

$$j(z, \bar{z}) = \frac{1}{2} \log \left[\frac{z}{(1-z)^2} \frac{\bar{z}}{(1-\bar{z})^2} \right] = \frac{1}{2} \mathcal{L}_0(z, \bar{z}) + \mathcal{L}_1(z, \bar{z})$$