Photoproduction of 3 jets in TMD factorization from the CGC

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QCD at high energies

The energy is the largest scale in the process

$$s \gg Q^2$$

Need to resum large logarithms

$$\ln \frac{s}{Q^2} \simeq \ln \frac{1}{x} \gg 1$$

Low-x evolution equation (BFKL) predicts very fast growth of the gluon distribution:

$$x\mathcal{G}(x, Q^2) \simeq \frac{1}{x^{2.77 \times \frac{\alpha_s N_c}{\pi}}}$$

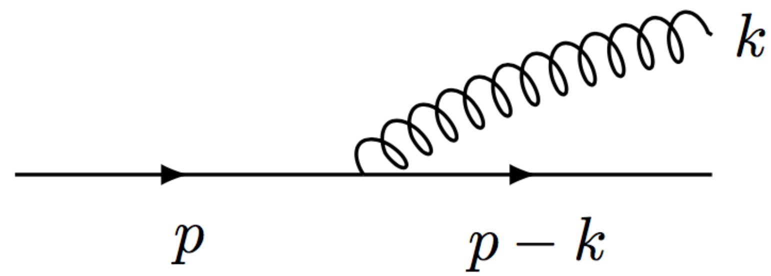
At high enough density, semiclassical regime is reached where

$$g_s A \sim 1$$

This regime is characterized by dynamically generated hard saturation scale Q_s, low-x evolution becomes nonlinear: BK-JIMWLK equations

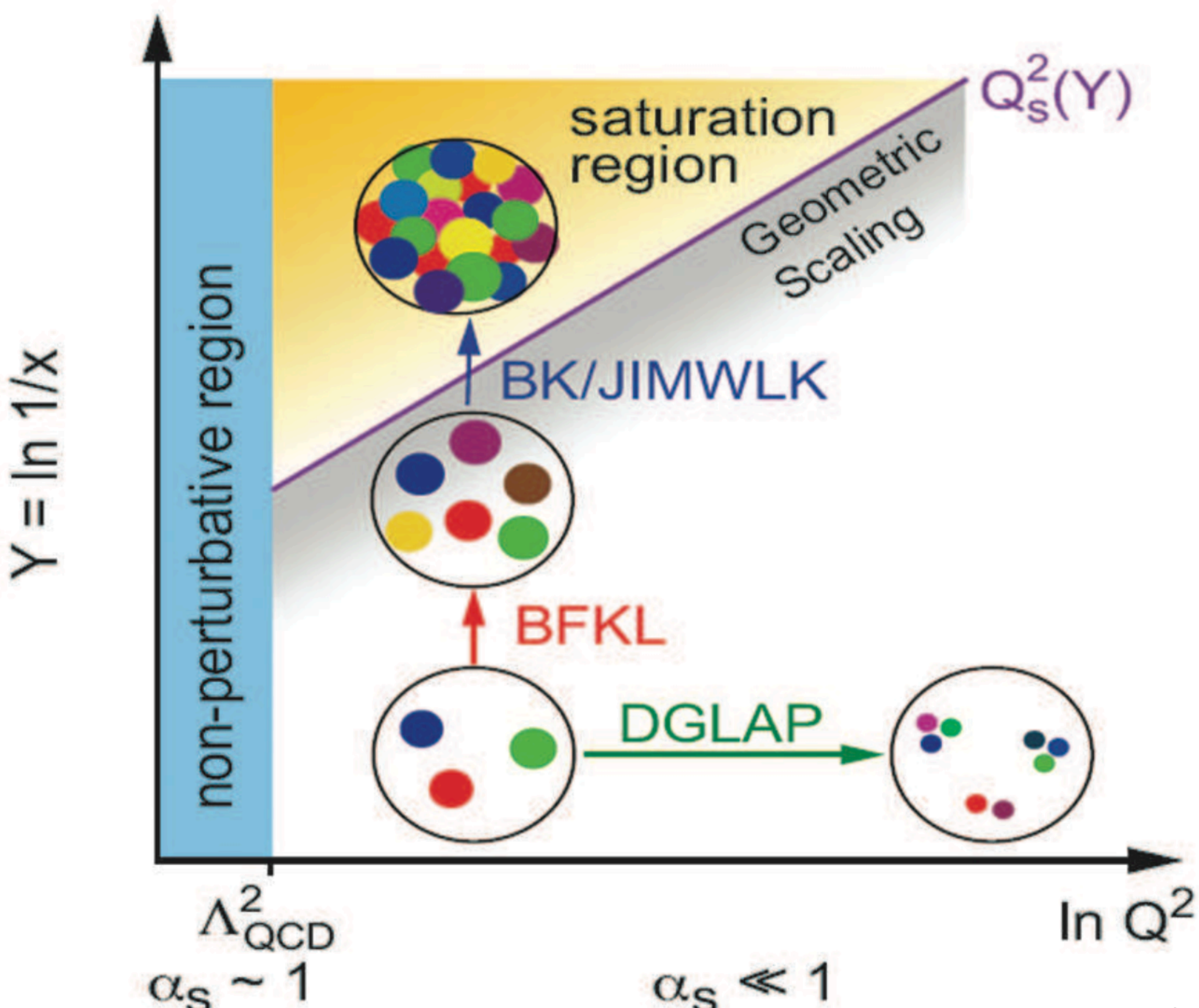
QCD at high energies

Bremsstrahlung law



$$\mathrm{d}P \simeq \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}x}{x} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

$$x = k^+/p^+$$



$$\alpha_s \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \alpha_s \ln \frac{Q^2}{\mu_0^2} \quad \text{collinear}$$

$$\alpha_s \int_x^1 \frac{\mathrm{d}z}{z} = \alpha_s \ln \frac{1}{x} \quad \text{soft}$$

Gluon TMDs at high energies

What if we measure another, smaller scale?

$$s \gg Q^2 \gg k^2 \sim Q_s^2$$

For 2→2 processes in unpolarized collisions at low x, at LO the CGC and TMD frameworks yield *exactly* the same result.

CGC framework (low-*x*)	TMD framework (Q>>k)
CGC calculation	Calculate collinear hard parts
Correlation limit	Insert gauge links (Bomhof, Mulders, Pijlman 2006)
	Take low-*x* limit
Low-*x* TMD expression	

Dominguez, Marquet, Xiao, Yuan (2011)

Three-jet photoproduction at low x

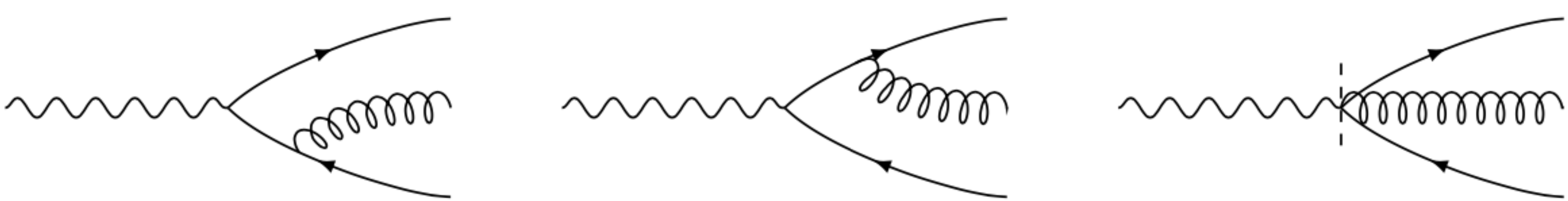
Cross section is given by:

$$2k_1^+ 2k_2^+ 2k_3^+ (2\pi)^9 2\pi\delta\Big(p^+ - \sum_{j=1}^{3} k_j^+\Big) 2p^+ \frac{\mathrm{d}\sigma^{\gamma A \to q g \bar{q} + X}}{\mathrm{d}^3\vec{k}_1\,\mathrm{d}^3\vec{k}_2\,\mathrm{d}^3\vec{k}_3}$$

$$= \frac{1}{2}\,{}_{\text{out}}\Big\langle (\boldsymbol{\gamma})\big[\vec{p}\big]_\lambda \Big| N_q(\vec{k}_1) N_g(\vec{k}_2) N_{\bar{q}}(\vec{k}_3) \Big| (\boldsymbol{\gamma})\big[\vec{p}\big]_\lambda \Big\rangle_{\text{out}}$$

Expansion of the photon Fock state

$$|\boldsymbol{\gamma}\rangle_{\text{dressed}} = Z_0 |\boldsymbol{\gamma}\rangle_0 + g_e Z_1 |\mathbf{q}\bar{\mathbf{q}}\rangle_0 + g_e g_s Z_2 |\mathbf{q}\bar{\mathbf{q}}\mathbf{g}\rangle_0 + \mathcal{O}$$

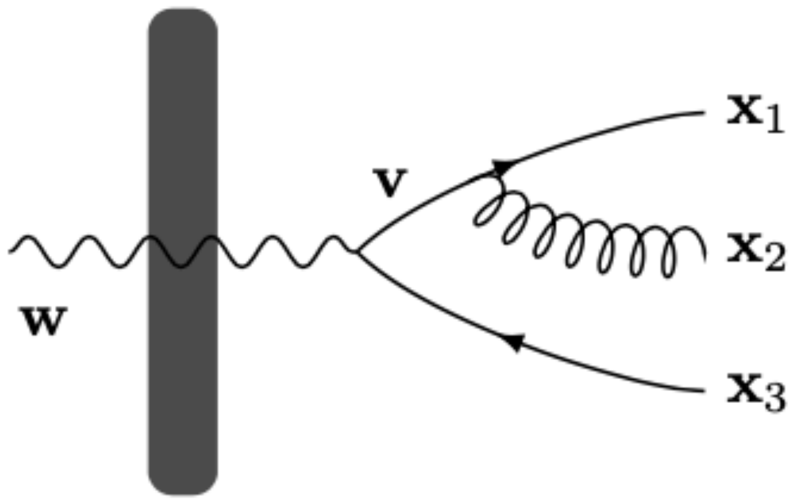


Eikonal approximation

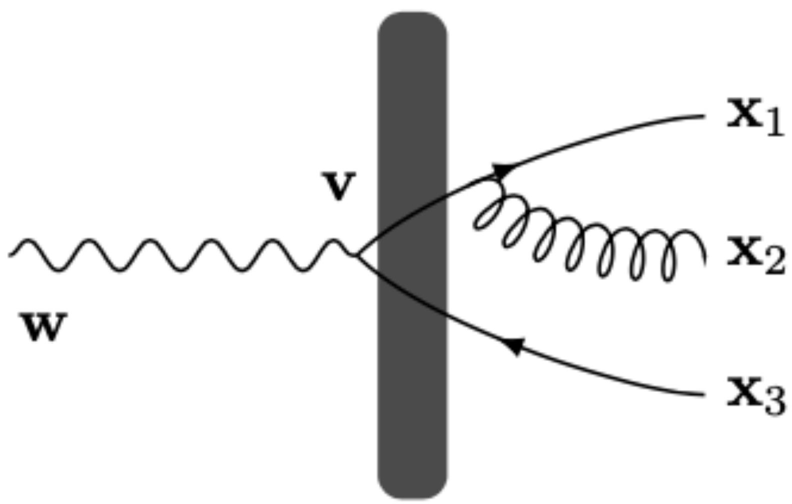
In our frame, dressed photon hits Lorentz contracted proton or nucleus target = ‘shockwave’

Wilson line:

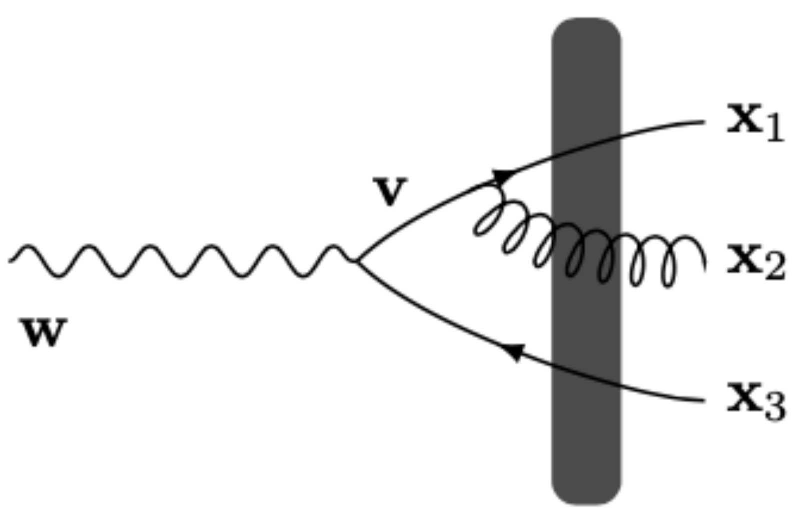
$$S_{F,A}(\mathbf{x}) = \mathcal{P} e^{ig_s \int \mathrm{d}x^+ A_c^-(\mathbf{x}^+,\mathbf{x}) t^c_{F,A}}$$



$$\mathbb{1}$$



$$t^c S_F(\mathbf{v}) S_F^\dagger(\mathbf{x}_3)$$



$$S_F(\mathbf{x}_1) t^d S_A^{dc}(\mathbf{x}_2) S_F^\dagger(\mathbf{x}_3)$$

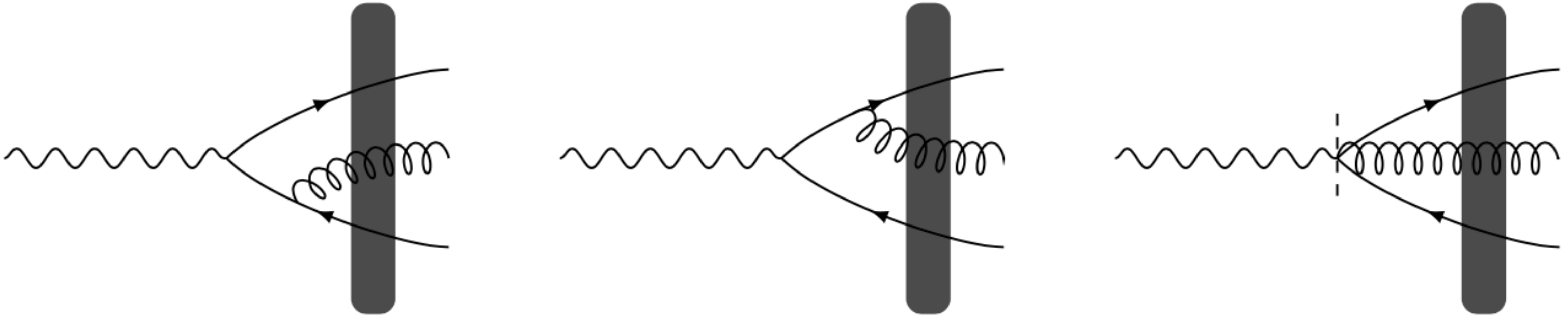
See Florian’s talk for subeikonal corrections

Outgoing photon state

$$
\begin{aligned}
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\rm out} &= g_e g_s \int \frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}k_2^+}{2\pi}\int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} |(\mathbf{q})[k_1^+,\mathbf{x}_1]_s^i;(\mathbf{g})[k_2^+,\mathbf{x}_2]_c^\eta;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]_{s'}^j\rangle_D \\
&\times\Bigg\{\Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij}S_A(\mathbf{x}_2)^{dc} - t^c_{ij}\Big) \\
&\quad\times\big(F_q^{(2)}+F_{\bar{q}}^{(2)}+F_C^{(2)}\big)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\eta\lambda}_{s's} \\
&-\int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij}-t^c_{ij}\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^++k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^\lambda_{\bar{s}s'} \\
&\quad\times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]^\eta_{s\bar{s}} \\
&-\int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij}-t^c_{ij}\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^++k_3^+,\mathbf{w}-\mathbf{v}]\Big]^\lambda_{s\tilde{s}} \\
&\quad\times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]^\eta_{s'\tilde{s}}\Bigg\}
\end{aligned}
$$

Outgoing photon state

$$
\begin{aligned}
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\text{out}} = g_e g_s \int \frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}k_2^+}{2\pi}\int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} & |(\mathbf{q})[k_1^+,\mathbf{x}_1]_s^i;(\mathbf{g})[k_2^+,\mathbf{x}_2]_c^\eta;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]_{s'}^j\rangle_D \\
\times \Bigg\{ & \left(\left[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\right]_{ij} S_A(\mathbf{x}_2)^{dc} - t_{ij}^c\right) \\
& \times \left(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\right)\left[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\right]_{s's}^{\eta\lambda} \\
& - \int_{\mathbf{v}}\left(\left[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\right]_{ij} - t_{ij}^c\right) F_\gamma^{(1)}\left[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\right]_{\bar{s}s'}^{\lambda} \\
& \quad \times F_q^{(1)}\left[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\right]_{s\bar{s}}^{\eta} \\
& - \int_{\mathbf{v}}\left(\left[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\right]_{ij} - t_{ij}^c\right) F_\gamma^{(1)}\left[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\right]_{s\tilde{s}}^{\lambda} \\
& \quad \times F_{\bar{q}}^{(1)}\left[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\right]_{s'\tilde{s}}^{\eta}\Bigg\}
\end{aligned}
$$



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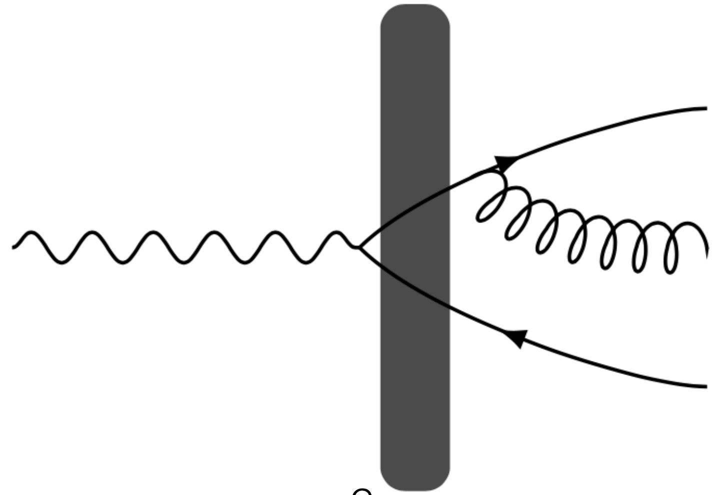
\times \Big\{ & \Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t^c_{ij}\Big) \\
& \times \big(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\big)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\eta\lambda}_{s's} \\
- & \int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} - t^c_{ij}\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\lambda}_{\bar{s}s'} \\
& \times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]^{\eta}_{s\bar{s}} \\
- & \int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} - t^c_{ij}\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]^{\lambda}_{s\tilde{s}} \\
& \times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]^{\eta}_{s'\tilde{s}}\Big\}
\end{aligned}
$$



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& \times\Big\{\Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij}S_A(\mathbf{x}_2)^{dc}-t^c_{ij}\Big) \\
& \quad\times\big(F_q^{(2)}+F_{\bar{q}}^{(2)}+F_C^{(2)}\big)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\eta\lambda}_{s's} \\
& -\int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij}-t^c_{ij}\Big)F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^++k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^\lambda_{\bar{s}s'} \\
& \quad\times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]^\eta_{s\bar{s}} \\
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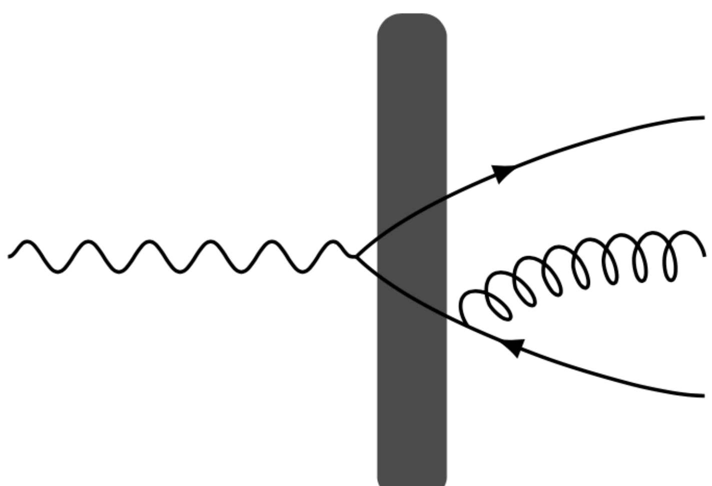
& \quad\times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]^\eta_{s'\tilde{s}}\Big\}
\end{aligned}
$$



Differential $\gamma p \to q\bar{q}g$ cross section

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A \to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1 \mathrm{d}^3\vec{k}_2 \mathrm{d}^3\vec{k}_3} = g_e^2 g_s^2 \frac{1}{k_2^+ p^+} 2\pi\delta\Big(p^+ - \sum_{i=1}^{3} k_i^+\Big) \times \Big\langle I_{qq} + I_{\bar{q}\bar{q}} + I_{CC} + 2I_{q\bar{q}} + 2I_{Cq} + 2I_{C\bar{q}} \Big\rangle_{x_A}$$

Differential $\gamma p \to q\bar{q}g$ cross section

$$
\begin{aligned}
\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2} \mathcal{M}_{qq}^{\bar\eta\bar\eta';\bar\lambda\bar\lambda'}\left(\bar\xi_3, \frac{\xi_2}{\bar\xi_3}\right) \int_{\mathbf{v}\mathbf{v}'} \prod_{i=1}^{3} \int_{\mathbf{x}_i\mathbf{x}_i'} e^{i\mathbf{k}_i\cdot(\mathbf{x}_i'-\mathbf{x}_i)} \\
\times A^{\bar\eta}(\mathbf{x}_1-\mathbf{x}_2) A^{\bar\eta'}(\mathbf{x}_1'-\mathbf{x}_2')\, \delta^{(2)}\left(\mathbf{v} - \frac{\xi_1}{\bar\xi_3}\mathbf{x}_1 - \frac{\xi_2}{\bar\xi_3}\mathbf{x}_2\right) \delta^{(2)}\left(\mathbf{v}' - \frac{\xi_1}{\bar\xi_3}\mathbf{x}_1' - \frac{\xi_2}{\bar\xi_3}\mathbf{x}_2'\right) \\
\times \Bigg\{ \left[W_1(\mathbf{x}_1.\mathbf{x}_2;\mathbf{x}_2',\mathbf{x}_1'|\mathbf{x}_2,\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_2') - \frac{1}{N_c^2} W_3(\mathbf{x}_1,\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_1') \right] \\
\times \mathcal{A}^{\bar\lambda}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar\xi_3}, \mathbf{x}_1 - \mathbf{x}_2\right) \mathcal{A}^{\bar\lambda'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar\xi_3}, \mathbf{x}_1' - \mathbf{x}_2'\right) \\
- \left[W_2(\mathbf{x}_1,\mathbf{x}_2|\mathbf{x}_2,\mathbf{x}_3;\mathbf{x}_3',\mathbf{v}') - \frac{1}{N_c^2} W_3(\mathbf{x}_1,\mathbf{x}_3;\mathbf{x}_3',\mathbf{v}') \right] \\
\times \mathcal{A}^{\bar\lambda}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar\xi_3}, \mathbf{x}_1 - \mathbf{x}_2\right) A^{\bar\lambda'}(\mathbf{x}_3' - \mathbf{v}') \\
- \left[W_2(\mathbf{x}_2',\mathbf{x}_1'|\mathbf{v},\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_2') - \frac{1}{N_c^2} W_3(\mathbf{v},\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_1') \right] \\
\times A^{\bar\lambda}(\mathbf{x}_3 - \mathbf{v}) \mathcal{A}^{\bar\lambda'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar\xi_3}, \mathbf{x}_1' - \mathbf{x}_2'\right) \\
- \left(1 - \frac{1}{N_c^2}\right) W_3(\mathbf{v},\mathbf{x}_3;\mathbf{x}_3',\mathbf{v}')\, A^{\bar\lambda}(\mathbf{x}_3 - \mathbf{v}) A^{\bar\lambda'}(\mathbf{x}_3' - \mathbf{v}') \Bigg\}
\end{aligned}
$$

Differential $\gamma p \to q\bar{q}g$ cross section

$$\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2} \mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\left(\bar{\xi}_3, \frac{\xi_2}{\bar{\xi}_3}\right) \int_{\mathbf{v}\mathbf{v}'} \prod_{i=1}^{3} \int_{\mathbf{x}_i\mathbf{x}_i'} e^{i\mathbf{k}_i\cdot(\mathbf{x}_i'-\mathbf{x}_i)}$$

$$\times A^{\bar{\eta}}(\mathbf{x}_1 - \mathbf{x}_2) A^{\bar{\eta}'}(\mathbf{x}_1' - \mathbf{x}_2')\, \delta^{(2)}\left(\mathbf{v} - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1 - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2\right) \delta^{(2)}\left(\mathbf{v}' - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1' - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2'\right)$$

$$\times \left\{\left[W_1(\mathbf{x}_1.\mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1' | \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2') - \frac{1}{N_c^2} W_3(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1')\right]\right.$$

$$\times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right)$$

$$+ \ldots$$

W's are combinations of dipoles and quadrupoles:

$$s(\mathbf{x}, \mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[S_F(\mathbf{x}) S_F^\dagger(\mathbf{y})\right]\right\rangle_{x_A},$$

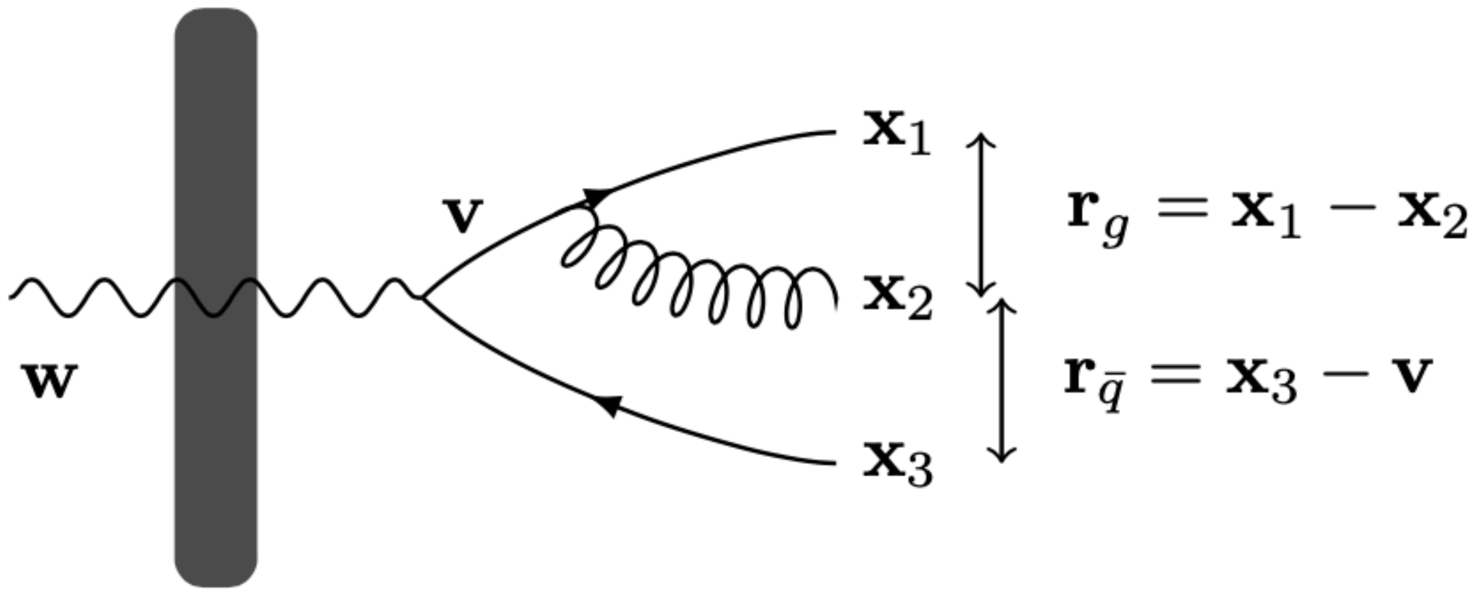
$$Q(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[S_F(\mathbf{x}) S_F^\dagger(\mathbf{y}) S_F(\mathbf{u}) S_F^\dagger(\mathbf{v})\right]\right\rangle_{x_A}$$

$\langle \ldots \rangle_{x_A}$ is the average over the semiclassical gluon fields of the target → contains the nonperturbative information on the hadron structure

Correlation limit

Defining the total transverse momentum: $\mathbf{q}_T \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$,
study the limit $|\mathbf{q}_T| \ll |\mathbf{k}_1| \sim |\mathbf{k}_2| \sim |\mathbf{k}_3|$

$$|\mathbf{k}_i| \gg 1 \leftrightarrow |\mathbf{r}_i| \ll 1$$



Perform Taylor expansion around small dipole sizes $\mathbf{r}$
→ derivatives enter Wilson line structures:

$$\mathrm{Tr}\langle S_F^\dagger(\mathbf{x})[\partial_i S_F(\mathbf{x})]S_F^\dagger(\mathbf{y})[\partial_j S_F(\mathbf{y})]\rangle_x$$

Emergence of gluon TMDs

Derivative on Wilson line → field strength

$$\partial_i S_F(\mathbf{x}) = \partial_i \mathcal{P} e^{ig_s \int_{-\infty}^{+\infty} \mathrm{d}x^+ A^-(x^+,\mathbf{x})}$$
$$= ig_s \int \mathrm{d}x^+ S_F(\mathbf{x}; -\infty, x^+) F^{i-}(x^+, \mathbf{x}) S_F(\mathbf{x}; x^+, +\infty)$$

After some algebra:

$$\mathrm{Tr}\langle S_F^\dagger(\mathbf{x})[\partial_i S_F(\mathbf{x})] S_F^\dagger(\mathbf{y})[\partial_j S_F(\mathbf{y})]\rangle_x$$
$$\lim \overset{x_A \to 0}{=} (ig_s)^2 \delta(0^+) \int \mathrm{d}r^+ e^{ixP^- r^+} \mathrm{Tr}\langle U_{\mathbf{r}}^{[+]} F^{i-}(r^+, \mathbf{r}) U_{\mathbf{r}}^{[+]\dagger} F^{j-}(0^+, \mathbf{0})\rangle_x$$

Mulders and Rodrigues (2001)

…and finally:

$$\int_{\mathbf{x}_2 \mathbf{x}_2'} e^{-i\mathbf{q}_T \cdot (\mathbf{x}-\mathbf{y})} \mathrm{Tr}\Big\langle S_F^\dagger(\mathbf{x}) \big[\partial_i S_F(\mathbf{x})\big] S_F^\dagger(\mathbf{y}) \big[\partial_j S_F(\mathbf{y})\big] \Big\rangle_{x_A}$$
$$= -g_s^2 \, (2\pi)^3 \, \frac{1}{4} \left[\frac{1}{2} \delta^{ij} \mathcal{F}_{gg}^{(3)}(x_A, \mathbf{q}_T) + \frac{1}{2} \left(2 \frac{\mathbf{q}_T^i \mathbf{q}_T^j}{\mathbf{q}_T^2} - \delta^{ij} \right) \mathcal{H}_{gg}^{(3)}(x_A, \mathbf{q}_T) \right]$$

$$\mathcal{F}_{gg}^{(3)}(x_A, \mathbf{q}_T) = f_1^g(x_A, \mathbf{q}_T)$$
$$\mathcal{H}_{gg}^{(3)}(x_A, \mathbf{q}_T) = \frac{\mathbf{q}_T^2}{2M_p^2} h_1^{g\perp}(x_A, \mathbf{q}_T)$$

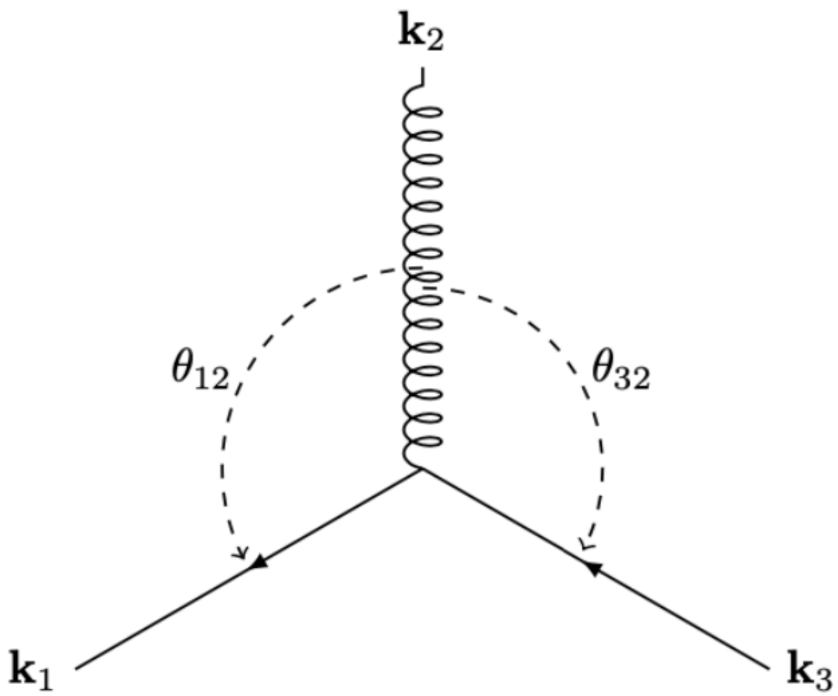
Unpolarized and linearly polarized gluon TMDs of the Weizsäcker-Williams type

Final result

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A\to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1\mathrm{d}^3\vec{k}_2\mathrm{d}^3\vec{k}_3}\bigg|_{\text{corr. limit}} = 2\pi\delta\Big(p^+ - \sum_{i=1}^{3} k_i^+\Big)\,[\mathrm{H}]_{ij}^{\text{total}} \times\left[\frac{1}{2}\delta^{ij}\mathcal{F}_{gg}^{(3)}(x_A,\mathbf{q}_T) + \frac{1}{2}\Big(2\frac{\mathbf{q}_T^i\mathbf{q}_T^j}{\mathbf{q}_T^2} - \delta^{ij}\Big)\mathcal{H}_{gg}^{(3)}(x_A,\mathbf{q}_T)\right]$$

$$\begin{aligned}[\mathrm{H}]_{ij}^{\text{total}} &= N_c\, g_e^2\, g_s^4\, \pi^3\, \frac{1}{k_2^+ p^+}\Big\{\mathcal{M}_{qq}^{\bar\eta\bar\eta';\bar\lambda\bar\lambda'}\Big(\bar\xi_3,\frac{\xi_2}{\bar\xi_3}\Big)\,[\mathrm{H}_{qq}]_{ij}^{\bar\lambda\bar\lambda';\bar\eta\bar\eta'} + \mathcal{M}_{\bar q\bar q}^{\bar\eta\bar\eta';\bar\lambda\bar\lambda'}\Big(\xi_1,\frac{\xi_2}{\bar\xi_1}\Big)\,[\mathrm{H}_{\bar q\bar q}]_{ij}^{\bar\lambda\bar\lambda';\bar\eta\bar\eta'} \\ &+2\,\mathcal{M}_{q\bar q}^{\bar\eta\bar\eta';\bar\lambda\bar\lambda'}(\xi_1,\xi_2)\,[\mathrm{H}_{q\bar q}]_{ij}^{\bar\lambda\bar\lambda';\bar\eta\bar\eta'} + \mathcal{M}_{CC}(\xi_1,\xi_2)\frac{1}{\xi_3^2\bar\xi_3^2}[\mathrm{H}_{CC}]_{ij} \\ &+2\,\mathcal{M}_{Cq}(\xi_1,\xi_2)\,\frac{2}{\xi_3\bar\xi_3}\,[\mathrm{H}_{Cq}]_{ij}^{\bar\lambda\bar\eta} + 2\,\mathcal{M}_{C\bar q}(\xi_1,\xi_2)\,\frac{2}{\xi_3\bar\xi_3}\,[\mathrm{H}_{C\bar q}]_{ij}^{\bar\lambda\bar\eta}\Big\}\end{aligned}$$

Final result in MV model

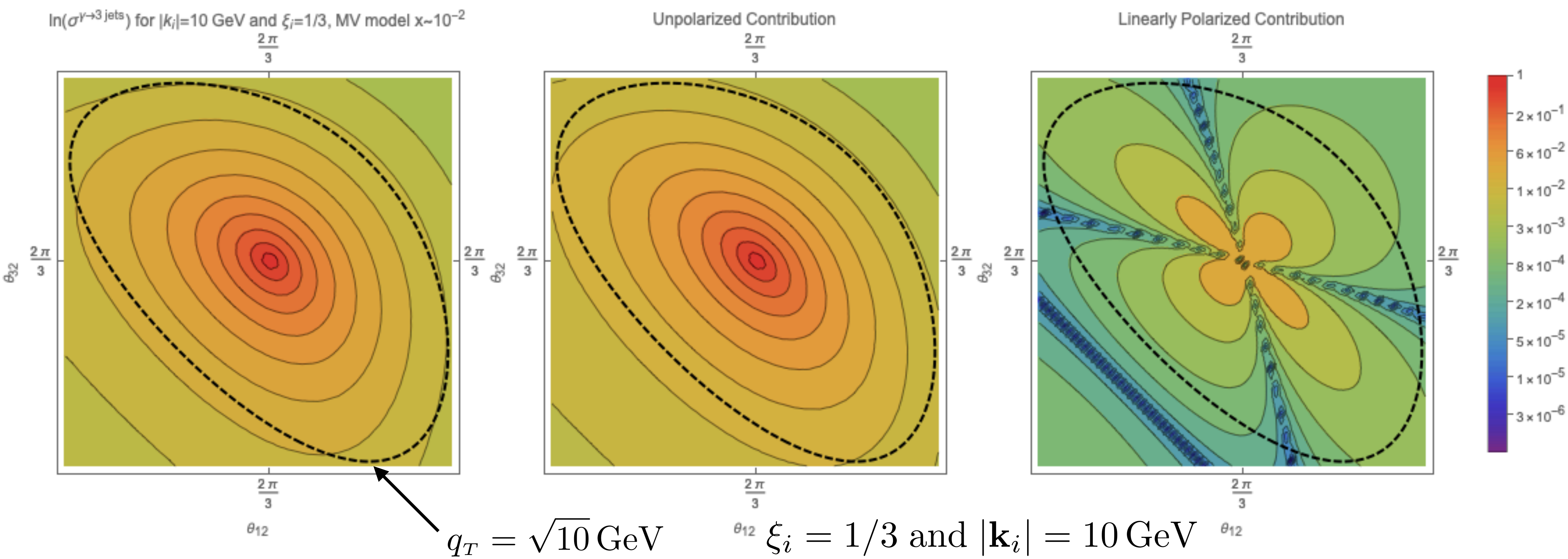


McLerran, Venugopalan (1994)

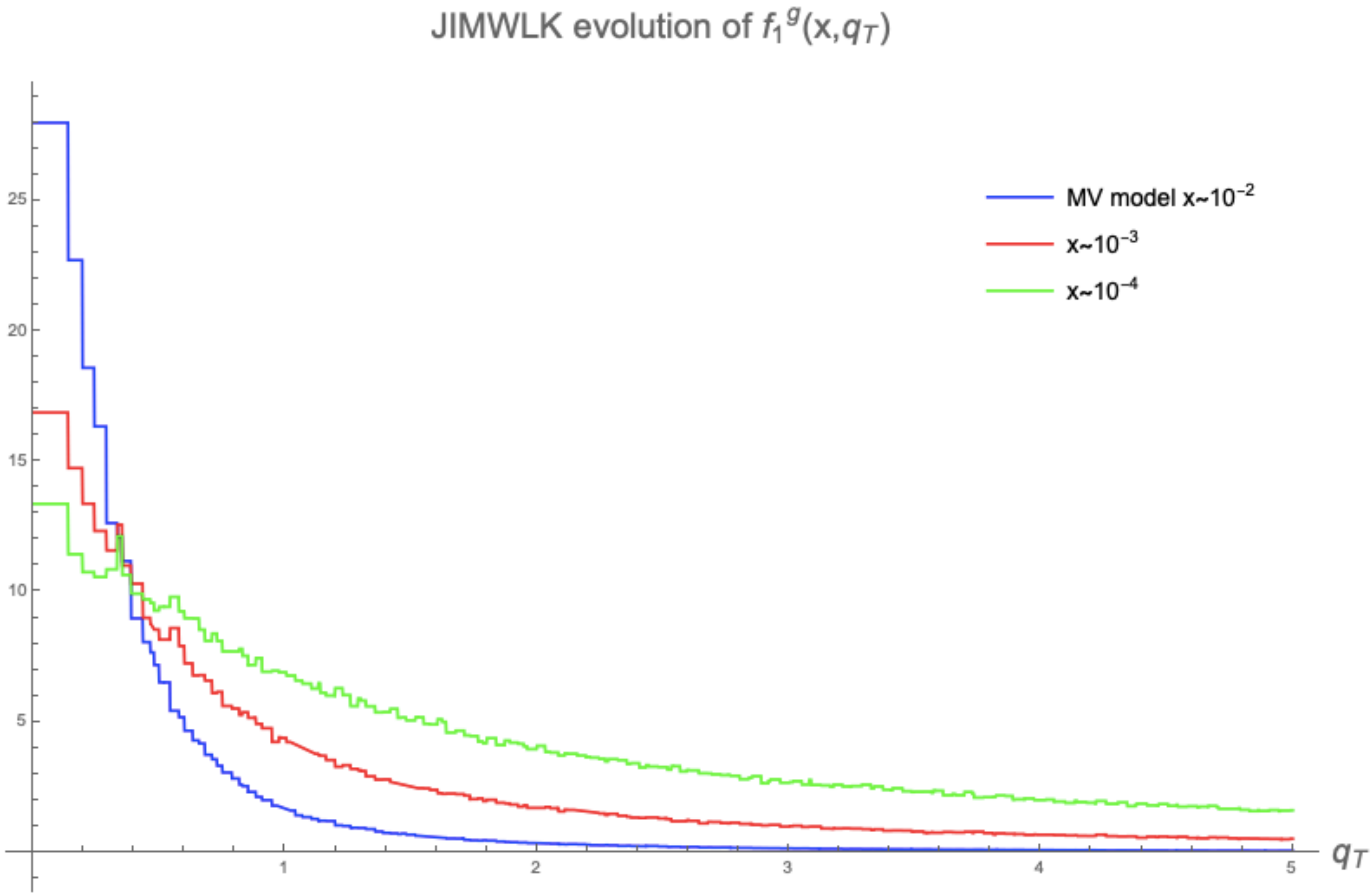
$$\mathcal{F}_{gg}^{(3)}(x, \mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3} \int \mathrm{d}r \frac{J_0(|q_T r|)}{r} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

$$\mathcal{H}_{gg}^{(3)}(x, \mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3} \int \mathrm{d}r \frac{J_2(|q_T r|)}{r \ln \frac{1}{r^2\Lambda^2}} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

Dominguez, Xiao, Yuan (2011)
Metz, Zhou (2011)



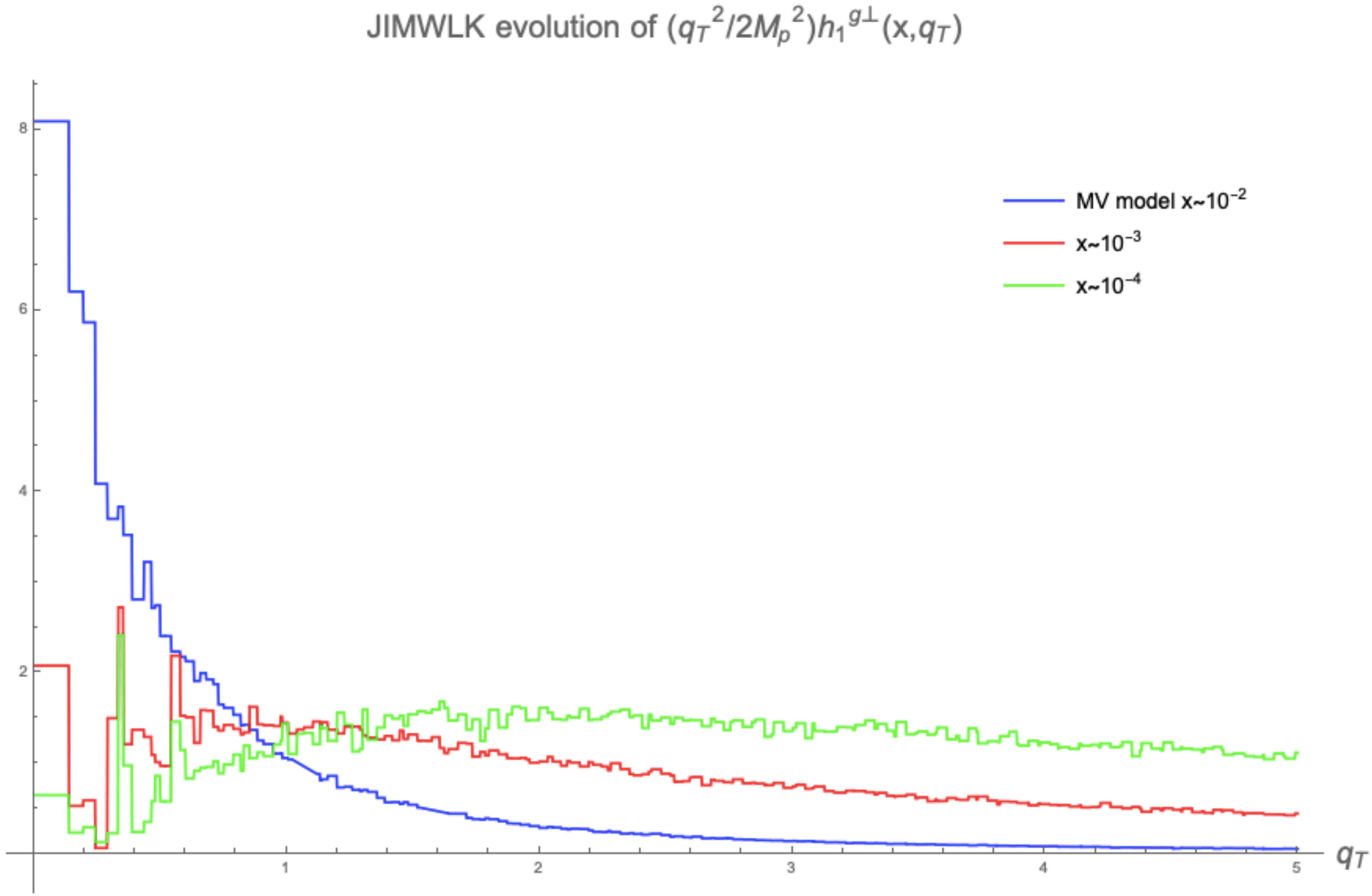
JIMWLK evolution of f_1^g and $h_1^{g\perp}$



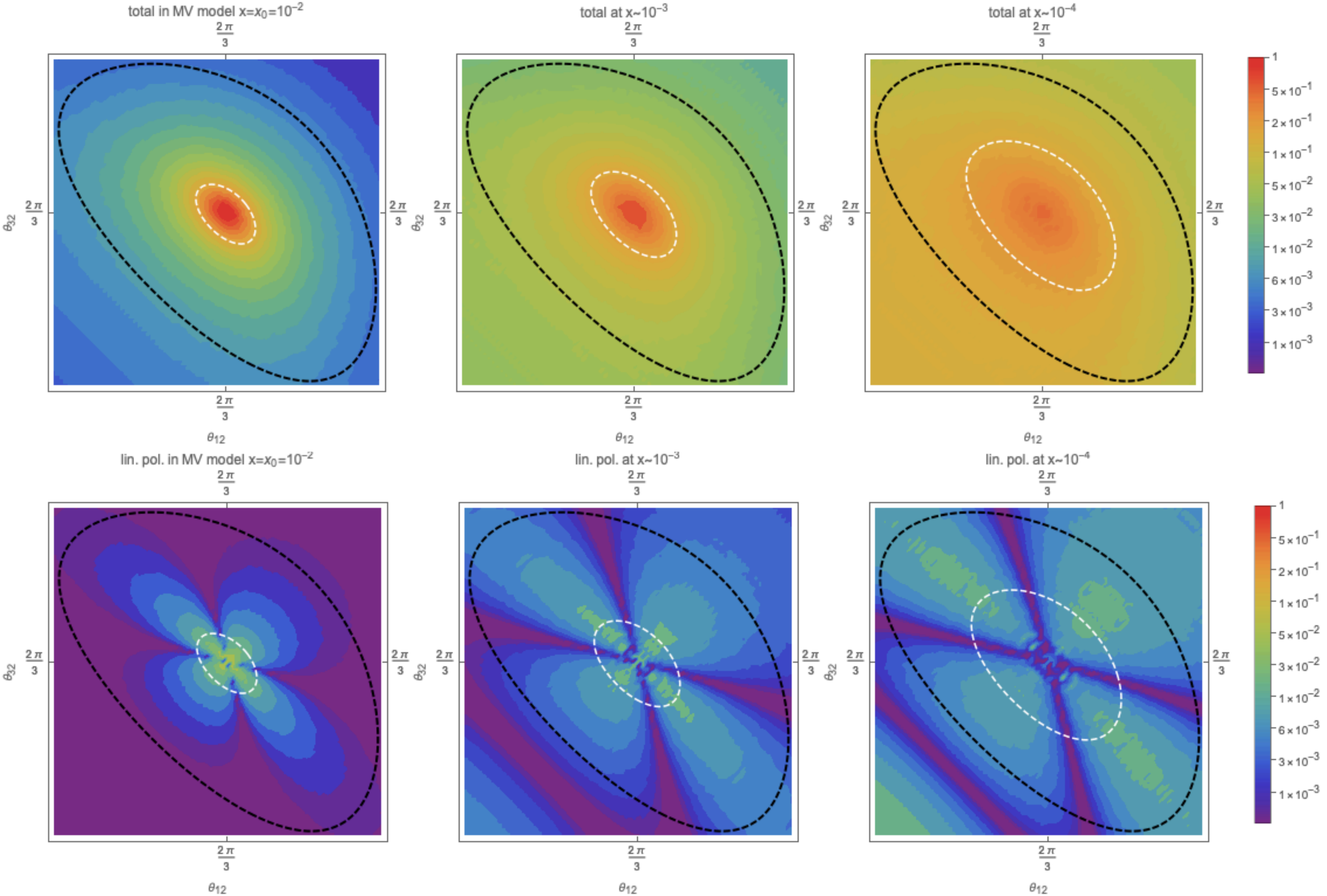
Langevin formulation of JIMWLK: Weigert (2002), Rummukainen; Weigert (2004); Lappi (2008)

Marquet, Petreska, Roiesnel (2016); Marquet, Roiesnel, PT (2018)

JIMWLK evolution of f_1^g and $h_1^{g\perp}$



JIMWLK evolution of final result



Conclusions and outlook

We studied the common limit of the CGC and TMD frameworks in the regime where they overlap *i.e.* low x and $q_T \ll Q$.

Proper understanding of this correspondence allows to apply CGC machinery to TMDs and vice versa. See Shu-Yi's talk for Sudakov resummations at low x. Giovanni talked about the efforts towards a unified evolution framework. Alternative 'improved TMD' framework proposed by Krzysztof and collaborators.

Renaud will explain how the correlation limit generalizes and the CGC resums both 'kinematic' and 'genuine' twist effects.

This work is situated in an effort to elucidate this correspondence beyond $2 \to 2$ processes. See also Altinoluk, Boussarie, Marquet, PT (2019). We are currently working on a simple NLO process $pA \to \text{photon} + \text{jet}$.