$$\frac{d}{dz}\hat{H}_{2\mathrm{d,i}}\mathcal{L}_{0,\sigma}(z,\bar{z}) = \frac{\hat{H}_{2\mathrm{d,i}}\mathcal{L}_{\sigma}(z,\bar{z})}{z}$$

$$\frac{d}{dz}\hat{H}_{2\mathrm{d,i}}\mathcal{L}_{1,\sigma}(z,\bar{z}) = \frac{\hat{H}_{2\mathrm{d,i}}\mathcal{L}_{\sigma}(z,\bar{z})}{1-z} - \frac{1}{4}\frac{\mathcal{L}_{1,\sigma}(z,\bar{z})}{z}$$

$$- \frac{1}{4}\frac{\mathcal{L}_{0,\sigma}(z,\bar{z}) + 2\mathcal{L}_{1,\sigma}(z,\bar{z}) - [\mathcal{L}_{0,\sigma}(w,\bar{w}) + \mathcal{L}_{1,\sigma}(w,\bar{w})]_{w,\bar{w}\to\infty}}{1-z}$$