Small-x resummation and its impact in PDF determination

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Context: collinear factorization

Collinear factorization: $$\sigma(x,Q^2) = \int_x^1 \frac{dz}{z}\, C_i\big(z,\alpha_s(Q^2)\big)\, f_i\Big(\frac{x}{z},Q^2\Big)$$

DGLAP evolution: $$\mu^2\frac{d}{d\mu^2} f_i(x,\mu^2) = \int_x^1 \frac{dz}{z}\, P_{ij}(z,\alpha_s(\mu^2))\, f_j\Big(\frac{x}{z},\mu^2\Big)$$

Heavy-quark matching: $$f_i^{[n_f+1]}(x,\mu_m^2) = \int_x^1 \frac{dz}{z}\, A_{ij}(z,\alpha_s(\mu_m^2))\, f_j^{[n_f]}\Big(\frac{x}{z},\mu_m^2\Big)$$

Any object with a perturbative expansion can exhibit a logarithmic enhancement:

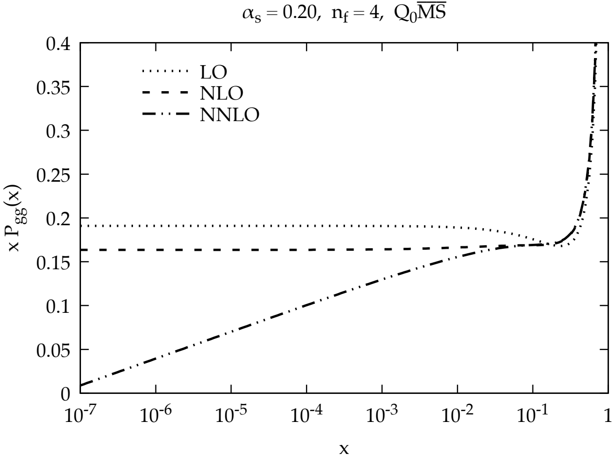
- observable: coefficient functions $C(x,\alpha_s)$
- evolution: splitting functions $P(x,\alpha_s)$ and matching conditions $A(x,\alpha_s)$

Small-x logarithms: single logs $\alpha_s^n \frac{1}{x}\log^k\frac{1}{x} \quad (0\le k\le n-1)$

When $\alpha_s \log\frac{1}{x} \sim 1$ perturbativity is spoiled $\to$ **all-order resummation needed**

In $\overline{\text{MS}}$ and related schemes, both coefficient $C(x,\alpha_s)$ and splitting $P(x,\alpha_s)$ functions, and also matching conditions $A(x,\alpha_s)$, are logarithmically enhanced at small x (in the singlet sector).

Only **singlet sector** affected: $\boldsymbol{P}_{\text{gg}}$, P_{gq}, P_{qg}, P_{qq}



fixed-order $xP_{\text{gg}}(x, \alpha_s)$ splitting function at small x:

LO:
$\alpha_s \times \text{const}$

NLO:
$\alpha_s^2 \left(\ln \frac{1}{x} + \text{const}\right)$

NNLO:
$\alpha_s^3 \left(\ln^2 \frac{1}{x} + \ln \frac{1}{x} + \text{const}\right)$

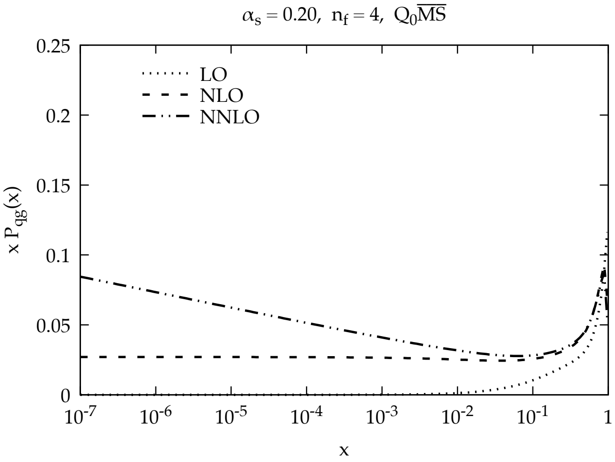
N^{3}LO:
$\alpha_s^4 \left(\ln^3 \frac{1}{x} + \ln^2 \frac{1}{x} + \ln \frac{1}{x} + \right.$

accidental zeros!

Logarithms start to grow for $x \lesssim 10^{-2}$ (for $Q \sim 5\text{GeV}$)
$\rightarrow$ very large resummation region!

Small-x logarithms in the splitting functions

Only **singlet sector** affected: P_{gg}, P_{gq}, $\boldsymbol{P_{qg}}$, P_{qq}



fixed-order $xP_{qg}(x, \alpha_s)$ splitting function at small x:

LO:
$\alpha_s \times 0$

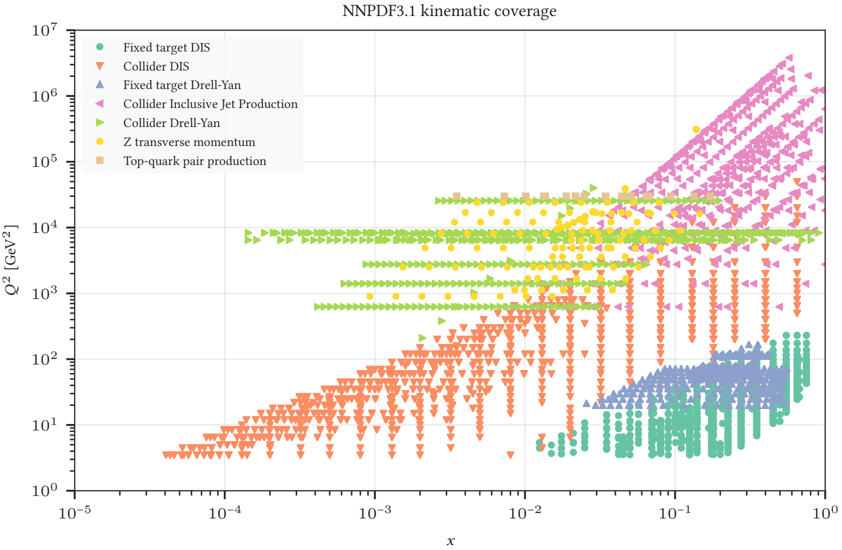
NLO:
$\alpha_s^2 \times \text{const}$

NNLO:
$\alpha_s^3 \left(\ln \frac{1}{x} + \text{const}\right)$

N^{3}LO:
$\alpha_s^4 \left(\ln^2 \frac{1}{x} + \ln \frac{1}{x} + \text{const}\right)$

P_{qg} and P_{qq} are NLL quantities, while P_{gg} and P_{gq} are LL
Small-x resummation is gluon-driven

Do we reach those x values?



The theory of small-x resummation

Ingredients for resummation

Small-x resummation is based on the interplay of

Collinear factorization: $$\sigma(x,Q^2)=\int_x^1\frac{dz}{z}\,C_i\big(z,\alpha_s(Q^2)\big)\,f_i\Big(\frac{x}{z},Q^2\Big)$$

DGLAP evolution: $$\mu^2\frac{d}{d\mu^2}f_i(x,\mu^2)=\int_x^1\frac{dz}{z}\,P_{ij}\big(z,\alpha_s(\mu^2)\big)\,f_j\Big(\frac{x}{z},\mu^2\Big)$$

with

k_t factorization: $$\sigma(x,Q^2)=\int_x^1\frac{dz}{z}\int_0^\infty dk_t^2\,\mathcal{C}_{\rm g}\big(z,k_t^2,\alpha_s\big)\,\mathcal{F}_{\rm g}\Big(\frac{x}{z},k_t^2\Big)$$

BFKL evolution: $$-x\frac{d}{dx}\mathcal{F}_{\rm g}(x,k_t^2)=\int_0^\infty\frac{dq_t^2}{k_t^2}\,\mathcal{K}\bigg(\frac{k_t^2}{q_t^2},\alpha_s(\cdot)\bigg)\,\mathcal{F}_{\rm g}\big(x,q_t^2\big)$$

- $\mathcal{F}_{\rm g}\big(x,k_t^2\big)$: unintegrated ($k_t$-dependent) PDF
- $\mathcal{C}_{\rm g}\big(z,k_t^2,\alpha_s\big)$: off-shell coefficient function

Consistency between equations allows to resum small-x logs:

- DGLAP + BFKL eqns $\to$ resum $P(x,\alpha_s)$
- collinear + k_t factorizations $\to$ resum $C(x,\alpha_s)$ and heavy quark matching $A(x,\alpha_s)$

The key: relation between unintegrated and integrated PDFs

The relation between collinear and unintegrated PDFs is (in Mellin space)

$$\mathcal{F}_{\mathsf{g}}\big(N, k_t^2\big) = R(N, \alpha_s)\, \frac{d}{dk_t^2} f_{\mathsf{g}}(N, k_t^2)$$

where R is a scheme-dependent function. In $\overline{\mathsf{MS}}$ we have

$$R_{\overline{\mathsf{MS}}}(N, \alpha_s) = 1 + \mathcal{O}(\alpha_s^3)$$

In small-x theory, one usually defines the $Q_0\overline{\mathsf{MS}}$ scheme by

$$R_{Q_0\overline{\mathsf{MS}}}(N, \alpha_s) = 1,$$

which is equal to $\overline{\mathsf{MS}}$ up to N^{3}LO, such that

$$\mathcal{F}_{\mathsf{g}}\big(N, k_t^2\big) = \frac{d}{dk_t^2} f_{\mathsf{g}}(N, k_t^2) \qquad (Q_0\overline{\mathsf{MS}})$$

This relation is valid up to NLL (at least)
From now on we work in $Q_0\overline{\mathsf{MS}}$

In the small-x and large-Q^2 limit (in N Mellin space)

$$\sigma(N,Q^2) = C\big(N,\alpha_s(Q^2)\big)\, f\big(N,Q^2\big) \qquad \text{(collinear factorization)}$$

$$= \int_0^\infty dk_t^2\, \mathcal{C}\big(N,k_t^2,\alpha_s\big)\, \mathcal{F}\big(N,k_t^2\big) \qquad (k_t \text{ factorization})$$

In the $Q_0\overline{\text{MS}}$ scheme

$$\mathcal{F}\big(N,k_t^2\big) = \frac{d}{dk_t^2} f(N,k_t^2) = \frac{d}{dk_t^2} U\left(N,\frac{k_t^2}{Q^2}\right) f(N,Q^2)$$

$$U\left(N,\frac{k_t^2}{Q^2}\right) = \exp \int_{Q^2}^{k_t^2} \frac{d\nu^2}{\nu^2}\gamma(N,\alpha_s(\nu^2))$$

with *resummed* anomalous dimension $\gamma(N,\alpha_s)$, so that [MB,Marzani,Peraro 1607.02153]

$$C(N,\alpha_s) = \int_0^\infty dk_t^2\, \mathcal{C}\left(N,\frac{k_t^2}{Q^2},\alpha_s\right) \frac{d}{dk_t^2} U\left(N,\frac{k_t^2}{Q^2}\right)$$

Equivalent to previous formulations, but easier numerical implementation and suitable for generalizations

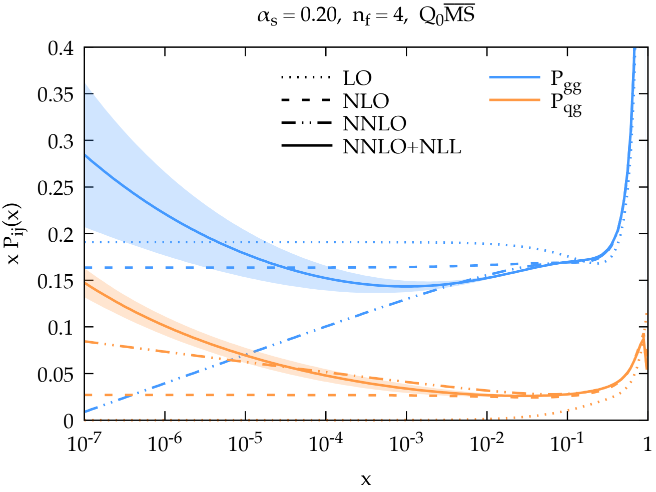
Small-x resummation of splitting functions

Formalism developed in the 90s-00s, known at LL and NLL, technical details rather complicated [Altarelli,Ball,Forte] [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Thorne,White]

Recent developments: [MB,Marzani,Peraro 1607.02153][MB,Marzani,Muselli 1708.07510] [MB,Marzani 1805.06460]

- technical improvements
- estimate uncertainty from subleading logs
- matching to NNLO (and N^3LO), allowing **NNLO+NLL phenomenology**
- prediction of the yet unknown **$\mathbf{N^3}$LO** splitting functions
- public code `HELL: High-Energy Large Logarithms`
  `www.ge.infn.it/~bonvini/hell`
- `HELL` interfaced to `APFEL` (`apfel.hepforge.org`) → PDF fits

Some representative HELL results: splitting functions

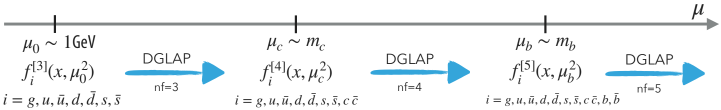


All-order behaviour rather different from fixed order (especially for $P_{\rm gg}$)

$P_{\rm gg} > P_{\rm qg}$ at resummed level (at NNLO they swap at some x)

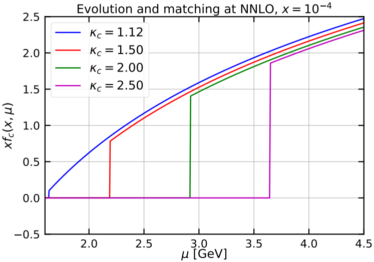
Matching conditions at the charm threshold

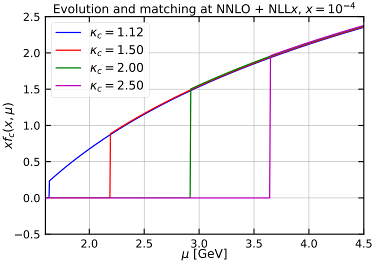
The number n_f of "active" flavours changes during the evolution (factorization scheme choice to resum large collinear mass logarithms from heavy quark pair production)



Matching relation between PDFs in schemes with different n_f

$$f_i^{[n_f+1]}(\mu^2) = \sum_{j=\text{light}} A_{ij}(m^2/\mu^2) \otimes f_j^{[n_f]}(\mu^2) \qquad A_{ij} = \text{perturbative matching coefficients}$$



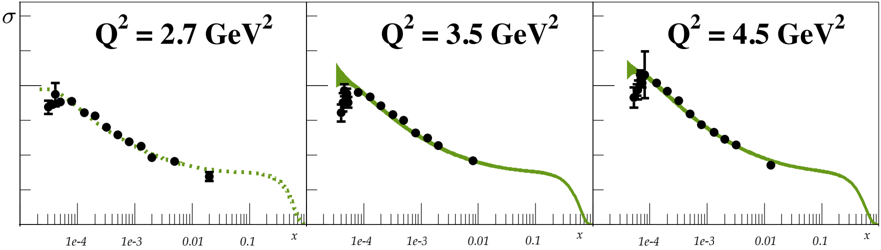


The perturbatively generated charm PDF is much less dependent on the (unphysical) matching scale when small-x resummation is included!

PDF fits with small-x resummation

Deep-inelastic scattering (DIS) data from HERA extend down to $x \sim 3 \times 10^{-5}$

Tension between HERA data at low Q^2 and low x with fixed-order theory



Also leads to a deterioration of the χ^2 when including low-Q^2 data

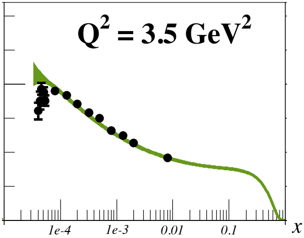
Attempts to explain this deviation with higher twists, phenomenological models, ...

Successful description of this region when including small-x resummation!

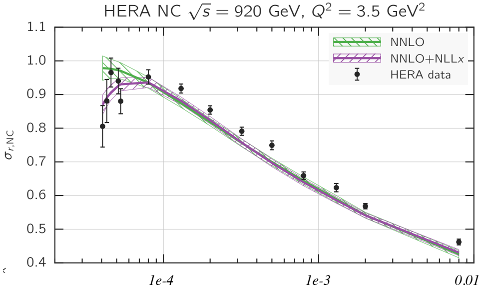
- NNPDF framework [Ball,Bertone,MB,Marzani,Rojo,Rottoli 1710.05935]
- xFitter framework [xFitterCollaboration+MB 1802.00064] [MB,Giuli 1902.11125]

$\chi^2/N_{\rm dat}$	NNLO	NNLO+NLLx
xFitter (default PDF parametrization)	**1.23**	**1.17**
xFitter (new PDF parametrization)	**1.16**	**1.14**
NNPDF3.1sx	**1.130**	**1.100**

Substantial improvement due to better description of small-x low-Q^2 HERA data



$\rightarrow$



The better description mostly comes from a larger resummed F_L

$$\sigma_{r,\rm NC} = F_2(x_{\rm Bj}, Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x_{\rm Bj}, Q^2) \qquad y = \frac{Q^2}{x_{\rm Bj}\, s}$$

Significantly improved description of the HERA data

χ^2/N_{dat}	NNLO	NNLO+NLLx
xFitter (default PDF parametrization)	**1.23**	**1.17**
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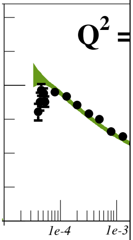
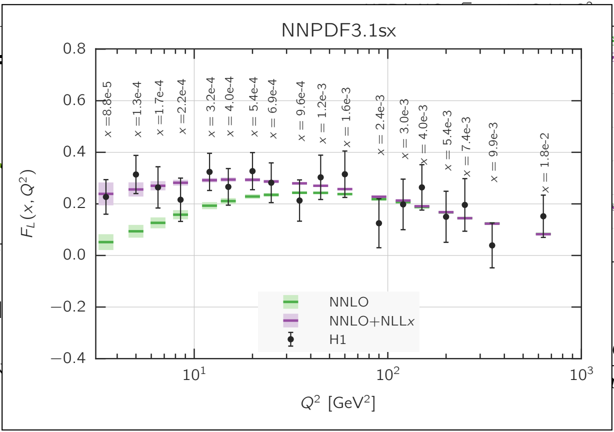
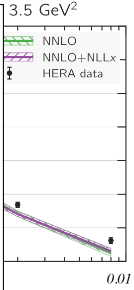


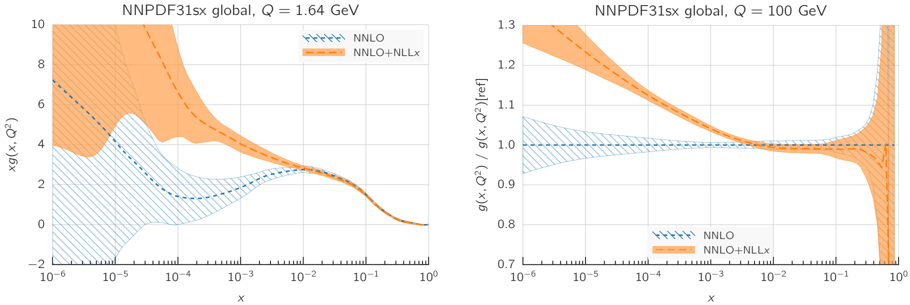




The better d

$\sigma_{r,\mathrm{N}}$ … $\frac{Q^2}{x_{\mathrm{Bj}} s}$

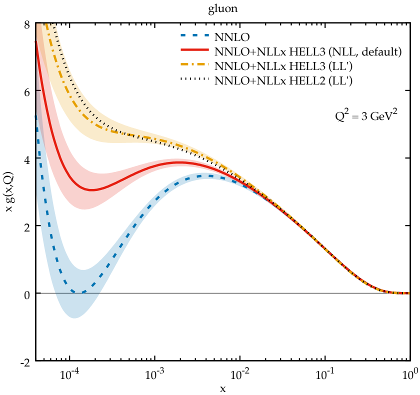
Fit results: impact on gluon PDF



Note: future higher energy colliders will probe smaller values of x $(x_{\rm min} \sim Q^2/s)$
$\to$ small-x resummation will be even more important in future!

First fit with `HELL 3.0` [MB,Giuli 1902.11125]

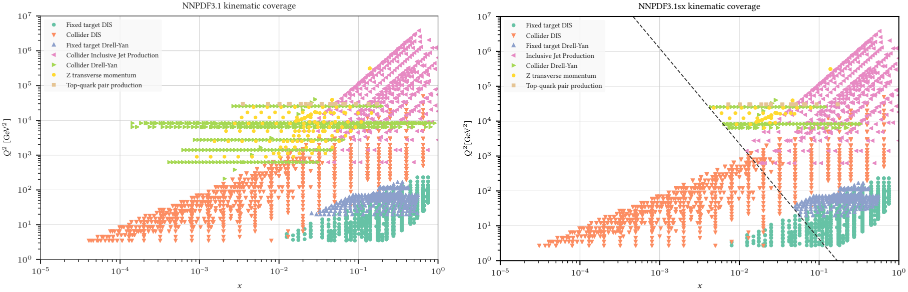
Red and yellow curves differ by subleading logs



We have full resummation for DGLAP evolution and DIS structure functions, but not hadron-hadron collider observables (yet)

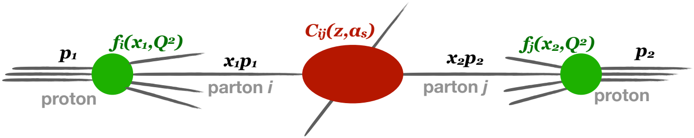
We cut those hadronic data potentially sensitive to small-x resummation, i.e.

$$\alpha_s(Q^2)\log\frac{1}{x} > H_{\rm cut} = 0.6 \quad \text{(default cut)}$$



The most important missing observable is Drell-Yan
Would provide a very important validation of the fit (low x but high Q^2)

LHC phenomenology



Challenges:

- two protons in the initial state ✓
- want to describe differential distributions ✓

Processes considered so far in HELL:

- $gg \to H$ (inclusive cross section) ✓
- $c\bar{c}$, $b\bar{b}$ pair production (fully differential) *(in progress)*
- Drell-Yan (fully differential) *(in progress)*
- $gg \to H$ (fully differential) *(should be straightforward)*

LHC observables in collinear and k_t factorization

Differential cross section in collinear factorization $\tau = Q^2/s, \quad y = Y - \frac{1}{2}\log\frac{x_1}{x_2}$

$$\frac{d\sigma}{dQ^2 dY\dots} = \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, \mathscr{L}_{ij}\left(\frac{\tau}{z}, \hat{y}, Q^2\right) \frac{dC_{ij}}{dy\dots}(z, Y-\hat{y}, \dots, \alpha_s)$$
$$\mathscr{L}_{ij}(x, \hat{y}, Q^2) = f_i(\sqrt{x}e^{\hat{y}}, Q^2)\, f_j(\sqrt{x}e^{-\hat{y}}, Q^2)\, \theta(e^{-2|\hat{y}|} - x)$$

and in k_t factorization [Caola,Forte,Marzani 1010.2743] [Muselli 1710.09376]

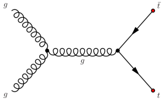
$$\frac{d\sigma}{dQ^2 dY\dots} = \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, \mathcal{L}_{\rm gg}\left(\frac{\tau}{z}, \hat{y}, k_1^2, k_2^2\right) \frac{d\mathcal{C}_{\rm gg}}{dy\dots}(z, Y-\hat{y}, k_1^2, k_2^2, \dots, \alpha_s)$$
$$\mathcal{L}_{\rm gg}(x, \hat{y}, k_1^2, k_2^2) = \mathcal{F}_{\rm g}(\sqrt{x}e^{\hat{y}}, k_1^2)\, \mathcal{F}_{\rm g}(\sqrt{x}e^{-\hat{y}}, k_2^2)\, \theta(e^{-2|\hat{y}|} - x)$$

Resummation can proceed as before, exploiting the relation between unintegrated and collinear PDFs. We found

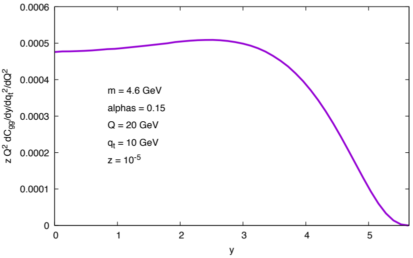
$$\frac{dC_{\rm gg}}{dy\dots}(z, y, \dots) = \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \int_z^1 \frac{dx}{x} \int d\hat{y}\, \frac{d\mathcal{C}_{\rm gg}}{dy\dots}(x, y-\hat{y}, k_1^2, k_2^2, \dots, \alpha_s)$$
$$\times \frac{d}{dk_1^2} U_+\left(\sqrt{\frac{z}{x}}e^{\hat{y}}, k_1^2, Q^2\right) \frac{d}{dk_1^2} U_+\left(\sqrt{\frac{z}{x}}e^{-\hat{y}}, k_2^2, Q^2\right) \theta\left(e^{-2|\hat{y}|} - \frac{z}{x}\right)$$

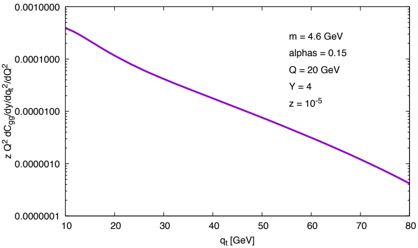
Transverse final-state variables do not play a role (except for kinematic constraints)

Fully differential heavy-quark pair production [MB,Silvetti, in preparation]



Preliminary parton-level purely-resummed results for $\frac{d\sigma}{dQ^2\, dY\, dq_t}$ (pair kinematics)



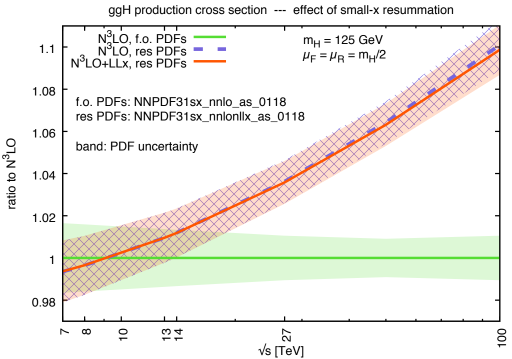


LHCb data sensitive to very small $x \to$ useful to constrain the PDFs

Extension to single-heavy-quark kinematics ongoing

Impact of resummation in ggH at LHC and future colliders

$gg \to H$ inclusive cross section [MB,Marzani 1802.07758] [MB 1805.08785]



ggH cross section at FCC-hh can be $\sim 10\%$ larger than expected with NNLO PDFs!

At LHC $+1\%$ effect (plus another 1% from threshold resummation)

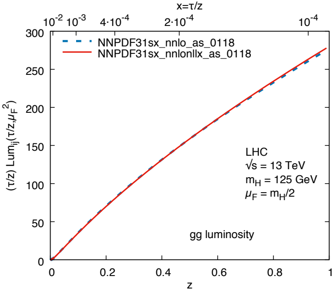
Let's consider again the collinear factorization formula

$$\frac{d\sigma}{dQ^2 dY...} = \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, f_i\left(\sqrt{\frac{\tau}{z}} e^{\hat{y}}, Q^2\right) f_j\left(\sqrt{\frac{\tau}{z}} e^{-\hat{y}}, Q^2\right) \frac{dC_{ij}}{dy...}(z, Y - \hat{y}, ..., \alpha_s)$$

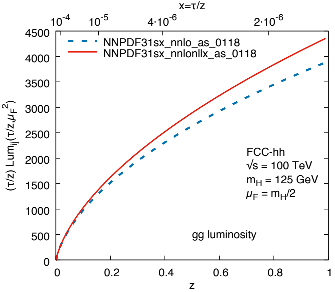
The small z integration region, where logs in C are large, is weighted by the PDFs at large momentum fractions $x = \sqrt{\frac{\tau}{z}} e^{\pm\hat{y}}$
Since PDFs die fast at large x, especially the gluon, the small-z region is suppressed!

Rather, the large z region is enhanced by the gluon-gluon luminosity
In that region, the difference between fixed-order and resummed PDFs is large

LHC:



FCC:



Concluding remarks

Interesting progress and results:

- resummation stabilizes perturbative behaviour
- improved PDF fits
- large impact on gluon PDF
- exploration of LHC application just started

Limitations $\rightarrow$ outlook:

- for $c\bar{c}, b\bar{b}$ we considered only quark-level final states
  $\rightarrow$ add hadronization and refit PDFs with LHCb data
- at forward rapidities one parton is at large x
  $\rightarrow$ combine with threshold resummation
- low accuracy (two log orders known for $P_{\rm gg}$ only, for everything else just one)
  $\rightarrow$ extension to the next logarithmic order

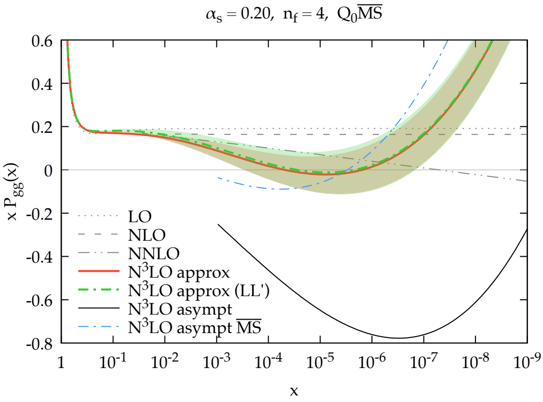
Backup slides

Towards N^{3}LO evolution

Recent impressive progress towards N^{3}LO splitting functions

[Davies,Vogt,Ruijl,Ueda,Vermaseren 1610.07477] [Moch,Ruijl,Ueda,Vermaseren,Vogt 1707.08315]

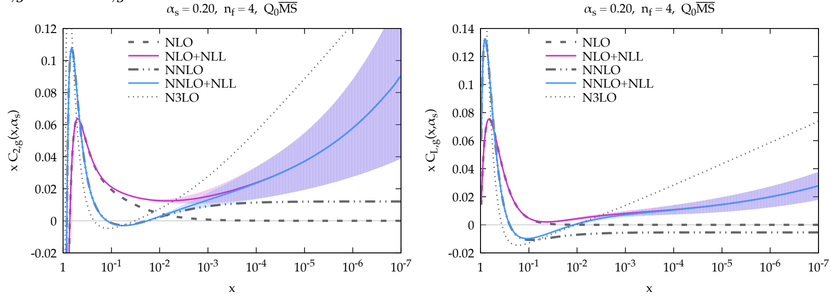
At small x, approximate predictions from NLLx resummation [MB,Marzani 1805.06460]



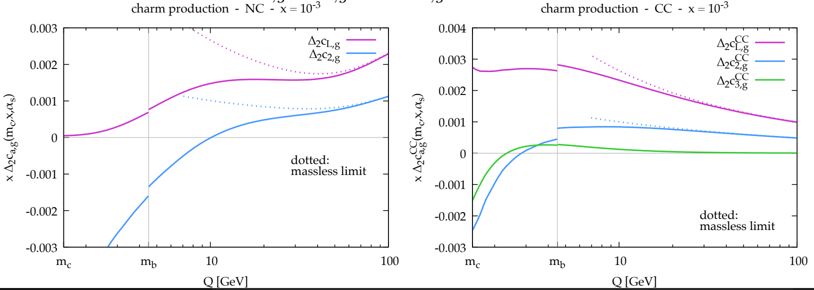
Large uncertainties from subleading logs

N^{3}LO splitting functions are much more unstable at small x $\rightarrow$ need resummation!

$F_{2,g}$ and $F_{L,g}$ massless DIS coefficient functions



resummed contribution to $F_{2,g}$, $F_{L,g}$ and $F_{3,g}$ DIS coeff.functs with mass effects



NNPDF and xFitter fits with small-x resummation

APFEL+HELL $\rightarrow$ make possible a PDF fit with small-x resummation

NNPDF3.1sx [1710.05935]	xFitter [1802.00064]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
VFNS: FONLL	VFNS: FONLL
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC (~ 4000 datapoints)	only HERA data (~ 1200 datapoints)
NLO, NLO+NLL, NNLO, NNLO+NLL	NNLO, NNLO+NLL

One interesting difference in the HERA data we include:

Lowest Q^2 HERA bins	NNPDF3.1/HERAPDF2.0	NNPDF3.1sx/xFitter
$Q^2 = 3.5$ GeV^2	included	included
$Q^2 = 2.7$ GeV^2	excluded	included
$Q^2 = 2.0$ GeV^2	excluded	excluded

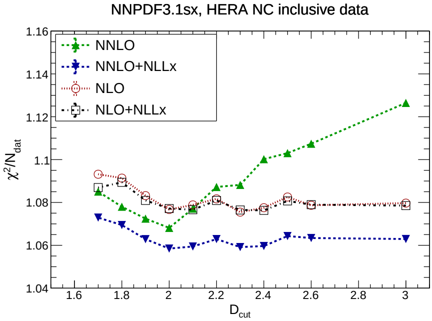
lower $Q^2 \rightarrow$ lower x

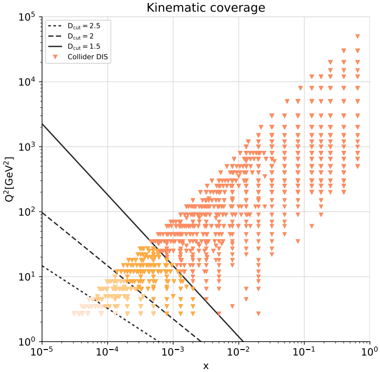
Fit results: χ^2 as quality estimator and the onset of BFKL dynamics

$\chi^2/N_{\rm dat}$	NLO	NLO+NLLx	NNLO	NNLO+NLLx
xFitter			**1.23**	**1.17**
NNPDF3.1sx	**1.117**	**1.120**	**1.130**	**1.100**
	these are similar		largest	smallest

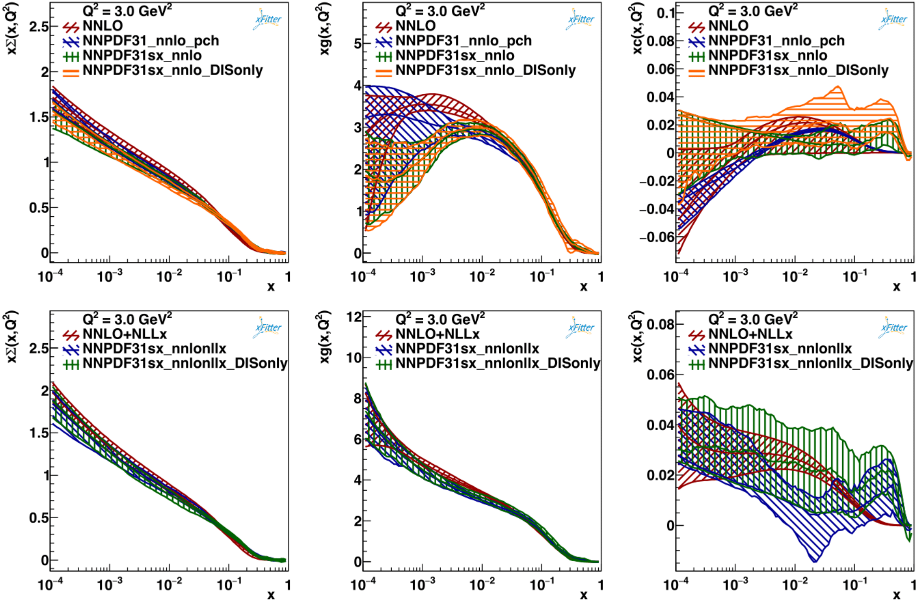
Hierarchy as expected from splitting function behaviour!

Mostly due to HERA data: we study the $\chi^2/N_{\rm dat}$ profile as we cut out HERA data at small x small Q^2

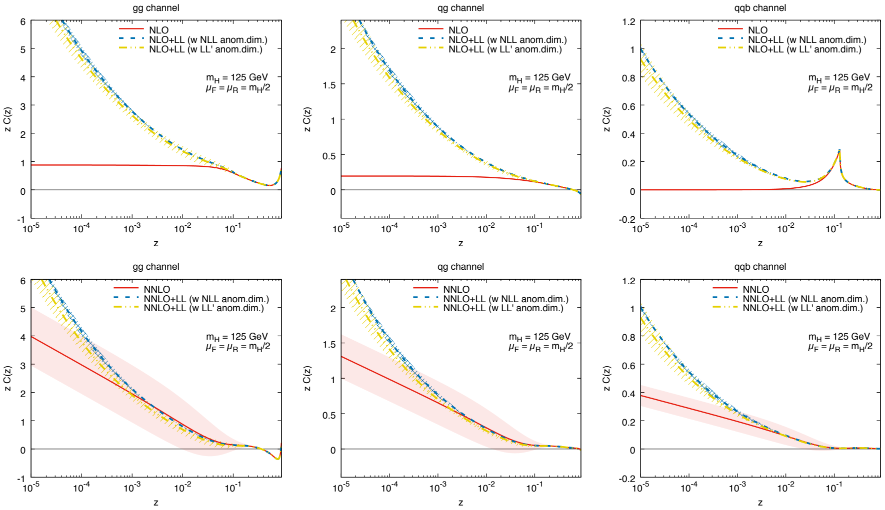




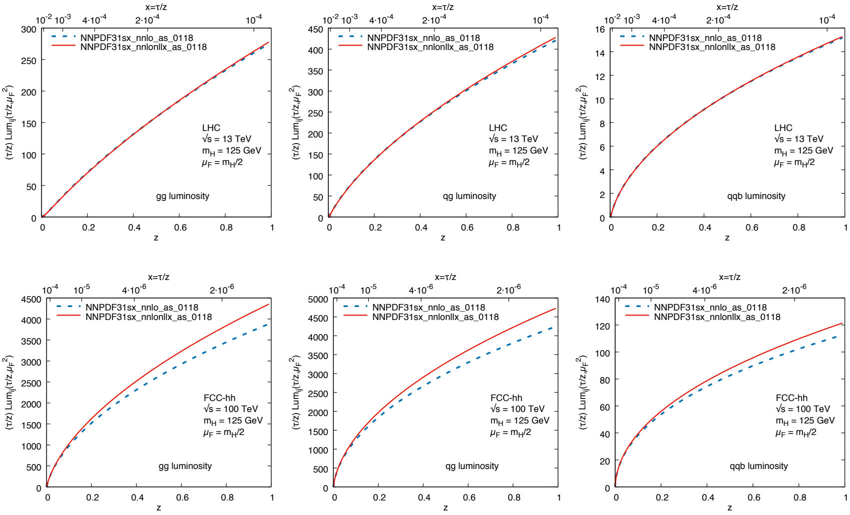
xFitter comparison



Higgs production: parton-level results



Parton luminosities for ggH



$$\sigma(\tau, Q^2) = \sigma_0(Q^2) \sum_{i,j=g,q} \int_\tau^1 \frac{dz}{z}\, C_{ij}(z, \alpha_s)\, \mathscr{L}_{ij}\left(\frac{\tau}{z}, \mu_F^2\right) \qquad \tau = \frac{m_H^2}{s}$$

Higgs production: LHC and future colliders