Combining QED and QCD transverse-momentum resummation for vector boson production at hadron colliders

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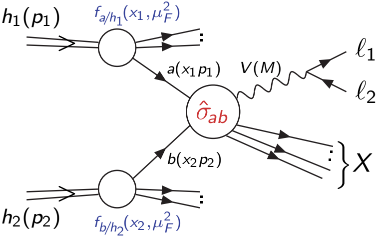
$\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\to\; F(M) + X}$$

where $\quad F = V, V_1 V_2, \gamma\gamma, H, HH$
and $\quad V \to l_1 l_2, V_1 V_2 \to 4l, H \to \gamma\gamma/4l, \ldots$

pQCD factorization formula ($q_T \sim M \gg \Lambda_{QCD}$):

$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{M}\right)^p.$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \overset{q_T \ll M}{\sim} c_0 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \cdots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} \overset{q_T \to 0}{=} 0$$

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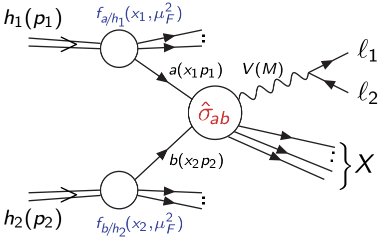

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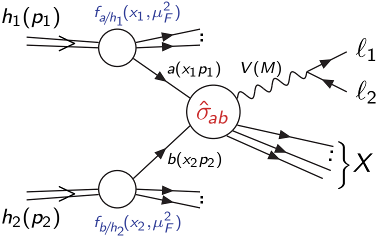

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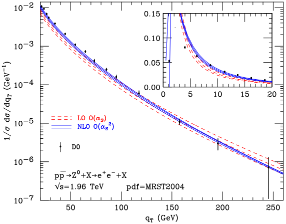

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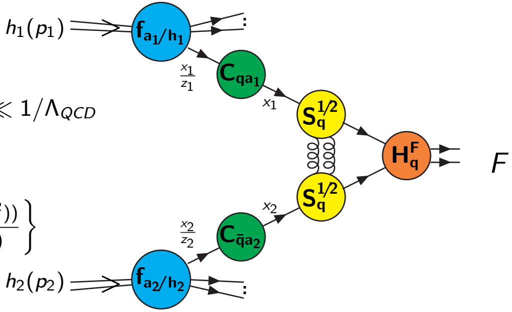
Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD}\ ,\quad b \gg 1/M\ ,\quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln\alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right] H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1,a_2}\int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

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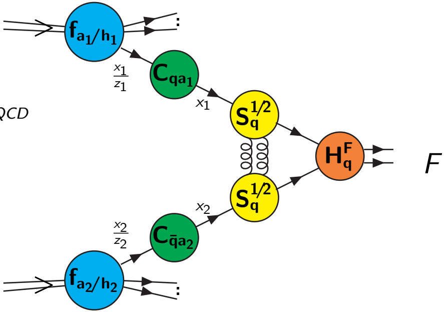

$$M \gg \Lambda_{QCD} \;, \quad b \gg 1/M \;, \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2)) \times \; \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \, \beta(\alpha_S(q^2)) \, \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} \, e^{i\mathbf{b}\cdot\mathbf{q_T}} \, S_q(M,b)$$

$$\times \int_{x_1}^{1} \frac{dz_1}{z_1} \, C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2} \, C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

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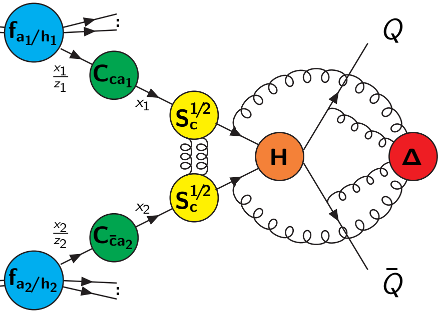
q_T resummation for heavy-quark hadroproduction

[Catani,Grazzini,Torre('14)]

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c}}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}$$

$$\times\; S_c(M,b)\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\,\left[(\mathbf{H\Delta})C_1C_2\right]_{c\bar{c};a_1a_2}$$

$$\times\; f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\,,$$



- Main difference with colourless case: soft factor (colour matrix) $\mathbf{\Delta}(\mathbf{b},M;\mathbf{\Omega})$ which embodies soft (wide-angle) emissions from $Q\bar{Q}$ and from initial/final-state interference (no collinear emission from heavy-quarks). Its contribution starts at NLL.
- Soft radiation produce colour-dependent azimuthal correlations at small-q_T entangled with the azimuthal dependence due to gluonic collinear radiation.
- Explicit results for coefficients obtained up NLO and NNLL accuracy.
- Soft-factor $\mathbf{\Delta}(\mathbf{b},M;\mathbf{\Omega})$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins,Qiu('07)].

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}_{c\bar{c}\to F}^{(n)}(\hat{p}_1, \hat{p}_2; \{q_i\}),$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon),$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}),$$

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Hard factor is directly related to the all-loop virtual amplitude:

$$H_q^F(x_1p_1, x_2p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}{|\mathcal{M}_{q\bar{q}\to F}^{(0)}(x_1p_1, x_2p_2; \{q_i\})|^2},$$

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$\mathbf{q_T}$ resummation in QCD at partonic level

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2}$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)} \mathcal{H}_N(\alpha_S) \times \exp\{\mathcal{G}_N(\alpha_S, L)\}$$

with $L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

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$$\frac{d\hat\sigma}{dq_T^2} = \frac{d\hat\sigma^{(res)}}{dq_T^2} + \frac{d\hat\sigma^{(fin)}}{dq_T^2}$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat\sigma^{(res)}}{dq_T^2} = \frac{M^2}{\hat s} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat s$) we have:

$$\mathcal{W}_N(b,M) = \hat\sigma^{(0)} \, \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

$$\text{with} \quad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

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- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
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- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \rightarrow \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

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q_T resummation: numerical implementations

- We have implemented the calculation in the publicly available codes:

  DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables
  [Bozzi,Catani,de Florian,G.F.,Grazzini('06,'09,'11,'12)]

  DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions)
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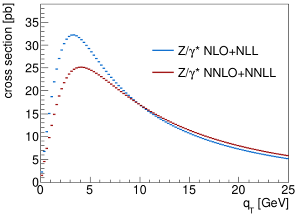
Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('19)]

Example calculation



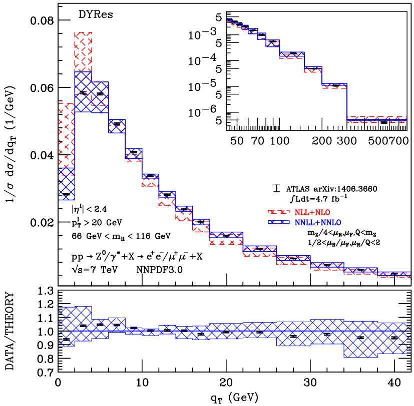
- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

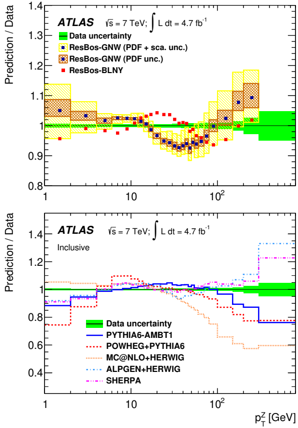
Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

q_T spectrum of Z boson: LHC data

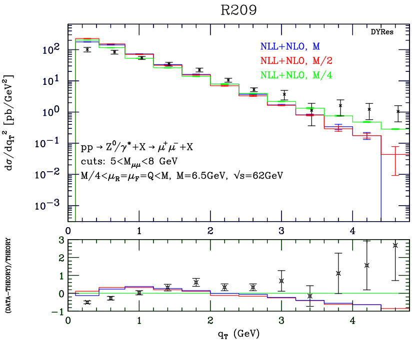


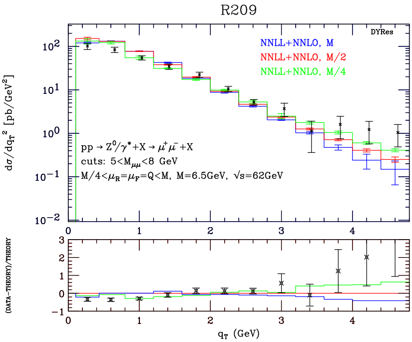


Left: NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with and ATLAS data (7 TeV).
Right Top: Ratios between ResBos predictions and ATLAS data.
Right Bottom: Ratios between various MC generators results and ATLAS data.

q_T spectrum of Z boson: ISR data

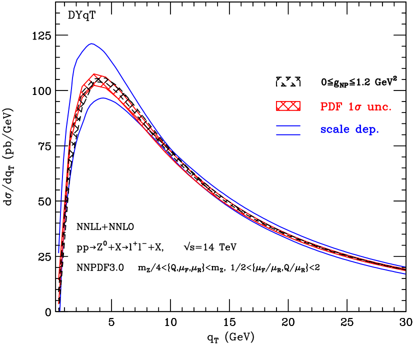
[F.Tognocchi (degree thesis '19)]





NLL+NLO (left) and NNLL+NNLO (right) bands for Z/γ^* q_T spectrum compared with and R209 data at CERN ISR ($\sqrt{s} = 62$ GeV, $M \simeq 6.5$ GeV).

PDF uncertainties and NP effects



NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\,GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T.
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3GeV$ (i.e. below the peak).

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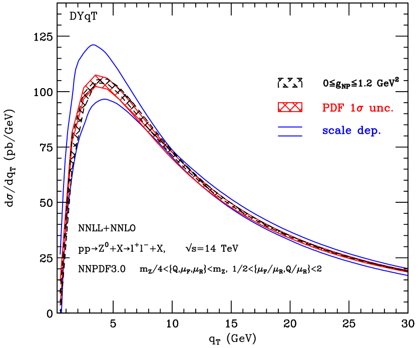

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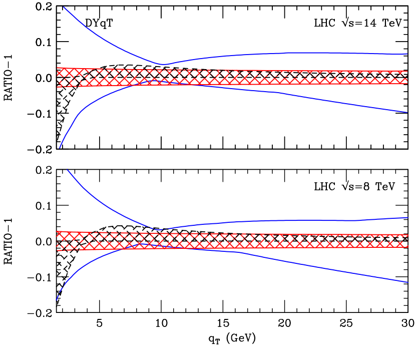

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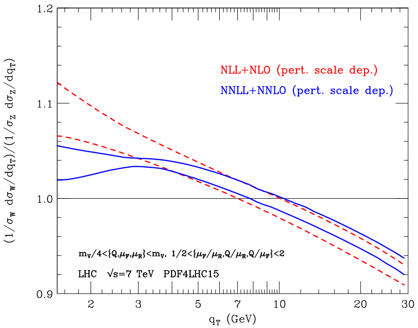
W/Z ratio: the q_T spectrum



Ratio of NNLL+NNLO and NLL+NLO results for W/Z q_T spectra at the LHC. Perturbative scale dependence.

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- *Correlated* ($\mu^W/M_W = \mu^Z/M_Z$) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for $q_T > 3\,GeV$).
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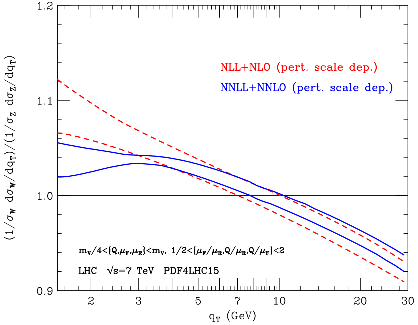

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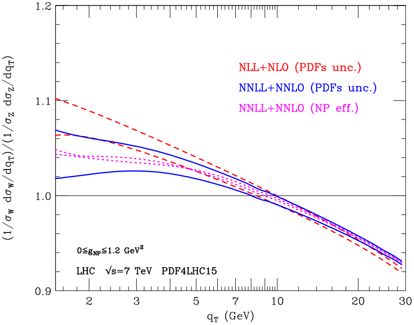

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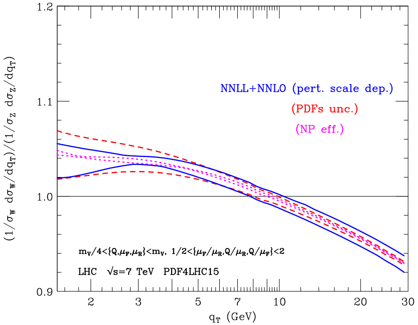


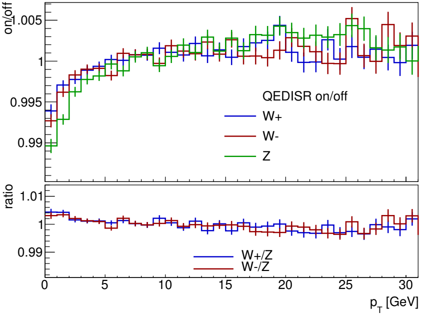
Ratio of NNLL+NNLO results for W/Z q_T spectra at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- Ratio of W/Z observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- *Correlated* ($\mu^W/M_W = \mu^Z/M_Z$) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for $q_T > 3\,GeV$).
- PDF uncertainty dominates at very small (q_T $q_T \lesssim 5\,GeV$).
- Non trivial interplay of perturbative and NP effects.

Combining QED and QCD q_T resummation

LHC measurements for DY process (e.g. M_W) sensitive to pure QED and mixed QCD-QED effects.

Pythia 8 QED ISR



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Combining QED and QCD q_T resummation

[Cieri,G.F.,Sborlini('18)]

We start considering QED contributions to the q_T spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell Z boson production

$$h_1 + h_2 \;\to\; Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\,\mathcal{H}'_N(\alpha_S,\alpha) \times \exp\left\{\mathcal{G}'_N(\alpha_S,\alpha,L)\right\}$$

$$\mathcal{G}'(\alpha_S,\alpha,L) = \mathcal{G}(\alpha_S,L) + L\,g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)$$

$$+\; g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

$$\mathcal{H}'(\alpha_S,\alpha) \;=\; \mathcal{H}(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}'^{(1)} + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^n \mathcal{H}_N'^{(n)} + \sum_{n,m=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n\left(\frac{\alpha}{\pi}\right)^m \mathcal{H}_N'^{F\,(n,m)}$$

LL QED ($\sim\alpha^n L^{n+1}$): $g'^{(1)}$; NLL QED ($\sim\alpha^n L^n$): $g'^{(2)}$, $\mathcal{H}'^{(1)}$;

LL mixed QCD-QED ($\sim\alpha_S^n\alpha^n L^{2n}$): $g'^{(1,1)}$;

The LL and NLL QED functions $g'^{(1)}$ and $g'^{(2)}$ has the same *functional* form of the QCD ones:

$$g'^{(1)}(\alpha L) = \frac{A_q'^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1-\lambda')}{\lambda'} \quad ,$$

$$g_N'^{(2)}(\alpha L) = \frac{\widetilde{B}_{q,N}'^{(1)}}{\beta_0'} \ln(1-\lambda') - \frac{A_q'^{(2)}}{\beta_0'^2} \left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda') \right)$$
$$+ \frac{A_q'^{(1)} \beta_1'}{\beta_0'^3} \left(\frac{1}{2} \ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'} \right) \quad ,$$

the *novel* LL mixed QCD-QED function reads:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)} \beta_{0,1}}{\beta_0^2 \beta_0'} h(\lambda, \lambda') + \frac{A_q'^{(1)} \beta_{0,1}'}{\beta_0'^2 \beta_0} h(\lambda', \lambda) \quad ,$$

$$h(\lambda, \lambda') = -\frac{\lambda'}{\lambda - \lambda'} \ln(1-\lambda) + \ln(1-\lambda') \left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'} \right) \right]$$
$$- \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'} \right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'} \right) ,$$

where $\lambda = \frac{1}{\pi} \beta_0 \, \alpha_S \, L$, $\lambda' = \frac{1}{\pi} \beta_0' \, \alpha \, L$, and β_0, β_0', β_1', $\beta_{0,1}$, $\beta_{0,1}'$ are the coefficients of the QCD and QED β functions.

Abelianization procedure

$$\frac{d\ln \alpha_S(\mu^2)}{d\ln \mu^2} = \beta(\alpha_S(\mu^2), \alpha(\mu^2)) = -\sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty} \beta_{n,m} \left(\frac{\alpha_S}{\pi}\right)^{n+1} \left(\frac{\alpha}{\pi}\right)^{m} ,$$

$$\frac{d\ln \alpha(\mu^2)}{d\ln \mu^2} = \beta'(\alpha(\mu^2), \alpha_S(\mu^2)) = -\sum_{n=0}^{\infty} \beta'_n \left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty} \beta'_{n,m} \left(\frac{\alpha}{\pi}\right)^{n+1} \left(\frac{\alpha_S}{\pi}\right)^{m} .$$

Novel QED coefficients obtained through an Abelianization algorithm

$$A_q'^{(1)} = e_q^2 , \qquad A_q'^{(2)} = -\frac{5}{9}\, e_q^2\, N^{(2)} \qquad \widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)} ,$$

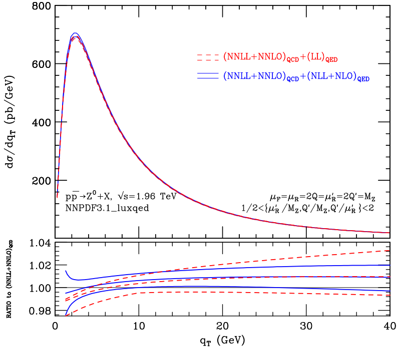
$$\text{with} \quad B_q'^{(1)} = -\frac{3}{2}\, e_q^2 , \quad N^{(n)} = N_c \sum_{q=1}^{n_f} e_q^n + \sum_{l=1}^{n_l} e_l^n ,$$

$$\gamma_{qq,N}'^{(1)} = e_q^2 \left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right) , \quad \gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\, e_q^2\, \frac{N^2+N+2}{N(N+1)(N+2)} .$$

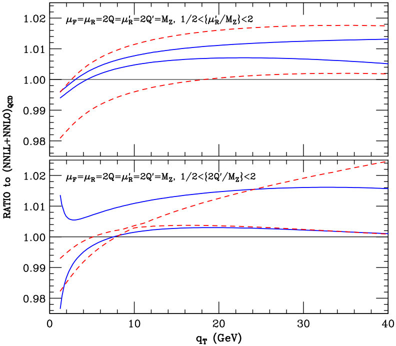
$$\mathcal{H}_{q\bar{q}\leftarrow q\bar{q},N}'^{F\,(1)} = \frac{e_q^2}{2} \left(\frac{2}{N(N+1)} - 8 + \pi^2\right) , \quad \mathcal{H}_{q\bar{q}\leftarrow \gamma q,N}'^{F\,(1)} = \frac{3\, e_q^2}{(N+1)(N+2)} ,$$

Resummed result *matched* with corresponding finite $\mathcal{O}(\alpha)$ term.

Combined QED and QCD q_T resummation for Z production at the Tevatron [Cieri,G.F.,Sborlini ('18)]

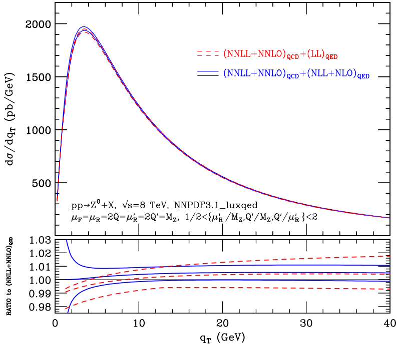


Z qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.



Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combined QED and QCD q_T resummation for Z production at the LHC [Cieri,G.F.,Sborlini ('18)]



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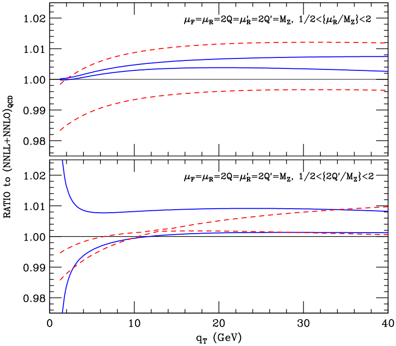


Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combining QED and QCD q_T resummation for W production

[Cieri,G.F.,Sborlini (in preparation)]

We next consider QED contributions to the q_T spectrum in the case of colourless and **charged** high mass systems, e.g. on-shell $W^\pm$ boson production

$$h_1 + h_2 \;\to\; W^\pm + X$$

- Initial state QED emissions sensitive to different quark charges ($q\bar{q}' \to W^\pm$):

$$2e_q^2 \to e_q^2 + e_{\bar{q}'}^2$$

- Final state QED emissions: *abelianizion* of QCD resummation formula q_T resummation for $t\bar{t}$ production [Catani,Grazzini,Torre('14)]:

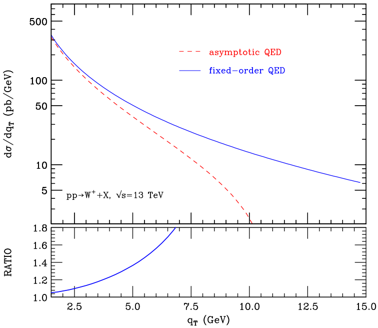
$$\Delta'(b,M) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

$$\text{with} \quad D'(\alpha) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n D'^{(n)}\,, \qquad \text{and} \qquad D'^{(1)} = -\frac{e^2}{2}\,.$$

- Factor $\Delta'(b,M)$ resums soft (non collinear) QED emissions from final state (and from initial-final interference). Effects from $D'(\alpha)$ start to contribute at NLL. Same functional dependence, in terms of $g'^{(i)}$ functions, as the $B'(\alpha)$ term.

Combined QED and QCD q_T resummation for W production

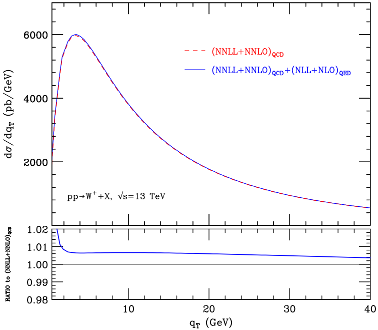
[S.Rota (degree thesis ’18)]



W qT spectrum at the LHC (13 TeV). $\mathcal{O}(\alpha)$ fixed-order QED results compared with the asymptotic expansion of the resummed result.

Combined QED and QCD q_T resummation for W production (preliminary)

[S.Rota (degree thesis '18)]



W qT spectrum at the LHC. NNLL QCD results combined with the NLL QED effects.

Conclusions

- Overview on q_T resummation formalism: hard-collinear factors and universality.
- NNLL+NNLO q_T-resummation for Drell-Yan production with full final-state kinematical dependence. Public numerical codes `DYqT` and `DYRes` available.
- New `DYTurbo` numerical code: significant enhancement in time performance and numerical precision.
- Extension of the QCD q_T resummation formalism to deal with the simultaneous QCD and QED emissions
- Phenomenological studies up to NLL+NLO for Z production at Tevatron and LHC: QED effects at $\mathcal{O}(+1\%)$ level. QED coupling scale ambiguity reduced by roughly a factor 2 including NLL+NLO corrections.
- Preliminary results for combined QCD and QED resummation from initial and final states and phenomenological study of $W^{\pm}$ production at the LHC.

Back up slides

Soft gluon exponentiation

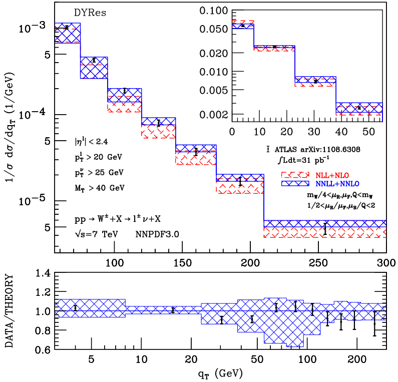
Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.
$$dw_n(q_1,\dots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$
- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

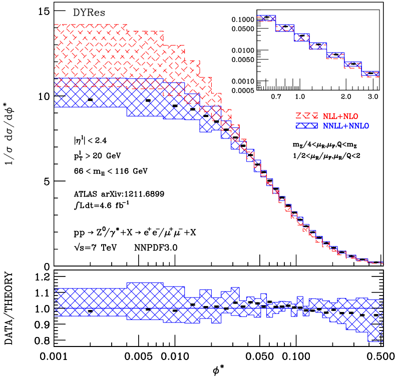
$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta^{(2)}\left(\mathbf{q_T}-\sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

DYRes results: q_T spectrum of W and ϕ^* spectrum of Z boson at the LHC

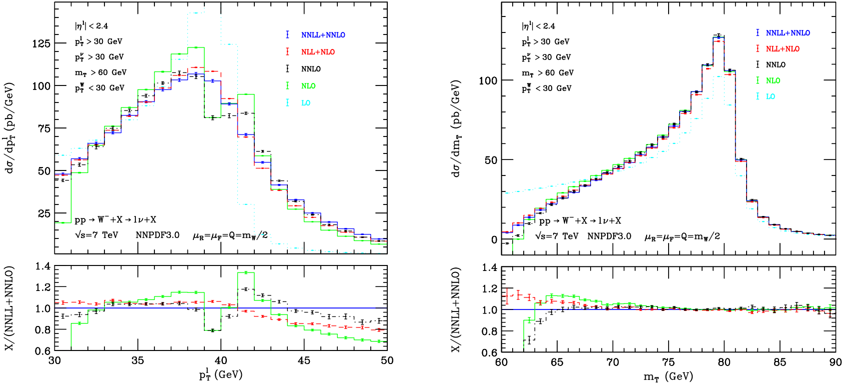


NLL+NLO and NNLL+NNLO bands for $W^{\pm}$ q_T spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.



NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

DYRes **results:** lepton kinematical distributions from W decay



Effect of q_T resummation on lepton p_T (left) and on the transverse mass (m_T) (right) distributions for W^- production at the LHC.
NLL+NLO and NNLL+NNLO results compared with LO, NLO and NNLO results.

Lower panel: ratio between various results and NNLL+NNLO result.

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('19)]

DYTurbo project

- Optimised version of DYNNLO, DYqT, DYRes with improvements in

Software	Numerical integration
Code profiling	Quadrature with interpolating functions
Loop vectorisation	Factorisation of integrals
Hoisting	Analytic integration
Loop unrolling	
Multi-threading	

- Achieved significant enhancement in time performance for a given numerical precision
- The main application is the measurement of the W mass at the LHC
- Other applications: PDF fits including qt-resummation for cross-section predictions, $\sin^2\theta_W$, $\alpha_s(m_Z)$
- Two main modes of operation: Vegas integration and Quadrature rules based on interpolating functions

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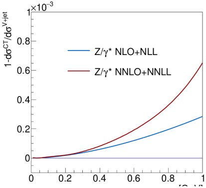
Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('19)]

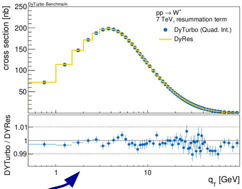
Closure tests and benchmark

- Matching conditions implies relation between the terms which can be used to test their numerical precision



$$\lim_{q_T \to 0} 1 - d\sigma^{\mathrm{CT(res)}}/d\sigma^{\mathrm{V+jet}} = 0$$

→ tested at 10^{-5}



- DYTurbo predictions fare benchmarked with DYRes at NNLL, and with other programs at NNLO

	SHERPA	DYNNLO	FEWZ	DYTurbo (Quad.)
$\sigma(pp \to W^+ \to l^+\nu)$ [pb]	3204 ± 4	3191 ± 7	3207 ± 2	3196 ± 7
$\sigma(pp \to W^- \to l^-\nu)$ [pb]	2252 ± 3	2243 ± 6	2238 ± 1	2248 ± 4
$\sigma(pp \to Z/\gamma \to l^+l^-)$ [pb]	502.0 ± 0.6	502.4 ± 0.4	504.6 ± 0.1	502.8 ± 1.0

Small differences between FEWZ and the other predictions are expected due to phase space with p_T^l symmetric cuts, and different subtraction scheme

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State of the art: q_T resummation

- Large q_T logarithms resummation in b-space [Parisi,Petronzio('79)],[Kodaira,Trentadue('82)],[Collins,Soper,Sterman('85)], [Altarelli et al.('84)],[Catani,d'Emilio,Trentadue('88)],[Catani,de Florian, Grazzini('01)],[Catani,Grazzini('10)],[Catani,Grazzini,Torre('14)]
- Various phenomenological studies [ResBos:Balasz,Yuan,Nadolsky et al.('97,'02)], [Ellis et al.('97)],[Kulesza et al.('02)],[Banfi et al.('12)], [Guzzi et al.('13)].
- Results for q_T resummation in the framework of Effective Theories and within p_T space formalisms: [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)],[Mantry,Petriello('10)], [Becher,Neubert('10)],[Chiu et al.('12)],[Dokshitzer,Diakonov,Troian('78)], [Frixione,Nason,Ridolfi('99)],[Erbert,Tackmann('17)],[Monni,Re,Torrielli('16)], [Bizon et al.('17,'18)].
- Studies within transverse-momentum dependent (TMD) factorization and TMD parton densities [D'Alesio,Murgia('04)],[Roger,Mulders('10)],[Collins('11)], [D'Alesio et al.('14)].
- Effective q_T-resummation obtained with Parton Shower algorithms POWHEG/MC@NLO combined with higher orders [Alioli et al.('13),[Hoeche et al.('14)],[Karlberg et al.('14)].

$\mathbf{q_T}$ resummation: $q\bar{q}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f\,\bar{q}_{f'} \to F$.

$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q}}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_q(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\;,$$

$b_0=2e^{-\gamma_E}$ $(\gamma_E=0.57\ldots)$, $x_{1,2}=\frac{M}{\sqrt{s}}\,e^{\pm y}$, $L\equiv\ln Mb$ [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]

$$S_q(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_q(\alpha_S(q^2))\,\ln\frac{M^2}{q^2}+B_q(\alpha_S(q^2))\right]\right\}\;.$$

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2} = H_q^F(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S(M^2))\;C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\;,$$

$$A_q(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)},\quad B_q(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S)=1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)},\qquad C_{qa}(z;\alpha_S)=\delta_{qa}\;\delta(1-z)+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\,.$$

LL$(\sim\alpha_S^nL^{n+1})$: $A_q^{(1)}$; NLL$(\sim\alpha_S^nL^n)$: $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL$(\sim\alpha_S^nL^{n-1})$: $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

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$$\times \sum_{a_1, a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $\quad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $\quad L \equiv \ln Mb$ [Collins, Soper, Sterman ('85)], [Catani, de Florian, Grazzini ('01)]

$$S_q(M, b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2))\, \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \;.$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q};a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{q a_1}(z_1; \alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\, a_2}(z_2; \alpha_S(b_0^2/b^2)) \;,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}, \quad B_q(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}, \qquad C_{qa}(z; \alpha_S) = \delta_{qa}\;\, \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z) \,.$$

LL($\sim \alpha_S^n L^{n+1}$): $A_q^{(1)}$; NLL($\sim \alpha_S^n L^n$): $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL($\sim \alpha_S^n L^{n-1}$): $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

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$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

[Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $\quad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $\quad L \equiv \ln Mb$

$$S_q(M, b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \;.$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q};a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{q a_1}(z_1; \alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\, a_2}(z_2; \alpha_S(b_0^2/b^2)) \;,$$

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LL$(\sim \alpha_S^n L^{n+1})$: $A_q^{(1)}$; NLL$(\sim \alpha_S^n L^n)$: $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL$(\sim \alpha_S^n L^{n-1})$: $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

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$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q}} \left[d\sigma^{(0)}_{c\bar{c},F}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\; S_q(M,b)$$

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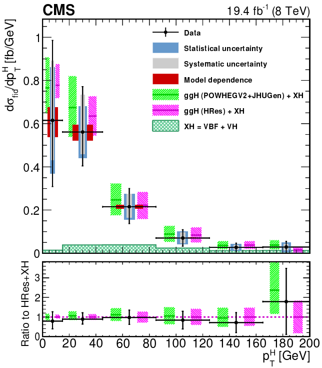
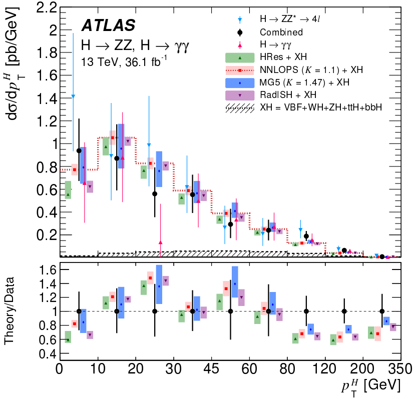
Higgs results: q_T-resummation with H boson decay

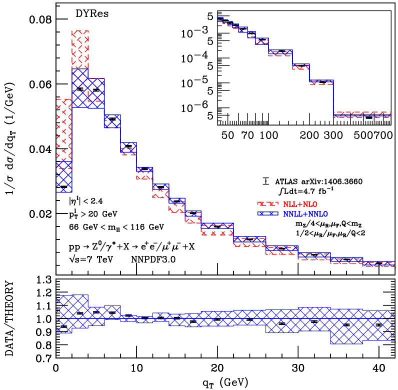


H q_T spectrum ($H \to WW$): theory predictions (HRes [de Florian,G.F.,Grazzini,Tommasini ('12]) compared with CMS data (from [CMS Coll.('16)]).
Lower panel: ratio to theory (HRes).

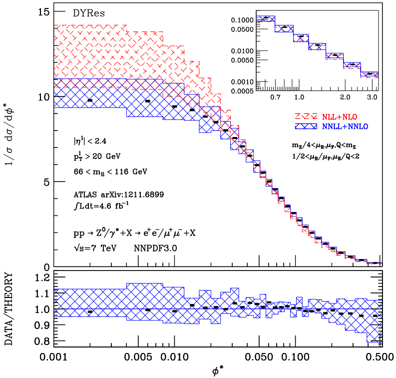


H q_T spectrum ($H \to \gamma\gamma$): various theory predictions (HRes [de Florian,G.F.,Grazzini, Tommasini('12])) compared with ATLAS data (from [ATLAS Coll.('18)]).
Lower panel: ratio to data).

DY results: q_T and ϕ^* spectrum of Z boson at the LHC



NLL+NLO and NNLL+NNLO bands from DYRes [Bozzi,Catani,G.F,de Florian,Grazzini('15)] for Z/γ^* q_T spectrum compared with ATLAS data. Lower panel: ratio with respect to the NNLL+NNLO central value.



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$\mathbf{q_T}$ recoil and angular distribution

- The dependence of the resummed cross section on the final state variables $\mathbf{\Omega}$ is

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) \;\; , \;\; \text{with} \;\; \int d\mathbf{\Omega}\, F(\mathbf{q_T}/M; \mathbf{\Omega}) = 1 \; .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the high-mass system due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) = F(\mathbf{0}/M; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T}/M) \;\; ,$$

  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T}/M)$ ambiguity start at $\mathcal{O}(\alpha_S^3)$ ($\mathcal{O}(\alpha_S^2)$) at NNLL+NNLO (NLL+NLO).
  - After integration over $\mathbf{\Omega}$ the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) high-mass system rest frame to generate final state momenta: e.g. the Collins–Soper rest frame.

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Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$).
- Explicit NNLO *analytic* calculations of the q_T cross section (at small-q_T): *(i)* SM Higgs boson production [Catani,Grazzini('07,'12)] and *(ii)* DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z)\,,\; C_{gq}^{(1)}(z) = \frac{C_F}{2}z\,,\; C_{qg}^{(1)}(z) = \frac{z}{2}(1-z)\,,$$

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$$H_q^{DY(1)} = C_F\left(\frac{\pi^2}{2}-4\right)\,,\; H_g^{H(1)} = C_A\pi^2/2 + \frac{11}{2}\,.$$

Analogous (longer) expressions for : $C_{qq}^{(2)}(z)\,, C_{qg}^{(2)}(z)\,, C_{gg}^{(2)}(z)\,, C_{gq}^{(2)}(z)\,, H_q^{DY(2)}\,, H_g^{H(2)}$.

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  (i) SM Higgs boson production [Catani,Grazzini('07,'12)] and
  (ii) DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z)\,,\ C_{gq}^{(1)}(z) = \frac{C_F}{2}z\,,\ C_{qg}^{(1)}(z) = \frac{z}{2}(1-z)\,,$$

$$C_{gg}^{(1)}(z) = C_{q\bar{q}}^{(1)}(z) = C_{qq'}^{(1)}(z) = C_{q\bar{q}'}^{(1)}(z) = 0\,,\ G_{ga}^{(1)}(z) = C_a\frac{1-z}{z} \quad (a=q,g)\,.$$

$$H_q^{DY(1)} = C_F\left(\frac{\pi^2}{2}-4\right)\,,\ H_g^{H(1)} = C_A\pi^2/2 + \frac{11}{2}\,.$$

Analogous (longer) expressions for : $C_{qq}^{(2)}(z)\,, C_{qg}^{(2)}(z)\,, C_{gg}^{(2)}(z)\,, C_{gq}^{(2)}(z)\,, H_q^{DY(2)}\,, H_g^{H(2)}\,.$

- Explicit independent computation of the hard-collinear coefficients in a TMD factorization approach in full agreement [Gehrmann,Lubbert,Yang('12,'14)].

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani,de Florian,Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$H_c^F(\alpha_S) \rightarrow H_c^F(\alpha_S) \, [\, h_c(\alpha_S)\,]^{-1} \,,$$

$$B_c(\alpha_S) \rightarrow B_c(\alpha_S) - \beta(\alpha_S) \, \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S} \,,$$

$$C_{cb}(z,\alpha_S) \rightarrow C_{cb}(z,\alpha_S) \, [\, h_c(\alpha_S)\,]^{1/2} \,.$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1 \,, \quad H_g^H(\alpha_S) \equiv 1 \,.$
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian,Grazzini('00)]).

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Hard factors at NNLO

- The previous all-order factorization formula was explicitly evaluated up to NNLO: we know the explicit expression of the *universal* subtraction operators up to two-loops $\tilde{I}_c^{(1)}(\epsilon)$, $\tilde{I}_c^{(2)}(\epsilon)$.
- We can straightforward apply the factorization formula to determine the NNLO hard-virtual factors from the knowledge of the two-loops amplitudes.
- E.g. diphoton production: we rederived the result for $H_q^{\gamma\gamma\,(1)}$ [Balazs et al.('98)] and (using the two-loop amplitudes [Anastasiou et al.('02)]) we obtained the $H_q^{\gamma\gamma\,(2)}$ [Catani,Cieri,de Florian,GF,Grazzini('12)]

$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2}\left\{(\pi^2-7) + \frac{\left((1-v)^2+1\right)\ln^2(1-v) + v(v+2)\ln(1-v) + (v^2+1)\ln^2 v + (1-v)(3-v)\ln v}{(1-v)^2+v^2}\right\} .$$

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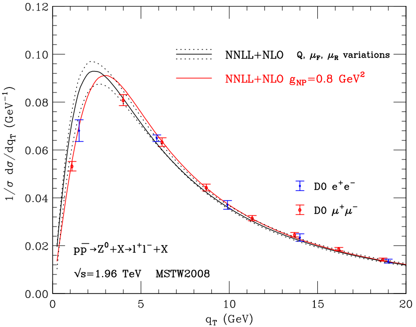
Non perturbative intrinsic k_T effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parameterized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$S_c(\alpha_S, \widetilde{L}) \rightarrow S_c(\alpha_S, \widetilde{L})\ S_{NP}$$

$$g_{NP} \simeq 0.8\ GeV^2$$ [Kulesza et al.('02)]

- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

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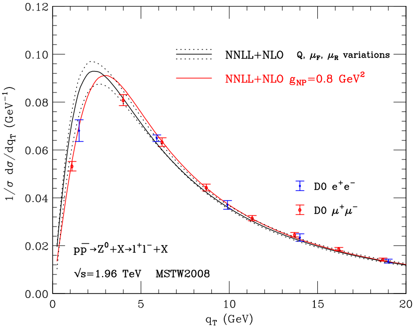

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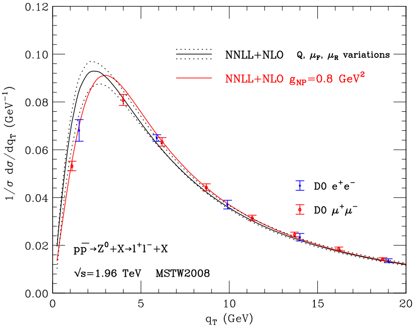

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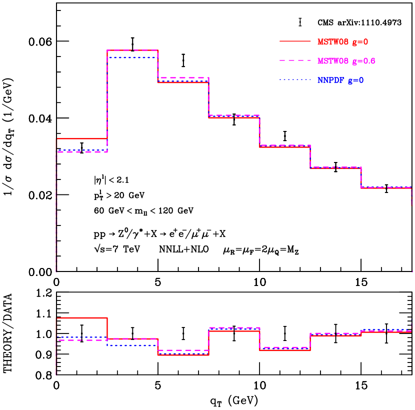

CMS data for the Z q_T spectrum.

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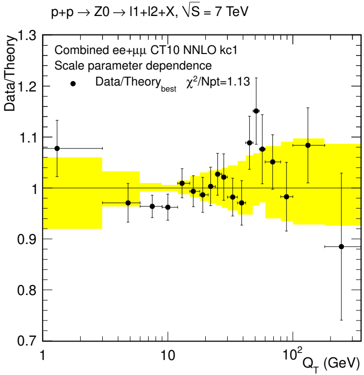
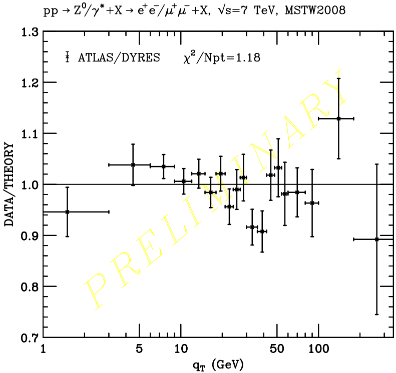


ATLAS ('11) data for the Z q_T spectrum compared with ResBos predictions with a Non Perturbative smearing parameter $g_{NP} = 1.1\ GeV^2$ [Guzzi,Nadolsky,Wang('13)].



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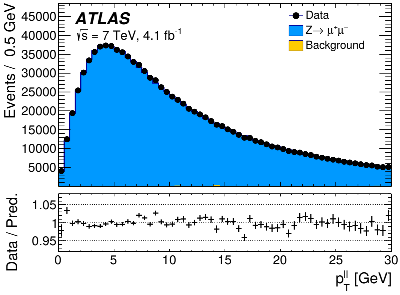
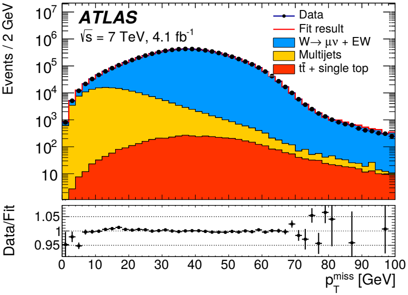
The Drell-Yan process: precise LHC measurements

LHC measurements for DY process reaches sub-percent precision.

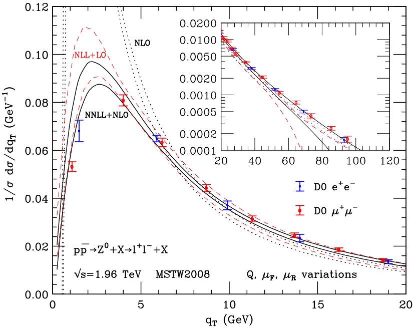


Z production at the LHC. Data and simulation comparison for lepton pair p_T distribution.



W production at the LHC. Data and simulation comparison for missing p_T distribution.

DYqT results: q_T spectrum of Z boson at the Tevatron



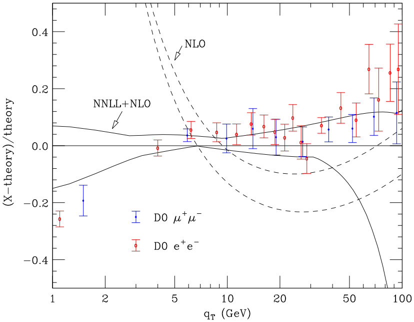
D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:

$$\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$$

- Significant reduction of scale dependence from NLL to NNLL for all q_T.
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
  The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.