

Heavy-quark production with k_T -factorization: The importance of the sea-quark distribution

Benjamin Guiot

REF 2019



UNIVERSIDAD TECNICA
FEDERICO SANTA MARIA

Based on Phys.Rev.D99 no.7, 074006 and arxiv 1910.09656

I will discuss:

- 1 The pt-distribution of one heavy quark in collinear factorization:
 - Difference between the Fixed-Flavor-Number scheme and the Variable-Flavor-Number scheme.
 - In the VFN scheme, the main contribution is $Qg \rightarrow Qg$, **not** $gg \rightarrow Q\bar{Q}$.
- 2 The situation is exactly the same in kt-factorization:
 - Explicit calculations with the parton-branching (PB) uPDFs.
- 3 Why some studies, including only $gg \rightarrow Q\bar{Q}$, are in agreement with data?
 - Study of different cases.
 - Emphasis on the KMR unintegrated PDFs.

H.Q. production within collinear factorization

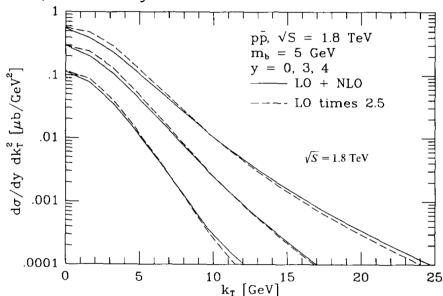
First results at NLO (1989) for H.Q. production obtained with a fixed-flavor-number scheme (**FFNS**):

- 3-flavor-number scheme: the proton contains only light flavors.
- 4-flavor-number scheme: includes also the charm quark (used for bottom production only).

Same slope for LO and NLO.

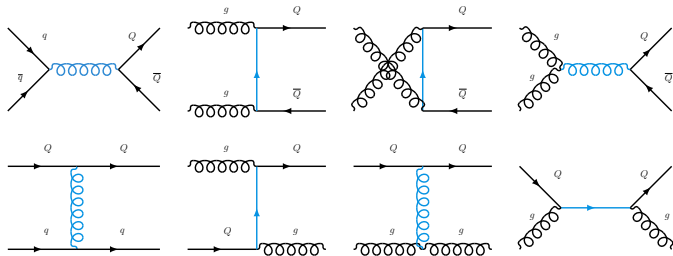
NLO bring a factor 2.5 in the hard cross section $\hat{\sigma}$.
(mainly due to the opening of the flavor excitation channel)

Taken from: *P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. B327.*



Known that the NLO FFNS calculations fail at $p_t \gg m_Q$. **Need for resummation of $\ln(p_t^2/m_Q^2)$ to all orders.**

Variable-flavor-number scheme (VFNS)



Includes the H.Q. densities \Rightarrow resums to all order the large $\ln(p_t^2/m_Q)$.

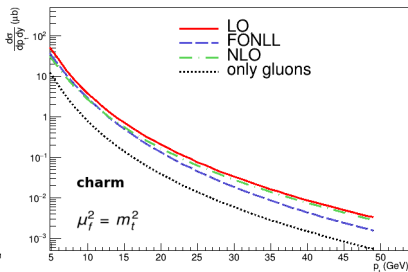
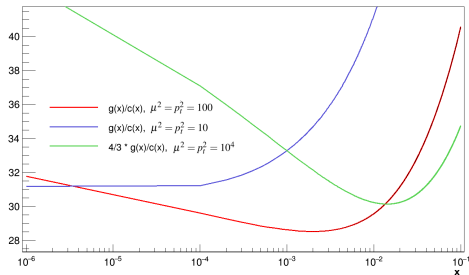
Includes the flavor excitation processes at LO. NLO corrections *do not* give a large factor K .

Used in modern calculations (see for instance FONLL calculations).

Main contribution to HQ production in the VFNS

Known at least since 2002 that the main process is $Qg \rightarrow Qg$ (*R. D. Field, "Sources of b quarks at the Fermilab Tevatron and their correlations", Phys. Rev. D 65, 094006*)

It's a 5 minutes calculation to check this is true: $g(x)/c(x) \sim 40\dots$
But $\sigma^{Qg \rightarrow Qg} / \sigma^{gg \rightarrow Q\bar{Q}} \sim 60$ ($y = 0$ and $p_t = 10$ GeV)



To summarize

FFNS: By definition ($Q(x, \mu) = 0, \forall \mu$), the main contribution is $gg \rightarrow Q\bar{Q}$ (in this discussion $q\bar{q} \rightarrow Q\bar{Q}$ is neglected).

VFNS: The main contribution is clearly $Qg \rightarrow Qg$.

*Remember that we discuss the production of **one** heavy quark.
The situation is different for quarkonia*

What should we expect for kt-factorization (in a VFN scheme)

1. Collinear and unintegrated PDFs are related:

$$f_i(x, Q^2) = \int^{Q^2} F_i(x, k_t^2; Q^2) dk_t^2$$

If the different F_i have a similar k_t shape, then:

$$g(x)/c(x) \sim F_g(x, k_t)/F_c(x, k_t)$$

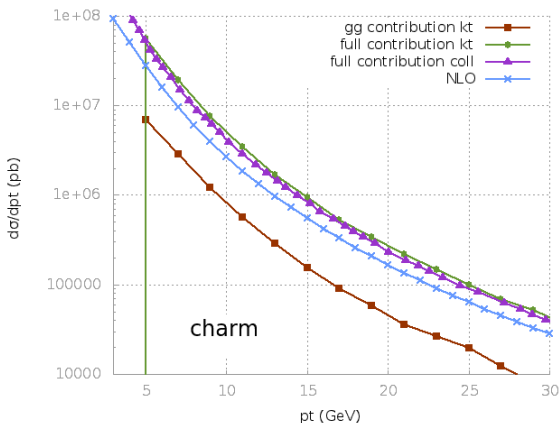
2. In some limits ($k_t \ll p_t$) the off-shell cross sections give back the usual partonic cross sections.

\Rightarrow No way that $\sigma^{Qg \rightarrow Qg} / \sigma^{gg \rightarrow Q\bar{Q}} \sim 60$ in coll. fact. but ~ 1 in kt-fact.

Similar to the collinear fact. case, one should expect $Qg \rightarrow Qg$ to be the main contribution.

Charm production with KaTie

- KaTie (*A. Van Hameren, arXiv:1611.00680*) is an event generator allowing the use of k_t -fact.
- We used the Parton-Branching (PB) uPDFs (*A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik, arXiv:1804.11152*).



Importance of this discussion

Abstract of a recent paper including only the gg contribution:

“The use of the KMR uPDF leads to a good description of the existing charm (D -meson) data already at the leading-order. On the other hand, a new Parton-Branching (PB) uPDF strongly underestimates the same experimental data. A direct inclusion of the higher-orders at tree-level leads to an overestimation of the data, especially for the KMR uPDF. This suggests a significant double-counting. We propose a simple method ...”

Belief that $gg \rightarrow Q\bar{Q}$ gives (always) the main contribution \Rightarrow Calculations include only the gg process (other contributions are even not mentioned) \Rightarrow Use a correct uPDFs set, and concludes it is not working.

I have shown that the PB uPDFs work correctly. Let's see what happens with KMR and other studies!

What about calculations based on the gg contribution and in agreement with data?

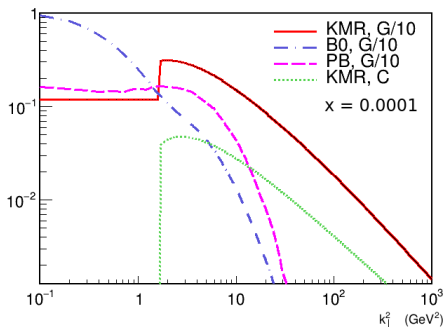
Two possibilities:

- Calculations done with a FFN scheme (for instance CCFM):
OK! *Note that in this case the gluon density extracted are larger than the one obtained with a VFN scheme.*
- Calculations done with a VFN scheme (for instance KMR):
Cannot be correct! After the inclusion of $Qg \rightarrow Qg$, the result overshoots the data.

Why is the gg contribution in agreement with data?

Some studies include an effective large K factor:

- 1 Wrong normalization of the unintegrated gluon density?
- 2 Too large factorization scale $\mu_F^2 > \hat{s}$. (see *B. Guiot, Phys.Rev. D99 (2019) no.7, 074006* for more details)
- 3 **Too large k_t tail (for $k_t > \mu$).** This is the case for KMR (with the angular-ordering cut-off).



$$\mu^2 = 10 \text{ GeV}^2$$

More details on the KMR/WMR prescriptions

Based on *B. Guiot, arXiv:1910.09656* and *K. Golec-Biernat and A. M. Stasto, Phys. Lett. B 781 (2018) 633-638*.

Not one, but several KMR/WMR prescriptions, leading to **significant differences**.

In *M. A. Kimber, A. D. Martin, M. G. Ryskin (2001)* the DGLAP equation is written:

$$\frac{\partial a(x, \mu^2)}{\partial \ln \mu^2} = \sum_{a'} \frac{\alpha_s}{2\pi} \left[\int_x^{1-\Delta} P_{aa'}(z) a' \left(\frac{x}{z}, \mu^2 \right) dz - a(x, \mu^2) \int_0^{1-\Delta} P_{a'a}(z) dz \right]$$

While, in *G. Watt, A. D. Martin, M. G. Ryskin (2003)* it is written:

$$\frac{\partial a(x, \mu^2)}{\partial \ln \mu^2} = \sum_{a'} \frac{\alpha_s}{2\pi} \left[\int_x^{1-\Delta} P_{aa'}(z) a' \left(\frac{x}{z}, k_t \right) dz - a(x, \mu^2) \int_0^{1-\Delta} z P_{a'a}(z) dz \right]$$

Modern studies with “KMR” uPDFs use the latter. These equations are not equivalent, I will use the terminology WMR for the latter.

Only the (2003) equation gives the correct DGLAP equation for quarks and gluon.

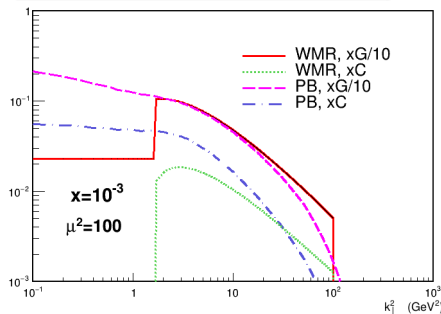
More details on the KMR/WMR prescriptions

Several prescriptions: two choices for the cut-off Δ .

Strong Ordering (SO)

$$\Delta = \frac{k_t}{\mu};$$

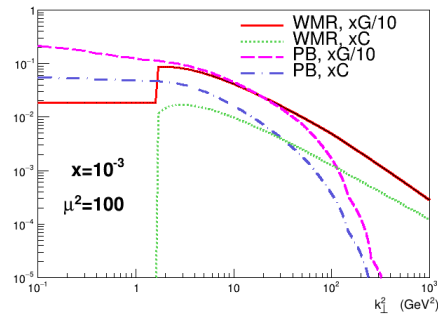
$$x < 1 - \Delta \Rightarrow k_t < \mu(1 - x) < \mu$$



Angular Ordering (AO)

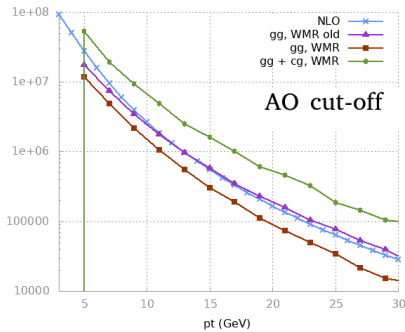
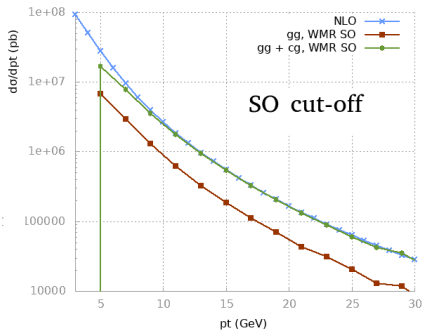
$$\Delta = \frac{k_t}{k_t + \mu};$$

$$k_t > \mu \text{ allowed, giving } T_a > 1$$



reminder: $T_a = \exp \left\{ - \int_{k_t^2}^{\mu^2} \frac{dp_t^2}{p_t^2} \sum_{a'} \int_0^{1-\Delta} dz z P_{a'a} \right\}$

Charm production with the SO ans AO cut-off



- SO cut-off: The sum gives a perfect agreement with NLO.
- As expected the gg contribution alone underestimate the cross section by a factor ~ 3 .
- AO: The sum overestimates completely NLO calculations. Due to the too large tail of the distribution at $k_t > \mu$.

The KMR/WMR parametrization uses the Sudakov factor:

$$T_a(\mu, k_t) = \exp \left\{ - \int_{k_t^2}^{\mu^2} \frac{dp_t^2}{p_t^2} \sum_{a'} \int_0^{1-\Delta(p_t)} dz z P_{a'a}(z, p_t) \right\},$$

with $P_{a'a}(z, \mu) = \frac{\alpha_s(\mu^2)}{2\pi} P_{a'a}^{\text{LO}}(z)$, and rewrite the DGLAP equation

$$\frac{\partial}{\partial \ln k_t^2} [T_a(\mu, k_t) f_a(x, k_t)] = T_a(\mu, k_t) \sum_{a'} \int_x^{1-\Delta} \frac{dz}{z} P_{aa'}(z, k_t) f_{a'} \left(\frac{x}{z}, k_t \right).$$

(be very careful with the k_t dependence of T_a !!!)

Def 1:

$$F_a(x, k_t^2, \mu^2) = \frac{1}{k_t^2} f_a(x, k_t^2, \mu^2) = \frac{1}{k_t^2} \frac{\partial}{\partial \ln k_t^2} [T_a(\mu, k_t) f_a(x, k_t)],$$

Def 2:

$$f_a(x, k_t^2, \mu) = T_a(\mu, k_t) \sum_{a'} \int_x^{1-\Delta} \frac{dz}{z} P_{aa'}(z, k_t) f_{a'} \left(\frac{x}{z}, k_t \right)$$

Issue with the definitions

In *Phys. Lett. B 781 (2018) 633-638*, it is mentioned that Def 1 and Def 2 give different results.

It is related to the fact that the previous relation is not correct when $k_t > \mu$ is allowed ([arXiv:1910.09656](https://arxiv.org/abs/1910.09656)).

In this case, T_a can be larger than 1, and what is generally done is:
 $\tilde{T}_a(k_t < \mu) = T_a(\mu, k_t)$; $\tilde{T}_a(k_t > \mu) = 1$.

These equations can be written:

$$\tilde{T}_a(\mu, k_t) = \Theta(\mu^2 - k_t^2)T_a(\mu, k_t) + \Theta(k_t^2 - \mu^2)$$

Then

$$\begin{aligned} \frac{\partial}{\partial \ln k_t^2} \left[\tilde{T}_a(\mu, k_t) f_a(x, k_t) \right] &= \tilde{T}_a(\mu, k_t) \sum_{a'} \int_x^{1-\Delta} \frac{dz}{z} P_{aa'}(z, k_t) f_{a'} \left(\frac{x}{z}, k_t \right) \\ &\quad - \Theta(k_t^2 - \mu^2) f_a(x, k_t) \sum_{a'} \int_0^{1-\Delta} dz z P_{a'a}(z, k_t) \end{aligned}$$

The issue is solved by changing Def 2 in agreement with the new relation:

Def 1:

$$F_a(x, k_t^2, \mu^2) = \frac{1}{k_t^2} f_a(x, k_t^2, \mu^2) = \frac{1}{k_t^2} \frac{\partial}{\partial \ln k_t^2} \left[\tilde{T}_a(\mu, k_t) f_a(x, k_t) \right],$$

Def 2 (new):

$$f_a(x, k_t^2, \mu^2) = \tilde{T}_a(\mu, k_t) \sum_{a'} \int_x^{1-\Delta} \frac{dz}{z} P_{aa'}(z, k_t) f_{a'}\left(\frac{x}{z}, k_t\right) \\ - \Theta(k_t^2 - \mu^2) f_a(x, k_t) \sum_{a'} \int_0^{1-\Delta} dz z P_{a'a}(z, k_t)$$

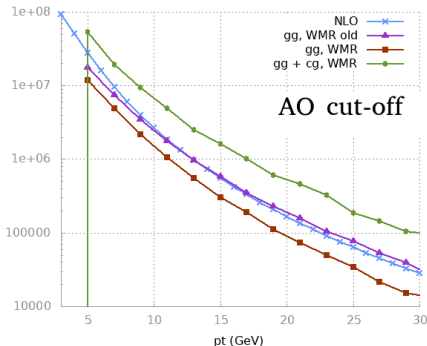
Angular-ordering KMR uPDFs are ill-defined

The condition $f_a(x, \mu^2) \simeq \int_0^{\mu^2} F(x, k_t^2; \mu^2) dk_t^2$ is not enough restrictive.

$F_a(x, k_t^2, \mu^2) \rightarrow F_a(x, k_t^2, \mu^2) + \Theta(k_t^2 - \mu^2)A(x, k_t^2, \mu^2)$ works the same \Rightarrow infinite number of non-equivalent definitions.

But it changes significantly the result for the cross section.

Not the case for the SO cut-off, since $F_a(x, k_t^2 > \mu^2) = 0$.



- **Main message:** In a VFN scheme, the main contribution to the p_t -distribution of one heavy quark is $Qg \rightarrow Qg$.
- True for all uPDFs discussed in this presentation: PB, SO KMR **AND** AO KMR uPDFs.
- Consequently, if the gg contribution is in agreement with data, the full result overestimates the cross section.
- This is the case for the AO KMR uPDFs. Due to the too large k_t tail at $k_t > \mu$ (see *F. Hautmann, L. Keersmaekers, A. Lelek, A. M. van Kampen, arXiv:1908.08524* for a related discussion).
- AO KMR uPDFs ill-defined: multiple non-equivalent definitions. **The SO KMR uPDFs should be preferred.**
- Drell-Yan Z-boson : $\mu^2 \simeq M_Z^2 \simeq 8000 \text{ GeV}^2$. The issue with the AO KMR uPDFs manifests only when $k_t^2 > \mu^2$.