RESUMMATION IN HEAVY QUARKONIUM PRODUCTION

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MOTIVATION

- Theoretical study of quarkonium production processes require knowledge of ***nonperturbative matrix elements***. Many of them have ***not been computed from first principles***.
- In order to to extract and test the unknown matrix elements, it is best to use processes that are well understood theoretically.
- Hard processes are well understood from collinear factorization. Precise calculations require resummation.
- In NREFTs, matching between nonlocal operators in QCD and local operators in NREFTs introduce ***singular distributions***.
- ***We need to compute evolution of fragmentation functions and distribution amplitudes with singularities.***

OUTLINE

- Resummation in inclusive production of heavy quarkonium: solving DGLAP equations with fragmentation functions with singularities
- Resummation in exclusive electromagnetic production: solving ERBL equations with light-cone distribution amplitudes with singularities

INCLUSIVE PRODUCTION IN NRQCD

- Inclusive cross section is given by a factorization formula

$$\sigma_H = \sum_n \sigma_{Q\bar{Q}(n)} \langle \mathcal{O}(n) \rangle_H$$

Bodwin, Braaten, Lepage, PRD51, 1125 (1995), PRD55, 5853 (1997)

n : spin and color state of $Q\bar{Q}$

$\sigma_{Q\bar{Q}(n)}$: perturbative cross section of $Q\bar{Q}$

$\langle \mathcal{O}(n) \rangle_H$: Nonperturbative matrix element, probability for $Q\bar{Q}$ in state n to evolve into quarkonium H.

- Matrix elements have known scalings in v, so sum over n is organized in powers of v.
- For J/ψ, n = ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$ up to $O(v^4)$.
- Usually, color octet matrix elements are obtained from data.

PRODUCTION IN NRQCD

- It is necessary to be able to compute cross sections of $Q\bar{Q}$ reliably as functions of p_T. Shapes of cross sections are well understood in terms of LP and NLP fragmentation.

$$\frac{d\sigma}{dp_T^2} = \sum_{i=g,q,\bar{q}} \frac{d\hat{\sigma}_i}{dp_T^2} \otimes D_i + \sum_n \frac{d\sigma_{Q\bar{Q}}}{dp_T^2} \otimes D_{Q\bar{Q}} + O(1/p_T^8)$$

Leading-power (LP) fragmentation
$(\sim 1/p_T^4)$

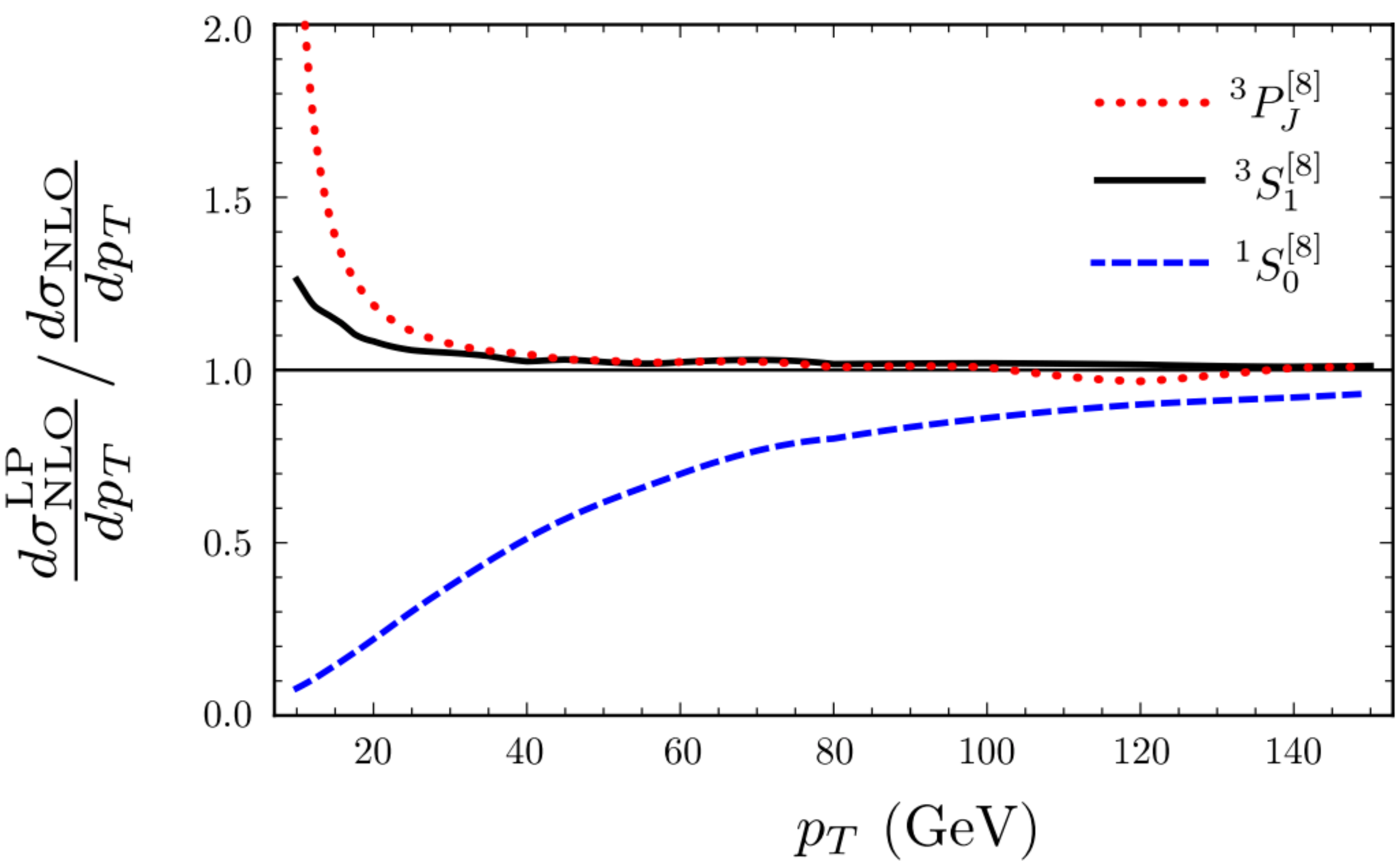
Next-to-leading-power (NLP) fragmentation
$(\sim 1/p_T^6)$

J.C.Collins and D.E.Soper, NPB194, 445 (1982)
Z.-B. Kang, J.-W. Qiu, G. Sterman, PRL108, 102002 (2012)
S. Fleming, A. K. Leibovich, T. Mehen, I. Z. Rothstein, PRD86, 094012 (2012)
Y.-Q. Ma, J.-W. Qiu, G. Sterman, H. Zhang, PRL113, 142002 (2014)

- At very large p_T, LP fragmentation dominates cross section, and DGLAP evolution becomes important.

HADROPRODUCTION IN NRQCD

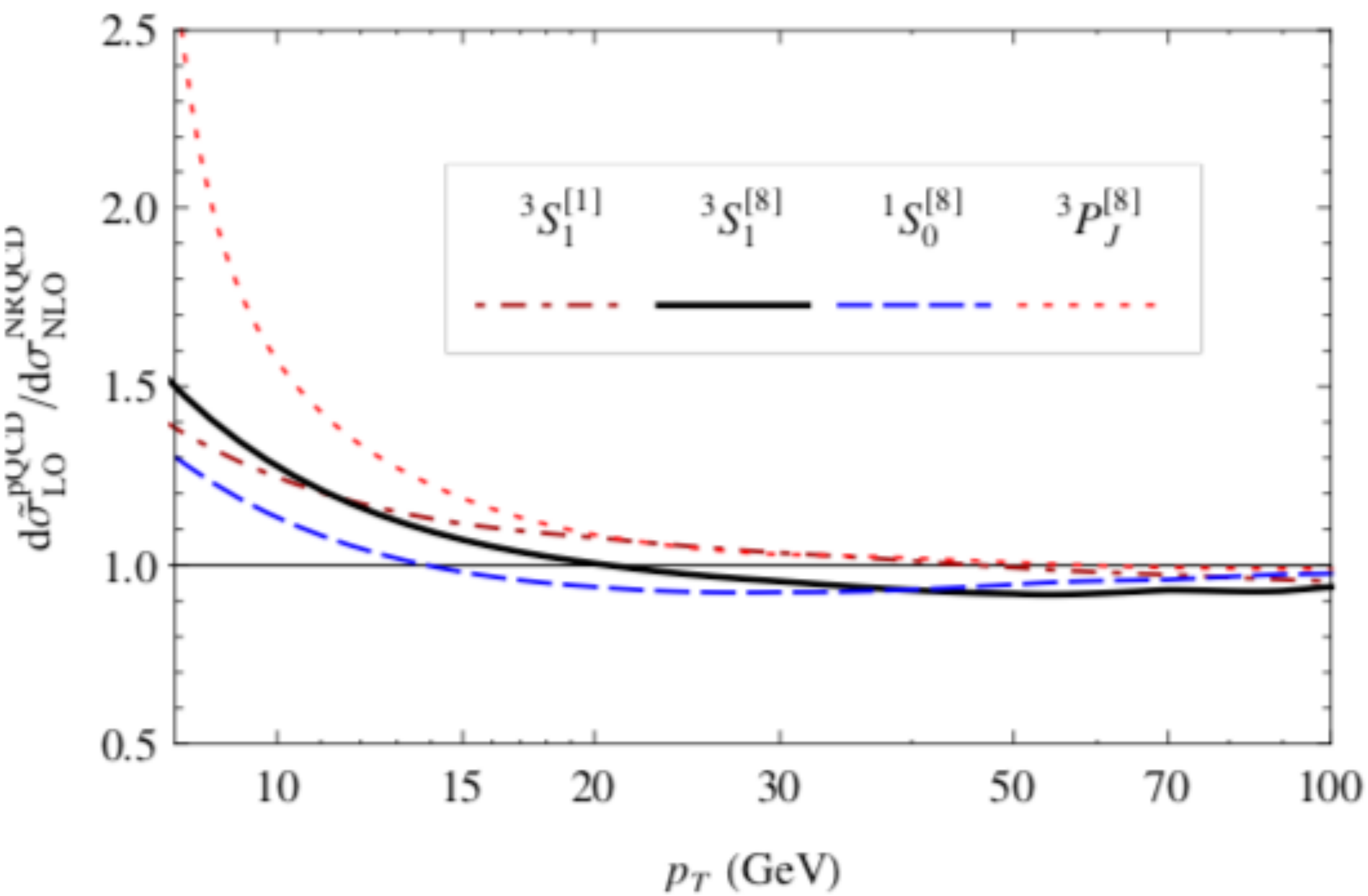
- Relative size of LP fragmentation contribution for inclusive $Q\bar{Q}$ hadroproduction at 7 TeV LHC



G. T. Bodwin, HSC, U.-R. Kim, J. Lee, PRL113, 022001 (2014)
G. T. Bodwin, K.-T. Chao, HSC, U.-R. Kim, J. Lee, Y.-Q. Ma, PRD93, 034041 (2016)

HADROPRODUCTION IN NRQCD

- LP+NLP fragmentation contributions describe inclusive $Q\bar{Q}$ cross sections well for $p_T > 10 - 15$ GeV at 7 TeV LHC



Y.-Q. Ma, J.-W. Qiu, G. Sterman, H. Zhang, PRL113, 142002 (2014)

FRAGMENTATION FUNCTIONS

- Fragmentation functions are vacuum expectation values of nonlocal operators that describe the evolution of an energetic parton into a hadron with momentum fraction z.
- Fragmentation functions for production of nonrelativistic $Q\bar{Q}$ can involve singular distributions in z. At $\mu_0 \approx m$,

$$\begin{aligned}
D_{g\to Q\bar{Q}({}^3S_1{}^{[8]})}(z,\mu_0) &= \frac{\pi\alpha_s(\mu_0)}{24m^3}\delta(1-z), \\
D_{g\to Q\bar{Q}({}^3P^{(8)})}(z,\mu_0) &= \frac{8\alpha_s^2(\mu_0)}{9(N_c^2-1)m^5}\frac{N_c^2-4}{4N_c}\left[\frac{1}{6}\delta(1-z)\left(1-6\log\frac{\mu_\Lambda}{2m_c}\right)\right. \\
&\quad \left. +\left(\frac{1}{1-z}\right)_+ + \frac{13-7z}{4}\log(1-z) - \frac{(1-2z)(8-5z)}{8}\right], \\
D_{g\to Q\bar{Q}({}^1S_0^{(8)})}(z,\mu_0) &= \frac{\alpha_s^2(\mu_0)}{8m^3}\frac{N_c^2-4}{4N_c}[3z-2z^2+2(1-z)\log(1-z)].
\end{aligned}$$

DGLAP EQUATION

- A standard way to solve the DGLAP equation is to use the Mellin transform. At leading logarithmic accuracy,

$$\tilde{D}(n,\mu_0) = \int_0^1 dz\, z^{n-1} D(z,\mu_0),$$

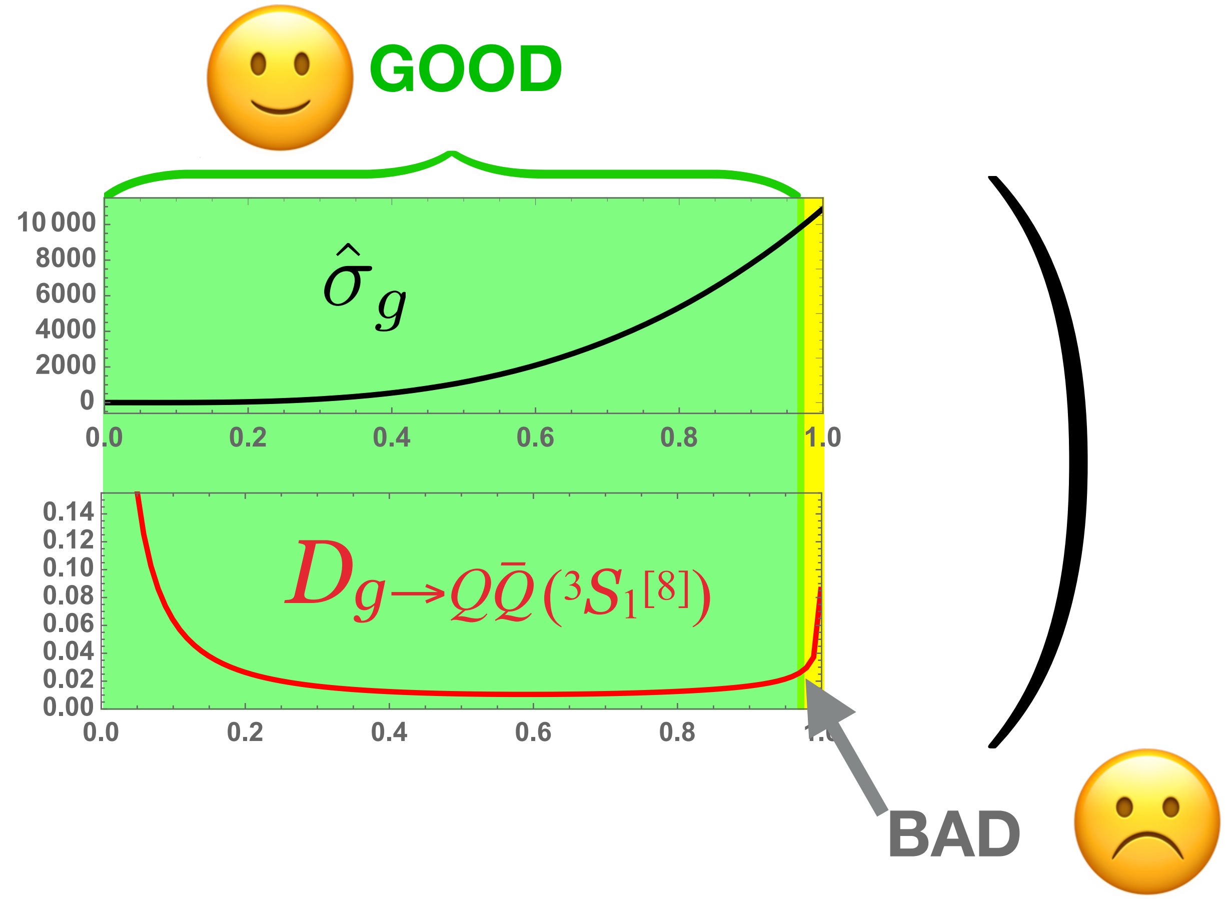
$$D(z,\mu) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, z^{-n}\tilde{D}(n,\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{2\tilde{P}_{gg}/b_0},$$

- The inversion integral can become ***ill-defined at $z=1$*** if the fragmentation function is a singular distribution at $z=1$. Hence the inversion integral can be computed for z very close to 1 but not at $z=1$.

CROSS SECTION WITH EVOLUTION

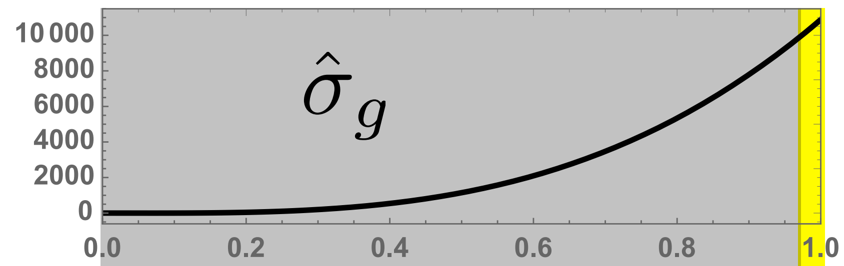
- The LP cross section is the single-parton cross section times the fragmentation function integrated over z. For gluon fragmentation into $Q\bar{Q}({}^3S_1^{[8]})$ (shown for $p_T=\mu=$20 GeV),

$$\sigma_{\mathrm{LP}} = \int_0^1 dz \left(\hat{\sigma}_g \times D_{g \to Q\bar{Q}({}^3S_1^{[8]})} \right)$$



- We need a better way to compute the bad integral.

🙁 $= \int_{1-\epsilon}^{1} dz \Bigg($



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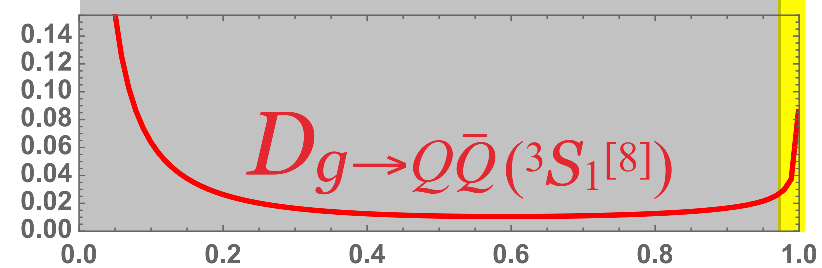
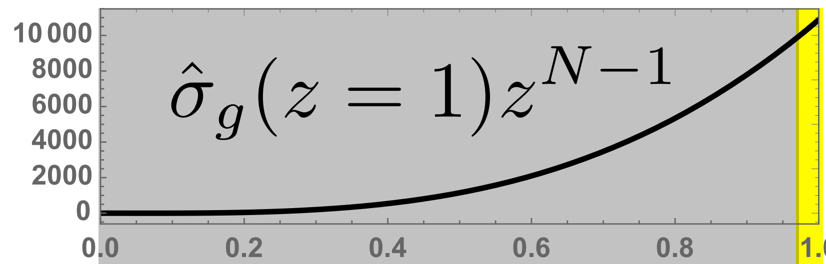


$\Bigg)$

For $1-\epsilon < z < 1$, $\hat{\sigma}_g(z) \approx \hat{\sigma}_g(1) z^{N-1}$ is a good approximation for some real number N.

☹ $\approx \int_{1-\epsilon}^{1} dz$ (



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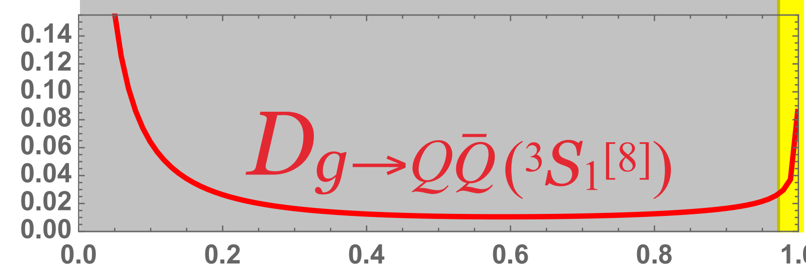
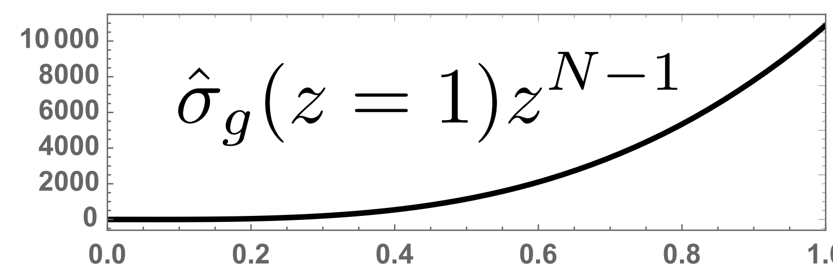


)

$= \left[\int_0^1 dz - \int_0^{1-\epsilon} dz \right]$ (



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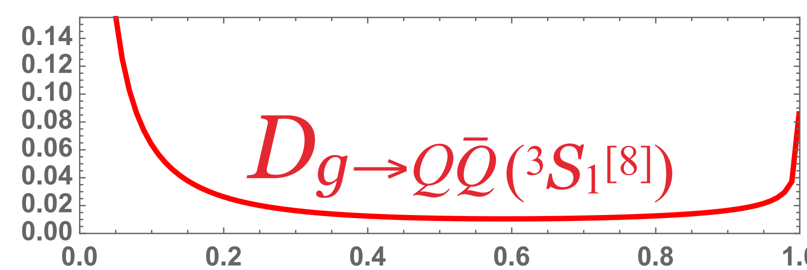

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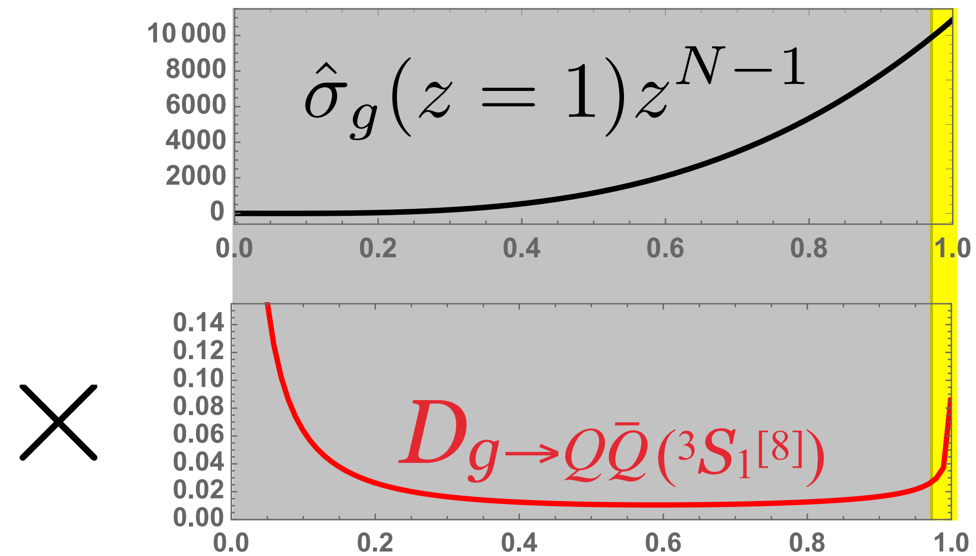
🙁 $\approx \int_{1-\epsilon}^{1} dz \Bigg($



$\Bigg)$

$$= \hat{\sigma}_g(z=1) \times \tilde{D}_{g\to Q\bar{Q}(^3S_1{}^{[8]})}(N,\mu)$$

Well-defined Mellin moment

$$-\hat{\sigma}_g(z=1) \times \int_0^{1-\epsilon} dz\, z^{N-1} D_{g\to Q\bar{Q}(^3S_1{}^{[8]})}(z,\mu)$$

Well-defined integral

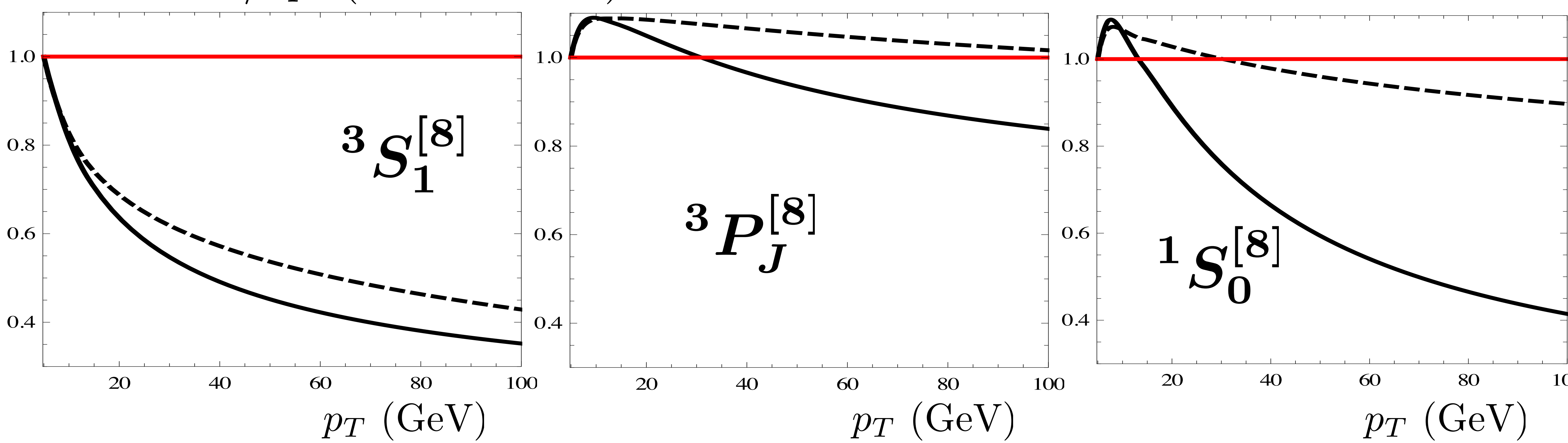
$$\boxed{\begin{aligned}\sigma_{\rm LP} \quad = \quad & \hat{\sigma}_g(z=1) \times \tilde{D}_{g\to Q\bar{Q}(^3S_1{}^{[8]})}(N,\mu) \\ & + \int_0^{1-\epsilon} dz\,[\hat{\sigma}_g(z) - \hat{\sigma}_g(z=1)z^{N-1}] D_{g\to Q\bar{Q}(^3S_1{}^{[8]})}(z,\mu)\end{aligned}}$$

- In practice, we put N=5 and $\epsilon = 10^{-6}$, and include quark-gluon mixing. It is straightforward to extend to NLL accuracy.

EFFECTS OF DGLAP EVOLUTION

- Modification of p_T-shape by evolution from μ_0=3 GeV to μ=p_T : Resummation modifies the shape differently for each channel.

—— : $\dfrac{d\sigma/dp_T(\text{with evolution})}{d\sigma/dp_T(\text{no evolution})}$ - - - - : perturbative calculation to order $\alpha_s \log(\mu/\mu_0)$



- When μ is not too large, numerical results agree well with perturbative calculation of DGLAP evolution to order $\alpha_s \log(\mu/\mu_0)$

EFFECTS OF DGLAP EVOLUTION

- Without DGLAP evolution, shapes of cross sections at large p_T are given by combinations of $1/p_T{}^4$ (LP) and $1/p_T{}^6$ (NLP). Hence, without resummation, it is ***impossible to constrain all three matrix elements*** from large p_T hadroproduction data.

Y.-Q. Ma, K. Wang, K.-T. Chao, PRL106, 042002 (2011)

- DGLAP evolution modifies the shapes of LP cross sections differently for each channel, leading to ***stronger constraints*** for the NRQCD matrix elements from LHC data.

G. T. Bodwin, HSC, U.-R. Kim, J. Lee, PRL113, 022001 (2014)
G. T. Bodwin, K.-T. Chao, HSC, U.-R. Kim, J. Lee, Y.-Q. Ma, PRD93, 034041 (2016)

EXCLUSIVE PRODUCTION

- For large $\sqrt{s}$, exclusive production amplitude factorizes into the perturbative hard part and the light-cone distribution amplitude (LCDA) at leading power in $m/\sqrt{s}$.

$$i\mathcal{A} = \int_0^1 dx\, T_H(x,\mu) \times f(\mu)\phi(x,m,\mu)$$

Lepage and Brodsky, PRD22, 2157 (1980)
Chernyak, Zhitnitsky, Phys.Rept.112, 173 (1984)

- LCDA describe the evolution of $Q\bar{Q}$ into a hadron.

- In NRQCD, LCDAs involve ***singular distributions***. Higher order corrections involve *derivatives of delta functions and complicated plus distributions*. For example, for a pseudoscalar quarkonium,

$$\phi_P(x,\mu_0) = \delta(x - \tfrac{1}{2}) + O(\alpha_s, v^2)$$

EXCLUSIVE PRODUCTION

- At tree level, ***NLP fragmentation function*** in the ***color singlet*** channel is just the ***square of the quarkonium LCDA*** at the scale of the heavy quark mass.
- If NRQCD factorization is valid, corrections to this relation is no worse than $O(\alpha_s)$ at the scale of the heavy quark mass.
- Hence, exclusive production at large $\sqrt{s}$ is an indirect probe for ***NLP fragmentation in inclusive production in the color singlet channel.***

EVOLUTION OF LCDA

- In NRQCD factorization, LCDA can be computed at the scale of the heavy quark mass.
- To compute exclusive production cross sections accurately, it is necessary to evolve the LCDA to the hard scale.
- Evolution of LCDA is governed by ERBL equation.

$$\mu^2 \frac{\partial}{\partial \mu^2} f_P(\mu)\phi_P(x,\mu) = C_F \frac{\alpha_s(\mu)}{2\pi} V(x,y) f_P(\mu)\phi_P(x,\mu)$$

Efremov and Radyushkin, PLB94, 245 (1980)
Lepage and Brodsky, PRD22, 2157 (1980)

- Standard way of solving the ERBL equation is to expand in terms of Gegenbauer polynomials.

EVOLUTION OF LCDA

- Gegenbauer expansion of LCDA:

$$|\phi\rangle = \sum_n |n\rangle\langle n|\phi\rangle$$

$$|n,x\rangle = N_n C_n^{(3/2)}(2x-1),$$
$$\langle n,x| = N_n w(x) C_n^{(3/2)}(2x-1),$$
$$N_n = \frac{4(2n+3)}{(n+1)(n+2)},$$
$$w(x) = x(1-x)$$

- Orthonormality: $\langle n|m\rangle = \delta_{nm}$
- Completeness: $\sum_n |n,x'\rangle\langle n,x| = \delta(x-x')$
- Solution to ERBL equation is given by

$$|\phi(\mu)\rangle = \sum_{m,n} |m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

Evolution matrix

- Production amplitude with resummation is given by

$$i\mathcal{A} = \sum_{m,n} \langle T_H|m\rangle\langle m|U|n\rangle\langle n|\phi(\mu_0)\rangle$$

EVOLUTION OF LCDA

$$i\mathcal{A} = \sum_{m,n} \langle T_H | m \rangle \langle m | U | n \rangle \langle n | \phi(\mu_0) \rangle$$

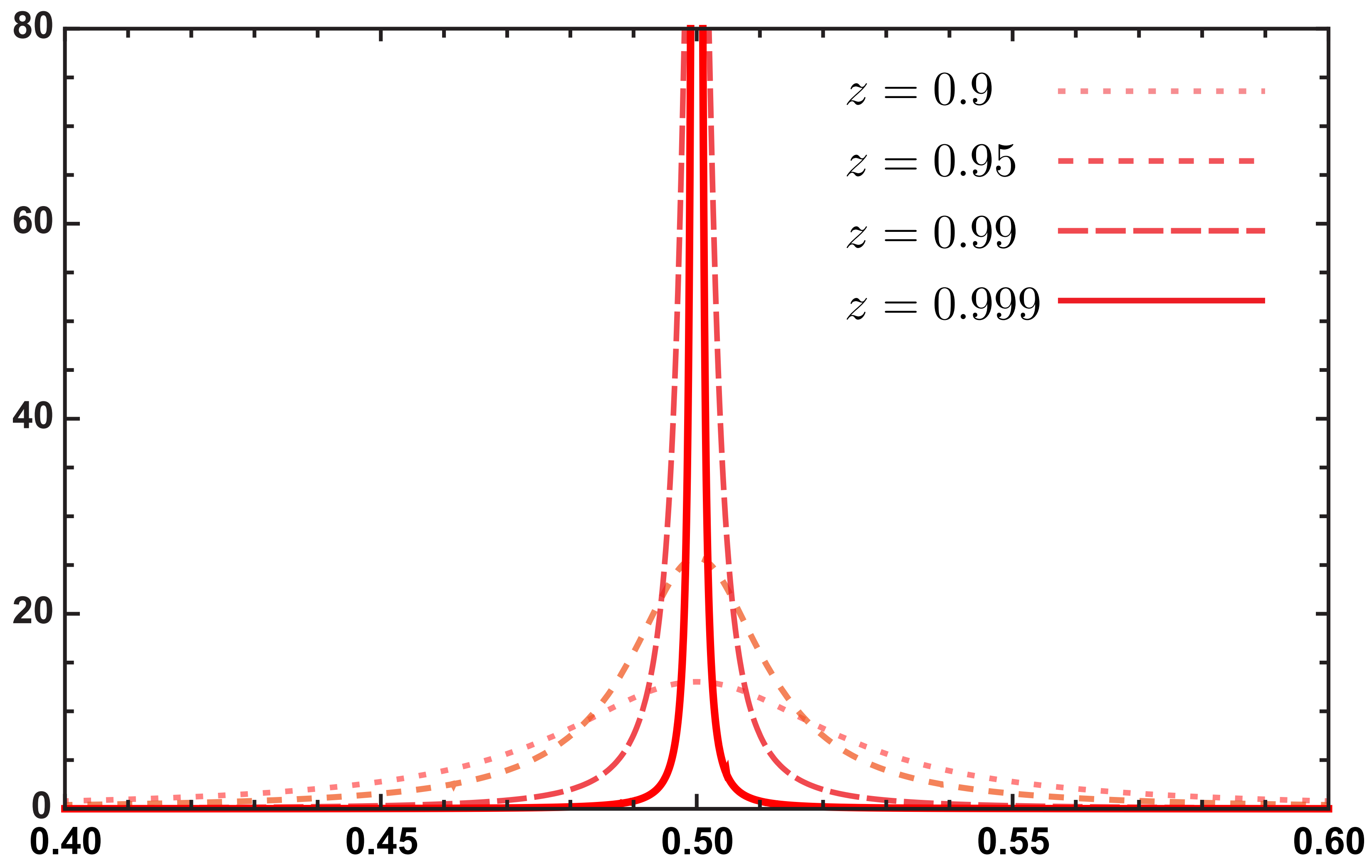
- When $\phi(x, \mu)$ is a singular distribution, **this sum *does not converge*** or *converges very slowly*.
- One way to define that divergent series is to use Abel sum.

$$\sum_n |n, x'\rangle \langle n, x| \to \lim_{z \to 1^-} \sum_n z^n |n, x'\rangle \langle n, x|$$

- This replaces the delta function $\delta(x - x')$ with a sequence of ordinary functions.

EVOLUTION OF LCDA

- Abel sum of the Gegenbauer expansion of $\delta(x - \frac{1}{2})$



EVOLUTION OF LCDA

- Now the series converges and we can compute the resummed amplitude

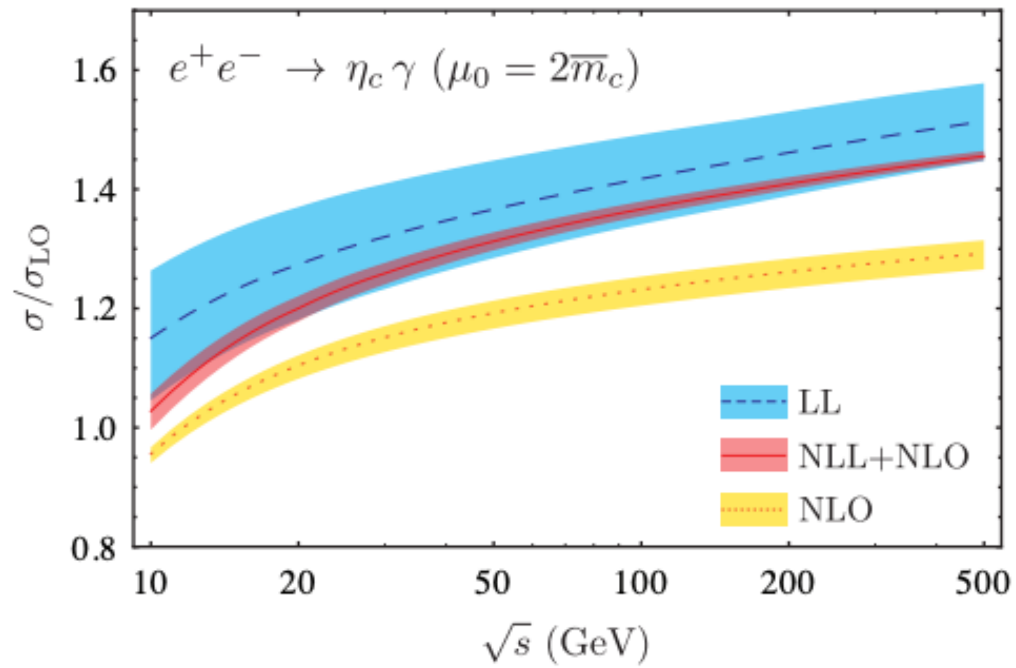
$$i\mathcal{A} = \lim_{z \to 1^-} \sum_{m,n} z^n \langle T_H | m \rangle \langle m | U | n \rangle \langle n | \phi(\mu_0) \rangle$$

- Numerical values of the series can be computed efficiently by using Padé approximants.
- This technique have been used for NLL resummation in $H \to J/\psi + \gamma$, $Z \to J/\psi + \gamma$, and $e^+e^- \to \eta_c + \gamma$.

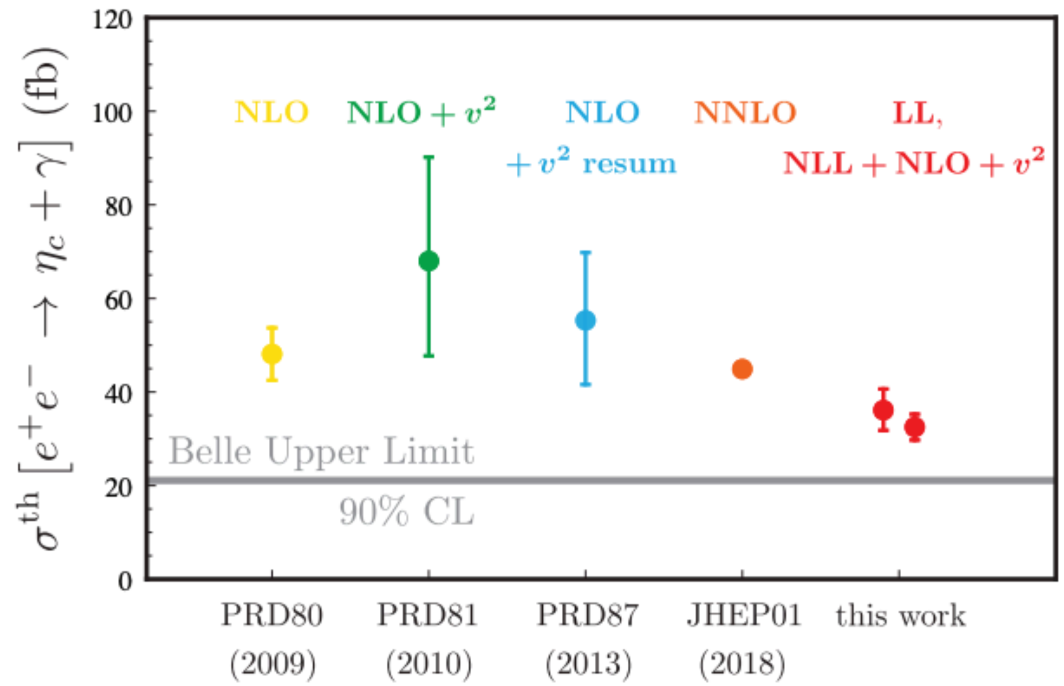
Bodwin, HSC, Ee, Lee, PRD95 (2017) 054018, PRD96 (2017) 116014, PRD97 (2018) 016009
Brambilla, HSC, Lai, Shtabovenko, Vairo, PRD100 (2019) 054038
HSC, Ee, Kang, Kim, Lee, Wang, JHEP 1910 (2019) 162

η_c PRODUCTION IN LEPTON COLLIDERS

- Resummation in the process $e^+e^- \to \eta_c + \gamma$ mildly increases the cross section.
  The theory prediction is above the measured upper limit.



Belle, PRD 98, 092015 (2018)



HSC, Ee, Kang, Kim, Lee, Wang, JHEP 1910 (2019) 162

- This may imply that the color singlet NLP fragmentation could be overestimating the inclusive η_c cross section.

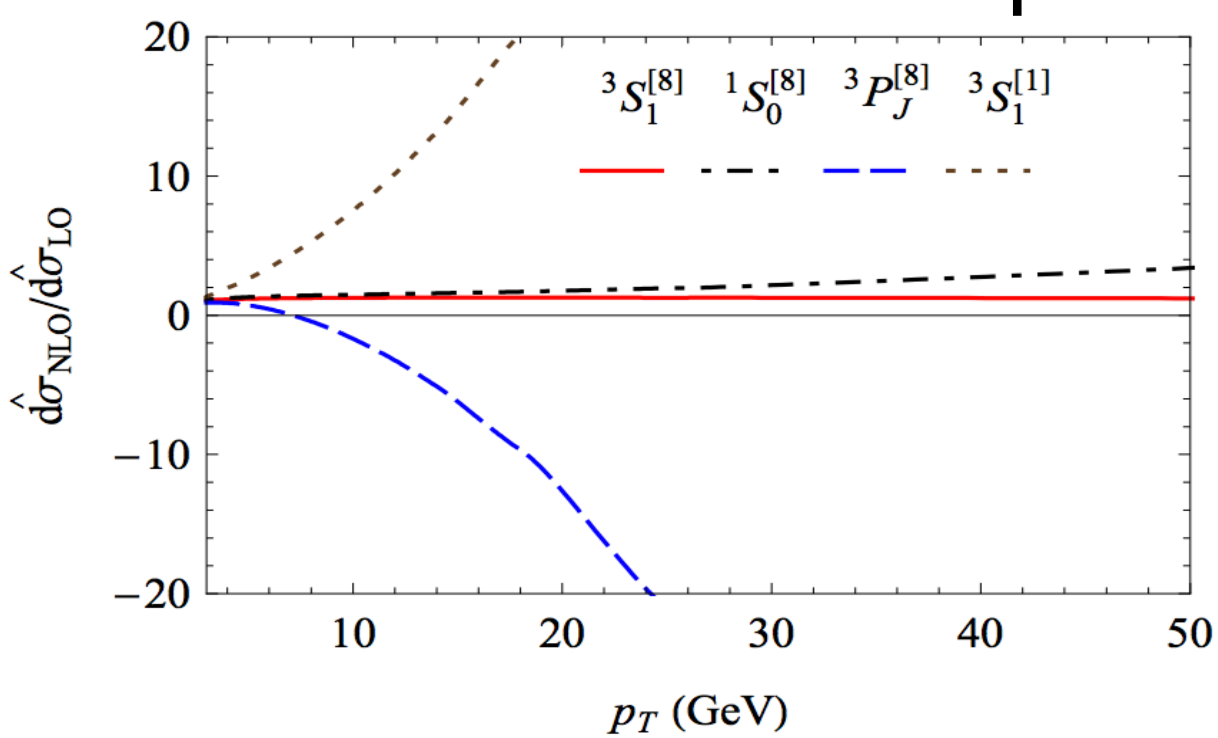
SUMMARY

- We developed methods to solve evolution equations for quarkonium fragmentation functions and light-cone distribution amplitudes that involve singular distributions.
- DGLAP evolution gives significant corrections for inclusive production processes at large p_T.
- ERBL evolution is necessary to make precise theoretical predictions for exclusive production processes.
- Exclusive production processes can serve as indirect probes for inclusive production at moderately large p_T through NLP fragmentation.

BACKUP

HADROPRODUCTION IN NRQCD

- Perturbative $Q\bar{Q}$ cross sections are generally available up to NLO accuracy in α_s. NLO cross sections can have shapes that are very different from LO, because new fragmentating contributions become available at NLO accuracy and give rise to K factors that depend strongly on p_T.



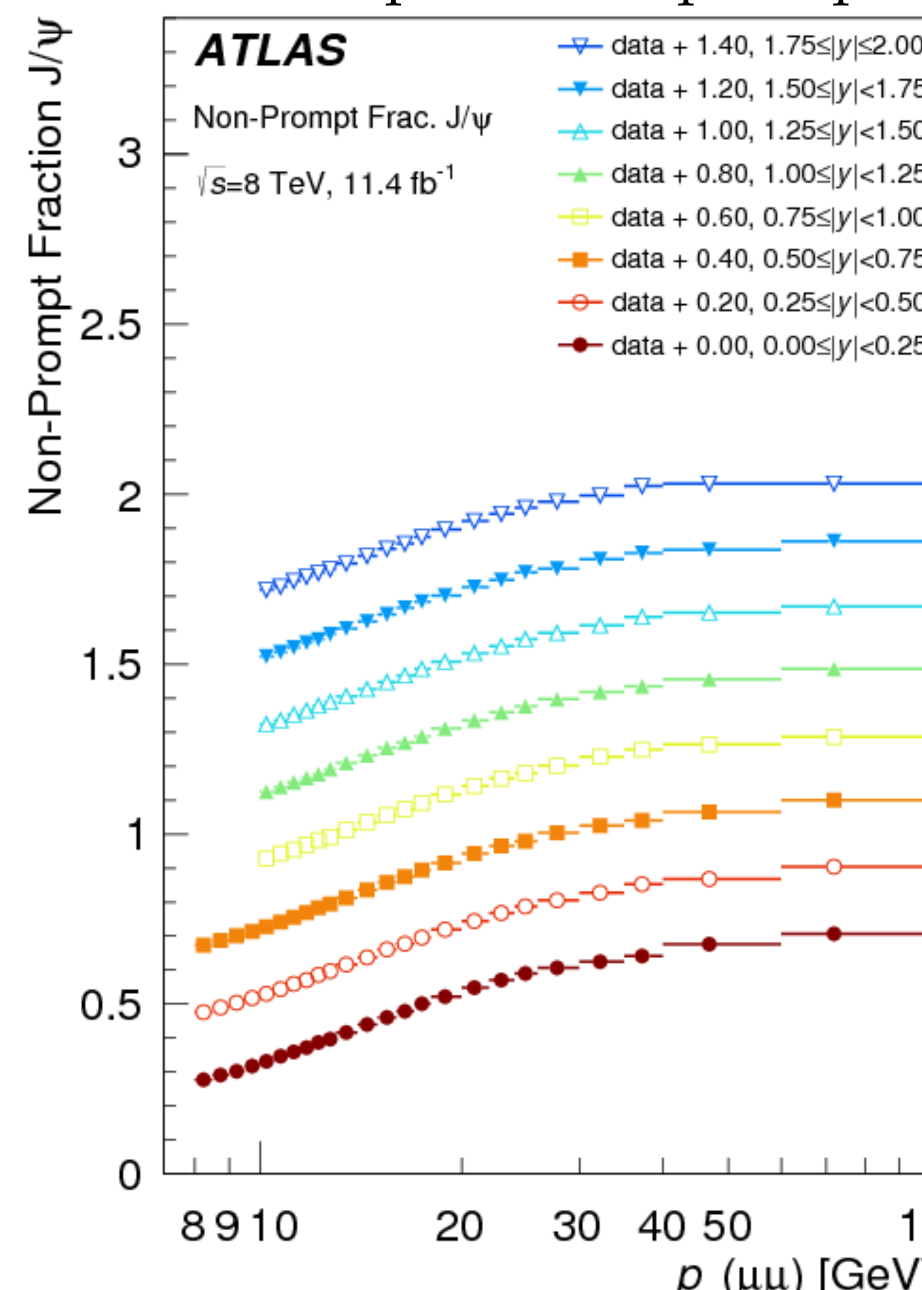
Ma, Wang, Chao, PRL106, 042002 (2011)

- We can understand this from QCD factorization theorems that apply to the leading and next-to-leading power contributions in the expansion in powers of $1/p_T$

QUARKONIUM PRODUCTION

- Fragmentation does not provide a good description of prompt quarkonium production unless p_T is extremely large.
- Comparison of p_T distributions of prompt and non-prompt J/ψ cross sections show that the fragmentation mechanism would only dominate the charmonium cross section when $p_T \gtrsim 100$ GeV.

$$\frac{\text{Non-prompt}}{\text{Prompt} + \text{Non-prompt}}$$



Atlas, EPJC 76(5), 1-47, (2016)