A model calculation of the transverse-momentum-dependent gluon density in the proton

Francesco Giovanni Celiberto

in collaboration with Alessandro Bacchetta, Marco Radici, Pieter Taels

Università degli Studi di Pavia & Sezione INFN di Pavia under the "**3DGLUE**" MIUR FARE grant (n. **R16XKPHL3N**)





HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS





Istituto Nazionale di Fisica Nucleare Sezione di Pavia



Gluon TMDs: a largely unexplored territory Gluon TMDs



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang, Phys. Rev. Lett. 108 (2012) 032002]
[W.J. den Dunnen, J.P. Lansberg, C. Pisano, M. Schlegel, Phys. Rev. Lett. 112 (2014) 21200]
[J.P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, Phys. Lett. B 784 (2018) 217]
[A. Bacchetta, D. Boer, C. Pisano, P. Taels, arXiv:1809.02056 [hep-ph]]
[U. D'Alesio, C. Flore, F. Murgia, C. Pisano, P. Taels, arXiv:1811.02970 [hep-ph]]

Motivation

* Need for a flexible model, suited to phenomenology

* Correctly reproduces gluon collinear PDF...

* ...allows us to predict unpol. and pol. gluon TMDs

Consistent framework for quark TMDs





Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not\!\!\!P + M_H)\frac{1+\gamma^5 \not\!\!\!S}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}^{\rho*}_g\mathcal{Y}_{g\sigma}(\not\!\!\!P - \not\!\!\!p + M_X)\right]$$

$$\mathcal{Y}_{g}^{\mu} = g_{1}(p^{2})\gamma^{\mu} + i\frac{g_{2}(p^{2})}{2M_{H}}\sigma^{\mu\nu}p_{\nu}$$



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$$\mathcal{Y}_{g}^{\mu} = g_{1}(p^{2})\gamma^{\mu} + i\frac{g_{2}(p^{2})}{2M_{H}}\sigma^{\mu\nu}p_{\nu}$$
Selection out of 12 Dirac structures

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12 Dirac structures

Gluon TMDs at twist-2



gluon pol.

T-even

T-odd

Gluon TMDs at twist-2



State of the art



First calculation of leading-twist *T*-even quark TMDs with scalar and axial-vector di-quarks

[R. Jakob, P. J. Mulders, and J. Rodrigues, Nucl. Phys. A626, 937 (1997)]



Gluon TMD PDFs and FFs

[P.J. Mulders, J. Rodrigues, Phys. Rev. **D63** (2001) 094021] [J. Rodrigues, PhD thesis (2001)]

Complete calculation of all the leading-twist TMDs with scalar di-quarks

[S. Meissner, A. Metz, and K. Goeke, Phys. Rev. D76, 034002 (2007)]

Inclusion of different axial-vector di-quark polarization states and nucleon-parton-spectator form factors

(fit to PDF parametrizations) [A. Bacchetta, F. Conti, M. Radici, Phys. Rev. **D78** (2008) 074010] (application on azimuthal asymmetries) [A. Bacchetta, M. Radici, F. Conti, M. Guagnelli, Eur. Phys. J. **A45** (2010) 373-388]

How to improve the description?

Spectator-system spectral-mass function

$F(x, \boldsymbol{p}_T^2) = \int \mathrm{d}M_X \, \boldsymbol{\rho}_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti, Phys.Rev. **D84** (2011) 034007]



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$$\rho_X\left(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, b, c, C, M_D, \sigma\}\right) = \mu^{2b} \left[\frac{A}{B + \mu^{2c}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - M_D)^2}{\sigma^2}}\right]$$

low-*x* (high- μ^2) tail $\propto (b - c)$ **2***N*-quark contribution

$$\mu^2 = M_X^2 - (M_H - m_{q/g})^2$$

moderate-*x* trend tri-quark contribution

Spectral function vs (b - c)



Spectral function vs (b - c)









Calculate TMDs from parton correlators

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- Weight TMDs over *M_X* via spectral function

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- **Fit** model parameters to reproduce f_1 and g_1 **collinear** PDFs

Unpolarized gluon PDF $xf_1^g(x)$



X

Unpolarized gluon PDF

 $xf_1^g(x)$



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Helicity gluon PDF $xg_1^g(x)$ 0.2 0.1 0.0 -0.1 -0.2 $Q_0^2 = 2.69 \,\mathrm{GeV}^2$ ----- NNPDFpol1.1 -0.3 0.005 0.010 0.500 0.001 0.050 0.100

X

Helicity gluon PDF $xg_1^{g}(x)$







Unpolarized gluon TMD

 $xf_1^{g}(x, p_T^{2})$











Worm-gear gluon TMD

 $xg_{1T}^{g}(x, p_{T}^{2})$



Worm-gear gluon TMD

 $xg_{1T}^{g}(x, p_{T}^{2})$










Bottom line

* Each TMD shows a distinctive *x*- and p_T -behavior

Data on gluon TMDs will exclude many replicas
and constrain parameters not yet so well constrained
by collinear PDFs





 $f_1(x, p_x, p_y)$

 ρ -densities



 $f_1(x, p_x, p_y)$



[**U**/+]

 $\frac{1}{2} \left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$

p-densities



p-densities



[unpol/unpol] ρ -density



$[\leftrightarrow/unpol] \rho - density (BM)$



 $[\cup / \rightarrow] \rho$ -density (WG)



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- Use of models to estimate impact of new data



Parton densities: an overview

p_T - integrated

Collinear PDFs

- Inclusive processes
- $p_T \sim \text{hardest scale}$



GPDs

- Exclusive processes
- Skewness effects



 p_T - unintegrated

TMDs

- (Semi-)inclusive processes
- $P_T <<$ hardest scale



UGDs

- High-energy factorization (**BFKL**)
- Small *x*, large p_T



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Effective vertices



Quark and gluon correlators

Scalar-diquark spectator

$$\Phi_s = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_s^2}{(p^2 - m_q^2)^2} (\not p + m_q) \frac{1 + \gamma^5 \$}{2} (\not p + M_H) (\not p + m_q)$$

Axial-vector-diquark spectator

$$\Phi_a = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_a^2}{(p^2 - m_q^2)^2} (\not p + m_q) \frac{\gamma^5 \gamma_\mu}{\sqrt{2}} \frac{1 + \gamma^5 \$}{2} (\not p + M_H) \frac{\gamma^5 \gamma_\nu}{\sqrt{2}} (\not p + m_q) d_T^{\mu\nu} (P - p)$$

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$$\mathcal{Y}_{g}^{\mu} = g_{1}(p^{2})\gamma^{\mu} + i\frac{g_{2}(p^{2})}{2M_{H}}\sigma^{\mu\nu}p_{\nu}$$

Quark and gluon correlators

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$$\mathcal{Y}_{g}^{\mu} = g_{1}(p^{2})\gamma^{\mu} + i\frac{g_{2}(p^{2})}{2M_{H}}\sigma^{\mu\nu}p_{\nu}$$
$$40$$
Selection out of 12 Dirac structures

Spectator-model gluon TMDs (1)

$$\begin{aligned} \hat{f}_{1}^{g}(x, \boldsymbol{p}_{T}^{2}) &= \left[4g_{1}^{2}M_{H}^{2}(x^{2}(M_{X} - M_{H}(1-x))^{2} + \boldsymbol{p}_{T}^{2}((x-2)x+2)) \right. \\ &- 4g_{1}g_{2}M_{H}x^{2}(M_{H} + M_{X})((M_{X} - M_{H}(1-x))^{2} + \boldsymbol{p}_{T}^{2}) \\ &+ g_{2}^{2}(\boldsymbol{p}_{T}^{2}x(M_{H}^{2}(3x-2) + 2M_{H}M_{X}x + M_{X}^{2}(x+2)) \\ &+ x^{2}(M_{H} + M_{X})^{2}(M_{X} - M_{H}(1-x))^{2} + 2(\boldsymbol{p}_{T}^{2})^{2}) \right] \\ &+ (2\pi)^{3}M_{H}^{2}x(L_{g}^{2}(m_{g}^{2}) + \boldsymbol{p}_{T}^{2})^{2} \end{aligned}$$

$$\hat{g}_{1L}^{g}(x, \boldsymbol{p}_{T}^{2}) = \left[(2g_{1}M_{H} - g_{2}(M_{H} + M_{X}))(2g_{1}M_{H}(x(M_{X} - M_{H}(1 - x))^{2} - \boldsymbol{p}_{T}^{2}(x - 2)) \right. \\ \left. + g_{2}(-M_{H}^{2}x^{3}(M_{H} + M_{X}) + \boldsymbol{p}_{T}^{2}x(M_{X} - 3M_{H}) + 2\boldsymbol{p}_{T}^{2}(M_{H} - M_{X}) \right. \\ \left. + 2M_{H}x^{2}(M_{H} - M_{X})(M_{H} + M_{X}) - x(M_{H} - M_{X})^{2}(M_{H} + M_{X}))) \right] \\ \left. + (4(2\pi)^{3}M_{H}^{2}(L_{g}^{2}(m_{g}^{2}) + \boldsymbol{p}_{T}^{2})^{2} \right]$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

Spectator-model gluon TMDs (2)

$$g_{1T}^{g}(x, \boldsymbol{p}_{T}^{2}) = \left[((2g_{1}M_{H} - g_{2}(M_{H} + M_{X}))((M_{X} - M_{H}(1 - x))) \\ (-2g_{1}M_{H}(1 - x) - g_{2}M_{X}x) - g_{2}\boldsymbol{p}_{T}^{2}) \right] \\ /(2(2\pi)^{3}M_{H}(L_{X}^{2}(m_{g}^{2}) + \boldsymbol{p}_{T}^{2})^{2})$$

$$h_1^{\perp g}(x, \mathbf{p}_T^2) = \left[(4g_1^2 M_H^2 (1-x) + g_2^2 (x(M_X^2 - M_H^2 (1-x)) + \mathbf{p}_T^2)) \right] \\ /((2\pi)^3 x (L_X^2 (m_g^2) + \mathbf{p}_T^2)^2)$$

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Spectator-model quark TMDs scalar di-quark (1)

$$\begin{split} f_1^{q(s)}(x, \boldsymbol{p}_T^2) &= \frac{g_s^2}{(2\pi)^3} \frac{\left[(m+xM)^2 + \boldsymbol{p}_T^2\right](1-x)}{2\left[\boldsymbol{p}_T^2 + L_s^2(m^2)\right]^2} \\ g_{1L}^{q(s)}(x, \boldsymbol{p}_T^2) &= \frac{g_s^2}{(2\pi)^3} \frac{\left[(m+xM)^2 - \boldsymbol{p}_T^2\right](1-x)}{2\left[\boldsymbol{p}_T^2 + L_s^2(m^2)\right]^2} \\ g_{1T}^{q(s)}(x, \boldsymbol{p}_T^2) &= \frac{g_s^2}{(2\pi)^3} \frac{M\left(m+xM\right)(1-x)}{\left[\boldsymbol{p}_T^2 + L_s^2(m^2)\right]^2} \\ h_{1L}^{\perp q(s)}(x, \boldsymbol{p}_T^2) &= -g_{1T}^{q(s)}(x, \boldsymbol{p}_T^2) \\ L_X^2(m^2) &= xM_X^2 + (1-x)m^2 - x(1-x)M_H^2 \end{split}$$

Spectator-model quark TMDs scalar di-quark (2)

$$\begin{split} h_{1T}^{q(s)}(x, \boldsymbol{p}_{T}^{2}) &= f_{1}^{q(s)}(x, \boldsymbol{p}_{T}^{2}) \\ h_{1T}^{\perp q(s)}(x, \boldsymbol{p}_{T}^{2}) &= -\frac{g_{s}^{2}}{(2\pi)^{3}} \frac{M^{2} \left(1 - x\right)}{\left[\boldsymbol{p}_{T}^{2} + L_{s}^{2}(m^{2})\right]^{2}} \\ h_{1}^{q(s)}(x, \boldsymbol{p}_{T}^{2}) &= h_{1T}^{q(s)}(x, \boldsymbol{p}_{T}^{2}) + \frac{\boldsymbol{p}_{T}^{2}}{2M^{2}} h_{1T}^{\perp q(s)}(x, \boldsymbol{p}_{T}^{2}) \\ &= \frac{g_{s}^{2}}{(2\pi)^{3}} \frac{(m + xM)^{2} \left(1 - x\right)}{2\left[\boldsymbol{p}_{T}^{2} + L_{s}^{2}(m^{2})\right]^{2}} = \frac{1}{2} \left(f_{1}^{q(s)}(x, \boldsymbol{p}_{T}^{2}) + g_{1}^{q(s)}(x, \boldsymbol{p}_{T}^{2})\right) \end{split}$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

Spectator-model quark TMDs axial-vector di-quark (1)

$$\begin{split} f_1^{q(a)}(x, \boldsymbol{p}_T^2) &= \frac{g_a^2}{(2\pi)^3} \; \frac{\boldsymbol{p}_T^2 \; (1+x^2) + (m+xM)^2 \; (1-x)^2}{2 \left[\boldsymbol{p}_T^2 + L_a^2(m^2) \right]^2 \; (1-x)} \\ g_{1L}^{q(a)}(x, \boldsymbol{p}_T^2) &= \frac{g_a^2}{(2\pi)^3} \; \frac{\boldsymbol{p}_T^2 \; (1+x^2) - (m+xM)^2 \; (1-x)^2}{2 \left[\boldsymbol{p}_T^2 + L_a^2(m^2) \right]^2 \; (1-x)} \\ g_{1T}^{q(a)}(x, \boldsymbol{p}_T^2) &= \frac{g_a^2}{(2\pi)^3} \; \frac{M x \; (m+xM)}{\left[\boldsymbol{p}_T^2 + L_a^2(m^2) \right]^2} \end{split}$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

Spectator-model quark TMDs axial-vector di-quark (2)

$$\begin{split} h_{1L}^{\perp q(a)}(x, \boldsymbol{p}_{T}^{2}) &= g_{1T}^{q(a)}(x, \boldsymbol{p}_{T}^{2})/x \;, \\ h_{1T}^{q(a)}(x, \boldsymbol{p}_{T}^{2}) &= -\frac{g_{a}^{2}}{(2\pi)^{3}} \frac{x \, \boldsymbol{p}_{T}^{2}}{[\boldsymbol{p}_{T}^{2} + L_{a}^{2}(m^{2})]^{2} \, (1-x)} \\ h_{1T}^{\perp q(a)}(x, \boldsymbol{p}_{T}^{2}) &= 0 \\ h_{1T}^{q(a)}(x, \boldsymbol{p}_{T}^{2}) &= h_{1T}^{q(a)}(x, \boldsymbol{p}_{T}^{2}) \\ L_{X}^{2}(m^{2}) &= x M_{X}^{2} + (1-x) m^{2} - x (1-x) M_{H}^{2} \end{split}$$

Nucleon-parton-spectator effective vertex

$$g_X(p^2) = \begin{cases} g_X^{p.l.} & \text{pointlike} \\ g_X^{dip} \frac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2} & \text{dipolar} \\ g_X^{exp} e^{(p^2 - m^2)/\Lambda_X^2} & \text{exponential} \end{cases}$$

Our parametrization

$$F^{u_{v}}(x, \boldsymbol{p}_{T}^{2}) = \int dM_{s}^{u_{v}} \rho_{s}\left(M_{s}^{u_{v}}\right) \hat{F}_{s}^{u_{v}}(x, \boldsymbol{p}_{T}^{2}) + \int dM_{a}^{u_{v}} \rho_{a}\left(M_{a}^{u_{v}}\right) \hat{F}_{a}^{u_{v}}(x, \boldsymbol{p}_{T}^{2})$$

$$F^{d_v}(x, \boldsymbol{p}_T^2) =$$

 $\int \mathrm{d}M_a^{d_v} \rho_a\left(M_a^{d_v}\right) \hat{F}_a^{d_v}(x, \boldsymbol{p}_T^2)$

$$F^{\text{sea}}(x, \boldsymbol{p}_T^2) = \int \mathrm{d}M_s^{\text{sea}} \rho_s \left(M_s^{\text{sea}}\right) \hat{F}_s^{\text{sea}}(x, \boldsymbol{p}_T^2)$$

$$F^{g}(x, \boldsymbol{p}_{T}^{2}) = \int dM^{g} \rho_{g}(M^{g}) \hat{F}^{g}(x, \boldsymbol{p}_{T}^{2})$$

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scalar di-quark axial-vector di-quark

Our parametrization

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$$F^{g}(x, p_{T}^{2}) = \int dM^{g} \rho_{g}(M^{g}) \hat{F}^{g}(x, p_{T}^{2})$$

scalar di-quark axial-vector di-quark
spin-1/2

Unpolarized gluon PDF

 $xf_1^g(x)$


Fit specifics



$x_{\text{TOT}} = 0.42358 \ [x > 10^{-5}] + 0.00006 \ [x < 10^{-5}]$

$S_{\text{TOT}} = 0.339 \ [x > 10^{-5}] + 0.014 \ [x < 10^{-5}]$

[unpol/unpol] ρ -density



[unpol/unpol] ρ -density



 $[\cup /+] \rho$ -density



 $[U] + \rho$ -density



 $[U] + \rho$ -density



$[\leftrightarrow/unpol] \rho - density (BM)$



$[\leftrightarrow/unpol] \rho - density (BM)$



Unpolarized valence-down PDF



0.005 0.010 0.050 0.100 0.500

X

58

0.001

Helicity valence-down PDF



X

Unpolarized valence-up PDF

 $xf_1^{U_v}(\mathbf{x})$



60

Helicity valence-up PDF

 $xg_1^{u_v}(x)$



X

61