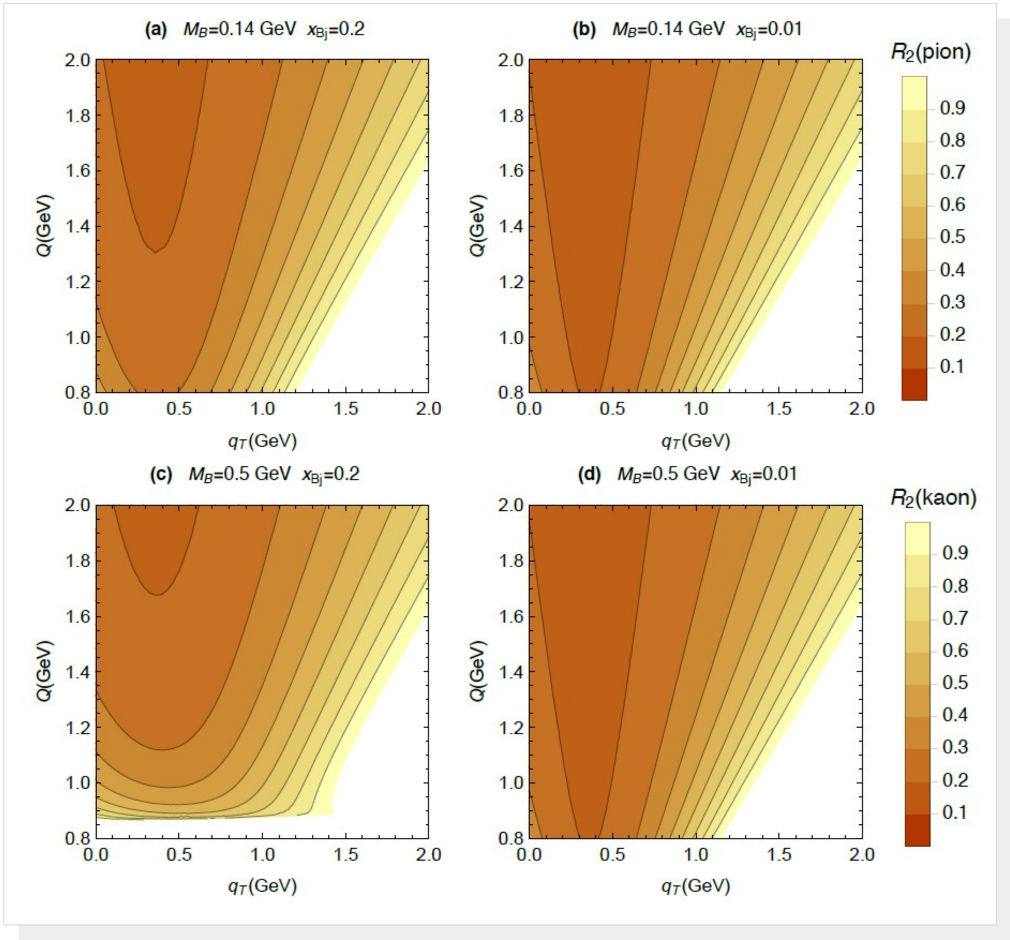
Transverse hardness ratio R_2

- R_2 maps out the applicability of large and small transverse momentum approximations.
- "large-q_T" grows with Q
- While the hadron is in the current region for most q_T values, the small transverse momentum region is much more limited.
- There is a broad intermediate region where the situation is not clear.
- The flavour of the final state hadron is decisive in determining the relevant factorization region.
- The flavour of the final state hadron has little effect on the transverse momentum hardness



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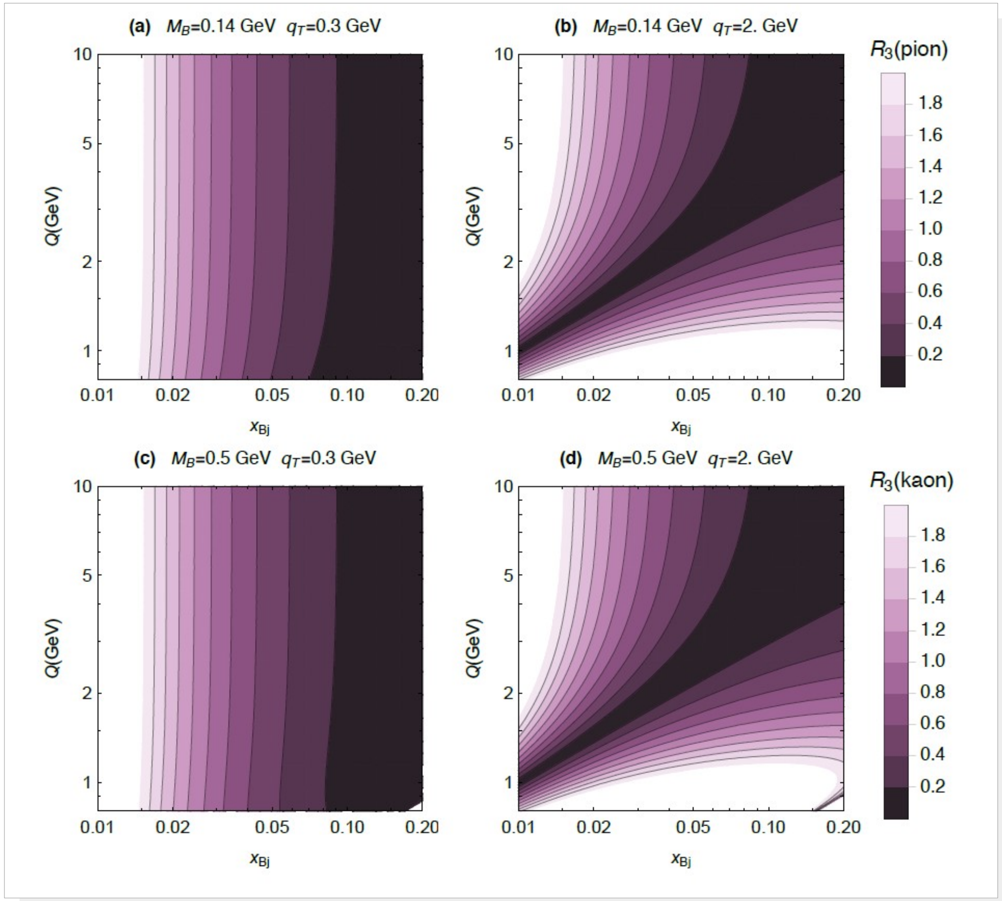
Spectator Virtuality ratio R_3

- R_3 helps us deciding whether a LO calculation is sufficient or whether higher order perturbative corrections are necessary.
- Large R_3 coupled with large R_2 signal a large-q_T region, where higher order pQCD corrections are relevant
- Small R_3 together with small R_2 indicate a TMD current region, which requires a TMD factorization scheme.
- At large transverse momentum there is a linear region in the Q versus x_{Bj} plane where the 2→2 process is the optimal description (R_3 is small) and low order QCD computations are applicable.



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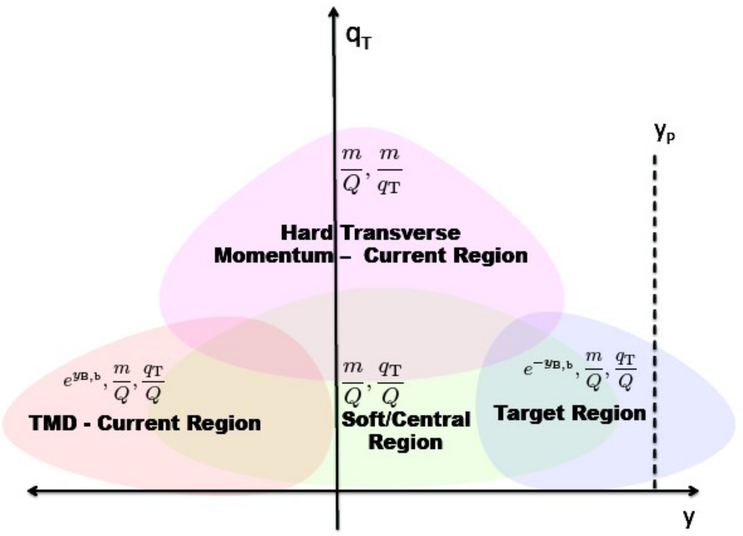
Summarizing ...

The strategy is the following:

- ***Collinearity R_1:*** *helps you determine which SIDIS data subset can be considered to belong to the* ***current*** *region or to the target / soft region*



- ***Transverse Hardness ratio R_2:*** *allows you to access whether your data subset sits in the* ***large*** *transverse momentum (current) region or in the* ***TMD*** *(current) region*

- ***Spectator Virtuality ratio R_3:*** *helps you decide whether a* ***LO*** *calculation is sufficient or whether* ***higher order*** *perturbative corrections are necessary*