BFKL resummation in heavy-quark pair hadroproduction

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Outline

Motivation

- Semi-hard collision process, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{QCD}, \qquad Q^2 \text{ a hard scale,}$$

$\nearrow$

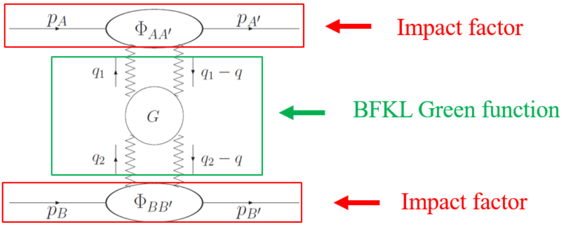
Regge kinematical region

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order resummation needed}$$

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach is the general framework for this resummation
  - Leading-logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
  - Next-to-leading-logarithm-Approximation (NLLA): $\alpha_s(\alpha_s \ln s)^n$
- However, experimental evidences of the BFKL dynamics are not conclusive, thus motivating proposal of new probes
- Here we propose the inclusive hadroproduction of two rapidity-separated heavy quarks as a new one

BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the Regge kinematical region
- Gluon Reggeization
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process indipendent) with the Impact factors of the colliding particles (process dipendent).



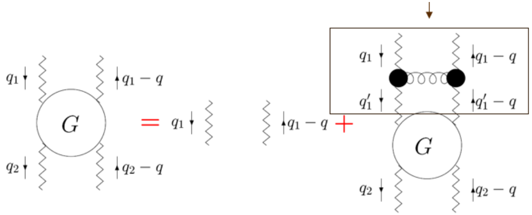
$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^{\,2}}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

BFKL resummation

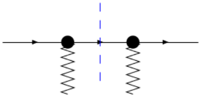
- $G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2)$$
$$+ \int \frac{d^{D-2} q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_{\omega}^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^{c}_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



BFKL resummation

- BFKL equation: $\vec{q}^{\,2} = 0$ and singlet color state representation
  [Ya. Ya. Balitsky, V. S. Fadin, E.A Kuraev, L.N Lipatov (1975)]

$$\text{Redefinition :} \qquad G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G^{(0)}_\omega(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \;, \quad \mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r \, \mathcal{K}(\vec{q}_1, \vec{q}_r) G(\vec{q}_r, \vec{q}_2)$$

- Elastic amplitude factorization:

$$\Im \mathcal{A}^{AB}_{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2} q_1 d^{D-2} q_2$$
$$\times \frac{\Phi^{(0)}_{AA}(\vec{q}_1, s_0)}{\vec{q}_1^{\,2}} \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi^{(0)}_{BB}(-\vec{q}_2, s_0)}{\vec{q}_2^{\,2}}$$

- Optical Teorem:

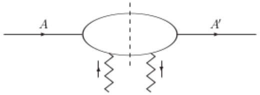
$$\sigma_{AB} = \frac{\Im \mathcal{A}^{AB}_{AB}}{s}$$

- Impact factor in the color singlet state:

$$\Phi^{(0)}_{PP} = \langle c c' | \hat{\mathcal{P}} | 0 \rangle \sum_{\{f\}} \int \frac{d s_{PR}}{2\pi} d\rho_f \Gamma^{c}_{\{f\}P} (\Gamma^{c'}_{\{f\}P})^*$$

BFKL resummation

Only few impact factors are known in the NLA so that a limited number of predictions can be built for total inclusive cross section and exclusive processes.



- $A = A' =$ quark, $B = B' =$ gluon [M. Ciafaloni and G. Rodrigo (2000)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
- A $= \gamma^*_L$, A'= V$_L$, with V$_L = \rho, \omega, \phi$ (forward)
  [D.Yu. Ivanov, M.I. Kotsky, A.Papa (2004)]
- $A = A' = \gamma^*$ (forward) [J. Bartels, S. Gieseke, C.F. Qiao (2001)]
  [J. Bartels, S. Gieseke, A. Kyrieleis (2002)]
  [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
  [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
  [J. Bartels, A. Kyrieleis (2004)]
  [I. Balitsky, G.A. Chirilli (2013)] [G.A. Chirilli, Yu.V. Kovchegov (2014)]

Partially inclusive processes

- A lot of more possibilities open for partially inclusive processes, with jets or identified particles in the final state, produced in the fragmentation regions or in the central one.
- Straightforward adaptation of the BFKL factorization: just restrict the summation over final states entering the definition of impact factors or Green function.
- Mueller-Navelet jet production
  1. NLO jet vertex

     [J. Bartels, D. Colferai, G.P. Vacca (2003)]

     [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]

     [D.Yu. Ivanov, A. Papa (2012)]

     [D. Colferai, A. Niccoli (2015)]
  2. Azimuthal correlations (full NLA)

     [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

     [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

     [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
  3. Compatible with CMS (7 TeV)

Partially inclusive processes

- Hadron-hadron production
  1. NLO hadron vertex [D.Yu. Ivanov, A. Papa (2012)]
  2. Azimuthal correlations (full NLA)

     [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2016,2017)]
- Hadron-jet production (full NLA)

  [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
- Three / four jet production (partial NLA)

  [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016,2017)]
- J/Ψ - jet production (partial NLA)

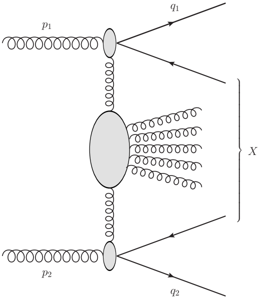
  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair - jet (partial NLA)

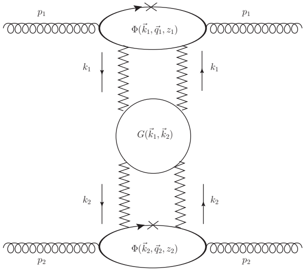
  [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- Higgs - jet (partial NLA)

  [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (in preparation)]

Heavy-quark pair hadroproduction: Partonic process

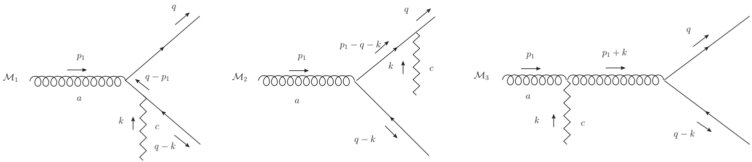
- Partonic process: $g(p_1) + g(p_2) \longrightarrow Q\text{−jet}(q_1) + Q\text{−jet}(\mathrm{q}_2) + X$
- Q = charm, bottom (detected in the fragmentation regions)
- Partial NLLA resummation
  1. BFKL Green function → NLLA
  2. Impact factor → LO





Heavy-quark pair hadroproduction: Impact factor

- Feynman diagrams



- LO impact factor

$$d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z) = \frac{\alpha_s^2\sqrt{N_c^2-1}}{2\pi N_c}\left[\left(m^2\left(R+\bar{R}\right)^2+\left(z^2+\bar{z}^2\right)\left(\vec{P}+\vec{\bar{P}}\right)^2\right)\right.$$

$$\left. -\frac{N_c^2}{N_c^2-1}\left(2m^2R\bar{R}+\left(z^2+\bar{z}^2\right)2\vec{P}\cdot\vec{\bar{P}}\right)\right]d^2\vec{q}\,dz\;,$$

$$R = \frac{1}{m^2+\vec{q}^{\,2}} - \frac{1}{m^2+(\vec{q}-z\vec{k})^2}\;, \qquad \bar{R} = \frac{1}{m^2+(\vec{q}-z\vec{k})^2} - \frac{1}{m^2+(\vec{q}-\vec{k})^2},$$

$$\vec{P} = \frac{\vec{q}}{m^2+\vec{q}^{\,2}} - \frac{\vec{q}-z\vec{k}}{m^2+(\vec{q}-z\vec{k})^2}, \qquad \vec{\bar{P}} = \frac{\vec{q}-z\vec{k}}{m^2+(\vec{q}-z\vec{k})^2} - \frac{\vec{q}-\vec{k}}{m^2+(\vec{q}-\vec{k})^2}.$$

Heavy-quark pair hadroproduction: Kinematics

- Sudakov decomposition:

$$q = zp_1 + \frac{m^2 + \vec{q}^{\,2}}{zW^2} p_2 + q_\perp, \qquad \bar{z} = 1 - z$$

$(p_1, p_2) \longrightarrow$ light cone basis

- Center of mass energy: $W^2 = (p_1 + p_2)^2 = 2p_1 p_2 = 4E_{g_1} E_{g_2}$
- Rapidities:

$$y_1 = \ln \left[\frac{2z_1 E_{g1}}{\sqrt{m^2 + \vec{q}_1^{\,2}}} \right] \qquad y_2 = -\ln \left[\frac{2z_2 E_{g2}}{\sqrt{m^2 + \vec{q}_2^{\,2}}} \right]$$

$$dz_1 dz_2 = \frac{e^{\Delta Y} \sqrt{m^2 + \vec{q}_1^{\,2}} \sqrt{m^2 + \vec{q}_2^{\,2}}}{W^2} dy_1 dy_2$$

$$\Delta Y \equiv y_1 - y_2 = \ln \frac{W^2 z_1 z_2}{\sqrt{(m^2 + \vec{q}_1^{\,2})(m^2 + \vec{q}_2^{\,2})}}$$

- Semi-hard kinematics:

$$\frac{W^2}{\sqrt{(m^2 + \vec{q}_1^{\,2})(m^2 + \vec{q}_2^{\,2})}} = \frac{e^{\Delta Y}}{z_1 z_2} \gg 1,$$

Heavy-quark pair hadroproduction: Observables

- Differential cross section:

$$\frac{d\sigma_{gg}}{dy_1 dy_2 d|\vec{q}_1| d|\vec{q}_2| d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\mathcal{C}_n\right], \qquad \varphi = \varphi_1 - \varphi_2 - \pi$$

- Azimuthal coefficients:

$$\mathcal{C}_n = \frac{|\vec{q}_1||\vec{q}_2|\sqrt{m^2+\vec{q}_1^{\,2}}\sqrt{m_2^2+\vec{q}_2^{\,2}}}{W^2} e^{\Delta Y}$$

$$\times \int_{-\infty}^{+\infty} d\nu \left(\frac{W^2}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu)+\frac{10}{3}+2\ln\frac{\mu_R^2}{\sqrt{s_1 s_2}}\right)\right)}$$

$$\times\, \alpha_s^4(\mu_R)\, c_1\left(n,\nu,\vec{q}_1^{\,2},z_1\right) c_2\left(n,\nu,\vec{q}_2^{\,2},z_2\right)\left\{1+\bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}}{c_1}+\frac{\bar{c}_2^{(1)}}{c_2}\right)\right.$$

$$\left. +\bar{\alpha}_s(\mu_R)\frac{\beta_0}{2N_c}\left(\frac{5}{3}+\ln\frac{\mu_R^2}{s_1 s_2}+f(\nu)\right)+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{W^2}{s_0}\right)\frac{\beta_0}{4N_c}\chi(n,\nu)f(\nu)\right\},$$

$$\bar{\alpha}_s = \frac{N_c\alpha_s}{\pi}, \qquad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

Heavy quark pair hadroproduction: Observables

- Projection of the LO impact factor onto the eigenfunctions of the leading-order BFKL kernel:

$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n,\nu,\vec{q},z)}{d^2\vec{q}\,dz} \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}}(\vec{k}^{\,2})^{i\nu-\frac{3}{2}} e^{in\theta}\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z)}{d^2\vec{q}\,dz} \equiv \alpha_s^2\, e^{in\varphi} c(n,\nu,\vec{q},z)$$

$$c_1\left(n,\nu,\vec{q}_1^{\,2},z_1\right) = \frac{1}{e^{in\varphi_1}\alpha_s^2}\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n,\nu,\vec{q}_1,z_1)}{d^2\vec{q}_1\ dz_1}$$

$$c_2\left(n,\nu,\vec{q}_2^{\,2},z_2\right) = \frac{1}{e^{-in(\varphi_2+\pi)}\alpha_s^2}\left[\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n,\nu,\vec{q}_2,z_2)}{d^2\vec{q}_2\ dz_2}\right]^*$$

- Universal part of the NLO impact factors:

$$\frac{\bar{c}_1^{(1)}}{c_1}+\frac{\bar{c}_2^{(1)}}{c_2} = \chi\left(n,\nu\right)\ln\frac{s_0}{\sqrt{\left(m_1^2+\vec{q}_1^{\,2}\right)\left(m_2^2+\vec{q}_2^{\,2}\right)}}\ .$$

  s_0 (arbitrary with NLLA) chosen as $\sqrt{s_1 s_2}$

- $f(\nu)$

$$i\frac{d}{d\nu}\ln\frac{c_1}{c_2} = 2\left[f(\nu)-\ln(\sqrt{s_1 s_2})\right]\ ,\qquad s_i = m_i^2+\vec{q}_i^{\,2}$$

Heavy quark pair hadroproduction: Observables

- From the gluon-initiated process to the one initiated by proton-proton:

$$d\sigma_{pp} = f_{g_1}(x_1, \mu_{F_1}) f_{g_2}(x_2, \mu_{F_2}) d\sigma_{gg} dx_1 dx_2 \ ,$$

$f_{g_i}, \ i = 1,2 \longrightarrow$ gluon collinear parton distribution functions
$x_i, \ i = 1,2 \longrightarrow$ gluon fractions of momenta
$\mu_{F_i}, \ i = 1,2 \longrightarrow$ factorization scales

- Hadronic cross section:

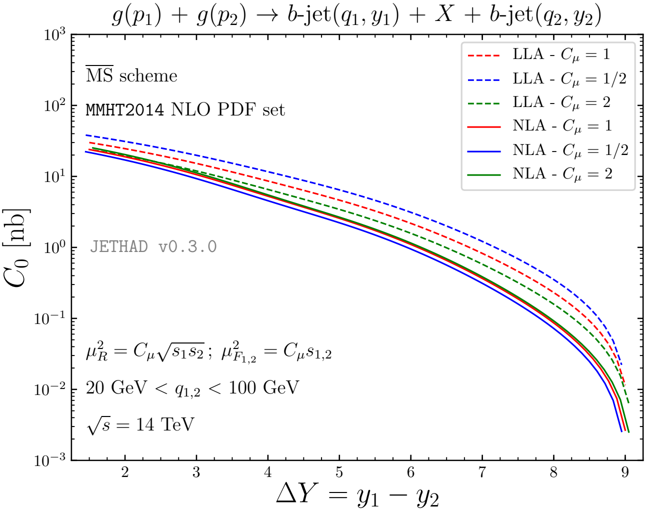
$$\frac{d\sigma_{pp}}{d(\Delta Y) d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2} \left[C_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) C_n \right] \ ,$$

where

$$C_n = \int_{q_{1,\min}}^{q_{1,\max}} d|\vec{q}_1| \int_{q_{2,\min}}^{q_{2,\max}} d|\vec{q}_2| \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \ \delta(y_1 - y_2 - \Delta Y)$$
$$\int_{e^{-(y_{1,\max} - y_1)}}^{1} dx_1 f_{g_1}(x_1, \mu_{F_1}) \int_{e^{-(y_{2,\max} + y_2)}}^{1} dx_2 f_{g_2}(x_2, \mu_{F_2}) \ \mathcal{C}_n$$

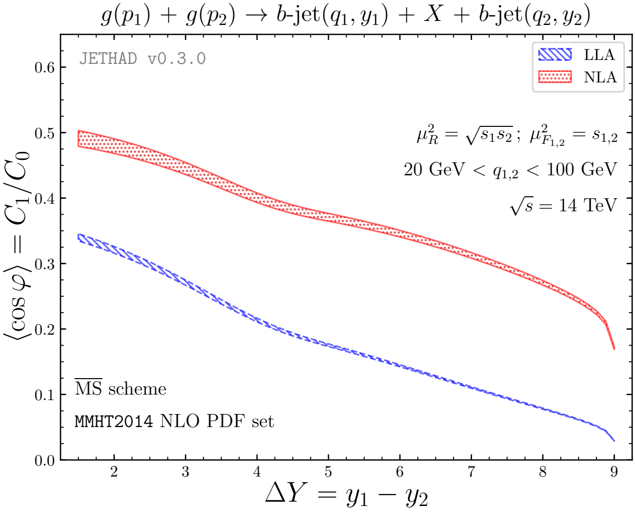
Heavy quark pair hadroproduction: Observables

- C_0
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\rm min} = 20$ GeV, $q_{\rm max} = 100$ GeV



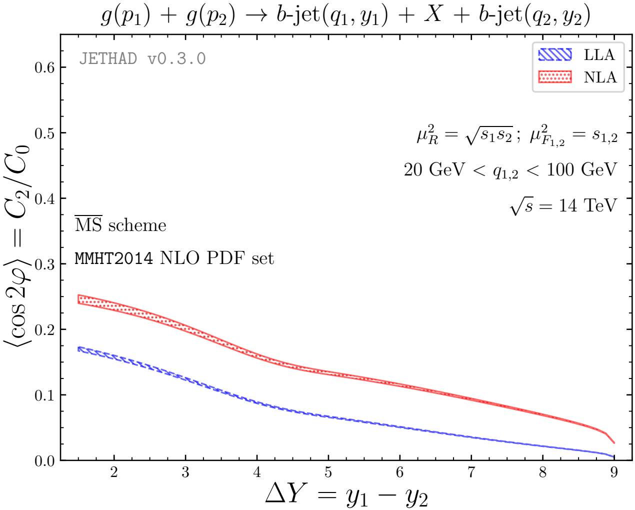
Heavy quark pair hadroproduction: Observables

- C_1/C_0
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



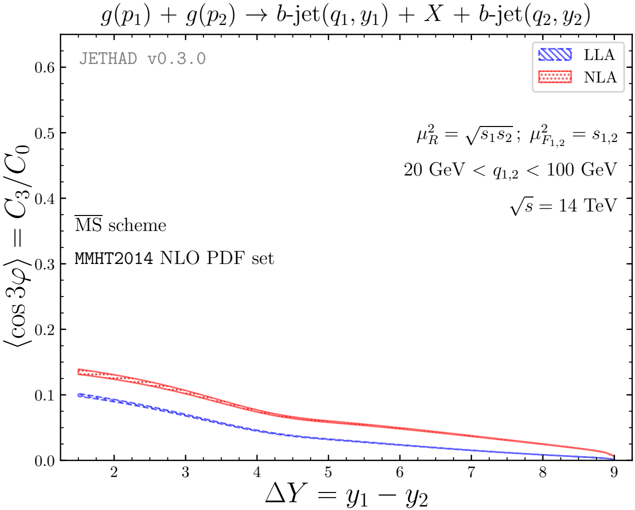
Heavy quark pair hadroproduction: Observables

- C_2/C_0
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



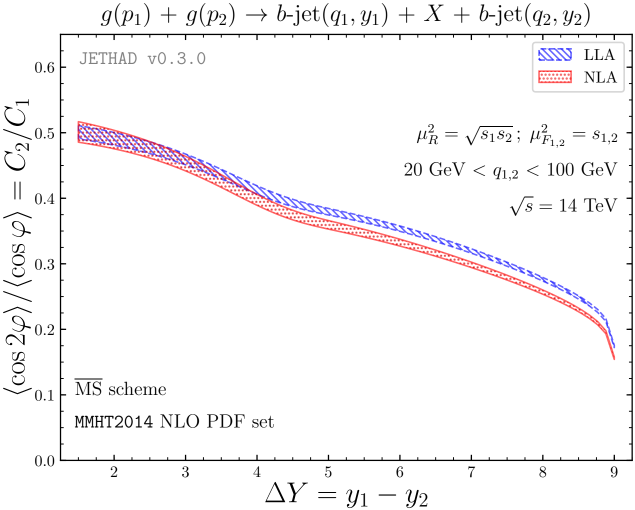
Heavy quark pair hadroproduction: Observables

- C_3/C_0
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



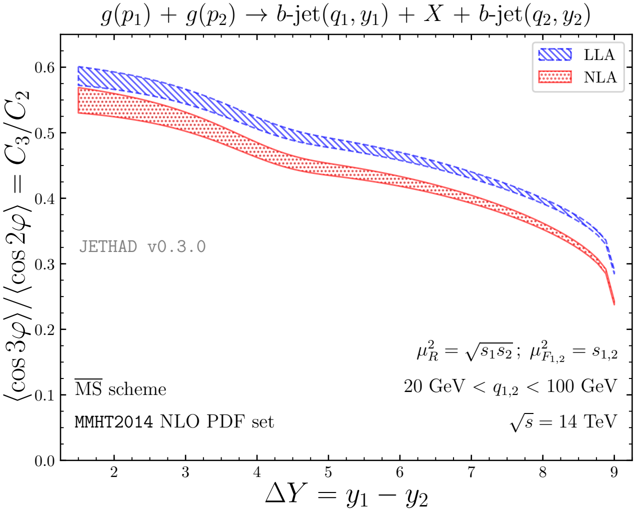
Heavy quark pair hadroproduction: Observables

- C_2/C_1
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



Heavy quark pair hadroproduction: Observables

- C_3/C_2
- b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



Conclusions and outlook

Conclusion

- Inclusive processes with jets and/or identified hadrons in the final state featuring large rapidity separation are a promising testfield for the search of BFKL dynamics in current and future colliders.
- Among them, the photo- or hadro-production of a pair of heavy quarks is an interesting new possibility.
- Theoretical predictions, including a relevant part of the energy resummation in the NLLA, are available for both photo- and hadro-production cases.

Future projects

- Inclusion of subleading corrections from the impact factors, needed to produce full-NLLA predictions.
- Production of mesons containing charm or bottom including into the theoretical analysis the quarks fragmentation.
- Single forward heavy-flavoured jet production, both in the LLA and in the NLLA, via the introduction of the small-x transverse-momentum-dependent gluon distribution (UGD).

Thank you for the attention

Backup

Backup: Before QCD

- Assumptions on S-matrix: Lorentz invariance, unitarity, analiticity

Cutkosky rules

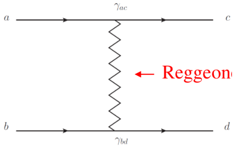
$$2\Im\mathcal{A}_{ab} = (2\pi)^4\delta^4(\sum_a p_a - \sum_b p_b)\sum_c \mathcal{A}_{ac}\mathcal{A}^\dagger_{cb}$$

Optical theorem

$$2\Im\mathcal{A}_{aa}(s,0) = F\sigma_{tot}$$

- Regge theory

$$s \gg |t| \Rightarrow \mathcal{A}(s,t) \xrightarrow[s\to\infty]{} \frac{\eta + e^{-i\pi\alpha(t)}}{2}\beta(t)s^{\alpha(t)}$$



- Definition of Reggeization

A particle of mass M and spin J is said to "Reggeize" if the amplitude, $\mathcal{A}$, for a process involving the exchange in the exchange in the t-channel of the quantum numbers of that particle behaves asymptotically in s as

$$\mathcal{A} \propto s^{\alpha(t)}$$

where $\alpha(t)$ is the trajectory and $\alpha(M^2) = J$, so that the particle itself lies on the trajectory.

Backup: Before QCD

- Postulates about the S-matrix
  1. The S-matrix is Lorentz invariant.
  2. The S-matrix is untitary:

Cutkosky rules

$$SS^\dagger = S^\dagger S = \mathbb{I} \qquad \Rightarrow \qquad 2\Im\mathcal{A}_{ab} = (2\pi)^4\delta^4(\sum_a p_a - \sum_b p_b)\sum_c \mathcal{A}_{ac}\mathcal{A}^\dagger_{cb}$$

$\swarrow$

Optical theorem

$$2\Im\mathcal{A}_{aa}(s,0) = F\sigma_{tot}$$

  3. The S-matrix is an analytic function of Lorentz invariants (regarded as complex variables), with only those singularities required by unitarity.

$\downarrow$

Dispersion relations that link the total amplitude to its imaginary part.

Backup: Regge theory

- Partial wave expansion

$$\mathcal{A}(s,t) = \sum_{l=0}^{\infty}(2l+1)a_l(t)P_l(1+\frac{2s}{t})$$

$\downarrow$ regarding l as a complex variable

$$\mathcal{A}(s,t) = \frac{1}{2i}\oint_C dl \frac{(2l+1)}{\sin \pi l} a^{(\eta)}(l,t)P_l(l,1+\frac{2s}{t})$$

Regge region: $s \gg |t|$ $\downarrow$ only simple poles in the l complex plane

$$\mathcal{A}(s,t) \xrightarrow[s\to\infty]{} \frac{\eta + e^{-i\pi\alpha(t)}}{2}\beta(t)s^{\alpha(t)}$$

$\alpha(t)$ is the position of the leading Regge pole.

- The amplitude can be seen as the exchange in a t-channel of an object with "angular momentum" $\alpha(t)$.

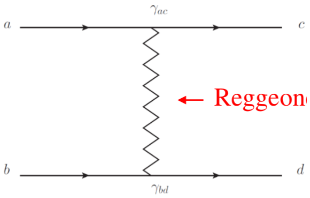
$\downarrow$

Reggeon

Backup: Regge theory

- The amplitude can be factorized as

1. Two Particle-Particle-Reggeon couplings ($\gamma_{ac}(t)$, $\gamma_{bd}(t)$)
2. A universal contribution from the Reggeon exchanged (it determines the s-asymptotic behaviour of the amplitude)



- A particle of mass M and spin J is said to "Reggeize" if the amplitude, $\mathcal{A}$, for a process involving the exchange in the exchange in the t-channel of the quantum numbers of that particle behaves asymptotically in s as

$$\mathcal{A} \propto s^{\alpha(t)}$$

where $\alpha(t)$ is the trajectory and $\alpha(M^2) = J$, so that the particle itself lies on the trajectory.

Backup: Pomeron

- Optical theorem $\longrightarrow \sigma_{tot} \propto s^{\alpha(0)-1}$
- In any scattering process in which there is charge exchange the cross-section vanishes asymptotically (Pomeranchuk theorem).
- Foldy e Peierls noticed the converse. If for a particular scattering process the cross-section does not fall as s increase then that process must be dominate by the exchange of vacuum quantum numbers.
- It is observed experimentally that total cross-sections do not vanish asymptotically (they rise slowly as s increase)

$$\downarrow$$

If we are to attribute this rise to the exchange of a single Regge pole then it follows that the exchange is that of a Reggeon whose intercept, $\alpha_P(0)$, is greater than 1, and which carries the quantum numbers of the vacuum

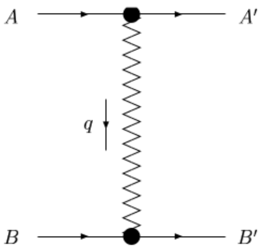
$$\downarrow$$

This trajectory is called the Pomeron

Backup: The Reggeized gluon

Elastic scattering process $A + B \longrightarrow A' + B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$- Reggeized gluon trajectory

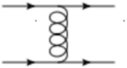
$$\Gamma^c_{A'A} = g\,\langle A'|T^c|A\rangle\,\Gamma_{A'A}$$

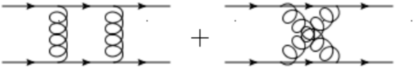
T^c- fundamental(quarks) or adjoint(gluons)

- LLA [Ya. Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}}\frac{N}{2}\int\frac{d^{D-2}k_\perp}{k_\perp^2(q-k)_\perp^2} = -g^2\frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon$$

Backup: The Reggeized gluon

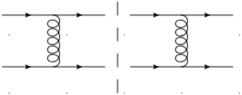




$$\mathcal{A}_0^8 = 8\pi\alpha_s \frac{s}{t}\delta_{\lambda_{A'}\lambda_A}\delta_{\lambda_{B'}\lambda_B} G_0^8 \qquad \mathcal{A}_1^8 = \mathcal{A}_0^8\,\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)$$

$$\mathcal{A}^8 = \mathcal{A}_0^8\left[1+\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)+\frac{1}{2}\left(\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)\right)^2+\ldots\right] \rightarrow \mathcal{A}^8 = \mathcal{A}_0^8\left(\frac{s}{\vec{q}^{\,2}}\right)^{\omega(t)}$$

$$\Im\mathcal{A}_1 = \frac{1}{2}\int d(P.S.^2)\mathcal{A}_0^8(k)\mathcal{A}_0^8(k-q)$$



The integration that appears in $\omega(t)$ is the residue of that over the phase space. The terms in the denominator come from the propagators.

- NLLA [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (1979)]

$$\Gamma_{A'A}^{(1)} = \delta_{\lambda_{A'}\lambda_A}\Gamma_{AA}^{(+)} + \delta_{\lambda_{A'}-\lambda_A}\Gamma_{AA}^{(-)}\,, \qquad \omega^{(2)}(t)$$

Backup: Sudakov decomposition

- Sudakov decomposition: $p = zp_1 + \lambda p_2 + p_\perp \qquad d^4p = \frac{s}{2} dz\, d\lambda\, d^2\vec{p}$

  (p_1, p_2)- light-cone basis of the initial particle momenta plane

$$p_1^\mu = \left(\frac{\sqrt{s}}{2}, \vec{0}, \frac{\sqrt{s}}{2}\right), \qquad p_2^\mu = \left(\frac{\sqrt{s}}{2}, \vec{0}, -\frac{\sqrt{s}}{2}\right), \qquad 2p_1 \cdot p_2 = s, \qquad p_1^2 = p_2^2 = 0$$

- For a generic process of collision between two particles A and B is convinient to choose the 4-vectors p_1 and p_2 such that they lie on the plane of momenta p_A and p_B. In this way the transverse component is transverse respect to the plane of collision.
- Sudakov decompositon for the initial particles:

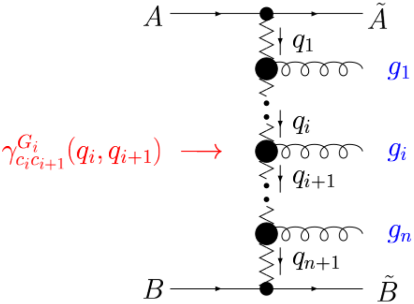
$$p_A = p_1 + \frac{m_A^2}{s} p_2 \qquad p_A^2 = m_A^2 \qquad\qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \qquad p_B^2 = m_B^2$$

- For massive particle one has to impose the on-shell condition:

$$q_1 = z_1 p_1 + \frac{m^2 + \vec{q}_1^{\,2}}{z_1 s} p_2 + q_{1\perp}$$

Backup: BFKL in LLA

Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



Leading logarithms resummation

↓

Multi-Regge kinematics (MRK)

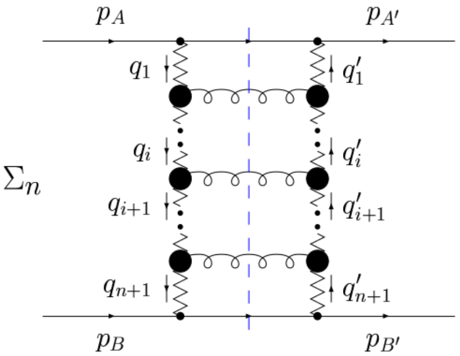
$$\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_0}\right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_0-energy scale, arbitrary in LLA.

Backup: BFKL in LLA

Elastic amplitude $A + B \to A' + B'$ in the LLA via s-channel unitarity

$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum_{n=0}^{\infty} \sum_{f} \int \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^{*} d\Phi_{\tilde{A}\tilde{B}+n}$$



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A})_{AB}^{A'B'} \qquad \mathcal{R} = 1(\text{singlet}), 8(\text{octect}), \ldots$$

Backup: Multi-Regge kinematics

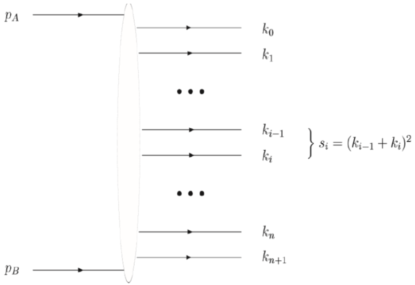
Multi-Regge kinematics

- Sudakov decomposition for the produced particles: $k_i = z_i p_1 + \lambda_i p_2 + k_{i\perp}$

- Transverse momenta of the produced particles are limited (i.e. not growing with s).
- Their Sudakov variables z_i and λ_i, are strongly ordered (in other words, the produced particles are strongly ordered in the rapidity space):

$$z_0 \gg z_1 \gg ... \gg z_n \gg z_{n+1}$$

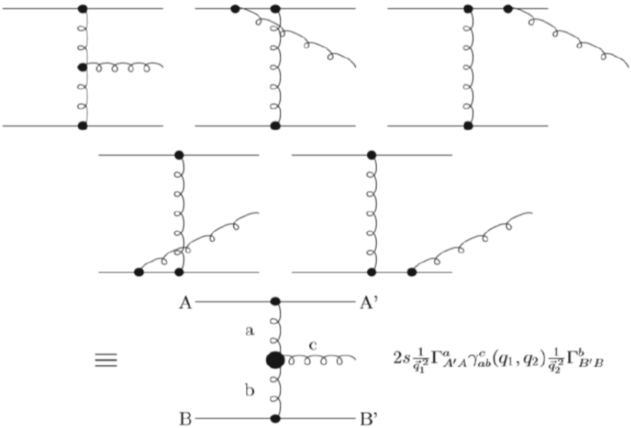
$$\lambda_{n+1} \gg \lambda_n \gg ... \gg \lambda_1 \gg \lambda_0$$



- Leading logarithms come from the integration over the longitudinal momenta of the produced particles
- In the LLA, where each added particle contributes only one $\ln s$, only this kinematics counts

Backup: Effective vertex

- Effective vertex for the production of a gluon



- Terms that contain fermions in intermediate states are suppressed in relation to those that involve the exchange of gluons
- "Vertical" gluons become Reggeized due to radiative corrections ("ladders within ladders").

Backup: Color decomposition

- Effective vertices

Coupling costant ↓ ; Lorentz structure ↙

$$\gamma^{G_i}_{c_i,c_{i+1}}(q_i,q_{i+1}) = gT^{d_i}_{c_i,c_{i+1}}\, e^*_\mu(k_i) C^\mu(q_i,q_{i+1})$$

↗ Matrix elements of $SU(N)$ generators (adjoint representation)

↑ Polarization 4-vector of the outgoing gluon

- Color decomposition

$$T^{d_i}_{c_i c_{i+1}} \left(T^{d_i}_{c'_i c'_{i+1}} \right)^* = \sum_{\mathcal{R}} c_{\mathcal{R}} \langle c_i c'_i | \hat{\mathcal{P}}_{\mathcal{R}} | c_{i+1} c'_{i+1} \rangle$$

$\hat{\mathcal{P}}_{\mathcal{R}}$- Projector of two-gluon color states in t-channel in the unitarity condition on the irriducible representations $\mathcal{R}$ of the color group.

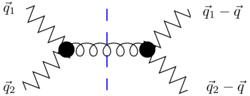
- "Real" part of the BFKL Kernel in the irriducible representation $\mathcal{R}$

$$\sum_{G_i} \gamma^{G_i}_{c_i c_{i+1}}(q_i,q_{i+1}) \left(\gamma^{G_i}_{c'_i c'_{i+1}}(q_i,q_{i+1}) \right)^* = \sum_{\mathcal{R}} c_{\mathcal{R}} \langle c_i c'_i | \hat{\mathcal{P}}_{\mathcal{R}} | c_{i+1} c'_{i+1} \rangle\, 2(2\pi)^{D-1} \mathcal{K}^{(\mathcal{R})}_r(\vec{q}_i,\vec{q}_{i+1};\vec{q})$$

$$\mathcal{K}^{(\mathcal{R})}_r(\vec{q}_i,\vec{q}_{i+1};\vec{q}\,) = -\frac{g^2 c_{\mathcal{R}}}{2(2\pi)^{D-1}} C^\mu(q_{i+1},q_i) C_\mu(q_{i+1}-q,q_i-q)$$

Backup: Kernel and BFKL Green function

- BFKL kernel in the irriducible representation $\mathcal{R}$



$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-(\vec{q}_1 - \vec{q}_2)^2\right)\right] \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q})$$

Kernel "virtual" part — Kernel "real" part

- Eigenvalues problem for the Kernel in the singlet case ($\mathcal{R} = 0$)

$$\mathcal{K} \bullet \phi_i(\vec{q}\,) = \omega_i \phi_i(\vec{q}\,) \longrightarrow$$

$$\omega_n(\nu) = \bar{\alpha}_s \chi_n(\nu) \longleftarrow \text{Eigenvalues}$$

$$\phi_\nu^n(\vec{q}\,) = \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{-\frac{1}{2}+i\nu} e^{in\theta} \longleftarrow \text{Eigenfunctions}$$

- BFKL Green function

$$G_\omega(\vec{q}_1, \vec{q}_2) = \sum_n \int d\nu \frac{\phi_\nu^n(\vec{q}_1)(\phi_\nu^n(\vec{q}_2))^*}{\omega - \omega_n(\nu)}$$

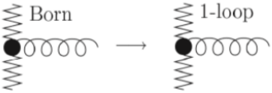
Backup: BFKL in the NLLA (MRK and QMRK)

- The BFKL Green function has been calculated up to the NLLA.
- For the next-to-leading logarithms resummation, two kinematics has to be take into account:
  1. Multi-Regge kinematics
  2. Quasi-multi-Regge kinematics
- Multi-Regge kinematics (one α_s more)

  The problem is reduced to the calculation of the two-loop contribution $\omega^{(2)}(t)$ to the gluon trajectory and to the PPR- and RRG- vertices.

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma^{c}_{P'P}(\text{Born}) \longrightarrow \Gamma^{c}_{P'P}(\text{1-loop})$

- $\gamma^{G_i}_{c_i c_{i+1}}(\text{Born}) \longrightarrow \gamma^{G_i}_{c_i c_{i+1}}(\text{1-loop})$

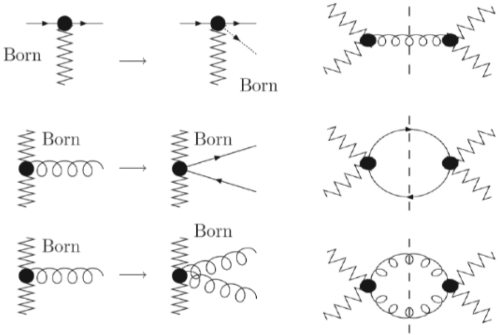


Backup: BFKL in the NLLA (MRK and QMRK)

- Quasi-multi-Regge kinematics (one $\ln s$ less)

  Any (but only one) pair of the produced particles can have a fixed (not increasing with s) invariant mass, i.e. components of this pair can have rapidities of the same order

- $\Gamma^{c\,(\text{Born})}_{P'P} \longrightarrow \Gamma^{c\,(\text{Born})}_{\{f\}P}$
- $\gamma^{G_i\,(\text{Born})}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\,(\text{Born})}_{c_i c_{i+1}}$
- $\gamma^{G_i\,(\text{Born})}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\,(\text{Born})}_{c_i c_{i+1}}$



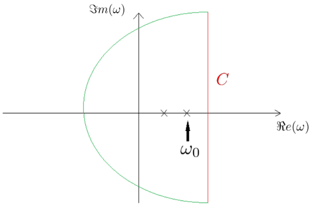
Backup: Mellin transform

- Mellin transform

$$\mathcal{F}(\omega) = \int_1^\infty d\left(\frac{s}{\vec{k}^{\,2}}\right)\left(\frac{s}{\vec{k}^{\,2}}\right)^{-\omega-1} f(s)$$

- Inverse Mellin transform

$$f(s) = \frac{1}{2\pi i}\int_{\mathcal{C}} d\omega \left(\frac{s}{\vec{k}^{\,2}}\right)^{\omega} \mathcal{F}(\omega)$$



- Example

$$f(s) = s^\alpha \ln\left(\frac{s}{\vec{k}^{\,2}}\right)^r \xrightarrow[\text{Mellin transform}]{} \mathcal{F}(\omega) = (\vec{k}^{\,2})^\alpha \frac{\Gamma(r+1)}{(\omega-\alpha)^{r+1}}$$

Impact factor calculation

- LO impact factor definition:

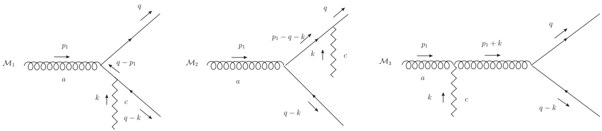
$$d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{q},\vec{k},z) = \frac{\langle cc'|\widehat{\mathcal{P}}|0\rangle}{2\left(N^2-1\right)} \sum_{\lambda_Q \lambda_{\bar{Q}} \lambda_G} \sum_{Q\bar{Q}a} \int \frac{ds_{gR}}{2\pi} d\rho_{\{Q\bar{Q}\}} \Gamma^{ca}_{g\to\{Q\bar{Q}\}}(q,k,z) \left(\Gamma^{ac'}_{g\to\{Q\bar{Q}\}}(q,k,z)\right)^*$$

- Color state projector and differential phase of space:

$$\langle cc'|\widehat{\mathcal{P}}|0\rangle = \frac{\delta^{cc'}}{\sqrt{N^2-1}}\,, \qquad \frac{ds_{gR}}{2\pi} d\rho_{\{Q\bar{Q}\}} = \frac{1}{2\left(2\pi\right)^3}\delta\left(1-z-\bar{z}\right)\delta^{(2)}\left(\vec{k}-\vec{q}-\vec{\bar{q}}\right)\frac{dz\,d\bar{z}}{z\bar{z}} d^2\vec{q}\, d^2\vec{\bar{q}}$$

- Gauge choice: $\epsilon(p_1)\cdot p_2 = 0$
- Non sense polarization for the Reggeon: $\epsilon_R^\mu = p_2^\mu/s$
- Amplitude:

$$\Gamma^{ca}_{g\to\{Q\bar{Q}\}} = ig^2\left(\tau^a\tau^c\right)\bar{u}\left(q\right)\left(mR\hat{\epsilon} - 2z\vec{P}\cdot\vec{\epsilon} - \hat{\vec{P}}\hat{\epsilon}\right)\frac{\hat{p}_2}{\hat{s}}v\left(\bar{q}\right)$$
$$+\, ig^2\left(\tau^c\tau^a\right)\bar{u}\left(q\right)\left(m\bar{R}\hat{\epsilon} - 2z\vec{\bar{P}}\cdot\vec{\epsilon} - \hat{\vec{\bar{P}}}\;\hat{\epsilon}\right)\frac{\hat{p}_2}{\hat{s}}v\left(\bar{q}\right)$$



Projected Impact factor

- Projected impact factor

$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n,\nu,\vec{q},z)}{d^2\vec{q}\,dz} \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}}(\vec{k}^{\,2})^{i\nu-\frac{3}{2}}e^{in\theta}\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z)}{d^2\vec{q}\,dz}$$

$$= \frac{\alpha_s^2\sqrt{N_c^2-1}}{2\pi N_c}\left\{m^2\left(I_3-2\frac{I_2(0)}{m^2+\vec{q}^{\,2}}\right)+(z^2+\bar{z}^2)\left(-m^2\left(I_3-2\frac{I_2(0)}{m^2+\vec{q}^{\,2}}\right)+\frac{I_2(1)}{m^2+\vec{q}^{\,2}}\right)\right.$$

$$-\frac{N_c^2}{N_c^2-1}\left[2m^2\left[\left(z^2+\bar{z}^2-1\right)\left(1-\left(z^2\right)^{\frac{1}{2}-i\nu}\right)\right]\frac{I_2(0)}{m^2+\vec{q}^{\,2}}+\left[2m^2(z^2+\bar{z}^2-1)\left(z^2\right)^{\frac{1}{2}-i\nu}\right]\right.$$

$$\left.\left.\left(I_3-\frac{I_4(0)}{(z^2)^{\frac{1}{2}-i\nu}}\right)-(z^2+\bar{z}^2)\left[(1-z)^2I_4(1)-\frac{\left(1-\left(z^2\right)^{\frac{1}{2}-i\nu}\right)}{m^2+\vec{q}^{\,2}}I_2(1)\right]\right]\right\}$$

$$\equiv \alpha_s^2\,e^{in\varphi}c(n,\nu,\vec{q},z)\ ,$$

where:

$$I_2(\lambda) = \frac{\left(\vec{q}^{\,2}\right)^{\frac{n}{2}}e^{in\varphi}}{\sqrt{2}}\frac{1}{(m^2+\vec{q}^{\,2})^{\frac{3}{2}+\frac{n}{2}-i\nu-\lambda}}\frac{\Gamma\left(\frac{1}{2}+\frac{n}{2}+i\nu+\lambda\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}-i\nu-\lambda\right)}{\Gamma(1+n)}$$

$$\times\frac{\left(\frac{1}{2}+\frac{n}{2}-i\nu-\lambda\right)}{\left(-\frac{1}{2}+\frac{n}{2}+i\nu+\lambda\right)}\,{}_2F_1\left(-\frac{1}{2}+\frac{n}{2}+i\nu+\lambda,\frac{3}{2}+\frac{n}{2}-i\nu-\lambda,1+n,\zeta\right)\ ,$$

Projected Impact factor

$$I_3 = \frac{\left(\vec{q}^{\,2}\right)^{\frac{n}{2}} e^{in\varphi}}{\sqrt{2}} \frac{1}{\left(m^2+\vec{q}^{\,2}\right)^{\frac{5}{2}+\frac{n}{2}-i\nu}} \frac{\Gamma\left(\frac{1}{2}+\frac{n}{2}+i\nu\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}-i\nu\right)}{\Gamma\left(1+n\right)} \frac{\left(\frac{1}{2}+\frac{n}{2}-i\nu\right)}{\left(-\frac{1}{2}+\frac{n}{2}+i\nu\right)}$$
$$\times\left(\frac{3}{2}+\frac{n}{2}-i\nu\right)\ {}_2F_1\left(-\frac{1}{2}+\frac{n}{2}+i\nu, \frac{5}{2}+\frac{n}{2}-i\nu, 1+n, \zeta\right)\ ,$$

$$I_4\left(\lambda\right) = \frac{\left(\vec{q}^{\,2}\right)^{\frac{n}{2}} e^{in\varphi}}{z^2\sqrt{2}} \frac{\left(\frac{3}{2}-i\nu-\lambda+\frac{n}{2}\right)}{\left(m^2+\vec{q}^{\,2}\right)^{\frac{5}{2}-i\nu-\lambda+\frac{n}{2}}} \frac{\Gamma\left(\frac{1}{2}+\frac{n}{2}+i\nu+\lambda\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}-i\nu-\lambda\right)}{\Gamma\left(1+n\right)}$$
$$\times\frac{\left(\frac{1}{2}+\frac{n}{2}-i\nu-\lambda\right)}{\left(-\frac{1}{2}+\frac{n}{2}+i\nu+\lambda\right)} \int_0^1 d\Delta\left(1+\frac{\Delta}{z}-\Delta\right)^n\left(1+\frac{\Delta}{z^2}-\Delta\right)^{-\frac{5}{2}+i\nu+\lambda-\frac{n}{2}}$$
$$\times\ {}_2F_1\left(-\frac{1}{2}+i\nu+\lambda+\frac{n}{2}, \frac{5}{2}-i\nu-\lambda+\frac{n}{2}, 1+n, \zeta\,\frac{\left(1+\frac{\Delta}{z}-\Delta\right)^2}{\left(1+\frac{\Delta}{z^2}-\Delta\right)}\right)\ ,$$

$\zeta \equiv \frac{\vec{q}^{\,2}}{m^2+\vec{q}^{\,2}}$; the azimuthal angles θ and φ are defined as $\cos\theta \equiv k_x/|\vec{k}|$ and $\cos\varphi \equiv q_x/|\vec{q}|$.

Integrals definition

$$I_1 \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}} \left(\vec{k}^{\,2}\right)^{i\nu-\frac{3}{2}} e^{in\theta} \ , \quad \text{for } n \neq 0 \ ,$$

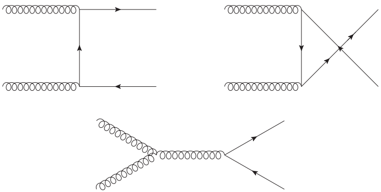
$$I_2\left(\lambda\right) \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}} (\vec{k}^{\,2})^{i\nu-\frac{3}{2}} e^{in\theta} \frac{(\vec{k}^{\,2})^{\lambda}}{m^2+(\vec{q}-\vec{k})^2}$$

$$I_3 \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}} (\vec{k}^{\,2})^{i\nu-\frac{3}{2}} e^{in\theta} \frac{1}{\left(m^2+(\vec{q}-\vec{k})^2\right)^2}$$

$$I_4\left(\lambda\right) \equiv \int \frac{d^2\vec{k}}{\pi\sqrt{2}} (\vec{k}^{\,2})^{i\nu-\frac{3}{2}} e^{in\theta} \frac{(\vec{k}^{\,2})^{\lambda}}{(m^2+(\vec{q}-\vec{k})^2)(m^2+(\vec{q}-z\vec{k})^2)} \ ,$$

"Box" $Q\bar{Q}$ mechanism

- "Box" cross section



$$M^2 = 4(m^2 + \vec{q}^{\,2})\cosh^2(\Delta Y/2)$$

$$t_1 = -\frac{M^2}{2}\left[1 - \tanh(\Delta Y/2)\right]$$

$$u_1 = -\frac{M^2}{2}\left[1 + \tanh(\Delta Y/2)\right]$$

$$\frac{d\sigma_{pp}}{d(\Delta Y)} = \frac{\pi\alpha_s^2}{s(N_c^2-1)} \int_{\min(q^2_{\min},\, \frac{s}{4\cosh^2(\Delta Y/2)} - m^2)}^{\min(q^2_{\max},\, \frac{s}{4\cosh^2(\Delta Y/2)} - m^2)} \frac{d\vec{q}^{\,2}}{M^2} \int_{\frac{M^2}{s}}^{1} \frac{dx}{x} f_{g_1}(x, \mu_{F_1}) f_{g_2}(M^2/xs, \mu_{F_2})$$

$$\times \left(C_F \frac{M^4}{t_1 u_1} - C_A\right)\left[\frac{t_1^2 + u_1^2}{M^4} + 4\frac{m^2}{M^2} - 4\frac{m^4}{t_1 u_1}\right]$$

- "Box" vs C_0

ΔY	Box $Q\bar{Q}$	LLA $C_\mu = 1/2$	LLA $C_\mu = 1$	LLA $C_\mu = 2$	NLA $C_\mu = 1/2$	NLA $C_\mu = 1$	NLA $C_\mu = 2$
1.5	33830.3	38.17(24)	30.01(21)	23.58(16)	22.25(26)	23.93(23)	25.19(27)
3.0	3368.86	18.118(98)	13.191(71)	9.838(61)	7.245(74)	8.205(76)	8.172(82)
4.5	124.333	6.996(33)	4.715(23)	3.276(16)	2.209(20)	2.411(17)	2.422(19)
6.0	3.19206	1.976(10)	1.2430(60)	0.8044(38)	0.4497(35)	0.4968(35)	0.4868(37)
7.5	0.0610921	0.3317(16)	0.19115(92)	0.11509(57)	0.05318(36)	0.05785(39)	0.05577(42)
9.0	0.000681608	0.02215(10)	0.011458(56)	0.006340(30)	0.002566(17)	0.002668(16)	0.002513(16)

Backup: Photoproduction

- Process:

$$\gamma(p_1) + \gamma(p_2) \longrightarrow Q\text{-jet}(q_1) + Q\text{-jet}(q_2) + X$$

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa, PLB 777 (2018) 141]

- The cross section has the same structure
- Impact factor:

$$d\Phi_{\gamma\gamma}^{\{Q\bar{Q}\}}(\vec{k},\vec{q},z) = \frac{\alpha\alpha_s e_q^2}{\pi}\sqrt{N_c^2-1}\left[m^2R^2 + \left(z^2+\bar{z}^2\right)\vec{P}^{\,2}\right] d^2\vec{q}\, dz\ ,$$

$$R = \frac{1}{m^2+\vec{q}^{\,2}} - \frac{1}{m^2+(\vec{q}-\vec{k})^2}\ , \qquad \vec{P} = \frac{\vec{q}}{m^2+\vec{q}^{\,2}} - \frac{\vec{q}-\vec{k}}{m^2+(\vec{q}-\vec{k})^2}$$

- PDF ($f_g(x_1, \mu_{F_1})$) $\longrightarrow$ EPA ($n(x)$)
- Color and coupling pre-factors enhance hadro-production cross section by some 10^3 respect to the photo-production one

Backup: Hadroproduction vs photoproduction

- C_0
- c-jet ($m_c = 1.2$ GeV), $\sqrt{s} = 3$ TeV, $q_{\min} = 1$ GeV, $q_{\max} = 10$ GeV