Azimuthal asymmetries in SIDIS and Drell-Yan processes: from high to low transverse momentum

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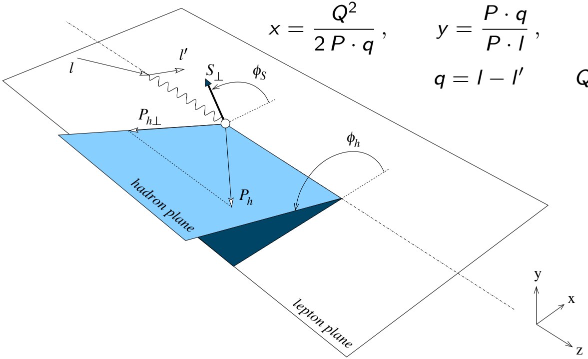
PLB 797 (2019) 134850

Unpolarized SIDIS

Differential cross section

$$\frac{d\sigma}{d\phi_h\, dP_{h\perp}^2} \propto \frac{\alpha^2}{Q^2} \left\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h} \right\}$$



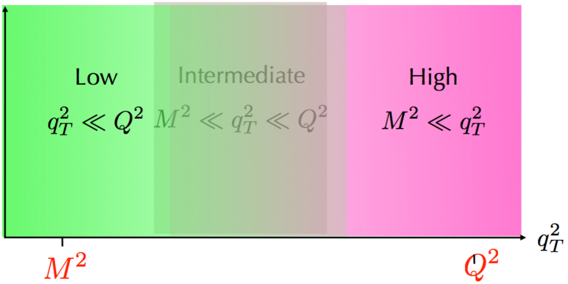
$$x = \frac{Q^2}{2\,P\cdot q}\,, \qquad y = \frac{P\cdot q}{P\cdot l}\,, \qquad z = \frac{P\cdot P_h}{P\cdot q}$$

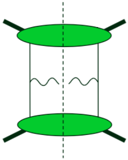
$$q = l - l' \qquad Q^2 = -q^2$$

Azimuthal asymmetries

$$A_{UU}^{\cos\phi_h} = \frac{\sqrt{2\,\varepsilon(1+\varepsilon)}\, F_{UU}^{\cos\phi_h} \cos\phi_h}{F_{UU,T} + \varepsilon F_{UU,L}}\,, \qquad A_{UU}^{\cos 2\phi_h} = \frac{\varepsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h}{F_{UU,T} + \varepsilon F_{UU,L}}$$

Three physical scales, two theoretical tools

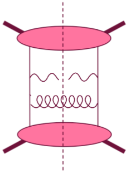




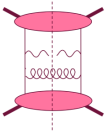
TMD

Do they describe the same dynamics or two competing mechanisms in the intermediate region?

(i.e., interpolation or sum?)

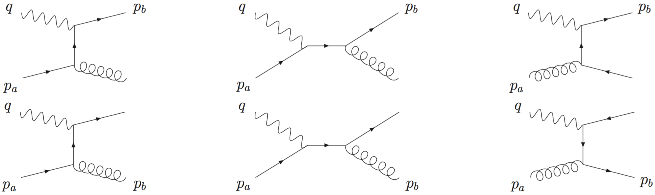


collinear PDF



collinear PDF

At leading twist and order α_S in collinear factorization

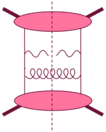


$F_{UU,T}$ given by the convolution of PDFs and FFs with hard scattering coefficients

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$

$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

Bacchetta, Boer, Diehl, Mulders, JHEP 0808 (2008) 023



collinear PDF

Expansion of delta function for small q_T/Q

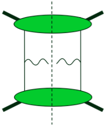
$$\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right) = \delta(1-\hat{x})\,\delta(1-\hat{z})\,\ln\frac{Q^2}{q_T^2} + \frac{\hat{x}}{(1-\hat{x})_+}\,\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x}) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\ln\frac{Q^2}{q_T^2}\right)$$

Meng, Olness, Soper, PRD 54 (1996) 1919

Extraction of leading behaviour for $M \ll q_T \ll Q$

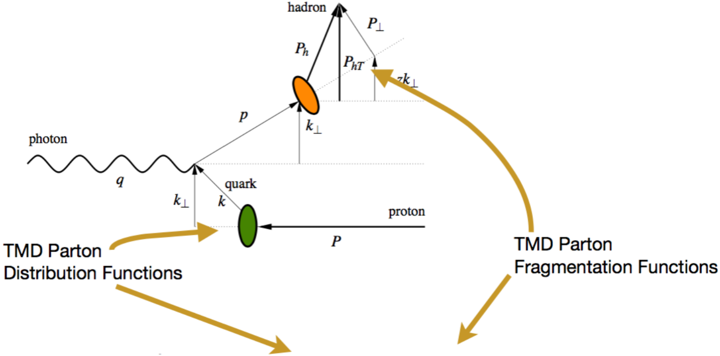
$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\big)(z) + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\,D_1^a(z)\Big]$$

$$L\left(\frac{Q^2}{q_T^2}\right) = 2C_F\ln\frac{Q^2}{q_T^2} - 3C_F$$

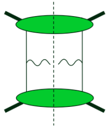


TMD

TMD factorization proven for $F_{UU,T}$. At leading twist and order α_S^0



$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a e_a^2 x \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;Q^2)\, D_1^{h/a}(z,\boldsymbol{P}_\perp^2;Q^2)\, \delta^2(z\boldsymbol{k}_T - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$



TMD

$$\widehat{f}_1^a(x, b_T^2; Q^2) \equiv \frac{1}{2\pi} \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b_T}\cdot\boldsymbol{k}_\perp} f_1^a(x, \boldsymbol{k}_\perp^2; Q^2)$$

$$\widehat{f}_1^a(x, b_T^2; Q^2) = \sum_{i=q,\bar{q},g} (C_{a/i} \otimes f_1^i)(x, b_T, \mu_b^2) \; e^{S(\mu_b^2, Q^2)} \left(\frac{Q^2}{Q_0^2}\right)^{g_K(b_T)} \widehat{f}_{1\mathrm{NP}}^a(x, b_T^2)$$

collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

Collins, Soper, Sterman, NPB 250 (85)
Collins, *Foundations of Perturbative QCD* (11)
Echevarria, Idilbi, Scimemi, JHEP 1207 (2012) 002

Factorization holds at all orders (with *subtracted* TMDs, scales set equal to Q)

$$F_{UU,T}(x, z, P_{hT}^2) = \mathcal{H}_{\mathrm{SIDIS}} \, 2\pi \sum_a e_a^2 x \int_0^\infty db_T \, b_T \, J_0\big(b_T |\boldsymbol{P}_{hT}|/z\big) \widehat{f}^a(x, b_T^2) \widehat{D}^a(z, b_T^2)$$

In the small-b_T region, $b_T \ll 1/M$, to $\mathcal{O}(\alpha_S)$ with $\mu_b = b_0/b_T$ and $b_0 = 2e^{-\gamma_E}$

$$\widehat{f}_1^a(x, b_T^2, Q^2) \approx \frac{1}{2\pi} \left\{ f_1^a(x, \mu_b^2) + \frac{\alpha_S}{\pi} \sum_i \left(C_{a/i}^{(1)} \otimes f^i \right)(x, \mu_b^2) \right\} e^{S(b_T^2, Q^2)}$$

$$S(b_T^2, Q^2) = -\frac{\alpha_S}{4\pi} C_F \left(\ln^2 \frac{Q^2 b_T^2}{b_0^2} - 3 \ln \frac{Q^2 b_T^2}{b_0^2} \right) + \mathcal{O}(\alpha_S^2)$$

The collinear PDF f_1^a is evolved from the scale μ_b to Q using DGLAP equation

$$f_1^a(x; \mu_b^2) = f_1^a(x; Q^2) - \frac{\alpha_s}{2\pi} \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g \right)(x) \ln \frac{Q^2 b_T^2}{b_0^2} + O(\alpha_s^2)$$

We recover the structure function at lowest order in the region $M \ll q_T \ll Q$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x\, e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) \left(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq} \right)(z) + \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g \right)(x)\, D_1^a(z) \Big]$$

Azimuthal asymmetries in SIDIS

New expression for the $\cos\phi$ structure function (TMD factorization assumed)

$$F_{UU}^{\cos\phi_h} = \frac{2MM_h}{Q}\,\mathcal{H}'_{\mathrm{SIDIS}}\,\mathcal{B}_1\left[\left(x\widehat{h}\,\widehat{H}_1^{\perp(1)} - \frac{M_h}{M}\,\widehat{f}_1\,\frac{\widehat{\tilde{D}}^{\perp(1)}}{z}\right) - \frac{M}{M_h}\left(x\widehat{f}^{\perp(1)}\widehat{D}_1 + \frac{M_h}{M}\,\widehat{h}_1^{\perp(1)}\,\frac{\widehat{\tilde{H}}}{z}\right)\right]$$

Cahn, PLB 78 (1978) 269
Anselmino *et al*, PRD 71 (2005) 074006
Bacchetta *et al.*, JHEP 0702 (2007) 093
Wei, Song, Chen, Liang, PRD 95 (2017) 074017

$$\mathcal{B}_n\left[\widehat{f}\widehat{D}\right] = 2\pi\sum_a e_a^2 x\int_0^\infty db_T\, b_T^{n+1} J_n\left(b_T|\boldsymbol{P}_{hT}|/z\right)\widehat{f}^a(x, b_T^2)\widehat{D}^a(z, b_T^2)$$

$$\widehat{f}^{(n)}(x, b_T^2) = n!\left(-\frac{2}{M^2}\frac{\partial}{\partial b_T^2}\right)^n \widehat{f}(x, b_T^2) \qquad \widehat{D}^{(n)}(z, b_T^2) = n!\left(-\frac{2}{M_h^2}\frac{\partial}{\partial b_T^2}\right)^n \widehat{D}(z, b_T^2)$$

QCD equations of motion lead to the relations

$$xf^\perp = x\tilde{f}^\perp + f_1$$

$$\frac{\tilde{D}^\perp}{z} = \frac{D^\perp}{z} - D_1$$

Wandzura–Wilczek approximation, the *pure twist-3* functions with a tilde are neglected: $x f^{\perp} \approx f_1, \quad x f_3^{\perp} \approx f_1$

The *whole* leading term of the low-q_T expansion of the collinear result

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Q q_T}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2\, f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + \ldots$$

$$L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln\frac{Q^2}{q_T^2} - 3C_F$$

matches the leading power behavior at small q_T/Q of the *collinear* LO results

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Q q_T}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a \otimes P'_{qq} + D_1^g \otimes P'_{gq}\big)(z) + \big(P'_{qq} \otimes f_1^a + P'_{qg} \otimes f_1^g\big)(x)\, D_1^a(z)\Big]$$

Bacchetta, Boer, Diehl, Mulders, JHEP 0808 (2008) 023

Azimuthal asymmetries in SIDIS

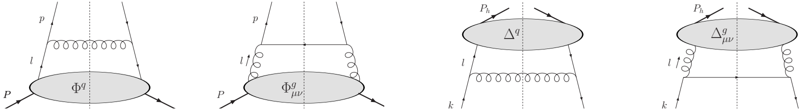
From low to intermediate q_T

We can obtain the *subtracted* functions $xf^{\perp}$ and $\tilde{D}^{\perp}$ at order α_S

$$xf^{\perp a}(x,k_\perp^2)\Big|_{k_\perp \neq 0} = \frac{\alpha_s}{4\pi^2 k_\perp^2}\left[\frac{1}{2}L\left(\frac{Q^2}{k_\perp^2}\right)f_1^a(x) + C_F f_1^a(x) + \sum_{i=a,g}(P'_{ai}\otimes f_1^i)(x,Q^2)\right]$$

$$\frac{1}{z}\,\tilde{D}^{\perp a}(z,P_\perp^2)\Big|_{P_\perp \neq 0} = -\frac{\alpha_s}{4\pi^2 P_\perp^2}\left[\frac{1}{2}L\left(\frac{z^2 Q^2}{P_\perp^2}\right)D_1^a(z) - C_F D_1^a(z) + \sum_{i=a,g}(D_1^i\otimes P'_{ia})(z)\right]$$

from the leading high-$k_\perp$ expressions of the *unsubtracted* $xf^{\perp}$ and $D^{\perp}$ (EOM)



Bacchetta, Boer, Diehl, Mulders, JHEP 0808 (2008) 023
Chen, Ma, PLB 768 (2017) 380

assuming the same prescription of the twist-2 unpolarized TMDs

By taking the Fourier transforms and the first derivatives w.r.t. b_T, we get

$$x\widehat{f}^{\perp(1)\,a}(x, b_T^2) = \frac{1}{M^2 b_T^2}\,\frac{\alpha_S}{4\pi^2}\left[C_F\left(\ln\frac{Q^2 b_T^2}{b_0^2} - \frac{3}{2}\right) f_1^a(x) + \left(P'_{ai}\otimes f_1^i\right)(x)\right]$$

$$\widehat{\tilde{D}}^{\perp(1)\,a}(z, b_T^2) = -\frac{1}{zM_h^2 b_T^2}\,\frac{\alpha_S}{4\pi^2}\left[C_F\left(2\ln\frac{Q^2 b_T^2}{b_0^2} - 3\right) D_1^a(z) + \left(D_1^i\otimes P'_{ai}\right)(z)\right]$$

By substituting in the LO TMD expression for the $\cos\phi$ structure function

$$F_{UU}^{\cos\phi_h} = \frac{2MM_h}{Q}\,\mathcal{B}_1\left[\left(x\widehat{h}\,\widehat{H}_1^{\perp(1)} - \frac{M_h}{M}\,\widehat{f}_1\,\frac{\widehat{\tilde{D}}^{\perp(1)}}{z}\right) - \frac{M}{M_h}\left(x\widehat{f}^{\perp(1)}\,\widehat{D}_1 + \frac{M_h}{M}\,\widehat{h}_1^{\perp(1)}\,\frac{\widehat{\tilde{H}}}{z}\right)\right]$$

we recover the leading power behavior at small qT /Q of the collinear LO results

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Qq_T}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,(D_1^a\otimes P'_{qq} + D_1^g\otimes P'_{gq})(z)\right.$$
$$\left. +(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g)(x)\,D_1^a(z)\right]$$

Bacchetta, Boer, Diehl, Mulders, JHEP 0808 (2008) 023

Azimuthal asymmetries in Drell-Yan

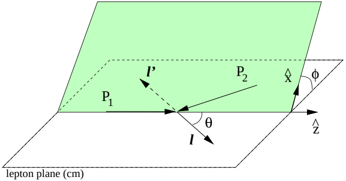
Azimuthal asymmetries in Drell-Yan

Gottfried-Jackson frame

$$F_{UU}^{\cos\phi}{}_{\mathrm{GJ}} = \frac{2M^2}{Q}\,\mathcal{H}_{\mathrm{DY}}\,\mathcal{B}_1^{\mathrm{DY}}\left[x_1 \widehat{f}^{\perp(1)\,a}(x_1, b_T^2)\widehat{f}_1^{\bar a}(x_2, b_T^2) - \widehat{f}_1^{a}(x_1, b_T^2)x_2\widehat{\tilde{f}}_1^{\perp\,(1)\,\bar a}(x_2, b_T^2)\right]$$

Lu, Schmidt, PRD 84 (2011) 114004

$$\mathcal{B}_n^{DY}\left[\widehat{f}^a\,\widehat{g}^{\bar a}\right] \equiv 2\pi\sum_a e_a^2\int_0^\infty db_T\, b_T^{n+1} J_n(b_T q_T)\,\widehat{f}^a(x_1, b_T^2)\,\widehat{g}^{\bar a}(x_2, b_T^2) + (1 \leftrightarrow 2)$$



In agreement with the known result in the region $M^2 \ll q_T^2 \ll Q^2$

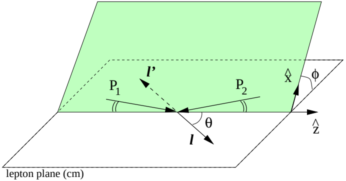
Boer, Vogelsang, PRD 74 (2006) 014004
Berger, Qiu, Rodriguez-Pedraza, PRD 76 (2007) 074006

$$F_{UU}^{\cos\phi}{}_{\mathrm{GJ}} = \frac{\alpha_s}{\pi^2 Q q_T}\sum_a e_a^2\left[L\left(\frac{Q^2}{q_T^2}\right) f_1^a(x_1)f_1^{\bar a}(x_2) + (P'_{qq}\otimes f_1^q)(x_1)f_1^{\bar a}(x_2)\right.$$

$$\left. + (P'_{qg}\otimes f_1^g)(x_1)f_1^{\bar a}(x_2) + f_1^a(x_1)(P^+_{qq}\otimes f_1^q)(x_2) + f_1^a(x_1)(P^+_{qg}\otimes f_1^g)(x_2)\right]$$

$$F_{UU}^{\cos\phi}{}_{\rm CS} = \frac{M^2}{Q}\,\mathcal{H}_{\rm DY}\,\mathcal{B}_1^{\rm DY}\left\{\left[x_1\widehat{f}^{\perp(1)\,a}(x_1,b_T^2) + x_1\widehat{\tilde{f}}^{\perp(1)\,a}(x_1,b_T^2)\right]\widehat{f}_1^{\bar a}(x_2,b_T^2)\right.$$
$$\left. -\widehat{f}_1^{\,a}(x_1,b_T^2)\left[x_2\widehat{f}_1^{\perp\,(1)\,\bar a}(x_2,b_T^2) + x_2\widehat{\tilde{f}}_1^{\perp\,(1)\,\bar a}(x_2,b_T^2)\right]\right\}$$

Lu, Schmidt, PRD 84 (2011) 114004



In agreement with the known result in the region $M^2 \ll q_T^2 \ll Q^2$

Boer, Vogelsang, PRD 74 (2006) 014004
Berger, Qiu, Rodriguez-Pedraza, PRD 76 (2007) 074006

$$F_{UU}^{\cos\phi}{}_{\rm CS} = -\frac{\alpha_s}{\pi Q q_T}\sum_a e_a^2\left[f_1^a(x_1)(\tilde{P}_{qq}\otimes f_1^{\bar q})(x_2) + f_1^a(x_1)(\tilde{P}_{qg}\otimes f_1^g)(x_2)\right.$$
$$\left. -(\tilde{P}_{qq}\otimes f_1^q)(x_1)f_1^{\bar a}(x_2) - (\tilde{P}_{qg}\otimes f_1^g)(x_1)f_1^{\bar a}(x_2)\right]$$

Logarithmic term absent also in Wandzura-Wilczek approximation

Summary

- Description of SIDIS and DY with measured q_T involves two frameworks: TMD factorization at low q_T and collinear factorization at high-q_T
- In the region $M \ll q_T \ll Q$ both are applicable. Depending on the observables the leading terms in the two approaches may coincide
- We propose a twist-3 TMD factorization formula, based on the parton model: $\cos\phi$ asymmetries in SIDIS and DY match the results at high q_T
- We solved the long-standing problem of the resummation of collinear results at high-q_T for the $\cos\phi$ asymmetry for these two processes
- Important step towards a full proof of TMD factorization at twist-three