TMD splitting functions in k_T-factorization and prospects for using them in the evaluation of TMD distribution functions

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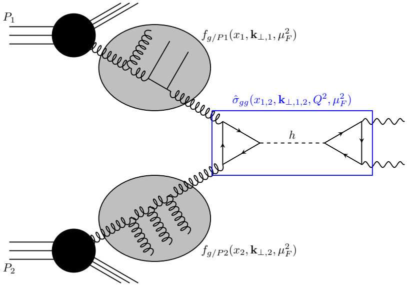
Overview

1. TMD Splitting functions
   - Introduction
   - Calculation
   - Limits
2. Parton Branching Implementation
   - Evolution equations
   - Results
3. Conclusions

$k_\perp$-factorization



Collinear factorization is commonly used
Some classes of processes require more general scheme

In high-energy (low x) limit $\rightarrow$ high-energy or $k_\perp$-factorization:

factorization in partonic cross-section and transverse momentum dependent PDFs (TMDs)

$$\sigma \propto \sum_{ij} \int dk_{\perp 1} \int dk_{\perp 2} \int_0^1 dx_1 dx_2 \hat{\sigma}_{ij}(x_{1,2}, k_{\perp 1,2}, Q^2, \mu_F^2) f_{i/P1}(x_1, k_{\perp 1}, \mu_F^2) f_{j/P2}(x_2, k_{\perp 2}, \mu_F^2)$$

BFKL equation for low-x evolution

Motivation

BFKL:

- Resums $\alpha_s \ln \frac{1}{x}$
- Only valid for small x
- Only gluon-gluon splittings
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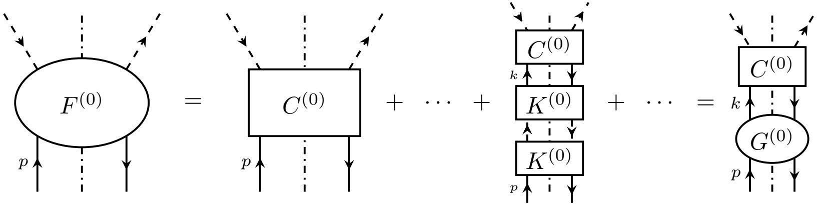
Goal of TMD Splitting Functions:

- Resummation in $\alpha_s \ln \frac{1}{x}$
- Full set of splitting functions
- Exact kinematics in both $k_\perp$ and x

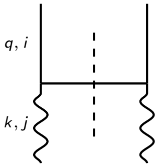
Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

Partonic cross-section can be expanded in 2PI-Kernels that do not contain collinear logarithms



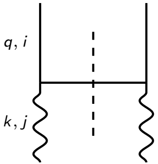
2PI Kernel:

$$\hat{K}_{ij} = z \int \frac{dq^2 d^{2+2\epsilon}\mathbf{q}_\perp}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 + q^2) \mathbb{P}_{j,\text{in}} \otimes \hat{K}_{ij}^{(0)} \otimes \mathbb{P}_{i,\text{out}}$$



$\hat{K}_{ij}^0$: Matrix element including outgoing propagators.
Projection operators $\mathbb{P}_{j,\text{in}}$, $\mathbb{P}_{i,\text{out}}$ that match the kinematics

TMD Splitting functions



- In high energy (small-x) limit, resummation of $\alpha_s \ln 1/x$ needed $\rightarrow$ 2GI
- From collinear to off-shell k: kinematics change ($k^\mu \rightarrow xp^\mu + \mathbf{k}_\perp$)

$\rightarrow$ appropriate choice of projection operators needed.

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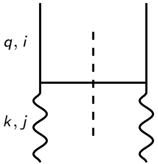

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Catani-Hautmann calculated $\tilde{P}_{qg}$ [NPB427 (1994) 475524, hep-ph/9405388]
After chosing appropriate projection operators and QCD-vertexes, all quark splittings and finally all splitting functions were calculated. [Gituliar, Hentschinski, Kutak JHEP 01 (2016) 181, 1511.08439], [Hentschinski, Kusina, Kutak, Serino EPJC 78 (2018) 174, 1711.04587]

$\tilde{P}_{ij}$: TMD splitting functions

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}_\perp^2}{\mu^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \frac{e^{-\epsilon\gamma_E}}{\mu^{2\epsilon}} \int \frac{d^{2+2\epsilon}\tilde{\mathbf{q}}_\perp}{\pi^{1+\epsilon}\tilde{\mathbf{q}}_\perp^2} \tilde{P}_{ij}(z, \mathbf{k}_\perp, \tilde{\mathbf{q}}_\perp) \Theta\left(\mu_F^2 - \frac{\tilde{\mathbf{q}}_\perp^2 + z(1-z)\mathbf{k}_\perp^2}{1-z}\right)$$

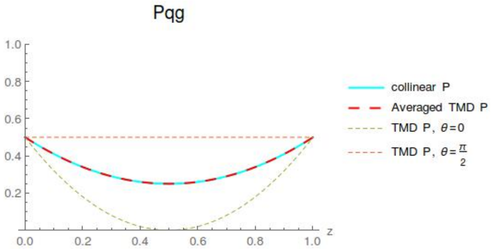
$$\tilde{\mathbf{q}}_\perp = \mathbf{q}_\perp - z\mathbf{k}_\perp$$

Collinear limit

$\bar{P}_{ij}$: Angular averaged TMD splitting functions

In limit $\mathbf{k}_\perp \to 0$

$\bar{P}_{qg} = T_R(z^2 + (1-z)^2)$
$\bar{P}_{gq} = C_F \frac{1+(1-z)^2}{z}$
$\bar{P}_{qq} = C_F \frac{1+z^2}{1-z}$
$\bar{P}_{gg} =$
$2C_A \left(\frac{1}{1-z} - 1 + \frac{1-z}{z} + z(1-z) \right)$

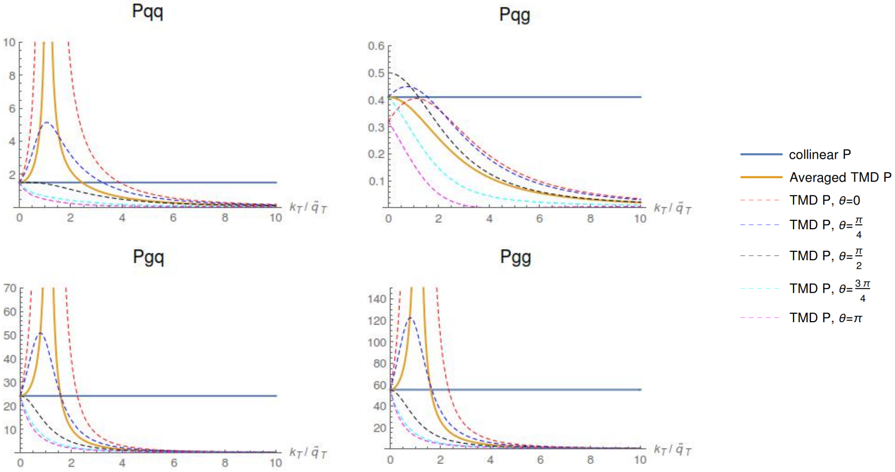


In DGLAP limit → DGLAP Splitting functions

$\tilde{P}_{qg}$ and $\tilde{P}_{gg}$ have angular dependence for small $\mathbf{k}_\perp$

Figure: $k_\perp/\tilde{q}_\perp = 10^{-5}$

$k_\perp$-dependence of TMD splitting functions



z=0.1, $\tilde{\mathbf{q}}_\perp = \mathbf{q}_\perp - z\mathbf{k}_\perp$, k/q incoming/outgoing momentum

Properties of TMD Splitting functions

- $\tilde{P}_{gq}$, $\tilde{P}_{qq}$ and $\tilde{P}_{gg}$ have singularity at $(1-z)\mathbf{k}_\perp = \tilde{\mathbf{q}}_\perp \to \mathbf{p}_\perp = 0$
  $\bar{P}_{gq}$, $\bar{P}_{qq}$ and $\bar{P}_{gg}$ have singularity at $(1-z)k_\perp = \tilde{q}_\perp$
- Anti-collinear limit (for fixed $\tilde{q}_\perp$): $\tilde{P}_{ij} \to 0$
  The high $k_\perp$-tail is important for small-x resummation
- Splitting functions are positive definite

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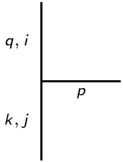
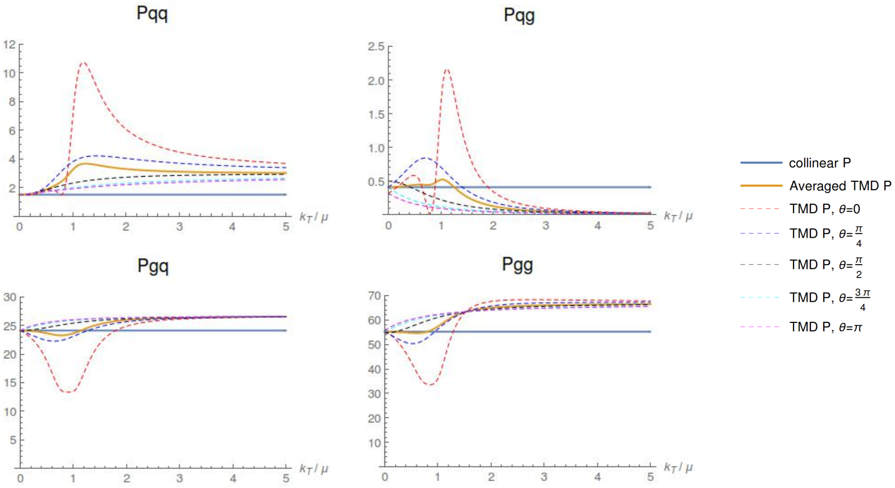
Can be useful to define splitting functions in different variables:

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}_\perp^2}{\mu^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \frac{e^{-\epsilon\gamma_E}}{\mu^{2\epsilon}} \int \frac{d^{2+2\epsilon}\mathbf{p}_\perp}{\pi^{1+\epsilon}\mathbf{p}_\perp^2} \tilde{P}_{ij}(z, \mathbf{k}_\perp, \mathbf{p}_\perp) \times \\ \times \Theta\left(\mu_F^2 - \frac{z\mathbf{p}_\perp^2}{1-z} - (\mathbf{k}_\perp - \mathbf{p}_\perp)^2\right)$$



$k_\perp$-dependence of $\tilde{P}_{ij}(z, \mathbf{k}_\perp, \mathbf{p}_\perp)$



$z = 0.1$, $\mu = \frac{p_\perp}{1-z}$ with $p_\perp$ transverse momentum of the emitted parton

Anti-collinear limit at fixed $p_\perp/(1-z)$

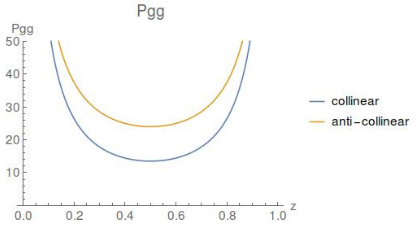
In limit $k_\perp \to \infty$ at fixed $p_\perp/(1-z)$:

$$\tilde{P}_{qg} = 0$$

$$\tilde{P}_{gq} = C_F \frac{2}{z} = P_{gq} + C_F(2-z)$$

$$\tilde{P}_{qq} = C_F \frac{2}{1-z} = P_{qq} + C_F(1+z)$$

$$\tilde{P}_{gg} = C_A \frac{2}{z(1-z)} = P_{gg} + 2C_A(2 - z(1-z))$$



$\tilde{P}_{qg}$ goes to zero, all other splitting functions rise.
There is no angular dependence.

Other limits

For $\tilde{P}_{gg}$:

[Hentschinski, Kusina, Kutak, Serino EPJC 78 (2018) 174, 1711.04587] examined also the BFKL and CCFM limit of the TMD splitting functions

The LO BFKL Kernels is recovered in the low x/high-energy limit

The CCFM gluon-gluon splitting functions is reobtained in the limit where the emitted gluon is soft ($\frac{p_\perp}{1-z} \to 0$)

Parton Branching Implementation

Parton Branching equations

Parton branching equations for TMDs:

[Hautmann, Jung, Lelek, Radescu, Zlebcik JHEP 01 (2018) 070, 1708.03279]

$$\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\times$$
$$\times \int_x^{z_M} dz P_{ab}^R(z)\tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, \mu'^2)$$

- Resolution scale z_M: resolvable $z < z_M$ and non-resolvable $z > z_M$ branchings

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Dynamical z_M, q_0 smallest emitted transverse momentum

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  Dynamical z_M, q_0 smallest emitted transverse momentum
- Sudakov form factor:
  $\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \; z \; P_{ba}^R(z)\right)$
  Interpretation: probability of an evolution without any resolvable branchings

Evolution equations with TMD Splitting functions

$P_{ab}(z) \rightarrow \tilde{P}_{ab}(z, \mathbf{k}_\perp, \mathbf{p}_\perp)$

- TMD splitting functions postitive definite
- q_0 : PB avoids the $\mathbf{p}_\perp \rightarrow 0$ singularity

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Sudakov form factor: probability of an evolution without any resolvable branchings

- Sudakov should depend on $\mathbf{k}_\perp$
- Should sum over all possible splittings $\rightarrow$ angular averaged splitting functions

$$\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ P_{ba}^R(z)\right) \rightarrow$$

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$$\times \int_x^{z_M} dz \tilde{P}_{ab}^R(z, \mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, a(z)\boldsymbol{\mu}'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, \mu'^2)$$

Iterative evolution equations

Iterative form of the evolution equation:

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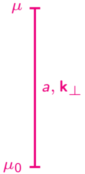

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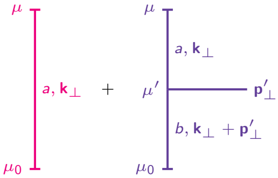

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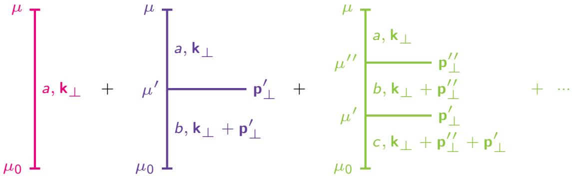

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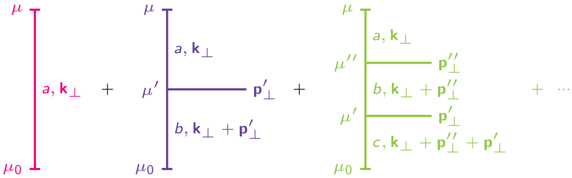

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Can be solved with MC methods, similar to the PB method

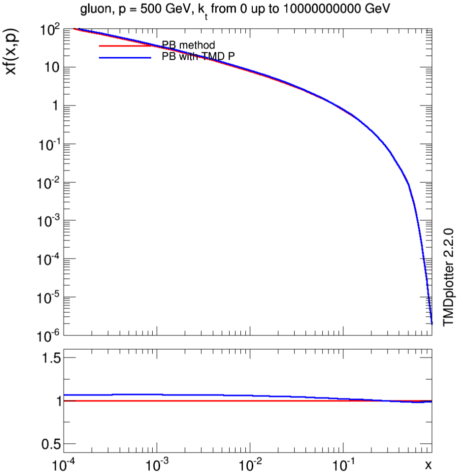
Current implementation: Collinear Sudakov form factor

Integrated TMDs

- Dynamical z_M, $q_0 = 1$ GeV
- Starting distribution: QCDNUM

Implementation with:

- PB method (LO)
- PB with TMD Splitting functions



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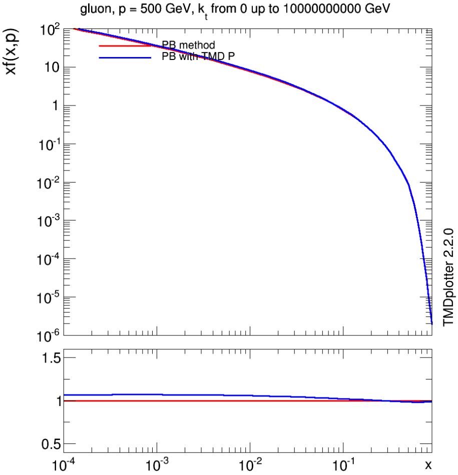
Integrated TMDs:

- Gluon: Increase for large range of x



Integrated TMDs

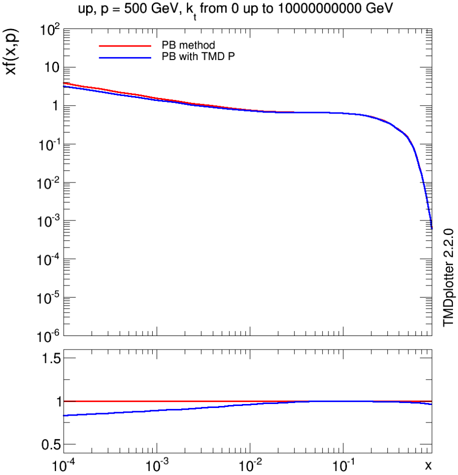
- Dynamical z_M, $q_0 = 1$ GeV
- Starting distribution: QCDNUM

Implementation with:

- PB method (LO)
- PB with TMD Splitting functions

Integrated TMDs:

- Gluon: Increase for large range of x
- Quarks: Decrease for low and intermediate x

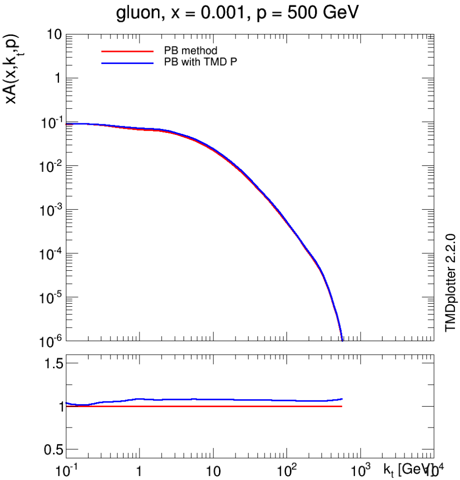


TMDs vs $\mathbf{k}_\perp$

- PB method (LO)
- PB with TMD Splitting functions

Small x:

- Gluon: increase in large region of $\mathbf{k}_\perp$

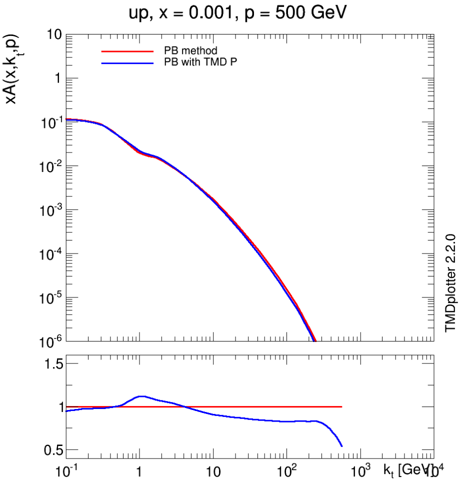


TMDs vs $\mathbf{k}_\perp$

- PB method (LO)
- PB with TMD Splitting functions

Small x:

- Gluon: increase in large region of $\mathbf{k}_\perp$
- Quarks: decrease for middle and high $\mathbf{k}_\perp$, drop for very large $\mathbf{k}_\perp$



TMDs vs $\mathbf{k}_\perp$

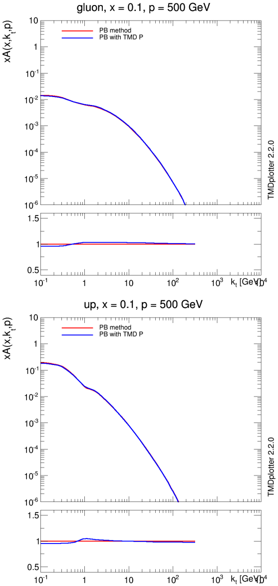
- PB method (LO)
- PB with TMD Splitting functions

Small x:

- Gluon: increase in large region of $\mathbf{k}_\perp$
- Quarks: decrease for middle and high $\mathbf{k}_\perp$, drop for very large $\mathbf{k}_\perp$

Large x:

- Only small effects



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  - Overview of the calculations
  - Correct limits

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- Implementation of TMD Splitting functions
  - Shown that an implementation based on the Parton Branching method is possible
  - First implementation with collinear Sudakov form factor

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  - First implementation with collinear Sudakov form factor

Thank you!

Backup

Momentum sum rule

Generalize DGLAP:

$$\frac{d\tilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz P_{ab}(z,\mu^2)\tilde{f}_b(\frac{x}{z},\mu^2) \to$$

$$\frac{d\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P_{ab}(z,\mathbf{k}_\perp+\mathbf{p}_\perp,\mathbf{p}_\perp,\mu^2)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+\mathbf{p}_\perp,\mu^2)$$

$\tilde{f}_a$:PDF, $\tilde{\mathcal{A}}_a$:TMD Note:

$$P_{ab}(z,\mathbf{k}_\perp+\mathbf{p}_\perp,\mathbf{p}_\perp,\mu^2) \neq \tilde{P}_{ab}(z,\mathbf{k}_\perp+\mathbf{p}_\perp,\mathbf{p}_\perp)$$

Momentum sum rule

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Assuming $\tilde{f}_a(x,\mu^2) = \int \frac{d^2\mathbf{k}_\perp}{\pi}\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)$ and $\int_0^1 dx\tilde{f}_a(x,\mu^2) = 1$:

Momentum sum rule: $\sum_a \int_0^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi}\ z\ P_{ab}(z,\mathbf{k}_\perp,\mathbf{p}_\perp,\mu^2) = 0$

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Momentum sum rule: $\sum_a \int_0^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi}\; z\; P_{ab}(z,\mathbf{k}_\perp,\mathbf{p}_\perp,\mu^2) = 0$

Equation with only real splitting functions:

$$\frac{d\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P^R_{ab}(z,\mathbf{k}_\perp+\mathbf{p}_\perp,\mathbf{p}_\perp,\mu^2)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+\mathbf{p}_\perp,\mu^2)$$

$$-\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)\sum_b \int_0^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P^R_{ab}(z,\mathbf{k}_\perp,\mathbf{p}_\perp,\mu^2)$$

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Evolution equations

Starting from:

$$\frac{d\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P^R_{ab}(z,\mathbf{k}_\perp+\mathbf{p}_\perp,\mathbf{p}_\perp,\mu^2)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+\mathbf{p}_\perp,\mu^2)$$
$$-\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)\sum_b \int_0^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P^R_{ab}(z,\mathbf{k}_\perp,\mathbf{p}_\perp,\mu^2)$$

- Introduce z_M: resolvable $z < z_M$ and non-resolvable $z > z_M$ branchings

Evolution equations

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$$- \tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)\sum_b \int_0^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P_{ab}^R(z,\mathbf{k}_\perp,\mathbf{p}_\perp,\mu^2)$$

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$$\frac{d\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2)}{d \ln \mu^2} = \sum_b \int_x^1 dz \int \frac{d^2\mathbf{p}_\perp}{\pi} P^R_{ab}(z, \mathbf{k}_\perp + \mathbf{p}_\perp, \mathbf{p}_\perp, \mu^2) \tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + \mathbf{p}_\perp, \mu^2)$$

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  $\rightarrow$ Angular ordering condition: $a(z) = 1 - z,\ z_M = 1 - q_0/\mu'$

Evolution equations

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- Introduce Sudakov form factor:
  $\Delta_a(\mu^2, \mathbf{k}_\perp) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\ z\ \bar{P}^R_{ba}(z, k_\perp/\left(a(z)\mu\right))\right)$

Evolution equations

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$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2,\mathbf{k}_\perp)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2}\frac{\Delta_a(\mu^2,\mathbf{k}_\perp)}{\Delta_a(\mu'^2,\mathbf{k}_\perp)}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\times$$

$$\times\int_x^{z_M} dz \tilde{P}^R_{ab}(z,\mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp, a(z)\boldsymbol{\mu}'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp + a(z)\boldsymbol{\mu}'_\perp,\mu'^2)$$