Dynamical resolution scale for TMDs at the LHC

Workshop on Resummation, Evolution, Factorization
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based on
Nucl. Phys. B949 (2019) 114795 [arXiv:1908.08524]
F. Hautmann, L. Keersmaekers, A. Lelek, A.M. van Kampen

Contents

Motivation

Transverse momentum dependent (TMD) parton distributions

- Soft gluon resummation needed in nonperturbative parts of QCD factorization theorems
- Examples of regions with logarithmic enhanced contributions to the cross section:

low q_T: $q_T \ll Q$

$\alpha_s^n \ln^m Q/q_T$

low-q_T factorization

high $\sqrt{s}$: $\sqrt{s} \gg M$

$(\alpha_s \ln \sqrt{s}/M)^n$

high-energy factorization

See e.g. R. Angeles-Martinez et al. TMD parton distribution functions: status and prospects, Acta Phys.Polon. B46 (2015) no.12, 2501-2534

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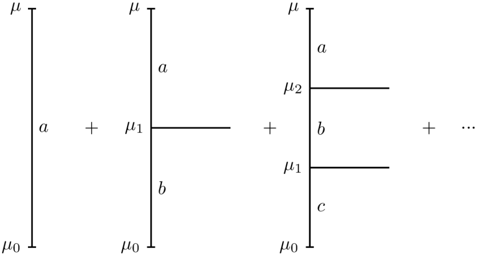
- Parton Branching (PB) method:
  Recently developed formalism for TMDs
  - Applicable over large range of transverse momenta
  - Applicable to exclusive final states and MC event generators
  - Connection with DGLAP evolution for collinear PDFs

See Hautmann, Jung, Lelek, Radescu, Zlebcik, "Collinear and TMD quark and gluon densities from parton branching", JHEP 1801 (2018) 070

The parton branching method: introduction

Evolution of TMDs $\tilde{\mathcal{A}}_a(x,k,\mu^2) = x\mathcal{A}_a(x,k,\mu^2)$ from μ_0 up to μ:

$$\begin{aligned}\tilde{\mathcal{A}}_a(x,\boldsymbol{k},\mu^2) &= \Delta_a(\mu^2,\mu_0^2)\tilde{\mathcal{A}}_a(x,\boldsymbol{k},\mu_0^2)+ \\ &+\sum_b \Bigg[\int \frac{d^2\boldsymbol{\mu}'}{\pi\mu'^2}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a(\mu'^2,\mu_0^2)}\times \\ &\times\int_x^{z_M(\mu')} dz P_{ab}^{(R)}(z,\alpha_s(\underbrace{q_\perp}))\tilde{\mathcal{A}}_b\left(\frac{x}{z},\boldsymbol{k}+a(z)\boldsymbol{\mu}',\mu'^2\right)\Bigg].\end{aligned} \tag{1}$$



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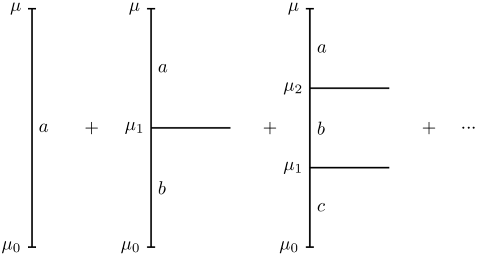
$P_{ab}^{(R)}(z,\alpha_s)$ real emission probability



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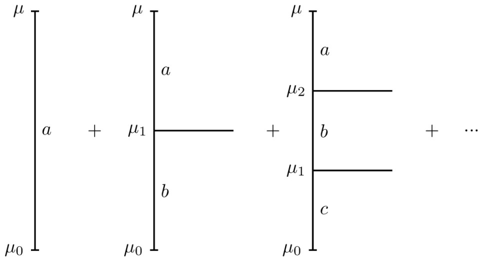
$P_{ab}^{(R)}(z,\alpha_s)$ real emission probability, $\Delta_a(\mu,\mu_0)$ Sudakov (no branching probability)



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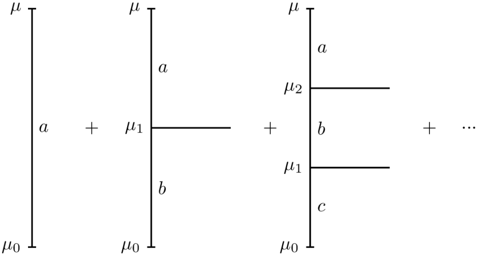
Features with ordering variables: $z_M(\mu')$, $a(z)$ and $b(z)$



The parton branching method: introduction

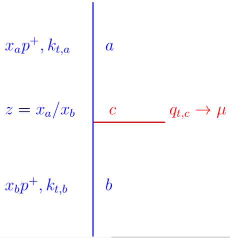
Properties of parton branching:

- Association μ' with emitted transverse momentum $q_\perp$:

$$q_\perp^2 = (1-z)^2 \mu'^2. \qquad (2)$$

Eq. (2) is **angular ordering** condition

[Hautmann, Jung, Lelek, Radescu, Zlebcik, JHEP 1801 (2018) 070]



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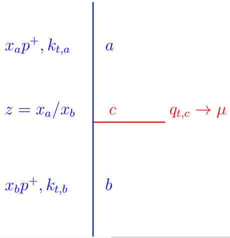

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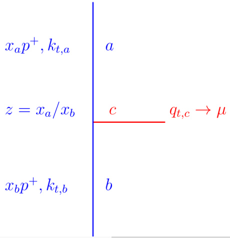

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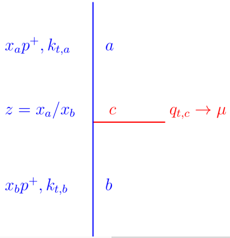

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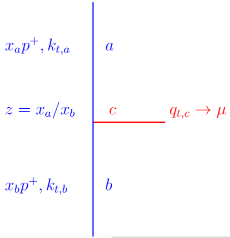
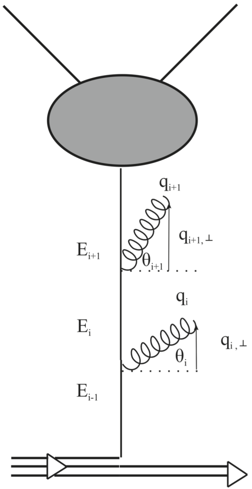
Focus in this work on z_M

[Hautmann, Jung, Lelek, Radescu, Zlebcik, JHEP 1801 (2018) 070]

[Hautmann, Lelek, Keersmaekers, van Kampen, Nucl. Phys. B949 (2019) 114795]



Angular ordering condition



- QCD colour coherence

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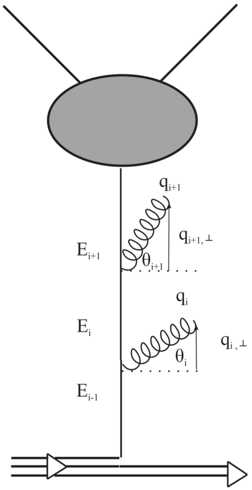

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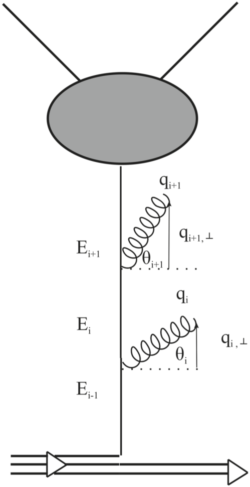
- Order branchings according to their angle θ_i:

$$|q_{\perp,i}| = (1 - z_i)|k_{i-1}| \sin\theta_i \tag{3}$$

Angular ordering condition



- QCD colour coherence
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$$|q_{\perp,i}| = (1 - z_i)|k_{i-1}| \sin\theta_i \tag{3}$$

- Associate the evolution variable μ' with the rescaled transverse momentum $\bar{q}_\perp$:

$$\mu' = \bar{q}_\perp = \frac{q_\perp}{1 - z} \tag{4}$$

$\Rightarrow$ angular ordering condition

Dynamical resolution scale

Resolution parameter z_M separates resolvable (yellow regions) from non-resolvable (grey regions) color radiation

- Past PB calculations: fixed z_M close to 1
- **New: dynamical $z_M(\mu')$**

$$\boxed{z_M(\mu') = 1 - \frac{q_0}{\mu'}} \quad (q_0 = q_{\perp,\text{min}}) \tag{5}$$

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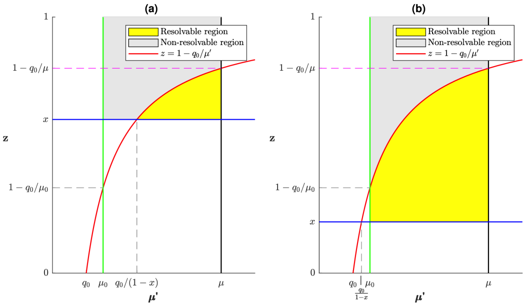

- When $x < 1 - q_0/\mu_0$ (case (b)): evolution goes down to μ_0
- When $x > 1 - q_0/\mu_0$ (case (a)): evolution goes down to $q_0/(1-x)$

Dynamical resolution scale

$$\boxed{z_M(\mu') = 1 - \frac{q_0}{\mu'}}$$

PB equation for integrated distributions $\tilde{f}_a(x,\mu) = \int d^2\boldsymbol{k}\tilde{\mathcal{A}}_a(x,\boldsymbol{k},\mu^2)$

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^1 dz \Theta(\underbrace{1 - q_0/\mu'}_{z_M(\mu')} - z)$$
$$\times \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a(\mu'^2,\mu_0^2)} P_{ab}^{(R)}\left(z, \alpha_s\left((1-z)^2\mu'^2\right)\right) \tilde{f}_b\left(\frac{x}{z},\mu'^2\right). \qquad (6)$$

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- in limit $z_M \to 1$ & $\alpha_s(q_\perp) \to \alpha_s(\mu')$: recover **DGLAP** (see [JHEP 1801 (2018) 070] for detailed numerical calculation)

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- in limit $z_M \to 1$ & $\alpha_s(q_\perp) \to \alpha_s(\mu')$: recover DGLAP (see [JHEP 1801 (2018) 070] for detailed numerical calculation)
- with dynamic z_M: agreement with **CMW**

Comparison with CMW [Catani, Marchesini, Webber, Nucl.Phys. B349 (1991) 635-654]

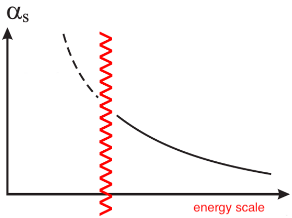
Equation (6) coincides with Markov process based evolution equation from [Marchesini & Webber, Nucl.Phys. B310 (1988) 461-526]

Scale of α_s: avoidence of the Landau pole

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^1 dz \Theta(\underbrace{1 - q_0/\mu'}_{z_M} - z)$$

$$\times \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a(\mu'^2,\mu_0^2)} P_{ab}^{(R)}\left(z, \alpha_s\big(\underbrace{(1-z)^2\mu'^2}_{\text{scale of strong coupling}}\big)\right)\tilde{f}_b\left(\frac{x}{z},\mu'^2\right).$$

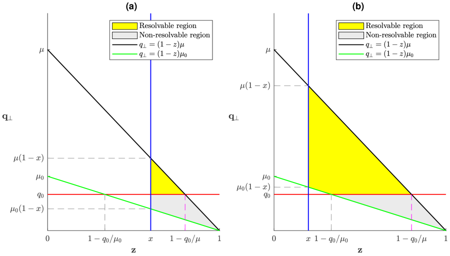
- Scale of strong coupling: $\alpha_s((1-z)^2\mu'^2) = \alpha_s(q_\perp^2)$
- Lowest scale in α_s corresponds to minimal $q_\perp$: $q_{\perp,\mathrm{min}}$
- $q_{\perp,\mathrm{min}} = q_0$, set $q_0 = 1$ GeV $\Longrightarrow$ perturbative calculations are always valid!



$\Lambda_{QCD} \simeq 0.2$ GeV

PB: map evolution variable to transverse momentum

Change phase space variable μ' to $q_\perp$ with angular ordering condition



- Evolution in $q_\perp$ down to q_0 (Red line) for all z (case a)
- When $x < z < 1 - q_0/\mu_0$ (case b): $q_{\perp,\mathrm{min}} = (1-z)\mu_0$ (Green line)

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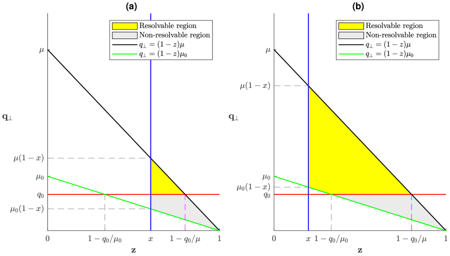

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- When $x < z < 1 - q_0/\mu_0$ (case b): $q_{\perp,\mathrm{min}} = (1-z)\mu_0$ (Green line)
  - Subtraction term arises in evolution equation:

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int \frac{dq_\perp^2}{q_\perp^2} \int_x^1 dz \Big[\Theta(q_\perp^2 - q_0^2)\Theta(\mu^2(1-x)^2 - q_\perp^2)$$

$$\times \Theta(1 - q_\perp/\mu - z) - \Theta(q_\perp^2 - q_0^2)\Theta(\mu_0^2(1-x)^2 - q_\perp^2)\Theta(1 - q_\perp/\mu_0 - z)\Big]$$

$$\times \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^{(R)}\left(z, \alpha_s(q_\perp^2)\right) \tilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right) \quad (7)$$

Single emission $\leftrightarrow$ multiple emission

	Multiple emission	**Single emission**
(z, μ')	CMW/PB	
$(z, q_\perp)$	PB	KMRW

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	Multiple emission	Single emission
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Kimber Martin Ryskin Watt (KMRW) approach constructs TMDs in a **single step**

[Kimber, Martin, Ryskin, Phys.Rev. D63 (2001) 114027]

[Watt, Martin, Ryskin, Eur.Phys.J. C31 (2003) 73-89]

[Martin, Ryskin, Watt, Eur.Phys.J. C66 (2010) 163-172]

$$\tilde{f}_a(x,\mu^2) = T_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{q^2_{\perp,M}} \frac{dq_\perp^2}{q_\perp^2} \Big\{ \underbrace{T_a(\mu^2,q_\perp^2) \int_x^{1-C(q_\perp,\mu)} dz P_{ab}^{(R)}\left(z,\alpha_s(q_\perp^2)\right) \tilde{f}_b\left(\frac{x}{z},q_\perp^2\right)}_{\text{TMD: } \tilde{f}_a(x,q_\perp^2,\mu)} \Big\} \quad (8)$$

[Golec-Biernat, Stasto, Phys.Lett. B781 (2018) 633-638], [Guiot, e-Print: arXiv:1910.09656]

Ordering condition sets cut-off in $q_\perp$ and z integration

- Strong ordering (SO): $C(q_\perp,\mu) = \frac{q_\perp}{\mu}$, $q_{\perp,M} = \mu(1-x)$
- Angular ordering (AO): $C(q_\perp,\mu) = \frac{q_\perp}{q_\perp+\mu}$, $q_{\perp,M} = \frac{\mu(1-x)}{x}$

Comparison KMRW and PB

KMRW

$$\tilde{f}_a(x,\mu^2) = T_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{q_{\perp,M}^2} \frac{dq_\perp^2}{q_\perp^2} \int_x^{1-C(q_\perp,\mu)} dz T_a(\mu^2,q_\perp^2) P_{ab}^{(R)}\left(z,\alpha_s(q_\perp^2)\right)\tilde{f}_b\left(\frac{x}{z},q_\perp^2\right) \quad (9)$$

Parton branching (angular ordering & dynamic z_M)

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{q_0^2}^{\mu^2(1-x)^2} \frac{dq_\perp^2}{q_\perp^2} \int_x^{1-\frac{q_\perp}{\mu}} dz \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a\left(\frac{q_\perp^2}{(1-z)^2},\mu_0^2\right)} P_{ab}^{(R)}\left(z,\alpha_s(q_\perp^2)\right)\tilde{f}_b\left(\frac{x}{z},\frac{q_\perp^2}{(1-z)^2}\right) \quad (10)$$

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- KMRW has no rescaling in:
  a) initial distributions $\tilde{f}_b$, b) Sudakov form factors $T_a \leftrightarrow \Delta_a$

Comparison KMRW and PB

KMRW

$$\tilde{f}_a(x,\mu^2) = T_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{q^2_{\perp,M}} \frac{dq_\perp^2}{q_\perp^2} \int_x^{1-C(q_\perp,\mu)} dz\, T_a(\mu^2, q_\perp^2) P_{ab}^{(R)}\left(z, \alpha_s(q_\perp^2)\right) \tilde{f}_b\left(\frac{x}{z}, q_\perp^2\right) \quad (9)$$

Parton branching (angular ordering & dynamic z_M)

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{q_0^2}^{\mu^2(1-x)^2} \frac{dq_\perp^2}{q_\perp^2} \int_x^{1-\frac{q_\perp}{\mu}} dz \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a\left(\frac{q_\perp^2}{(1-z)^2},\mu_0^2\right)} P_{ab}^{(R)}\left(z,\alpha_s(q_\perp^2)\right)\tilde{f}_b\left(\frac{x}{z},\frac{q_\perp^2}{(1-z)^2}\right) \quad (10)$$

- KMRW has no rescaling in:
  a) initial distributions $\tilde{f}_b$, b) Sudakov form factors $T_a \leftrightarrow \Delta_a$
- Differences in phase space limits (mostly pronounced in AO)

Remarks on the Sudakov form factor

Parton branching

Sudakov is no resolvable branching probability:

$$\Delta_a(\mu^2, \mu_0^2) = \exp\left\{ - \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{1-q_0/\mu'} dz \; z \; P_{ba}^{(R)}(\alpha_s((1-z)^2\mu'^2), z) \right\}. \quad (11)$$

Product of two is:

$$\Delta_a(\mu^2, \tilde{\mu}^2)\Delta_a(\tilde{\mu}^2, \mu_0^2) = \Delta_a(\mu^2, \mu_0^2) \quad (12)$$

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KMRW

Sudakov of KMRW:

$$T_a(\mu^2, \mu_0^2) = \exp\left\{ -\sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \int_0^{1-C(q_\perp,\mu)} dz \; z \; P_{ba}^{(R)}(z, q_\perp) \right\}. \quad (13)$$

Product of two is:

$$T_a(\mu^2, k_\perp^2)\, T_a(k_\perp^2, \mu_0^2) = \quad (14)$$

$$= T_a(\mu^2, \mu_0^2) \times \exp\left\{ \sum_b \int_{\mu_0^2}^{k_\perp^2} \frac{dq_\perp^2}{q_\perp^2} \int_{1-C(q_\perp,k_\perp)}^{1-C(q_\perp,\mu)} dz \; z \; P_{ba}^{(R)}(z, \alpha_s(q_\perp^2)) \right\}$$

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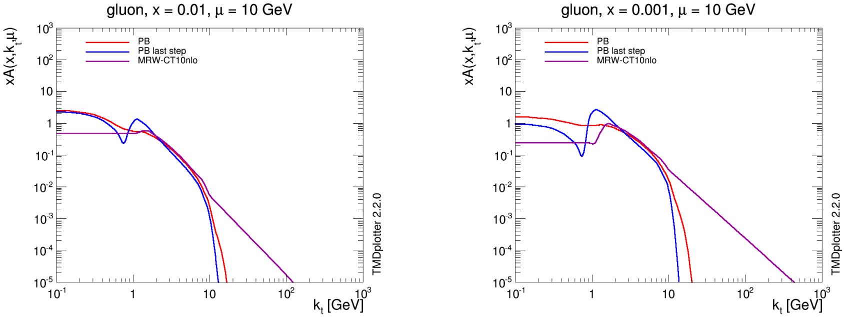
$\Rightarrow$ differences in the treatment of virtual and non-resolvable emissions!

Numerical comparison: TMDs

Gluon TMD[1] versus $k_\perp$, $\mu = 10$ GeV



MRW-CT10nlo is KMRW TMD constructed from integrated PDF CT10nlo

[1] PB last step is an invented way to simulate PB with a single emission

Numerical comparison: TMDs

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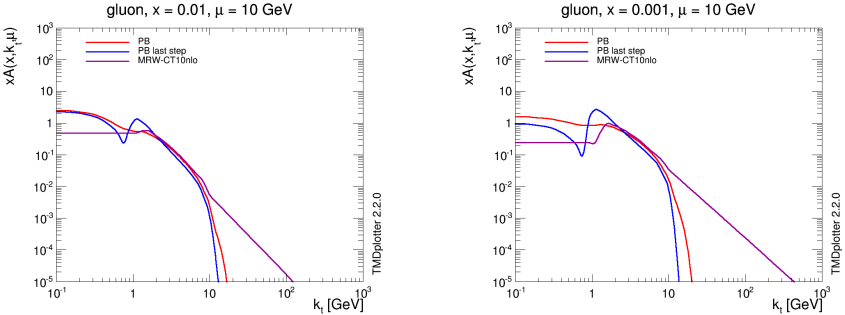

MRW-CT10nlo is KMRW TMD constructed from integrated PDF CT10nlo

- low $k_\perp$: bump at lower scale from single emission

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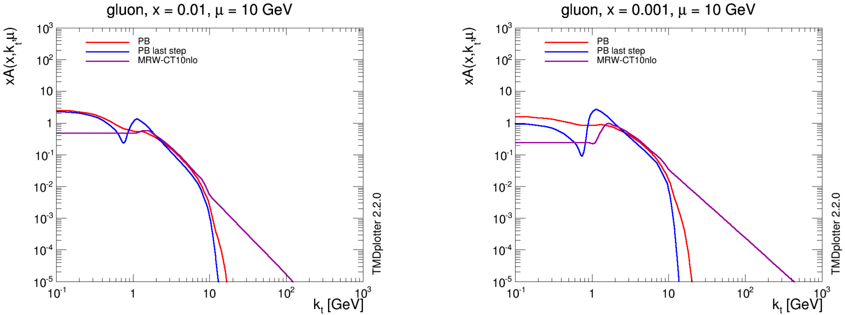

MRW-CT10nlo is KMRW TMD constructed from integrated PDF CT10nlo

- low $k_\perp$: bump at lower scale from single emission
- middle $k_\perp$: agreement KMRW and PB

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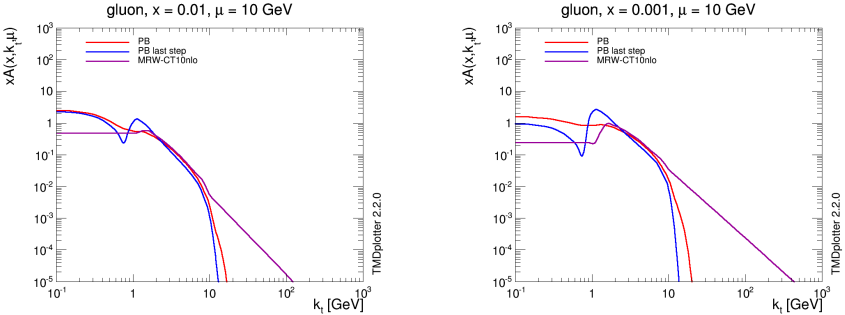


MRW-CT10nlo is KMRW TMD constructed from integrated PDF CT10nlo

- low $k_\perp$: bump at lower scale from single emission
- middle $k_\perp$: agreement KMRW and PB
- high $k_\perp$ tail from radiative effects + Sudakov

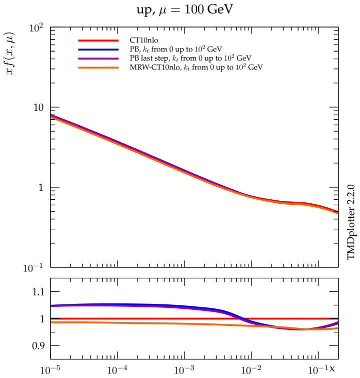
[1] PB last step is an invented way to simulate PB with a single emission

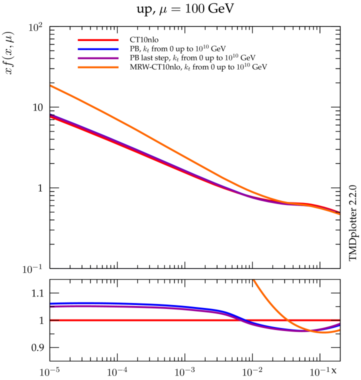
Numerical comparison: integrated TMDs

Up quark TMD integrated over $k_\perp$, $\mu = 100$ GeV

$$xf(x,\mu) = \int_0^{\mu} dk_\perp \tilde{A}(x,k_\perp,\mu)$$

$$xf(x,\mu) = \int_0^{\infty} dk_\perp \tilde{A}(x,k_\perp,\mu)$$

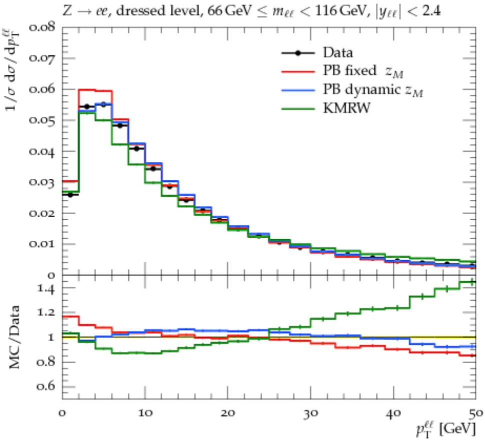




- KMRW integrated over full $k_\perp$ domain deviates strongly from CT10nlo

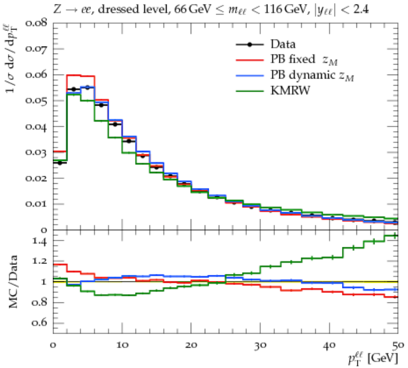
Z boson p_T spectrum

- Compare simulations with LHC data (ATLAS, $\sqrt{s} = 8$ GeV [Eur.Phys.J. C76 (2016) no.5, 291])
- Compare PB fixed z_M, PB dynamical z_M and KMRW



Z boson p_T spectrum

Use of dynamical resolution scale improves shape of DY spectrum!



- Fixed z_M: model for freezing scale in α_s
- Dynamic z_M: $q_\perp > q_0$, automatically avoids Landau pole region
- KMRW does not describe shape, both at low and high p_T

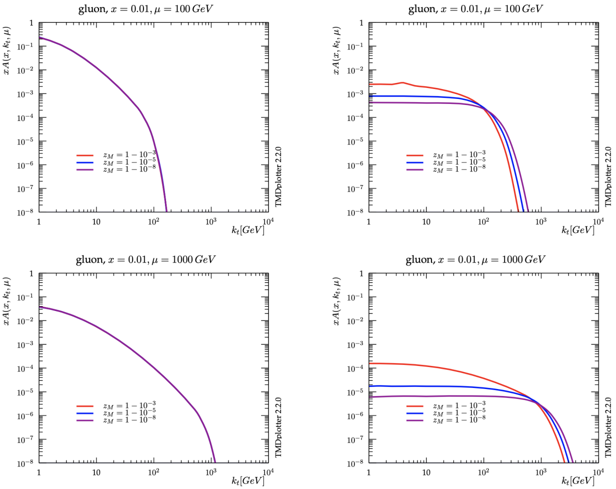
Conclusion

PB approach for TMDs takes into account soft-gluon emissions ($z \to 1$) and all transverse momenta ($q_\perp$) from branchings in QCD evolution

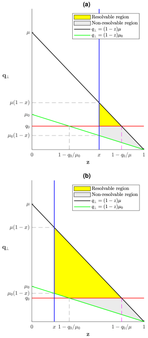
- We performed studies of PB TMDs with dynamical z_M for the first time
- PB with dynamical resolution scale:
  - Coincides with CMW at integrated level
  - Differs significantly from KMRW both at low and high $k_\perp$
- We found sensitivity of the low p_T DY spectrum at LHC to dynamical resolution scale
- *Future: fit PB TMDs with dynamic z_M to DIS and DY*

Backup

Ordering condition, resolution scale dependence



PB mapping μ' to $q_\perp$



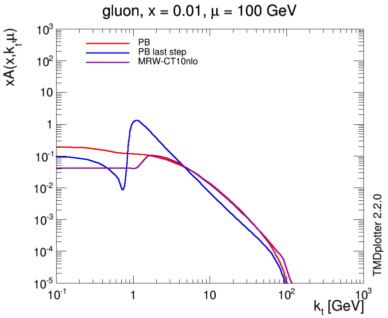
high x

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{q_0^2}^{\mu^2(1-x)^2} \frac{dq_\perp^2}{q_\perp^2} \int_x^1 dz \\ \times \Theta(1 - q_\perp/\mu - z)\Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) \\ \times P_{ab}^{(R)}\left(z, \alpha_s(q_\perp^2)\right)\tilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right) \quad (15)$$

low x (with subtraction term)

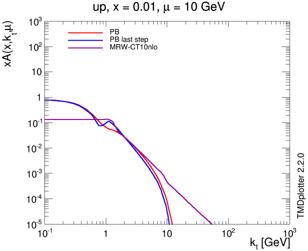
$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{q_0^2} \frac{dq_\perp^2}{q_\perp^2} \int_x^1 dz \\ \times \left[\Theta(\mu^2(1-x)^2 - q_\perp^2)\Theta(1 - q_\perp/\mu - z) - \Theta(\mu_0^2(1-x)^2 - q_\perp^2)\Theta(1 - q_\perp/\mu_0 - z)\right] \\ \times \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^{(R)}\left(z, \alpha_s(q_\perp^2)\right)\tilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right) \quad (16)$$

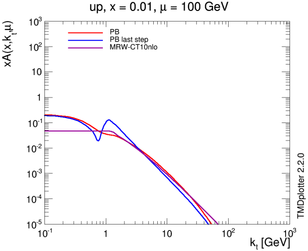
Gluon TMDs $\mu = 100$ GeV



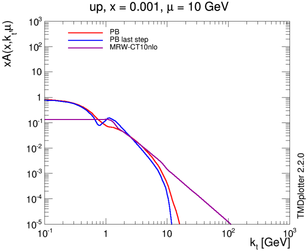
Up quark TMDs

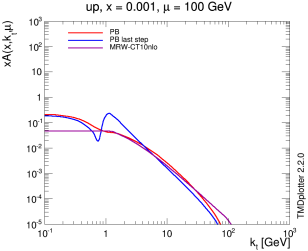
$x = 0.01$





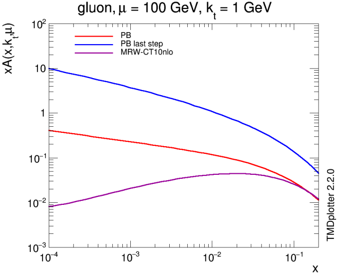
$x = 0.001$

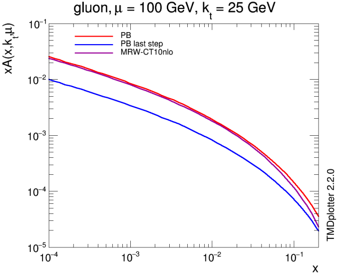


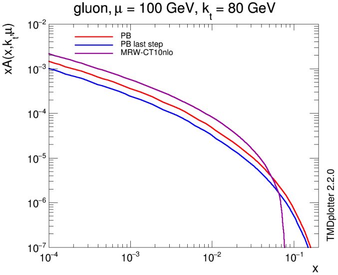


TMDs versus x

Gluon TMD versus x, $\mu = 100$ GeV







Middle $k_\perp$ (~ 25 GeV) $\rightarrow$ good agreement PB and KMRW