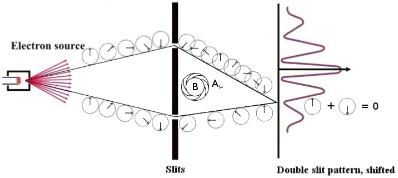
TMDs and DPDs on the lattice

G. Bali, T. Bhattacharya, P. Bruns, L. Castagnini, M. Diehl, M. Engelhardt, J. Gaunt, J. Green, B. Gläßle, P. Hägler, X. Ji, L.-C-. Jin, B. Lang, Yi. Liu, Yu. Liu, B. Musch, J. Negele, A. Schäfer, P. Wein, D. Ostermeier, A. Pochinsky, S. Syritsyn, Y-B. Yang, B. Yoon, J. Zhang, Y. Zhao, C. Zimmermann, ...

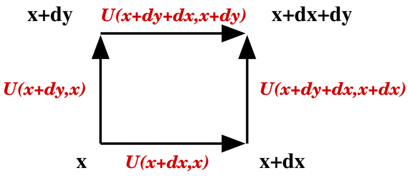
- Moments of TMDs from the lattice
- LPC: A quasiTMD proposal
- DPDs, a link between EIC and RHIC+LHC physics
- Lattice results for DPDs
- Conclusion

TMD simulations on the lattice are very non-trivial $\Rightarrow$ a long history. Like General Relativity local gauge theories have non-trivial parallel transport. TMDs and DPDs are sensitive to these.

Aharanov Bohm effect

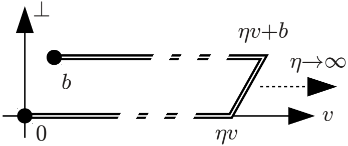


generic gauge links



TMDs are related to correlators of the type

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{U}[0, \eta v, \eta v + b, b]\, q(b)\, |P, S\rangle$$



We simulate for spatial, not light-like separations, but the limit $\hat{\zeta} \to \infty$ of

$$\hat{\zeta} := \frac{v \cdot P}{\sqrt{v^2}\sqrt{P^2}}$$

reproduces the light-cone behavior. For fixed target experiments $\zeta \sim O(1)$

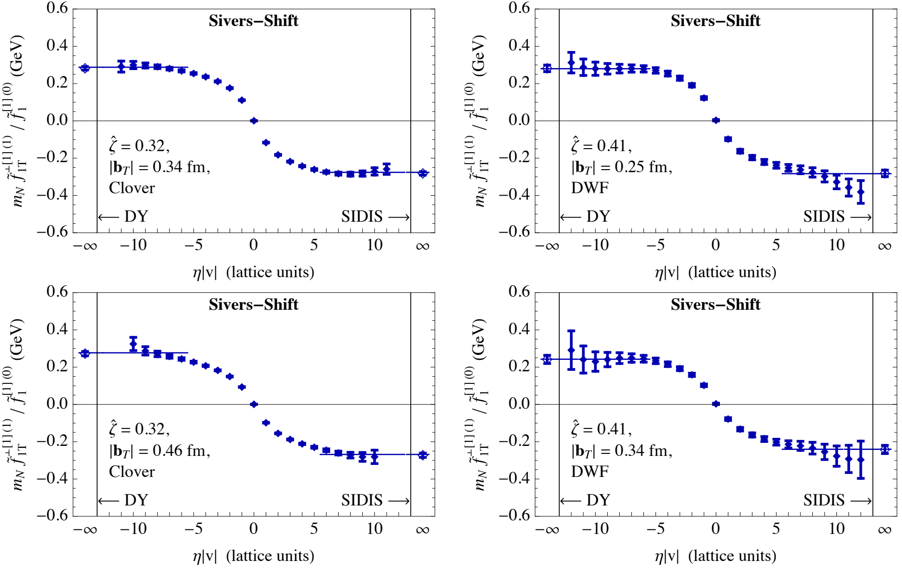
We used RBC/UKQCD (domain wall) and W&M (Clover) ensembles, $N_f = 2 + 1$

ID	Clover	DWF
Fermion Type	Clover	Domain-wall
Geometry	$32^3 \times 96$	$32^3 \times 64$
$a(\mathrm{fm})$	0.11403(77)	0.0840(14)
$m_\pi(\mathrm{MeV})$	317(2)(2)	297(5)
# confs.	967	533
# meas.	23208	4264

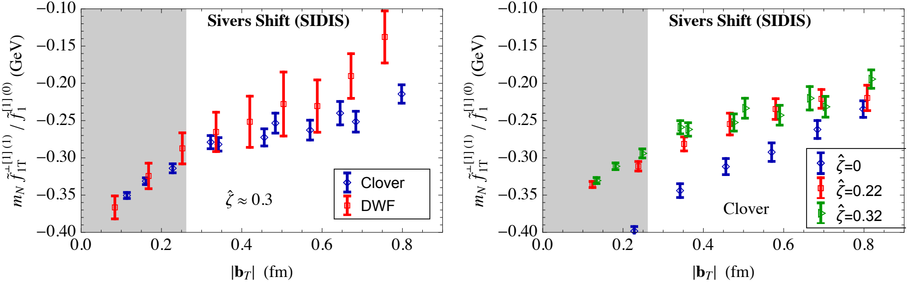
only connected diagrams, i.e. $u - d$

Presemt status arXiv:1706.03406: Yoon, Engelhardt, Gupta, Bhattacharya, Green, Musch, Negele, Pochinsky, AS, Syritsyn

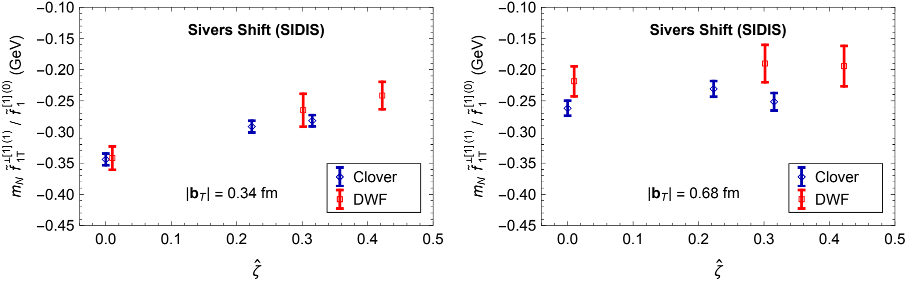
$$
\begin{aligned}
\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b,P,S,\ldots) &= \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)\cdot\mathcal{S}\cdot Z_{\text{TMD}}\cdot Z_2 \\
\Phi^{[\Gamma]}(x,\boldsymbol{k}_T,P,S,\ldots) &= \int\frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2}\int\frac{d(b\cdot P)}{2\pi P^+}e^{ix(b\cdot P)-i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_\perp}\;\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}\Big|_{b^+=0} \\
\Phi^{[\gamma^+]} &= f_1-\frac{\epsilon_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N}f_{1T}^\perp \\
\Phi^{[\gamma^+\gamma^5]} &= \Lambda g_1+\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N}g_{1T} \\
\Phi^{[i\sigma^{i+}\gamma^5]} &= \boldsymbol{S}_i h_1+\frac{(2\boldsymbol{k}_i\boldsymbol{k}_j-\boldsymbol{k}_\perp^2\delta_{ij})\boldsymbol{S}_j}{2m_N^2}h_{1T}^\perp+\frac{\Lambda\boldsymbol{k}_i}{m_N}h_{1L}^\perp+\frac{\epsilon_{ij}\boldsymbol{k}_j}{m_N}h_1^\perp \\
\tilde{f}^{[m](n)}(\boldsymbol{b}_\perp^2,\ldots) &= n!\left(-\frac{2}{m_N^2}\partial_{\boldsymbol{b}_\perp^2}\right)^n\int_{-1}^{1}dx\,x^{m-1}\int d^2\boldsymbol{k}_\perp \\
&\times\; e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_\perp}f(x,\boldsymbol{k}_\perp^2) \\
\langle\vec{k}_y\rangle_{TU}(\boldsymbol{b}_\perp^2;\ldots) &= m_N\frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_\perp^2;\ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_\perp^2;\ldots)}
\end{aligned}
$$



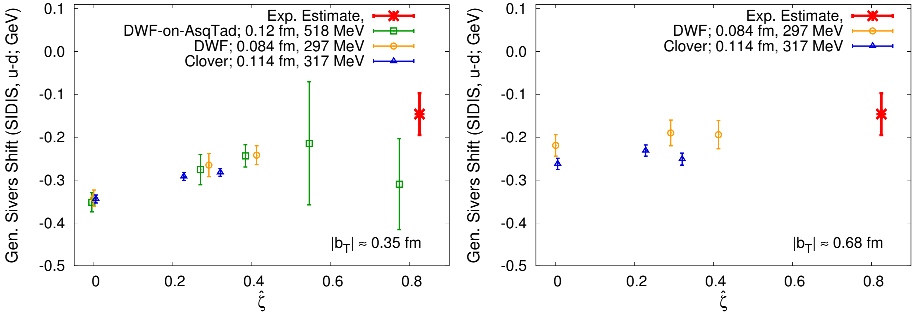
Sivers shift



Sivers shift



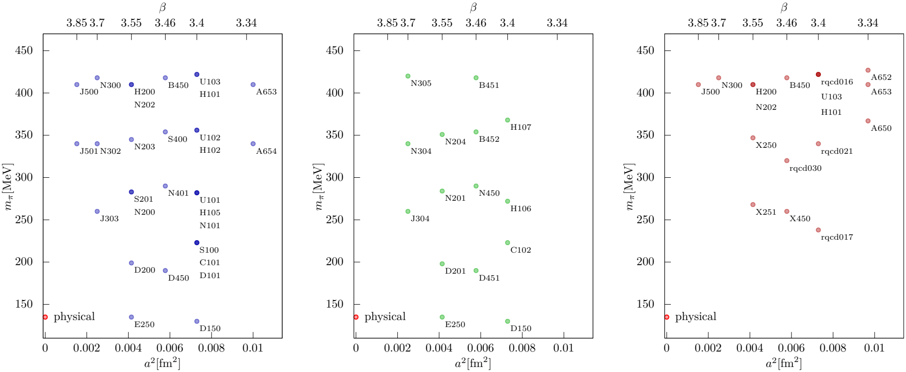
Sivers shift



Comparison with experiment

Presently we analyze non-perturbative evolution of TMDs with double ratios (submitted April 2018, accepted September 2018 but still not finished, LQCD project are always long term projects!) A. Vladimirov

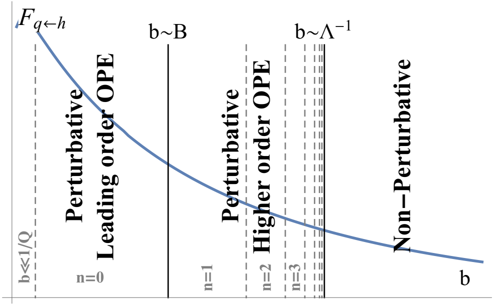
CLS ensembles



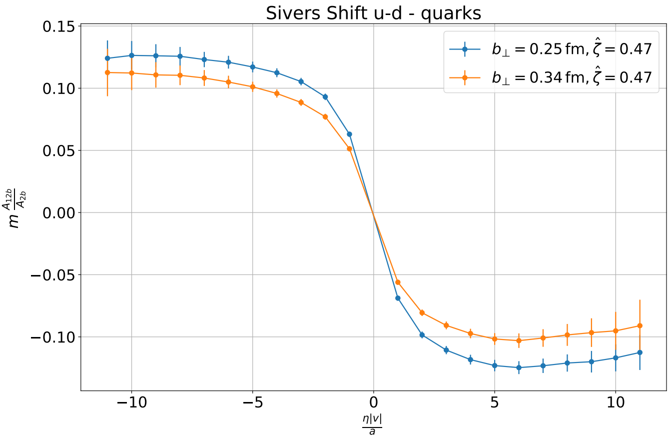
$$\frac{f^{[m,n]}(\zeta)}{f^{[m,n]}(\zeta')} \quad , \quad \frac{f^{[m,n]}(\zeta)}{f^{[m',n]}(\zeta')}$$

All of these ratios of double moments should give

$(\zeta/\zeta')^{-D_{NP}(\mu,\vec{b}_\perp)}$ The perturbative part of D has been calculated



One first result: H101, $a = 0.084$ fm, $m_\pi = 422$ MeV, 2000 configs



The CLS ensembles are also used by LPC (Lattice Parton Collaboration, formerly LP^3 for the study of quasi-distributions, including quasi-TMDs

Xiangdong Ji, Yizhuang Li and Yu-Shen Liu

arXiv:1910.11415 soft factor from HQET

$$
\begin{aligned}
W(Y,Y',T,T',b_\perp) &= \frac{1}{N_c}\,\langle\Omega|\mathrm{Tr}\mathcal{T}\mathcal{U}|\Omega\rangle \\
&\xrightarrow{T,T'\to\infty'} \frac{1}{N_c}\Phi^\dagger(\vec{b}_\perp)\,S(y,y',b_\perp)\,\Phi(\vec{b}_\perp)\,e^{-iE'T'-iET} \\
J(v,v',\vec{b}_\perp) &= \eta^\dagger_{v'}(\vec{b}_\perp)\eta_v(\vec{b}_\perp)\psi^\dagger_{v'}(0)\psi_v(0) \\
\Phi(\vec{b}_\perp) &= \lim_{T\to\infty'}\ {}_v\langle\bar{Q}Q,\vec{b}_\perp|\mathcal{O}_v(T,\vec{b}_\perp)|\Omega\rangle \\
S(Y,Y',b_\perp) &= {}_{v'}\langle\bar{Q}Q,\vec{b}_\perp|J(v,v',\vec{b}_\perp)|\bar{Q}Q,\vec{b}_\perp\rangle_v
\end{aligned}
$$

arXiv:1911.03840 quasi-TMDs $\rightarrow$ TMDs using this factor We will test whether the results for the double moments agree.

first try of a similar idea: arXiv:0710.4423 Musch, Hägler, AS, M. Göckeler, D. Renner, J. Negele (LHPC) LATTICE 2007

...

arXiv:1011.1213 Musch, Hägler, Negele, AS
Renormalization a la H. Dorn et al Fortschr. Phys. **34** (1986) 11

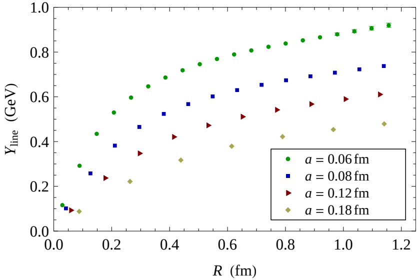
$$\begin{aligned} W(R,T) &= c(R)e^{-V(R)\,T} + \text{higher excitations} \\ W^{\text{ren}}(R,T) &= e^{-\delta m\,(2R+2T)-4\nu(90^\circ)}\,W(R,T) \\ V^{\text{ren}}(R) &= V(R) + 2\,\delta m \end{aligned}$$

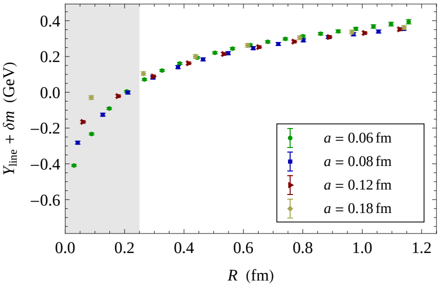
Renormalization condition: fixing δm to get

$$V_{\text{string}} = \sigma R - \frac{\pi}{12R}$$

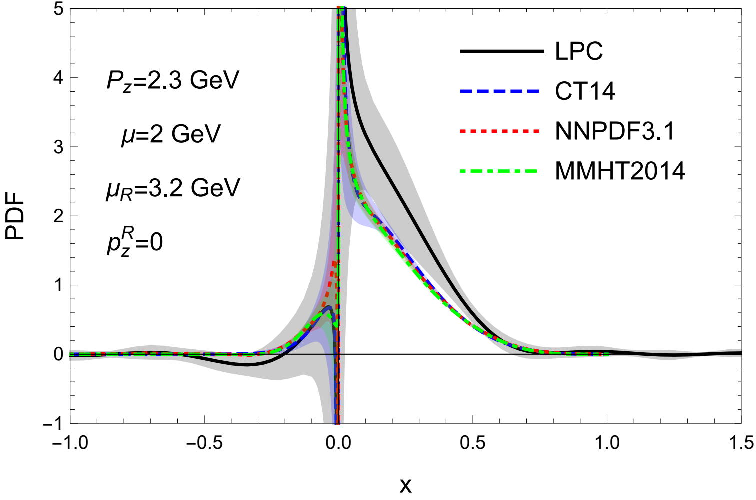
test

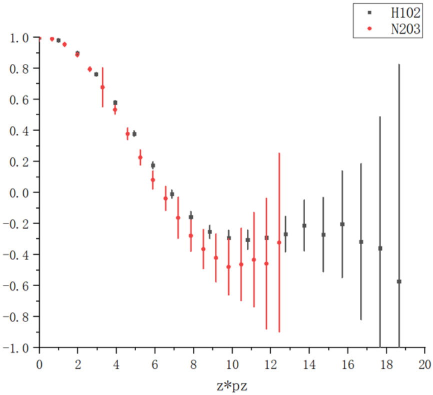
$$Y_{\text{line}}(R) \equiv -\frac{1}{a} \ln \frac{\left\langle\!\left\langle \text{tr}_c\, \mathcal{U}^{\text{lat}}[\mathcal{C}_{l'}] \right\rangle\!\right\rangle_{\text{Landau-gauge}}}{\left\langle\!\left\langle \text{tr}_c\, \mathcal{U}^{\text{lat}}[\mathcal{C}_{l}] \right\rangle\!\right\rangle_{\text{Landau-gauge}}}$$

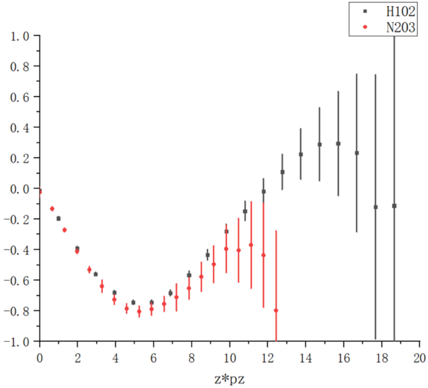




a first example for quasi-PDFs from CLS endembles: $u(x) - d(x)$ H102, $a = 0.086$ fm, $m_\pi = 356$ MeV



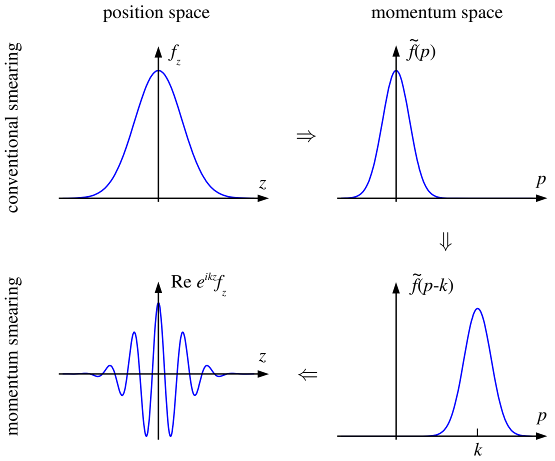




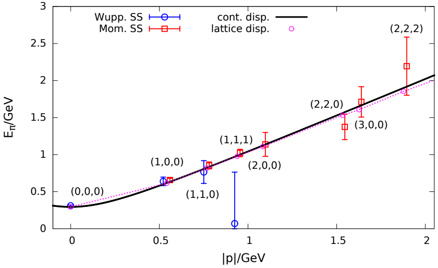
NP renormalized 3pt/2pt; left real part, right imaginary part; H102: $a = 08636$ fm; N203: $a = 0.06426$ fm

Whatever you do, reaching large momenta is the key

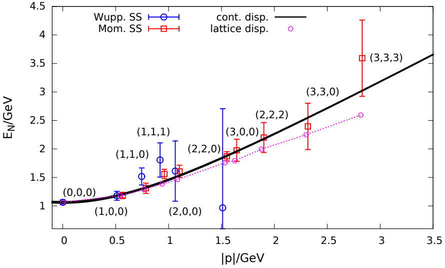
momentum dependent smearing Musch: 1602.05525



- modified smearing works great



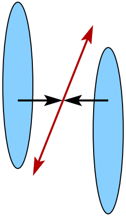
pion



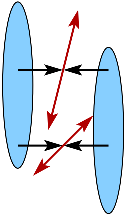
nucleon

But it is no silver bullet. The larger P^z the more problematic become excited state admixtures

Full use of discovery potential requires a better of the "underlying event", example: Double hard interactions



Single parton interaction

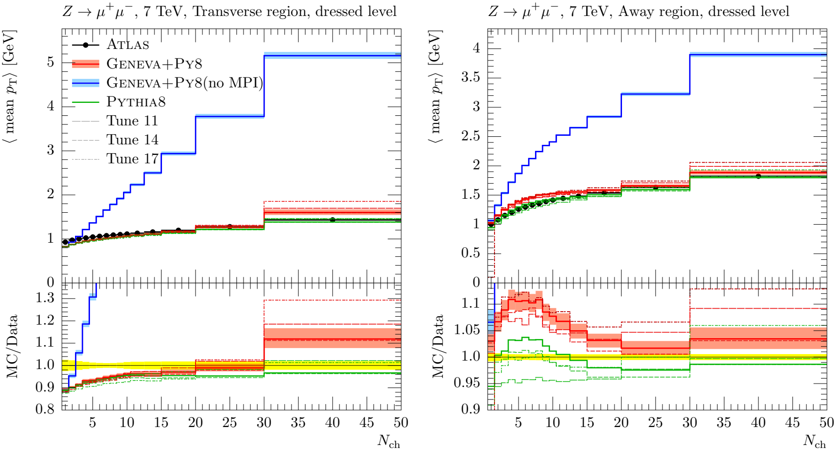


Double parton interaction

Naive (?) assumption

$$d\sigma_{DPS} = \frac{d\sigma_{SPS} d\sigma_{SPS}}{2\sigma_{eff}}$$

Two plots from Alioli, Bauer, Guns, Tackmann 1605.07192
mean charged particle p_T as function of N_{ch}.
Events with large N_{ch} have a higher MPI contribution. MPIs produce many particles with p_T of O(1 GeV).



Can the pocket formular be correct?

TMDs and DPDs are intimately connected. E.g. A. Vladimirov arXiv:1608.04920 soft factor TMDs $\Rightarrow$ soft factor DPDs.
The main question: Is the product form

$$\begin{aligned} C^{ij}_{\Gamma\Gamma'}(\vec{y}) &= \langle \pi^+(p')|\bar{q}(0,\vec{y})\Gamma q(0,\vec{y})\;\bar{q}'(0,\vec{0})\Gamma' q'(0,\vec{0})|\pi^+(p)\rangle \\ &= \sum_X \langle \pi^+(p')|\bar{q}(t,\vec{y})\Gamma q(t,\vec{y})|X\rangle\;\langle X|\bar{q}'(t,\vec{0})\Gamma' q'(t,\vec{0})|\pi^+(p)\rangle \\ &\overset{?}{\approx} \int \frac{d^3k}{(2\pi)^3 2k^0}\;\langle \pi^+(p')|\bar{q}(t,\vec{y})\Gamma q(t,\vec{y})|\pi^+(k)\rangle \\ &\qquad \langle \pi^+(k)|\bar{q}'(t,\vec{0})\Gamma' q'(t,\vec{0})|\pi^+(p)\rangle \end{aligned}$$

a good approximation or not ?

Quark-quark correlations in the pion; RQCD; $N_f = 2$, Clover-Wilson fermions, down to nearly physical mass ($m_\pi = 150$ MeV).

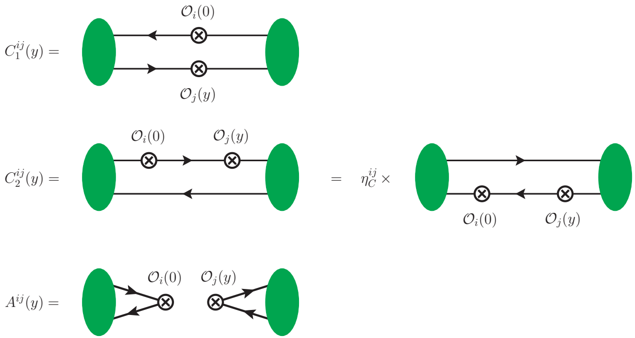
Many direct Tests of naive factorization, e.g., Integrals based on

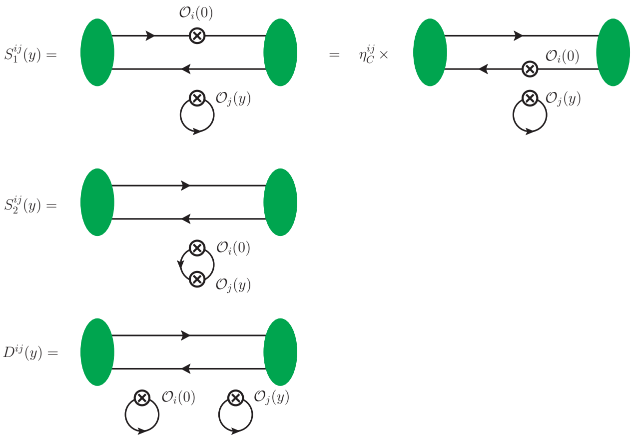
$$\int \; 4pt - correlator \;\; \stackrel{?}{=} \;\; \int \; (formfactor)^2$$

$$\begin{aligned}
\int_{-\infty}^{\infty} d(\vec{p}_\perp \cdot \vec{y}_\perp)\, A_{VV}(y^2, py) &= \frac{1}{2p^+} \int_{-\infty}^{\infty} dy^- \; \langle \pi(p)| V^+(0) V^+(y) |\pi(p)\rangle \Big|_{y^+=0} \\
\int_{-\infty}^{\infty} d(py) A_{VV}^{ud}(y^2, yp) &\stackrel{?}{\approx} -\int \frac{d^2 r_\perp}{(2\pi)^2}\, e^{-\mathrm{i}\vec{y}_\perp \cdot \vec{r}_\perp} |F_V(r^2)|^2
\end{aligned}$$

Quark-quark correlations in the pion are sizeable. On theoretical grounds they should be stronger in nucleons.

Lattice correlations we measure





$S_1^{ij}(y) =$
$\mathcal{O}_i(0)$
$\mathcal{O}_j(y)$
$= \eta_C^{ij} \times$
$\mathcal{O}_i(0)$
$\mathcal{O}_j(y)$
$S_2^{ij}(y) =$
$\mathcal{O}_i(0)$
$\mathcal{O}_j(y)$
$D^{ij}(y) =$
$\mathcal{O}_i(0)$
$\mathcal{O}_j(y)$

pion matrix elements for $N_f = 2$

$$
\begin{aligned}
\langle \pi^+ | \mathcal{O}_i^{uu}(0) \mathcal{O}_j^{dd}(y) | \pi^+ \rangle &= C_1^{ij}(y) + \left[S_1^{ij}(y) + S_1^{ji}(-y) \right] + D^{ij}(y) \\
\langle \pi^+ | \mathcal{O}_i^{uu}(0) \mathcal{O}_j^{uu}(y) | \pi^+ \rangle &= \left[C_2^{ij}(y) + C_2^{ji}(-y) \right] + \left[S_1^{ij}(y) + S_1^{ji}(-y) \right] \\
&+ D^{ij}(y) + S_2^{ij}(y) \\
\langle \pi^0 | \mathcal{O}_i^{uu}(0) \mathcal{O}_j^{dd}(y) | \pi^0 \rangle &= \left[S_1^{ij}(y) + S_1^{ji}(-y) \right] + D^{ij}(y) \\
&- \tfrac{1}{2} \left[A^{ij}(y) + A^{ji}(-y) \right] \\
\langle \pi^0 | \mathcal{O}_i^{uu}(0) \mathcal{O}_j^{uu}(y) | \pi^0 \rangle &= C_1^{ij}(y) + \left[S_1^{ij}(y) + S_1^{ji}(-y) \right] + D^{ij}(y) \\
&+ \left[C_2^{ij}(y) + C_2^{ji}(-y) \right] + \tfrac{1}{2} \left[A^{ij}(y) + A^{ji}(-y) \right] \\
&+ S_2^{ij}(y) \\
\langle \pi^+ | \mathcal{O}_i^{du}(0) \mathcal{O}_j^{ud}(y) | \pi^+ \rangle &= 2 C_2^{ij}(y) + A^{ij}(y) + S_2^{ij}(y) \\
\langle \pi^0 | \mathcal{O}_i^{du}(0) \mathcal{O}_j^{ud}(y) | \pi^0 \rangle &= -C_1^{ij}(y) + \left[C_2^{ij}(y) + C_2^{ji}(-y) \right] + S_2^{ij}(y) \\
\langle \pi^- | \mathcal{O}_i^{du}(0) \mathcal{O}_j^{du}(y) | \pi^+ \rangle &= 2 C_1^{ij}(y) + \left[A^{ij}(y) + A^{ji}(-y) \right] \\
\sqrt{2} \langle \pi^0 | \mathcal{O}_i^{du}(0) \mathcal{O}_j^{uu}(y) | \pi^+ \rangle &= C_1^{ij}(y) + \left[C_2^{ij}(y) - C_2^{ji}(-y) \right] + A^{ij}(y)
\end{aligned}
$$

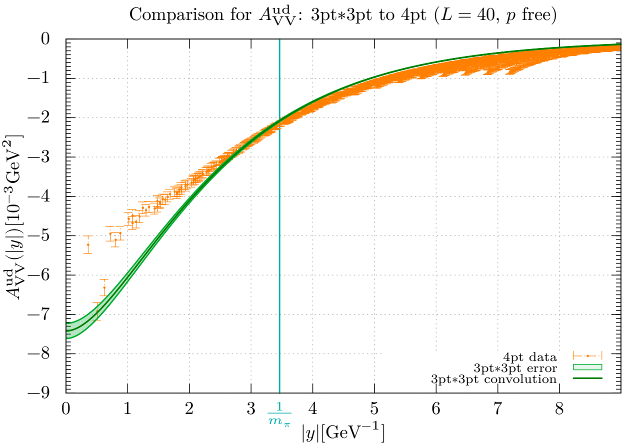
The RQCD ensembles used; $N_f = 2$, Clover-Wilson fermions, down to nearly physical mass ($m_\pi = 150$ MeV). N number of configurations, $N_{\rm sm}$ number of Wuppertal smearing iterations, $t_{\rm f}$ sink-source time differences. The error of the pion mass combines statistical and systematic errors.

Ensemble	β	a [fm]	κ	V	m_π [GeV]	Lm_π	N	$N_{\rm sm}$	$t_{\rm f}/a$
IV	5.29	0.071	0.13632	$32^3 \times 64$	0.2946(14)	3.42	2023	400	15
V	5.29	0.071	0.13632	$40^3 \times 64$	0.2888(11)	4.19	2025	400	15

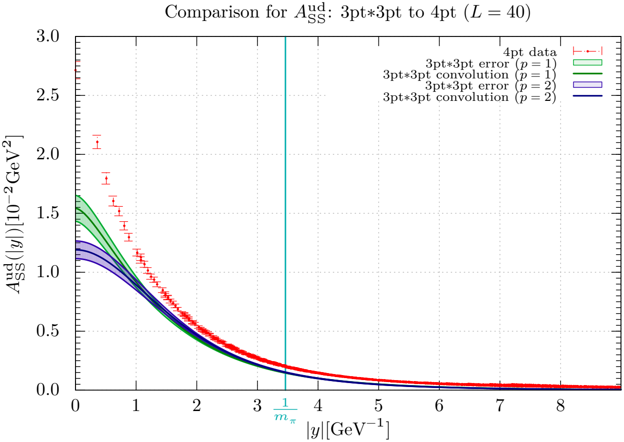
Direct Tests of naive factorization: Integrals based on

$$\int 4pt - correlator \stackrel{?}{=} \int (formfactor)^2$$

Direct tests of naive factorization: Test 1 VV case



Direct tests of naive factorization: Test 1 SS case

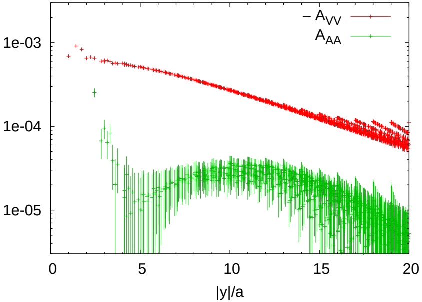


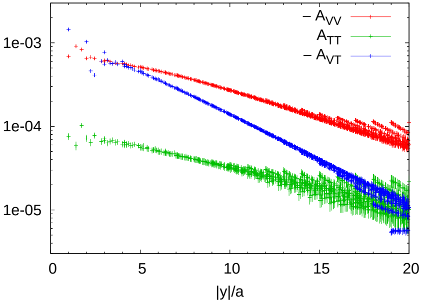
Spin correlations in the pion

AA: longitudinal spin correlation $u^{\uparrow}\bar{d}^{\uparrow} + u^{\downarrow}\bar{d}^{\downarrow} - u^{\uparrow}\bar{d}^{\downarrow} - u^{\downarrow}\bar{d}^{\uparrow}$

TT: transverse spin correlation $\vec{s}_u \cdot \vec{s}_d$

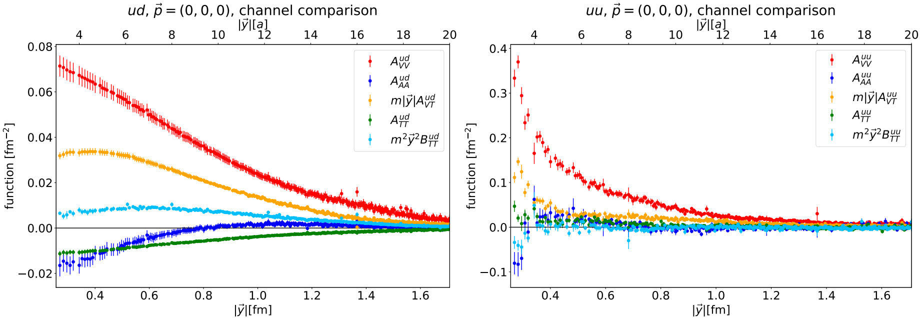
VT: $\vec{y} \cdot \vec{s}_{\bar{d}}$



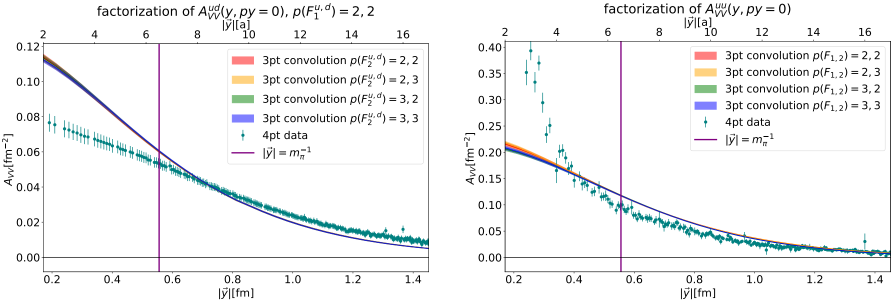


Is the “pocket formula” justified: **not really**

Now we (e.g. C. Zimmermann) have performed the same analysis for the nucleon and CLS ensembles with very similar results (preliminary):



signal over noise is comparable to pion case!



Corrections to pocket formula are comparable to pion case!

Conclusions

- TMDs are a vital part of the EIC physics case; DPDs are important for BSM searches at LHC
- Results for TMDs via quasi-TMDs could extend the outreach of lattice QCD calculations
- Calculations of DPDs and double moments of TMDs on the lattice are feasible
- First results for DPDs in the pion and nucleon show substantial quark correlations
- TMDs from LQCD has to become a precision tool by the time EIC starts to operate