NNLL resummations with ARES

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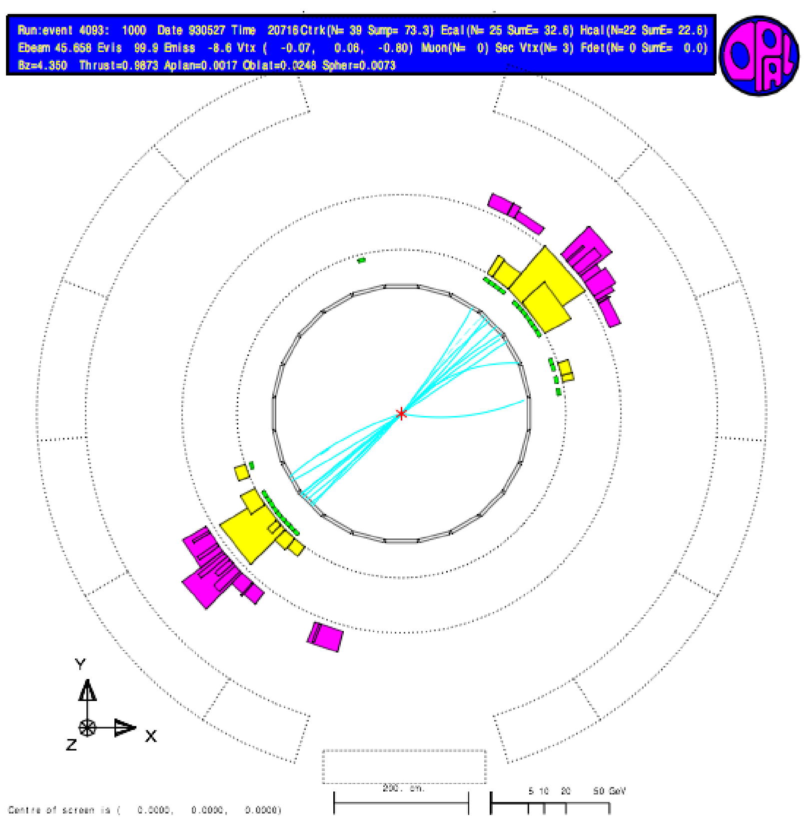
Outline

- Resummation of final-state observables
- Observable properties (rIRC safety)
- The ARES method for QCD resummation
- Recent results (including precise determination of α_s)
- Current work in progress and outlook

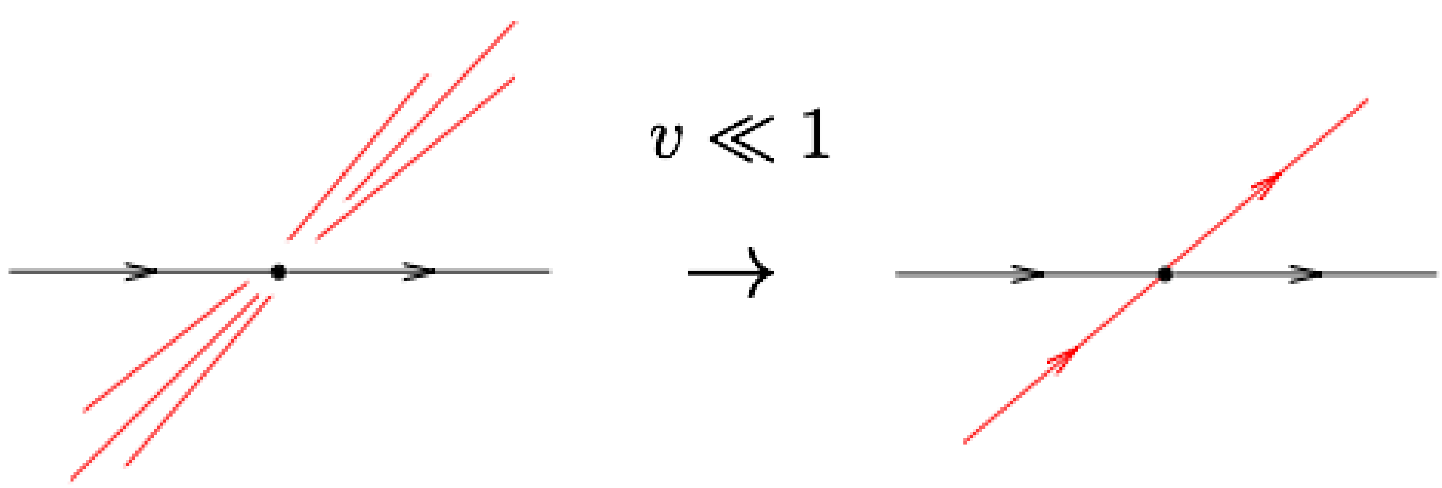
Final-state observables

- We consider a generic final-state observable, a function $V(p_1, \ldots, p_n)$ of all possible final-state momenta $p_1, \ldots, p_n$
- Examples: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$



$$v \ll 1 \quad \longrightarrow$$

The narrow-jet limit

- Selecting events close to the Born limit, i.e. $v \ll 1$, produces large logarithms of the resolution variable v due to incomplete real-virtual cancellations

$$\Sigma(v) \simeq 1 - C\,\frac{\alpha_s}{\pi}\ln^2\frac{1}{v} + \ldots$$

LO NLO

breakdown of perturbation theory!

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \dots\right]$$

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq e^{\overbrace{Lg_1(\alpha_s L)}_{\text{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}} + \ldots \right)$$

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All-order resummation

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$$\Sigma(v) \simeq e^{\overbrace{Lg_1(\alpha_s L)}^{}_{\mathrm{LL}}} \times \begin{pmatrix} \color{blue}{1} & \color{blue}{+} & \color{blue}{\alpha_s} & \color{blue}{+\dots} \\ \underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} & + & \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} & + \dots \end{pmatrix}$$

The ARES method

- NNLL corrections are often sizeable and important for precision physics
- The most important limitation is the analytical treatment of the observable in some (smartly defined) conjugate space
- The Automated Resummer for Event Shapes (ARES) is a semi-numerical approach that:
    - does not rely on analytical properties of the observable
    - is NNLL accurate and extendable to higher orders
    - is fully general for a very broad category of observables (~ all that can be possibly resummed at NNLL accuracy)
    - is flexible and automated (only input: observable’s routine in suitable limits)

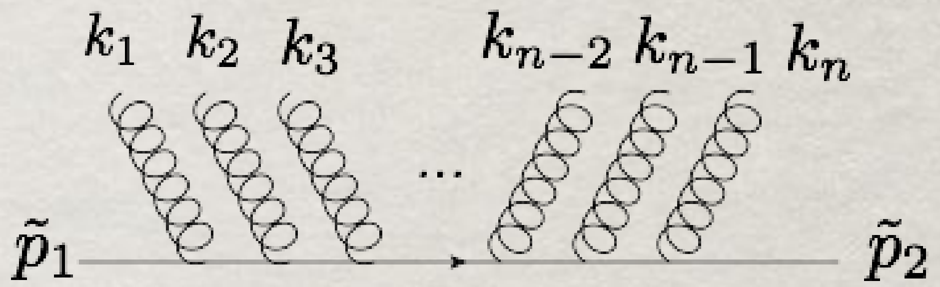
[AB McAslan Monni Zanderighi 1412.2126]

[AB El-Menoufi Monni 1807.11487]

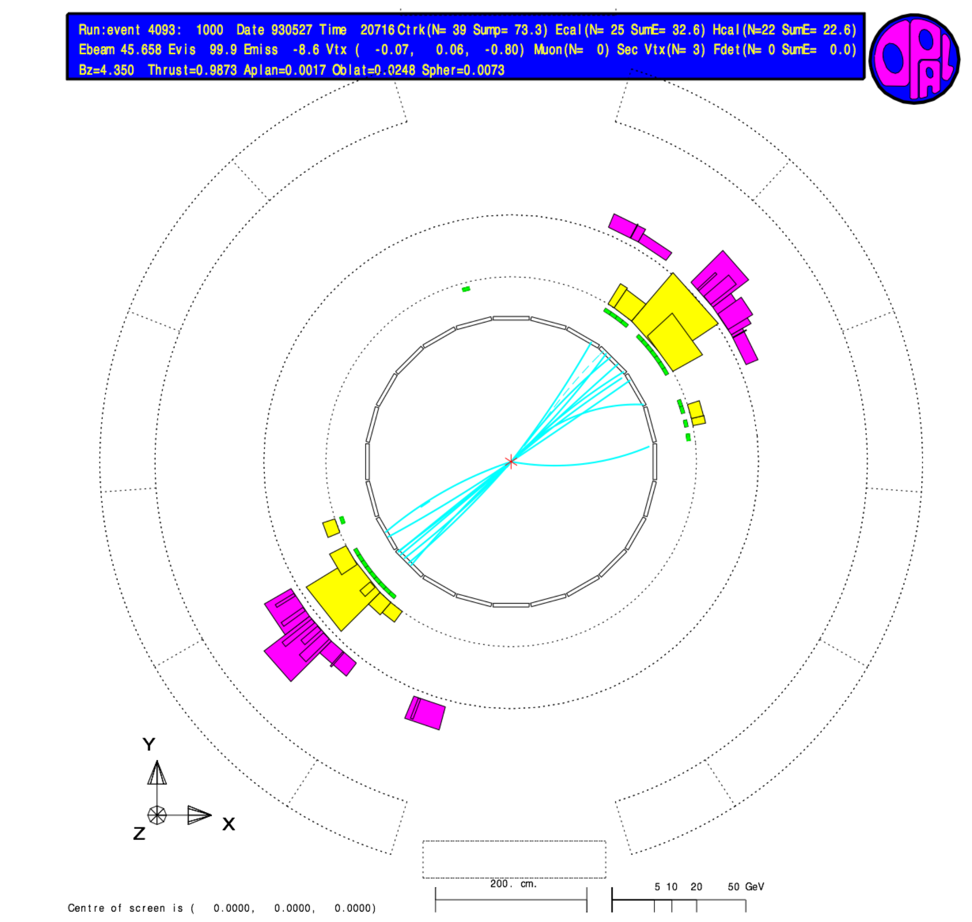
BASIC OBSERVABLE PROPERTIES

- We consider a generic infrared and collinear (IRC) safe observable

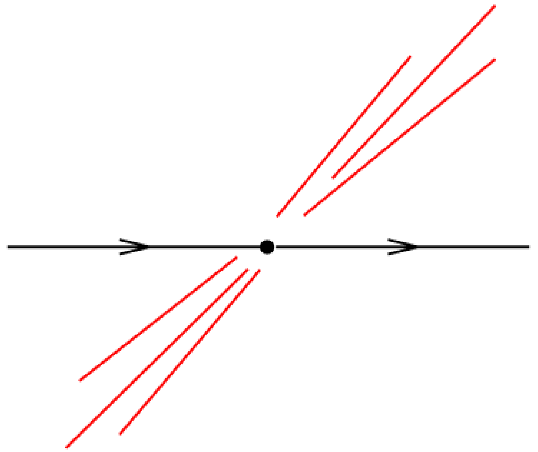
$$V(\{\tilde{p}\}, k_1, \ldots, k_n) \geq 0$$



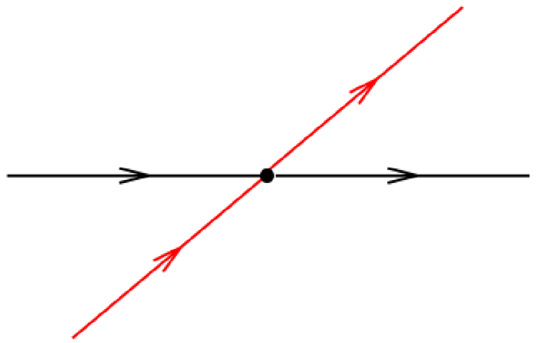
- In the Born limit, $V(\{\tilde{p}\}) = 0$
- In the limit $v \to 0$, quasi-Born kinematics, all secondary emissions $k_1, \ldots, k_n$ are soft and/or collinear



$$\Sigma(v) = \mathrm{Prob}[V(\{\tilde{p}\}, k_1, \ldots, k_n)] < v$$



$$v \ll 1$$



Recursive IRC safety

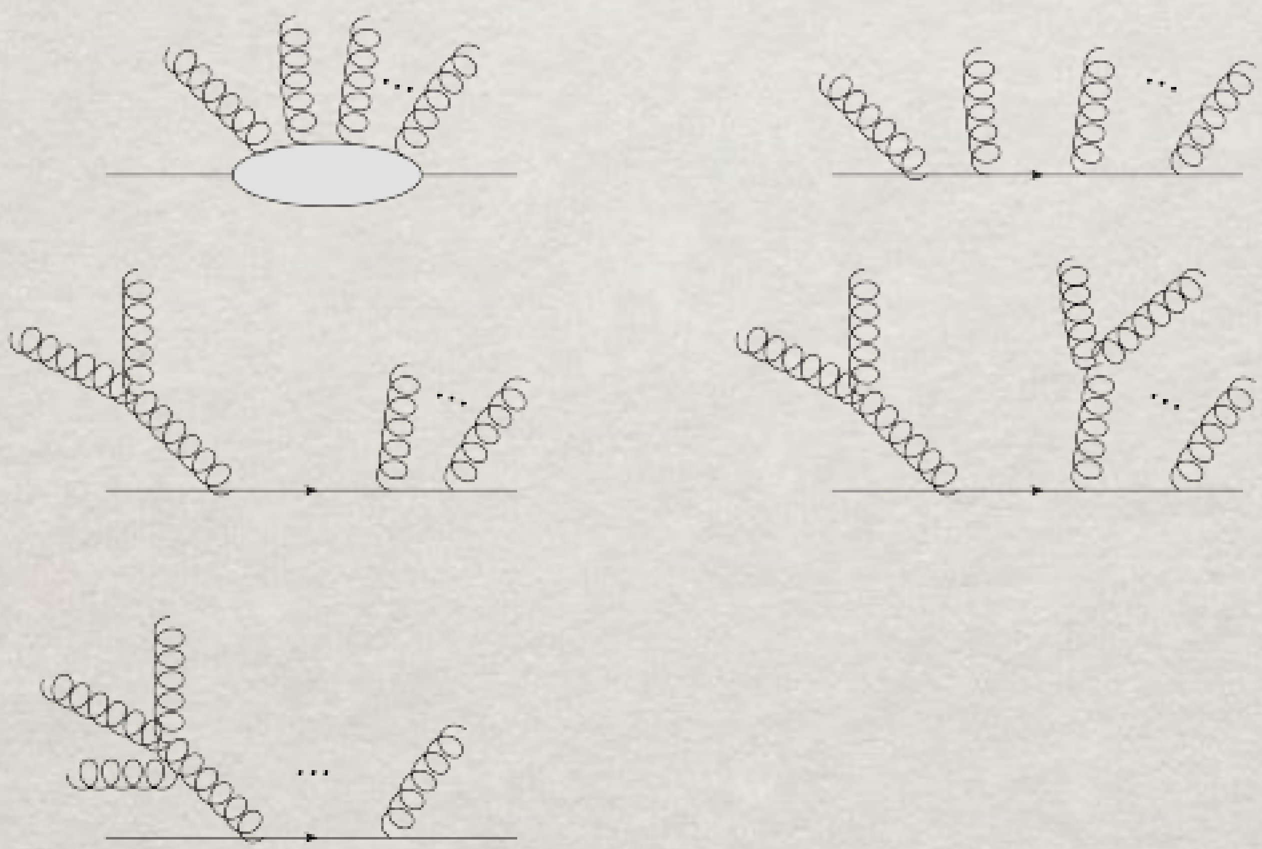
- We restrict ourselves to recursive IRC (rIRC) safe observables, for which
  - the observable scaling properties when we make all emissions simultaneously soft-collinear are the same with any number of secondary emissions
  - such scaling properties are unchanged after an infinitely soft emission or a perfect collinear splitting

- Examples of rIRC observables:
  - most global event shapes
  - Durham and Cambridge jet resolution parameters (no JADE and Geneva)
  - transverse momentum of the leading jet in Higgs or vector boson production

Implications of rIRC safety

- The only emissions that contribute to $\Sigma(v) = \mathrm{Prob}[V(\{\tilde{p}\}, k_1, \ldots, k_n)] < v$ in the limit $v \to 0$, up to powers of v, are those for which

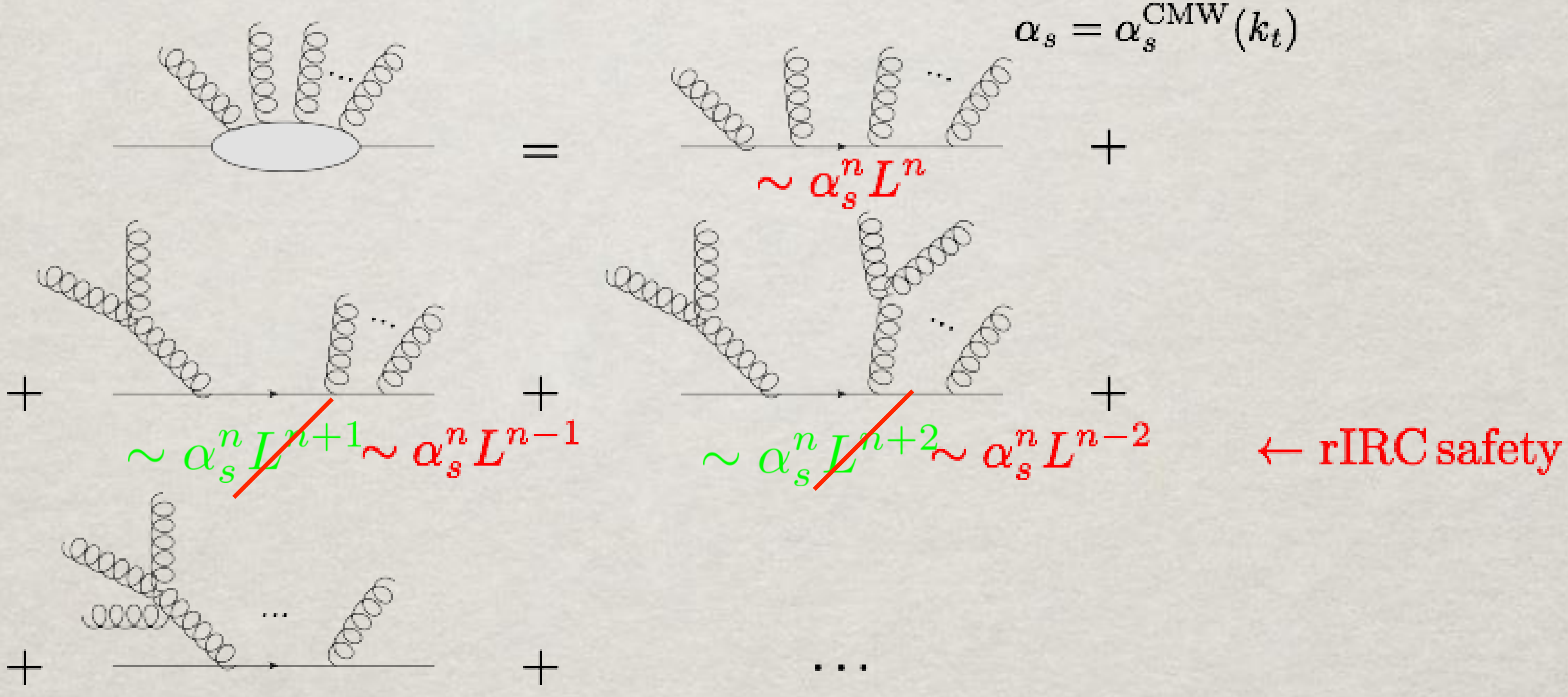
$$V(\{\tilde{p}\}, k_1) \sim V(\{\tilde{p}\}, k_2) \sim \cdots \sim V(\{\tilde{p}\}, k_n) \sim V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$$



- This, together with QCD coherence, is enough to establish the relative importance of soft-collinear contributions at all logarithmic orders

Relevant emissions

- At NLL the only relevant emissions are soft and collinear gluons widely separated in angle (rapidity separation $\sim \ln(1/v)$)



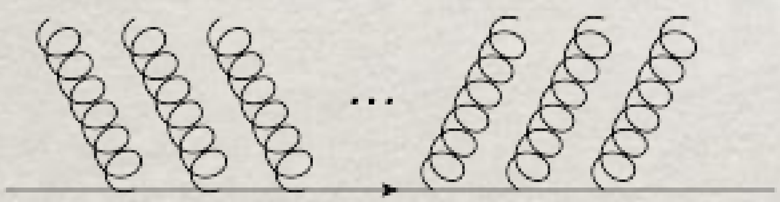
- Any other emission gives a contribution of relative order α_s

NLL RESUMMATION

- Unresolved emissions and virtual corrections result in a double-logarithmic Sudakov exponent, the radiator [AB Salam Zanderighi hep-ph/0407286]

$$\boxed{\Sigma(v) = e^{-R_{\mathrm{NLL}}(v)} \mathcal{F}_{\mathrm{NLL}}(v)}$$

- The effect of multiple soft and collinear gluons widely separated in angle is encoded in the single-logarithmic function $\mathcal{F}_{\mathrm{NLL}}(v)$



Measure defined by the soft-collinear ensemble

$$= \int dZ[\{R'_{\ell_i}, k_i\}] \qquad R' \equiv -v\frac{dR}{dv} = \sum_{\ell_i} R'_{\ell_i}$$

$$\boxed{\mathcal{F}_{\mathrm{NLL}}(v) = \left\langle \Theta\left(1 - \frac{V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) \right\rangle}$$

- The function $V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\}, \{k_i\})$ is the observable for soft and collinear emissions widely separated in angle. For jet rates, this is different from $V_{\mathrm{sc}}(\{\tilde{p}\}, \{k_i\})$, the observable in the soft-collinear limit

NNLL RESUMMATION

- The NNLL radiator, encoding the cancellation of unresolved real emissions and virtual corrections, can be computed for an arbitrary observable

[AB El-Menoufi Monni 1807.11487]

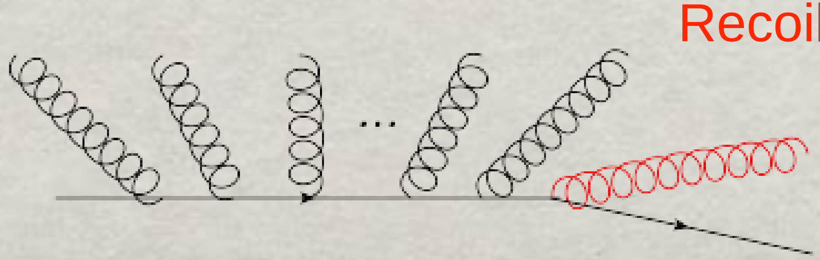
$$\boxed{\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]}$$

Clustering
(jet algorithms only)

Correlated emission

$$\boxed{\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}}$$

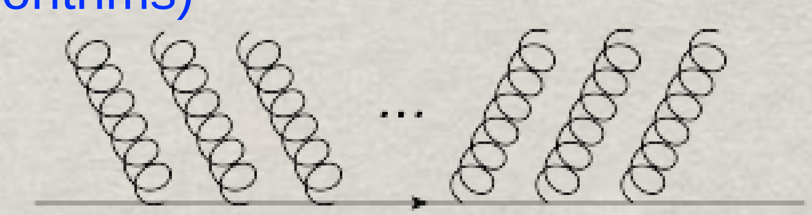
Hard collinear
Recoil



Running coupling
Rapidity
(jet algorithms)

Soft wide-angle

Soft-collinear
(event shapes)



NNLL RESUMMATION

- All NNLL corrections can be written in terms of finite integrals in four dimensions

$$\boxed{\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]}$$

Clustering
(jet algorithms only)

Correlated emission

[AB McAslan Monni Zanderighi 1412.2126]

$$\boxed{\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}}$$

Hard collinear
Recoil

Running coupling
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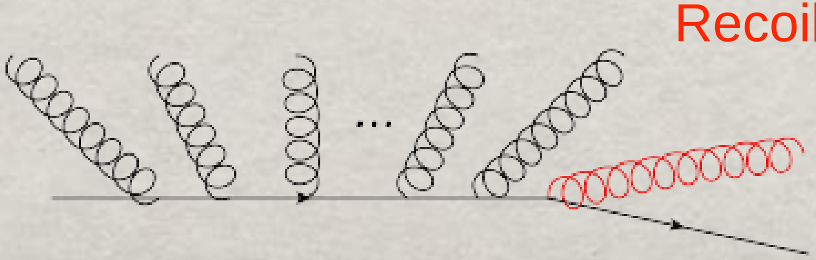
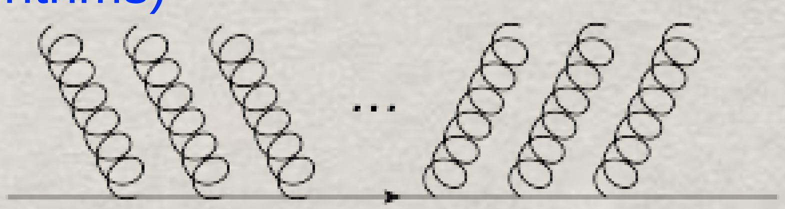




NNLL RESUMMATION

- Every NNLL correction requires to determine an approximate expression for the observable in the relevant kinematic limit

$$\boxed{\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]}$$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\}) \neq V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Clustering
(jet algorithms only)

Correlated emission

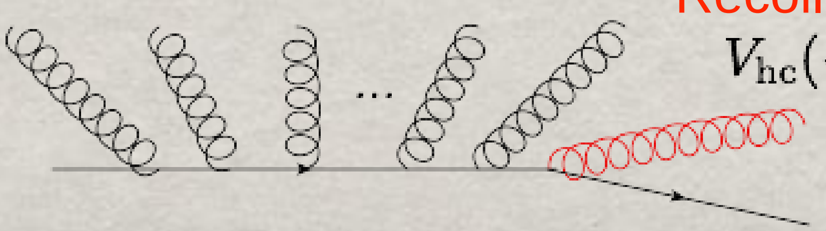
[AB McAslan Monni Zanderighi 1412.2126]

$$\boxed{\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}}$$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Hard collinear
Recoil

$V_{\mathrm{hc}}(\{\tilde{p}\},\{k_i\})$



$V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$

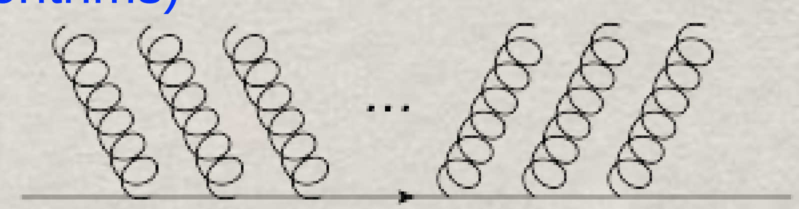
Running coupling
Rapidity
(jet algorithms)

$V_{\mathrm{wa}}(\{\tilde{p}\},\{k_i\})$

Soft wide-angle

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Soft-collinear
(event shapes)



CAESAR vs ARES

[AB Salam Zanderighi hep-ph/0407286]

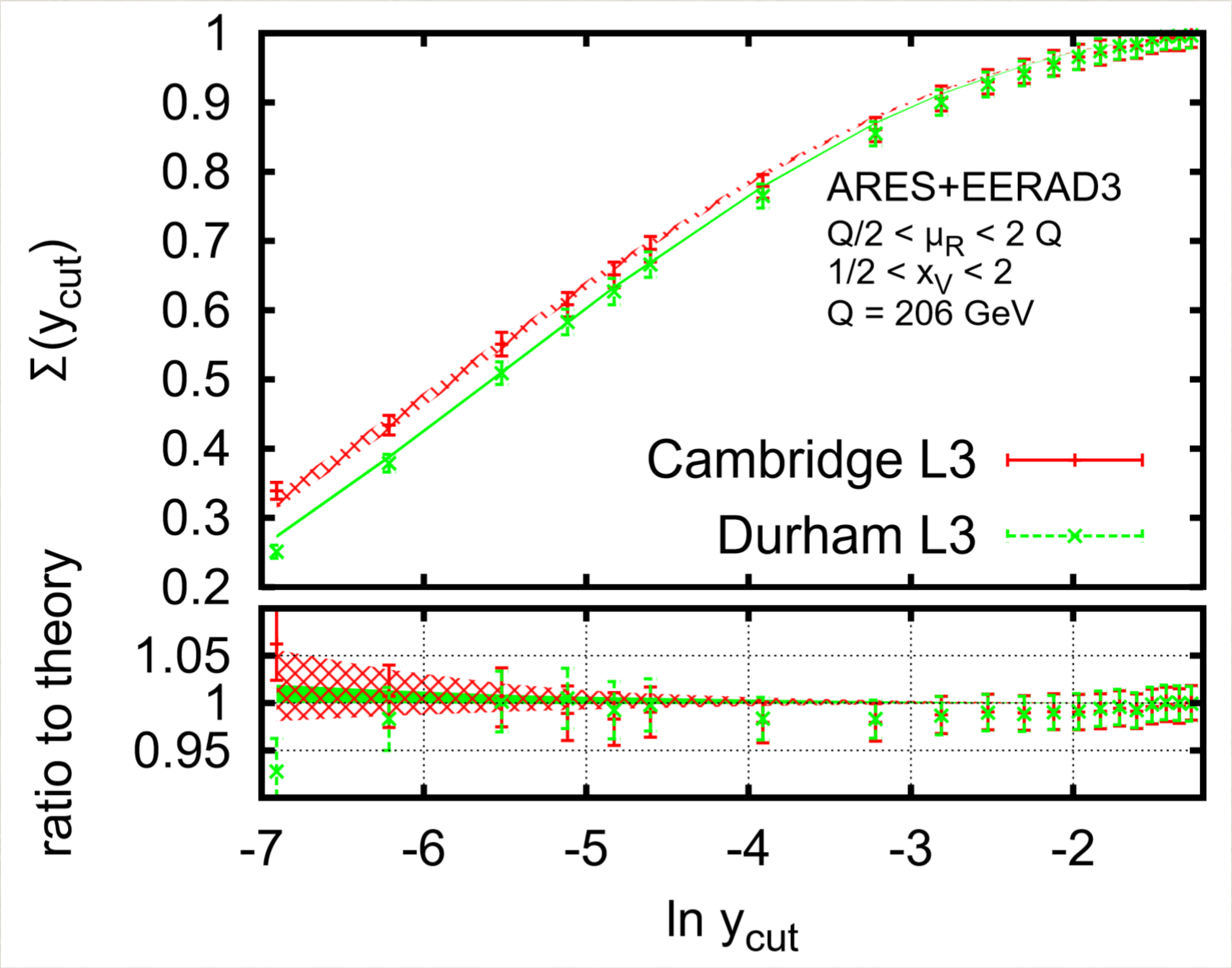
- Establishes the range in which actual emissions can be considered soft and collinear
- Uses the actual observable subroutine and computes its soft-collinear limit numerically
- Requires careful extrapolations to be extended to NNLL

[AB McAslan Monni Zanderighi 1412.2126]

- Generates emissions that are by construction soft and collinear (no energy-momentum conservation)
- Uses analytically determined soft and collinear limits of each observable
- Can be in principle extended to any logarithmic accuracy

Two-jet rate

- First-ever NNLL resummation of the two-jet rate for the Durham and Cambridge algorithms [AB McAslan Monni Zanderighi 1607.03111]



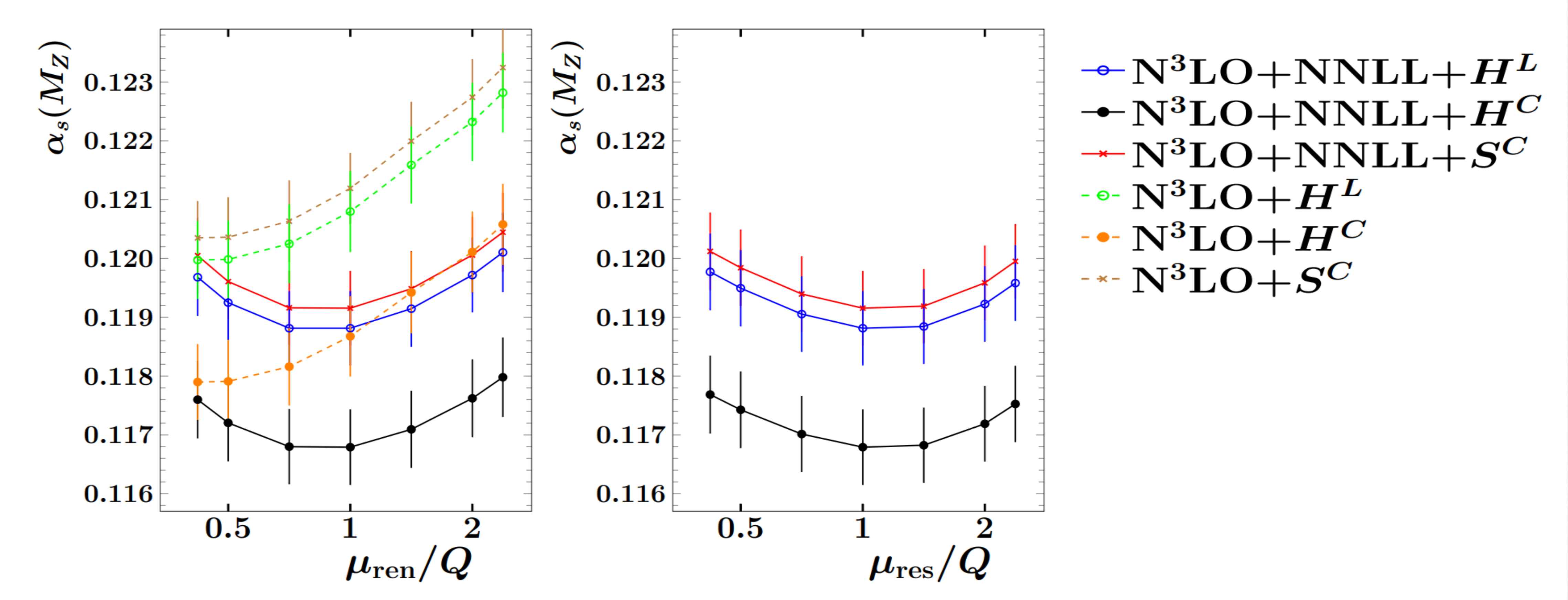
- NNLL resummation of the two-jet rate has been performed also for other rIRC safe jet algorithms (flavour k_t, angular-ordered Durham, inclusive k_t)

Strong coupling determination

- The NNLL resummation of the two-jet rate for the Durham algorithm leads to the most precise determination of $\alpha_s(M_Z)$ from jet observables

[Verbytskyi et al 1902.08158]

$$\alpha_s(M_Z) = 0.1188 \pm 0.0009(\mathrm{stat}) \pm 0.0009(\mathrm{exp}) \pm 0.0010(\mathrm{had}) \pm 0.0006(\mathrm{theo})$$



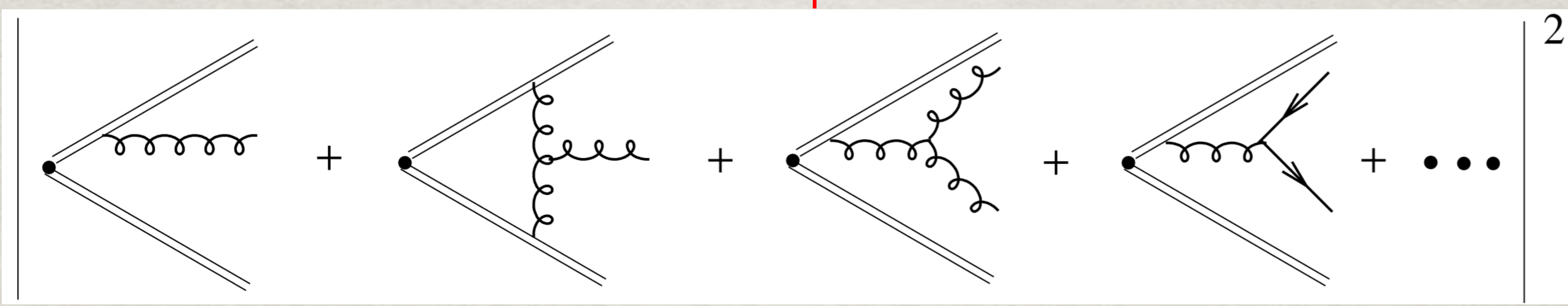
- For the first time, thanks to NNLL resummation accuracy, hadronisation uncertainties for jet observables are larger than perturbative uncertainties

WEB EXPONENTIATION

- Virtual corrections to two-leg processes in $d = 4 - 2\epsilon$ dimensions

[Magnea hep-ph/0006255]

$$\mathcal{H}(Q,\epsilon) = H(\alpha_s(Q)) \exp\left[-\int \frac{d^d k}{(2\pi)^d} \underbrace{w(k_t^2, k_t^2 + m^2, \epsilon)}_{\text{web(soft)}}\right] \exp\left[-\sum_\ell \int^Q \frac{dk}{k} \underbrace{\gamma_\ell(\alpha_s(k,\epsilon))}_{\text{hard collinear}}\right]$$



$\tilde{M}^2_{s,0}(k)$ $\tilde{M}^2_{s,1}(k)$ $\tilde{M}^2_{s,0}(k_1,k_2)$

- Soft virtual corrections arise from the exponentiation of correlated gluon emissions $\Rightarrow$ web exponentiation

[Gatheral PL133B (1983) 90]
[Frenkel Taylor NPB 246 (1984) 231]
[Gardi Smillie White 1304.7040]

- The web originates from eikonal matrix elements $\Rightarrow$ it depends only on variables that are invariant over rescaling of emitters momenta

Soft radiator

- IRC singularities cancel with contribution of real "unresolved" emissions

$$V_{\rm sc}(\{\tilde p\},k) < \delta v$$

$$\boxed{V_{\rm sc}(\{\tilde p\},k) = d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)}$$

observable for a soft emission k collinear to leg ℓ

$$\mathcal{H}(Q,\epsilon) \to H(\alpha_s(Q)) e^{-R_s(\delta v)} \exp\left[-\sum_\ell \int^Q \frac{dk}{k} \underbrace{\gamma_\ell(\alpha_s(k,\epsilon))}_{\text{hard collinear}}\right]$$

- The soft "radiator" $R_s(v)$ can be split in a "massless" contribution $R_s^0(v)$ and a "mass correction" $\delta R_s(v)$

[AB El-Menoufi Monni 1807.11487]

$$R_s^0(v) = \int \frac{d^4k}{(2\pi)^4} w(k_t^2, k_t^2+m^2) \times$$
$$\times \sum_\ell \Theta(\eta^{(\ell)}) \times \overbrace{\Theta\left(\ln\frac{Q}{k_t} - \eta^{(\ell)}\right)}^{\text{massless rapidity boundary}} \times \overbrace{\Theta\left(d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi) - v\right)}^{\text{resolution constraint}}$$

- The mass correction $\delta R_s(v)$ accounts for the exact rapidity bound due to the fact that the web momentum is massive, while the resolution variable is defined in the massless limit

Physical coupling

- The massless radiator involves integrating the web inclusively over its mass

$$\boxed{\int_0^\infty dm^2\, w(k_t^2, k_t^2+m^2) \equiv (4\pi)^2 \frac{2C_\ell}{k_t^2} \alpha_s^{\text{phys}}(k_t)}$$

- This integral gives a QCD "physical" coupling, an extension of CMW coupling beyond NLL accuracy

[Catani Marchesini Webber NPB349 (1991) 635]

[AB El-Menoufi Monni 1807.11487]

- This physical coupling is proportional to the cusp anomalous dimensions for a conformal theory, but it departs from it away from the conformal limit

[Catani De Florian Grazzini 1904.10365]

Hard-collinear radiator

- Split hard-collinear virtual corrections at the collinear scale $k \sim v^{\frac{1}{a+b_\ell}} Q$

[AB El-Menoufi Monni 1807.11487]

$$\mathcal{H}(Q,\epsilon) \to H(\alpha_s(Q)) e^{-R_s(\delta v)} \times$$
$$\times \exp\Bigg[\underbrace{-\sum_\ell \int_{v^{\frac{1}{a+b_\ell}} Q}^{Q} \frac{dk}{k} \gamma_\ell(\alpha_s(k,\epsilon))}_{\equiv R_{\rm hc}(v)}\Bigg] \left(1 - \sum_\ell \int^{v^{\frac{1}{a+b_\ell}} Q} \frac{dk}{k} \frac{\alpha_s(k,\epsilon)}{\pi} \gamma_\ell^{(0)}\right)$$

- The singularity of hard-collinear virtual corrections with $k \lesssim v^{\frac{1}{a+b_\ell}} Q$ is cancelled by unresolved hard-collinear emissions

$$\mathcal{H}(Q,\epsilon) \to H(\alpha_s(Q)) e^{-R_s(\delta v)} e^{-R_{\rm hc}(v)} \left(1 - \sum_\ell \frac{\alpha_s(v^{\frac{1}{a+b_\ell}} Q)}{2\pi} C^{(1)}_{{\rm hc},\ell}\right)$$

- This strategy can be generalised to soft-large angle virtual corrections in processes with more than two legs

- The remaining real corrections are finite and can be then computed in four dimensions, either analytically or with a Monte-Carlo procedure

Conclusions

- Novel general method for the resummation of any rIRC safe (global) observable in the two-scale regime
  - weak applicability conditions
  - cancellation of poles between real and virtual corrections performed analytically in dimensional regularisation
  - contributions of resolved real emissions formulated in terms of four-dimensional integrals (suitable for Monte Carlo implementation)
- Spin-offs
  - Very precise determination of the strong coupling from the two-jet rate
  - Transverse momentum resummations in direct space (RadISH) [see Paolo Torrielli's talk]
- Work in progress
  - Extension of the method to three-jet events
  - Initial-state radiation: DIS and hadron collisions

Conclusions

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Thank you for your attention!

Extra

Soft-collinear emissions

- Integration measure for soft and collinear emissions widely separated in angle

Measure defined by the soft-collinear ensemble

$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]$$

$$\langle G(\{k_i\})\rangle = \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]\, G(\{k_i\})$$
$$= \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(\int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \sum_{\ell_i} R'_{\ell_i} \int_0^{2\pi} \frac{d\phi_i^{(\ell_i)}}{2\pi} \int_0^1 d\xi_i^{(\ell_i)} \right) G(k_1, \dots, k_n)$$

- Probability to emit a soft and collinear gluon k_i with $V(\{\tilde{p}\}, k_i) = v\,\zeta_i$ off hard leg ℓ_i with azimuthal angle $\phi_i^{(\ell_i)}$ and rapidity fraction $\xi_i^{(\ell_i)}$

$$dP(R'_{\ell_i}, k_i) \sim \frac{R'_{\ell_i}}{R'} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'} \Theta\left(\zeta_{i-1} - \zeta_i\right) \Theta\left(\zeta_i - \epsilon\right) \frac{d\phi_i^{(\ell_i)}}{2\pi} d\xi_i^{(\ell_i)}$$

- The function $G(k_1, \dots, k_n)$ can be used to integrate out the largest of the ζ_i