



## Open issues in the realm of mesons up to 2 GeV

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Seminar

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# Outline



Symmetries of QCD

Conventional mesons

The state  $a_1(1260)$

Pseudotensor mesons and large isoscalar mixing

Excited vectors and the missing strange-antistrange  $\phi(1930?)$

Conclusions

# Symmetries of QCD

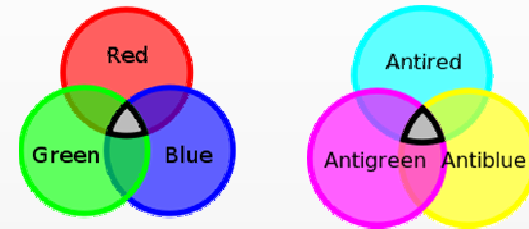


**Born** Giuseppe Lodovico Lagrangia  
25 January 1736  
Turin

**Died** 10 April 1813 (aged 77)  
Paris

# The QCD Lagrangian

Quark: u,d,s and c,b,t R, G, B

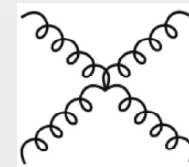
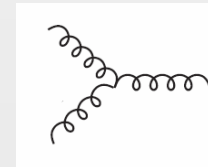
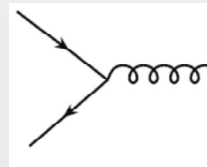


$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; i = u, d, s, \dots$$

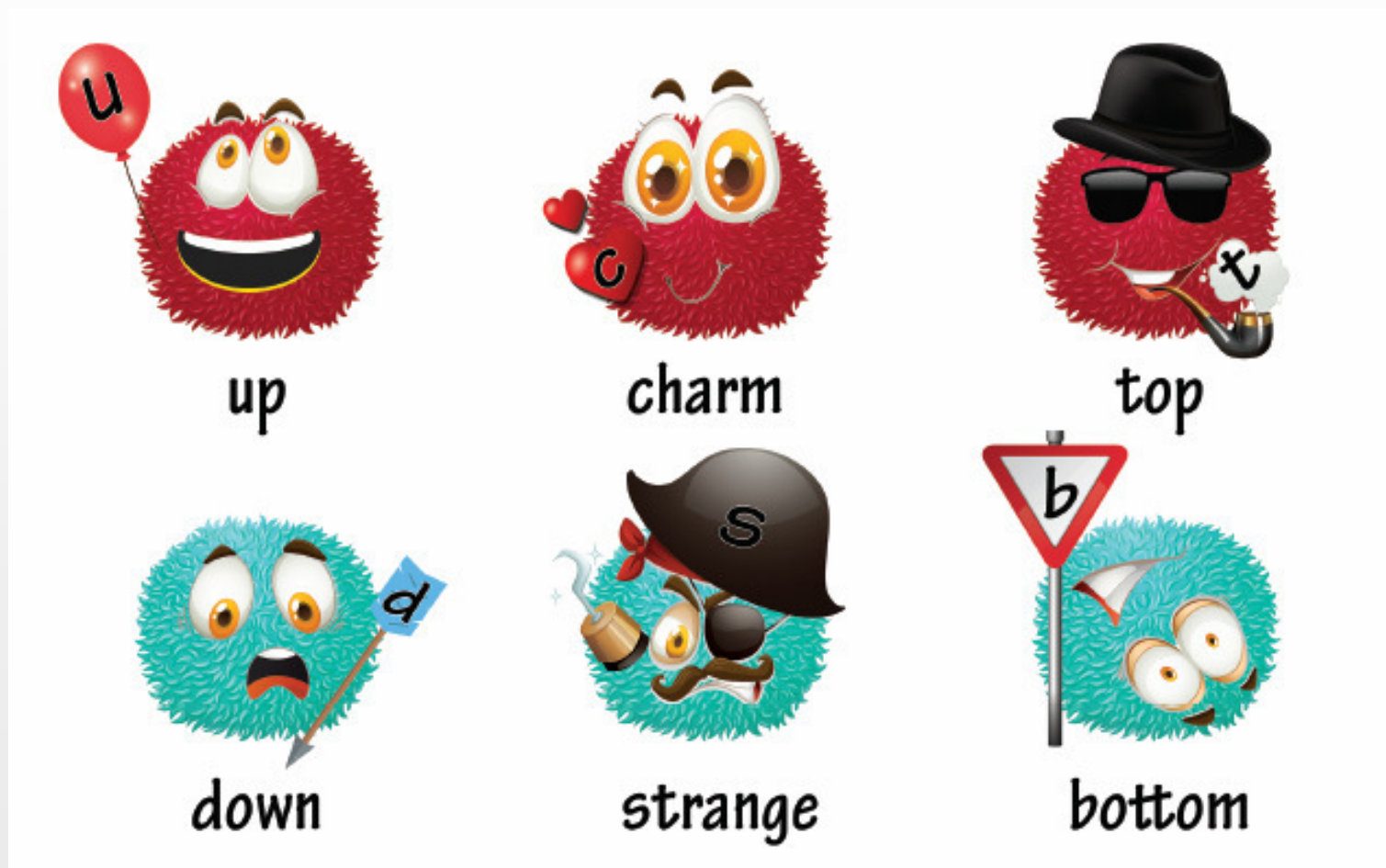
8 type of gluons (RG, BG, ...)

$$A_\mu^a; a = 1, \dots, 8$$

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$



Confinement: quarks never 'seen' directly. How they might look like ☺



Picture by Pawel Piotrowski

# Trace anomaly: the emergence of a dimension

**Chiral limit:**  $m_f = 0$

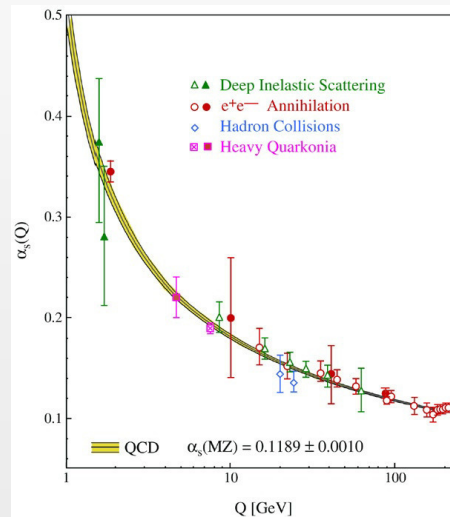
$$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$$

is a classical symmetry broken by quantum fluctuations (trace anomaly)

**Dimensional transmutation**

$$\Lambda_{YM} \approx 250 \text{ MeV}$$

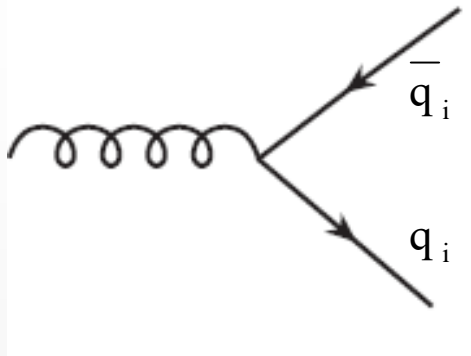
$$\alpha_s(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass:  $m_{gluon} = 0 \rightarrow m_{gluon}^* \approx 500 - 800 \text{ MeV}$

Gluon condensate:  $\langle G_{\mu\nu}^a G^{a,\mu\nu} \rangle \neq 0$

# Flavor symmetry



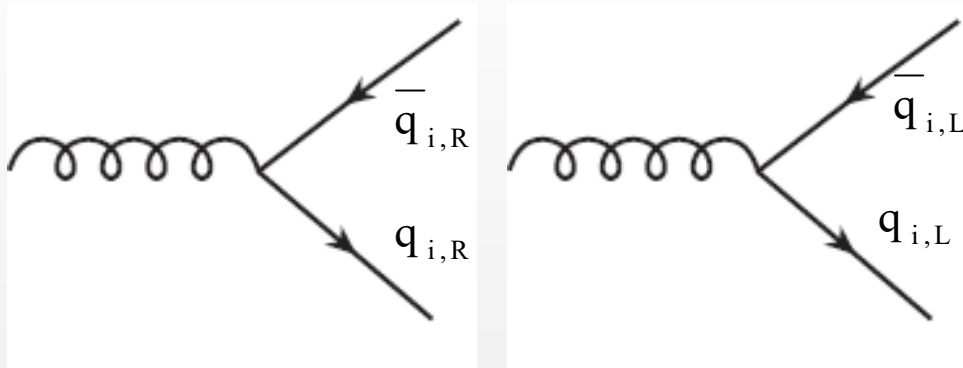
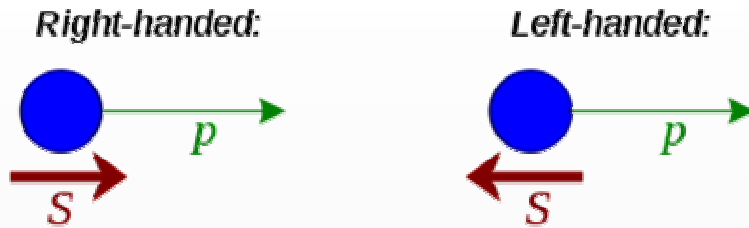
Gluon-quark-antiquark vertex

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^\dagger U = 1$$

# Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1 + \gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1 - \gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U_{ij}^R q_{j,R} + U_{ij}^L q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number      anomaly U(1)<sub>A</sub>

SSB into SU(3)<sub>v</sub>

Chiral (or axial) anomaly: explicitly broken by quantum fluctuations

$$\partial^\mu (\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr}(G_{\mu\nu} G_{\rho\sigma})$$

In the chiral limit ( $m_i=0$ ) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum



# Symmetries of QCD and breakings



**SU(3)<sub>color</sub>:** exact. Confinement: you never see color, but only white states.

**Dilatation invariance:** holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

**SU(3)<sub>R</sub> × SU(3)<sub>L</sub>:** holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to U(3)<sub>V=R+L</sub>

**U(1)<sub>A=R-L</sub>:** holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

# Conventional mesons: quark-antiquark fields

# Hadrons



The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‘colored’ state has been seen.

Confinement: physical states are “white” and are called hadrons.

Hadrons can be:

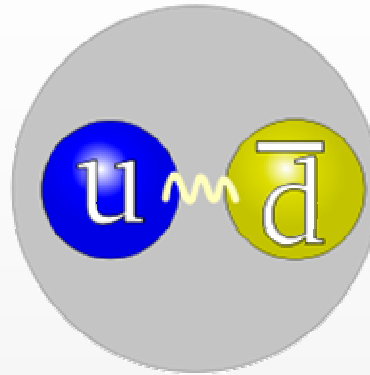
Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.

A quark-antiquark state is a conventional meson.

# Example of conventional quark-antiquark states: the $\rho$ and the $\pi$ mesons



Rho-meson

$$m_{\rho^+} = 775 \text{ MeV}$$

where

$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c} (|\pi^+\pi^0\rangle + \dots)$$

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Pion

$$m_{\pi^+} = 139 \text{ MeV}$$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD  
is a nonpert. phenomenon  
based on SSB  
(mentioned previously).

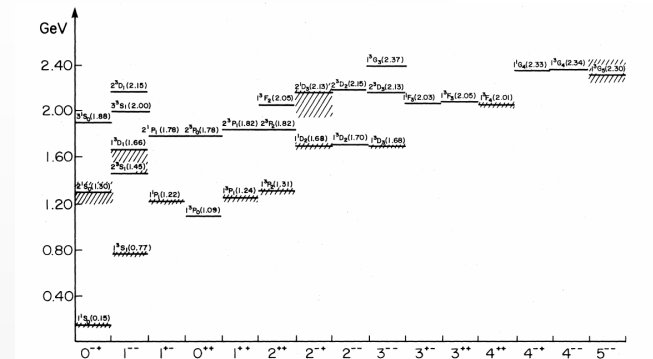
# Quark model(s) and their QFT extensions



Mesons in a Relativized Quark Model with Chromodynamics

S. Godfrey, N. Isgur

Phys.Rev. D32 (1985) **189-231**



QCD phenomenology based on a chiral effective Lagrangian

T. Hatsuda, T. Kunihiro

Phys.Rept. **247** (1994) 221-367

NJL: quark-based model with  
chiral symmetry and SSB  
chiral condensate  
Effective quark mass  
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions: Confinement  
dynamical symmetry breaking, and hadrons as relativistic bound  
states

R. Alkofer, L. von Smekal

Phys.Rept. **353** (2001) 281

DS:  
quarks and gluons propagators  
from QCD  
Condensates  
Effective quark and gluon masses  
Spectra of mesons as quarkonia  
(pion: ok) and baryons as qqg states

Baryons as relativistic three-quark bound states

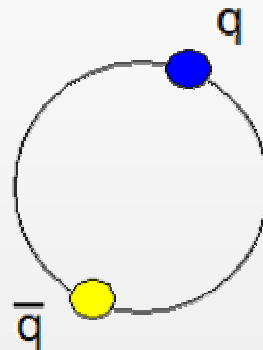
G. Eichmann et al.

Progr. Part. Nucl. Phys. **91** (2016) 1

# Conventional mesons

Quark: u,d,s,... R,G,B

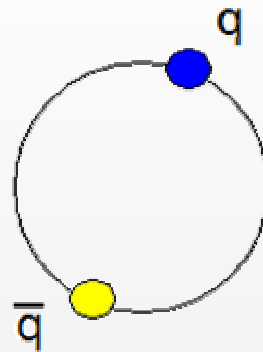
Quark-antiquark bound states: conventional mesons



$$|color\rangle = \sqrt{1/3} (\bar{R}R + \bar{B}B + \bar{G}G)$$

## Conventional mesons/2

Surely, with quark-antiquark states we can understand a lot of QCD, but definitely not everything.



$$P = -(-1)^L \quad C = (-1)^{L+S}$$

$$L, S \quad \longrightarrow \quad J = L + S \quad J^{PC}$$

$L = S = 0 \rightarrow J^{PC} = 0^{-+}$  pseudoscalar mesons

$$|\pi^+\rangle = |u\bar{d}\rangle |\text{space} : L = 0\rangle |\text{spin} : S = 0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$



$$|K^+\rangle = |u\bar{s}\rangle |\text{space} : L = 0\rangle |\text{spin} : S = 0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...

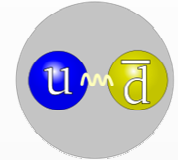
$$|D^0\rangle = |u\bar{c}\rangle |\text{space} : L = 0\rangle |\text{spin} : S = 0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...



$L = 0, S = 1 \rightarrow J^{PC} = 1^{--}$  vector mesons

$$|\rho^+\rangle = |u\bar{d}\rangle |space : L = 0\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$



...

$$|K^*(892)^+\rangle = |u\bar{s}\rangle |space : L = 0\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...

$$|D^{*0}\rangle = |u\bar{c}\rangle |space : L = 0\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...

$$|j/\Psi\rangle = |c\bar{c}\rangle |space : L = 0\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

$L = S = 1 \rightarrow J^{PC} = 0^{++}$  scalar mesons

$$|\sigma\rangle = |u\bar{u} + d\bar{d}\rangle |space : L = 1\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

corresponds to the resonance  $f_0(1370)$ .

...

...

$$|\chi_{c0}(1S)\rangle = |c\bar{c}\rangle |space : L = 1\rangle |spin : S = 1\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

# PDG quark-antiquark listing

$n^{2s+1}\ell_J$	$J^{PC}$	$l = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$l = \frac{1}{2}$ $u\bar{s}, d\bar{s}; \bar{d}s, -\bar{u}s$	$l = 0$ $f'$	$l = 0$ $f$
$1^1S_0$	$0^{-+}$	$\pi$	$K$	$\eta$	$\eta'(958)$
$1^3S_1$	$1^{--}$	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
$1^1P_1$	$1^{+-}$	$b_1(1235)$	$K_{1B}^\dagger$	$h_1(1380)$	$h_1(1170)$
$1^3P_0$	$0^{++}$	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
$1^3P_1$	$1^{++}$	$a_1(1260)$	$K_{1A}^\dagger$	$f_1(1420)$	$f_1(1285)$
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$
$1^3D_1$	$1^{--}$	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$
$1^3D_2$	$2^{--}$		$K_2(1820)$		

# Spontaneous symmetry breaking at the meson level

$$\pi = \pi^0 \equiv \sqrt{1/2}(\bar{u}u - \bar{d}d) \text{ neutral pion}$$

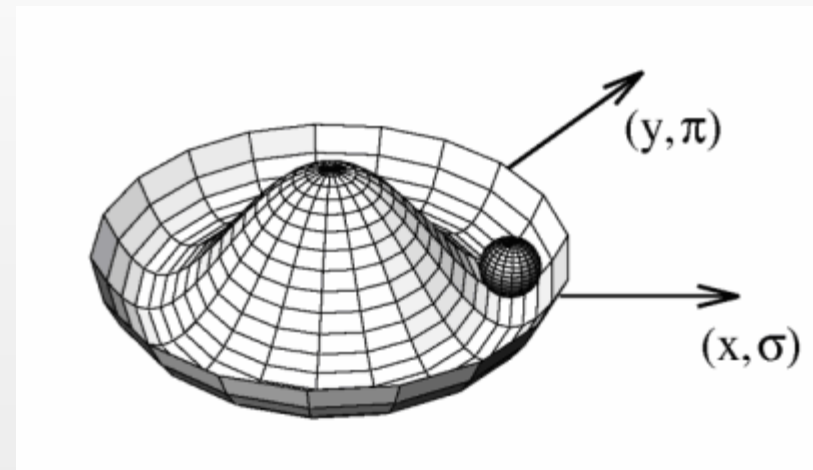
$$\sigma \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \equiv f_0(1370)$$

Chiral transformation:  $\sigma \leftrightarrow \pi$

$$V = \frac{m_0^2}{2}(\sigma^2 + \pi^2) + \frac{\lambda}{4}(\sigma^2 + \pi^2)^2$$

$m_0^2 < 0 \rightarrow$  Mexican hat

$$\text{SSB: } \langle \sigma \rangle \propto \langle \bar{u}u + \bar{d}d \rangle \neq 0$$

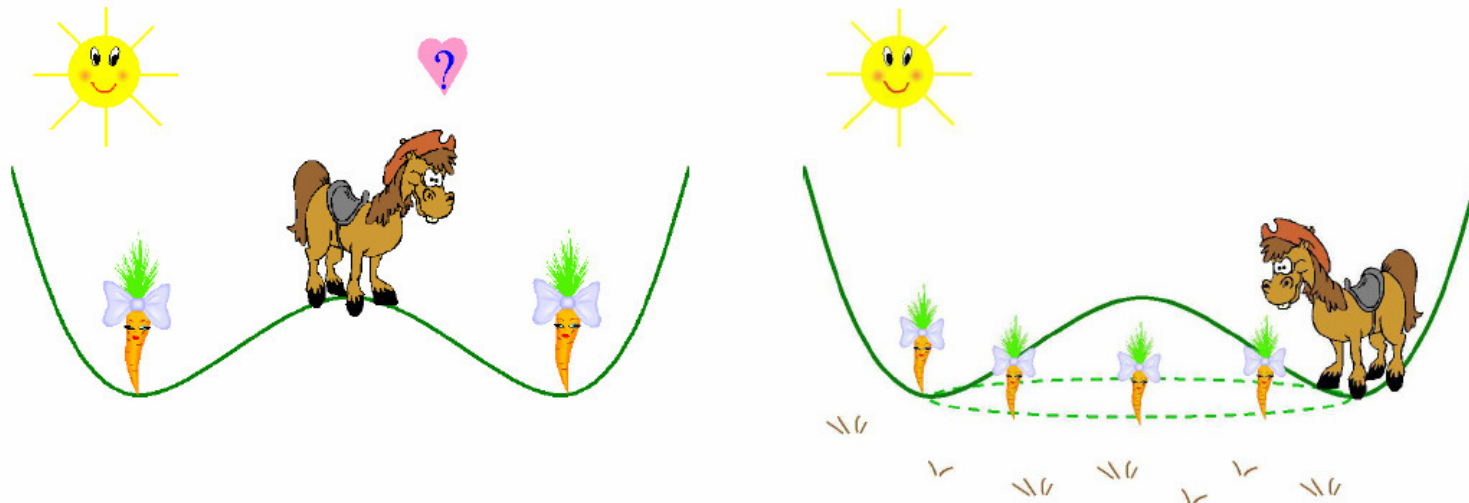


The mass of the pion vanishes, the mass of the sigma is large.

# The donkey of Buridan

**Jean Buridan** (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)

## Spontaneous Symmetry Breaking



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Picture taken from A. Pich, arXiv:0705.4264 [hep-ph],  
Cern-Claf Lecture on 'The Standard model of electroweak interactions'

# Chiral anomaly and $\eta$ $\eta'$ mesons



Pseudoscalar fields (L=S=0)

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos \theta_P & \sin \theta_P \\ -\sin \theta_P & \cos \theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

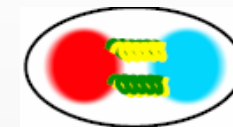
Mixing angle large (due to the anomaly, driven e.g. by instantons); non-perturbative phenomenon of QCD

# Non-conventional mesons: theoretical expectations

1) Glueballs



2) Hybrids



Compact diquark-antidiquark states



3) Four-quark states    Molecular states (a type of dynamical generation)



Companion poles (another type of dynamical generation)

**Today:** I shall not speak of those fields, but to detect them we need to understand q-antiq

# The state $a_1(1260)$



## The meson $a_1(1260)$

- $L=1$ ,  $S=1$  coupled to  $J=1$
- Chiral partner of the rho meson (hence a quark-antiquark state)
- ...but different interpretations exist (as rho-pion molecular state)
- Important to understand both for spectroscopy but also for chiral symmetry and its breaking

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

$a_1(1260)$

$$J^{PC} = 1^-(1^{++})$$

See also our review under the  $a_1(1260)$  in PDG 06, Journal of Physics **G33** 1 (2006).

### $a_1(1260)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b>1230 ± 40</b>				<b>OUR ESTIMATE</b>
<b>1245<sup>+10</sup><sub>-16</sub></b>				<b>OUR AVERAGE</b> Error includes scale factor of 1.1.

### $a_1(1260)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b>250 to 600</b>				<b>OUR ESTIMATE</b>
<b>389 ± 29</b>				<b>OUR AVERAGE</b> Error includes scale factor of 1.3.

# Axial-vector mesons

$n^{2s+1}\ell_J$	$J^{PC}$	$l = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$l = \frac{1}{2}$ $u\bar{s}, \bar{d}s; \bar{d}s, -\bar{u}s$	$l = 0$ $f'$	$l = 0$ $f$
$1^1S_0$	$0^{-+}$	$\pi$	$K$	$\eta$	$\eta'(958)$
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$1^3P_0$	$0^{++}$	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
$1^3P_1$	$1^{++}$	$a_1(1260)$	$K_{1A}^\dagger$	$f_1(1420)$	$f_1(1285)$
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$
$1^3D_1$	$1^{--}$	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$
$1^3D_2$	$2^{--}$		$K_2(1820)$		

# The eLSM: a chiral model of QCD



PHYSICAL REVIEW D 87, 014011 (2013)

## Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons

D. Parganlija,<sup>1,2,\*</sup> P. Kovács,<sup>2,3,†</sup> Gy. Wolf,<sup>3,‡</sup> F. Giacosa,<sup>2,§</sup> and D. H. Rischke<sup>2,4,||</sup>

<sup>1</sup>*Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria*

<sup>2</sup>*Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Strasse 1, D-60438 Frankfurt am Main, Germany*

<sup>3</sup>*Institute for Particle and Nuclear Physics, Wigner Research Center for Physics, Hungarian Academy of Sciences, H-1525 Budapest, Hungary*

<sup>4</sup>*Frankfurt Institute for Advanced Studies, Ruth-Moufang-Strasse 1, D-60438 Frankfurt am Main, Germany*

(Received 7 August 2012; published 8 January 2013)

PHYSICAL REVIEW D 90, 114005 (2014)

## Is $f_0(1710)$ a glueball?

Stanislaus Janowski,<sup>1</sup> Francesco Giacosa,<sup>1,2</sup> and Dirk H. Rischke<sup>1</sup>

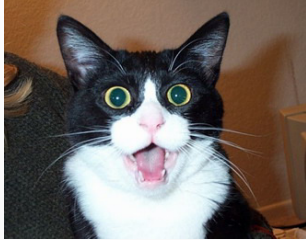
<sup>1</sup>*Institute for Theoretical Physics, Goethe University,*

*Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*

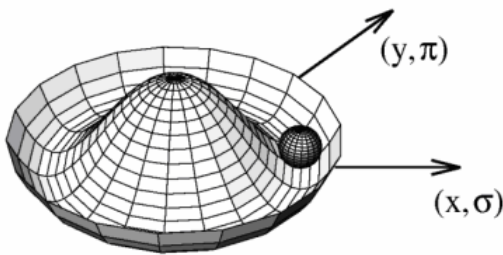
<sup>2</sup>*Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland*

(Received 26 August 2014; published 2 December 2014)

# Model of QCD – eLSM with scalar Glueball



$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left( G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right) + \text{Tr} [(D^\mu \Phi)^\dagger (D_\mu \Phi)] \\
 & - m_0^2 \left( \frac{G}{G_0} \right)^2 \text{Tr} [\Phi^\dagger \Phi] - \lambda_1 (\text{Tr} [\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\
 & + \left( \frac{G}{G_0} \right)^2 \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) ((L^\mu)^2 + (R^\mu)^2) \right] \\
 & - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} [H (\Phi^\dagger + \Phi)] \\
 & + c_1 [\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
 & + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]
 \end{aligned}$$

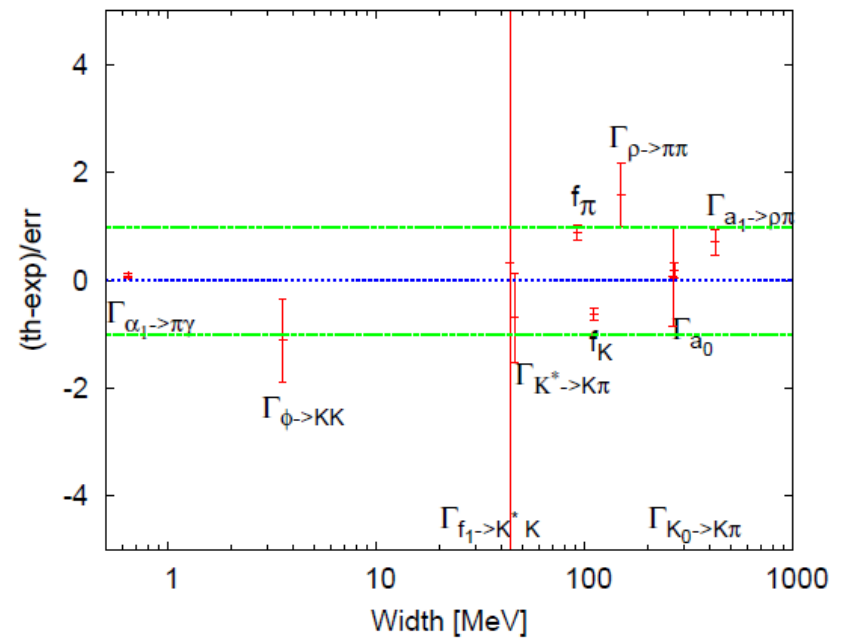
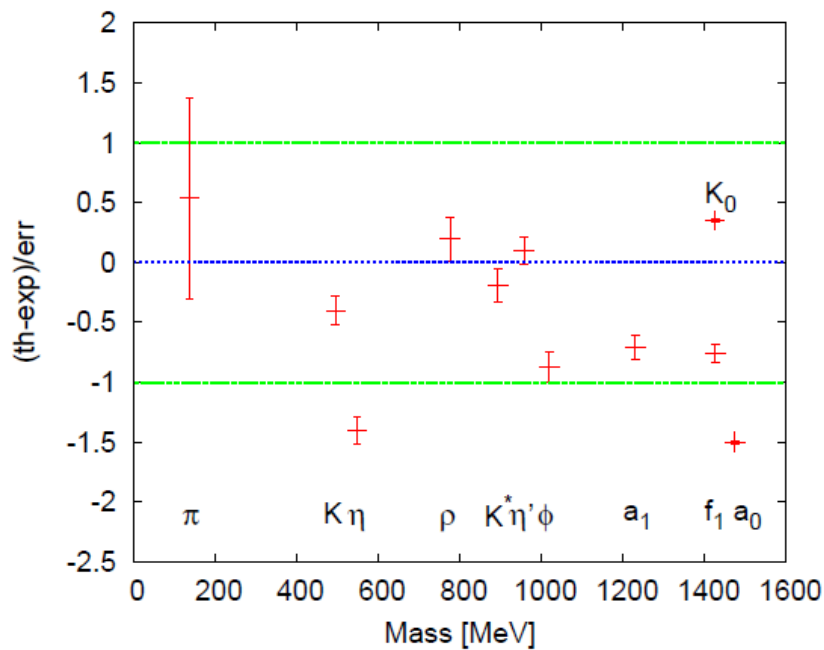


$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{*+} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{*0} \pm K_1^0 \\ K^{*-} \pm K_1^- & \bar{K}^{*0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011)**  
 D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

# Results of the eLSM (11 parameters, 21 exp. quantities)



arXiv:1208.0585

Overall phenomenology is good (many more quantities can be calculated)

Scalar mesons  $a_0(1450)$  and  $K_0(1430)$  above 1 GeV and are quark-antiquark states.

Importance of the (axial-)vector mesons on the phenomenology

Glueball:  $f_0(1710)$

$a_1 \rightarrow \rho\pi$



The decay of  $a_1$  into  $\rho\pi$  is poorly known experimentally (200-600 MeV quoted by the PDG).

The  $\rho$  further decays into two pions, finally one has a 3-pion decay.

Can one measure it better? (Presently, we have tau-decays. However, there is a strong mixing with non resonant background).

$a_1 \rightarrow \gamma\pi$

This is a very peculiar channel and crucial for chiral symmetry and the chiral condensate.

Namely, before the Mexican hat forces the formation of the chiral condensate, one has:

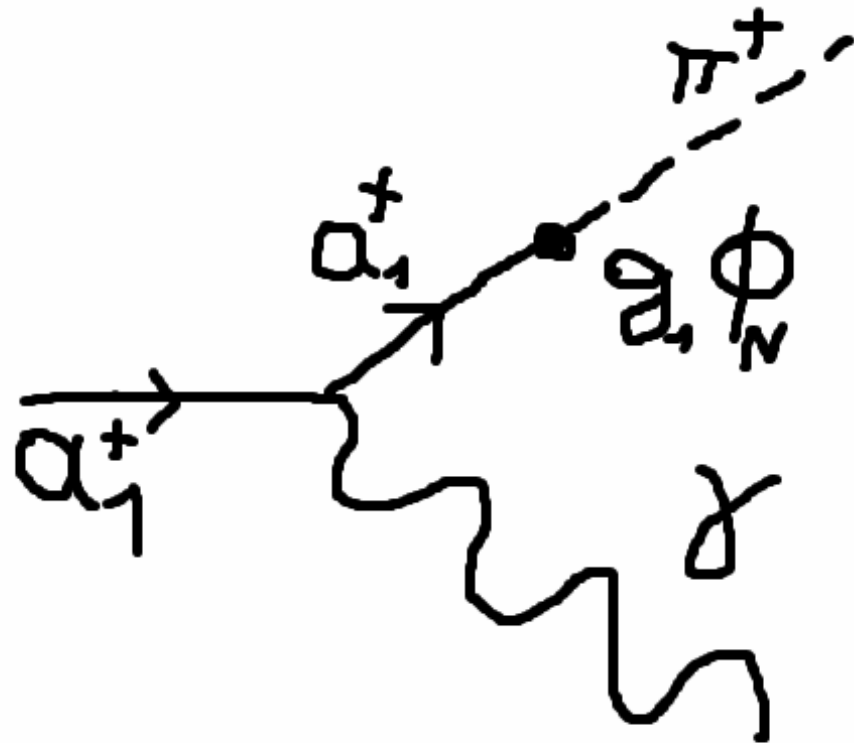
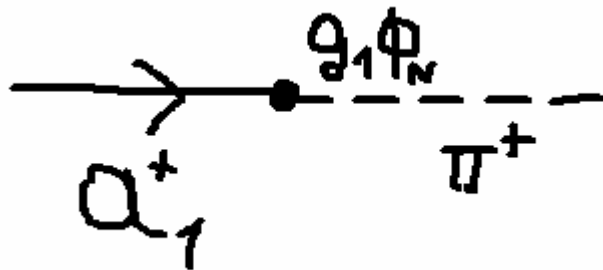
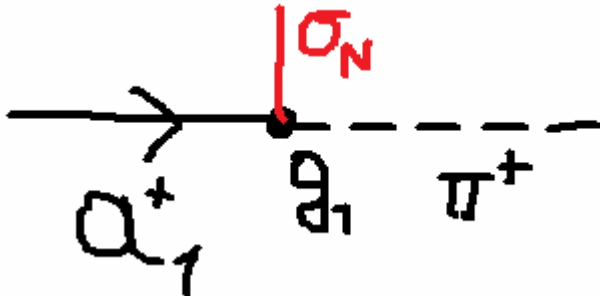
$$L_{\text{int}} = -g_1 \sigma_N \vec{a}_1^{\rightarrow\mu} \partial_{\mu} \vec{\pi}$$

But upon condensation of the chiral condensate:

$$\sigma_N \rightarrow \sigma_N + \phi_N$$
$$L_{\text{int}} = -g_{1N} \phi_N \vec{a}_1^{\rightarrow\mu} \partial_{\mu} \vec{\pi} + \dots$$

Mixing proportional  
to the chiral condensate

$a_1 \rightarrow \gamma\pi$ : intuitive origin





# Present experimental information



$\Gamma(\pi\gamma)$				$\Gamma_{12}$
<i>VALUE (keV)</i>	<i>DOCUMENT ID</i>	<i>TECN</i>	<i>COMMENT</i>	
<b>640 ± 246</b>	ZIELINSKI	84C	SPEC	200 $\pi^+ Z \rightarrow Z3\pi$

Need for more precise value for both the strong and the radiative decay of this state

# Pseudotensor mesons

## Phenomenology of pseudotensor mesons and the pseudotensor glueball

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**Abstract.** We study the decays of the pseudotensor mesons ( $\pi_2(1670)$ ,  $K_2(1770)$ ,  $\eta_2(1645)$ ,  $\eta_2(1870)$ ) interpreted as the ground-state nonet of  $1^1D_2$   $\bar{q}q$  states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of  $\pi_2(1670)$  and  $K_2(1770)$  can be well described, the decays of the isoscalar states  $\eta_2(1645)$  and  $\eta_2(1870)$  can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about  $-42^\circ$ , similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets: if confirmed, a deeper study of its origin will be needed in the future. Moreover, the  $\bar{q}q$  assignment of pseudotensor states predicts that the ratio  $[\eta_2(1870) \rightarrow a_2(1320)\pi]/[\eta_2(1870) \rightarrow f_2(1270)\eta]$  is about 23.5. This value is in agreement with Barberis *et al.*,  $(20.4 \pm 6.6)$ , but disagrees with the recent reanalysis of Anisovich *et al.*,  $(1.7 \pm 0.4)$ . Future experimental studies are necessary to understand this puzzle. If Anisovich's value is confirmed, a simple nonet of pseudoscalar mesons cannot be able to describe data (different assignments and/or additional states, such as an hybrid state, will be needed). In the end, we also evaluate the decays of a pseudoscalar glueball into the aforementioned conventional  $\bar{q}q$  states: a sizable decay into  $K_2^*(1430)K$  and  $a_2(1230)\pi$  together with a vanishing decay into pseudoscalar-vector pairs (such as  $\rho(770)\pi$  and  $K^*(892)K$ ) are expected. This information can be helpful in future studies of glueballs at the ongoing BESIII and at the future PANDA experiments.

ArXiv: 1608.08777

Francesco Giacosa

# Pseudotensor mesons

$n^{2s+1}\ell_J$	$J^{PC}$	$l = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$l = \frac{1}{2}$ $u\bar{s}, \bar{d}s; \bar{d}s, -\bar{u}s$	$l = 0$ $f'$	$l = 0$ $f$
$1^1S_0$	$0^{-+}$	$\pi$	$K$	$\eta$	$\eta'(958)$
$1^3S_1$	$1^{--}$	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
$1^1P_1$	$1^{+-}$	$b_1(1235)$	$K_{1B}^\dagger$	$h_1(1380)$	$h_1(1170)$
$1^3P_0$	$0^{++}$	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
$1^3P_1$	$1^{++}$	$a_1(1260)$	$K_{1A}^\dagger$	$f_1(1420)$	$f_1(1285)$
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$
$1^3D_1$	$1^{--}$	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$
$1^3D_2$	$2^{--}$		$K_2(1820)$		

L=2, S =0, out of which J =2

# Lagrangians and decays

Pseudotensor mesons:  $\{\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870)\}$   
Lagrangians based on flavour symmetry

$$\mathcal{L}_{TVP} = c_{TVP} \text{Tr} \{ T_{\mu\nu} [V^\mu, (\partial^\nu P)]_- \},$$

$$P = \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad V^\mu = \begin{pmatrix} \frac{\omega_N^\mu + \rho^{0\mu}}{\sqrt{2}} & \rho^{+\mu} & K^{*+\mu} \\ \rho^{-\mu} & \frac{\omega_N^\mu - \rho^{0\mu}}{\sqrt{2}} & K^{*0\mu} \\ K^{*-\mu} & \bar{K}^{*0\mu} & \omega_S^\mu \end{pmatrix},$$

$$T^{\mu\nu} = \begin{pmatrix} \frac{\eta_{2,N}^{\mu\nu} + \pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu} \\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu} - \pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu} \\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu} \end{pmatrix}.$$

$$\mathcal{L}_{TXP} = c_{TXP} \text{Tr} (T_{\mu\nu} \{X^{\mu\nu}, P\}_+)$$

$$P = \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad X^{\mu\nu} = \begin{pmatrix} \frac{f_{2,N}^{\mu\nu} + a_2^{0\mu\nu}}{\sqrt{2}} & a_2^{+\mu\nu} & K_2^{*+\mu\nu} \\ a_2^{-\mu\nu} & \frac{f_{2,N}^{\mu\nu} - a_2^{0\mu\nu}}{\sqrt{2}} & K_2^{*0\mu\nu} \\ K_2^{*- \mu\nu} & \bar{K}_2^{*0\mu\nu} & f_{2,S}^{\mu\nu} \end{pmatrix},$$

$$T^{\mu\nu} = \begin{pmatrix} \frac{\eta_{2,N}^{\mu\nu} + \pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu} \\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu} - \pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu} \\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu} \end{pmatrix}.$$

Tree-level decay widths:

$$\Gamma_{T \rightarrow VP}^{tl} = \frac{k_f}{8\pi m_T} \frac{g_{TVP}^2}{15} \left( 2 \frac{k_f^4}{m_V^2} + 5 k_f^2 \right) \Theta(m_T - m_V - m_P),$$

and

$$\Gamma_{T \rightarrow XP}^{tl} = \frac{k_f}{8\pi m_T} \frac{g_{TXP}^2}{45} \left( 4 \frac{k_f^4}{m_X^4} + 30 \frac{k_f^2}{m_X^2} + 45 \right) \Theta(m_T - m_X - m_P).$$

# Results for $I = 1$ and $I = 1/2$

Decay process	Theory (MeV)	Experiment (MeV)
$\pi_2(1670) \rightarrow \rho(770) \pi$	$80.6 \pm 10.8$	$80.6 \pm 10.8$
$\pi_2(1670) \rightarrow f_2(1270) \pi$	$146.4 \pm 9.7$	$146.4 \pm 9.7$
$\pi_2(1670) \rightarrow \bar{K}^*(892) K + c.c.$	$11.7 \pm 1.6$	$10.9 \pm 3.7$
$\pi_2(1670) \rightarrow \bar{K}_2^*(1430) K + c.c.$	0	
$\pi_2(1670) \rightarrow f_2'(1525) \pi$	$0.1 \pm 0.1$	
$\pi_2(1670) \rightarrow a_2(1320) \pi$	0	not seen
$\pi_2(1670) \rightarrow a_2(1320) \eta$	0	
$\pi_2(1670) \rightarrow a_2(1320) \eta'(958)$	0	
$K_2(1770) \rightarrow \rho(770) K$	$22.2 \pm 3.0$	
$K_2(1770) \rightarrow \bar{K}^*(892) \pi$	$25.5 \pm 3.4$	seen
$K_2(1770) \rightarrow \bar{K}^*(892) \eta$	$10.5 \pm 1.4$	
$K_2(1770) \rightarrow \bar{K}^*(892) \eta'(958)$	0	
$K_2(1770) \rightarrow \omega(782) K$	$8.3 \pm 1.1$	seen
$K_2(1770) \rightarrow \phi(1020) K$	$4.2 \pm 0.6$	seen
$K_2(1770) \rightarrow a_2(1320) K$	0	
$K_2(1770) \rightarrow \bar{K}_2^*(1430) \pi$	$84.5 \pm 5.6$	dominant
$K_2(1770) \rightarrow \bar{K}_2^*(1430) \eta$	0	
$K_2(1770) \rightarrow \bar{K}_2^*(1430) \eta'(958)$	0	
$K_2(1770) \rightarrow f_2(1270) K$	$5.8 \pm 0.4$	seen
$K_2(1770) \rightarrow f_2'(1525) K$	0	

Table 4: Decays of  $I = 1$  and  $I = 1/2$  pseudotensor states. The first two entries were used to determine the coupling constants of the model, see Eq. (3.2). The total decay widths are  $\Gamma_{\pi_2(1670)}^{\text{tot}} = (260 \pm 9)$  MeV and  $\Gamma_{K_2(1770)}^{\text{tot}} = (186 \pm 14)$  MeV.

ArXiv: 1608.08777

# Comments



The prediction for the decay channel  $\pi_2(1670) \rightarrow K^*(892) K$  is in agreement with the present value quoted by the PDG.

In our model  $\pi_2(1670)$  does not couple to the  $a_2(1320) \pi$  channel, see Eq. (A.2). Experimentally this decay could also be excluded to a good accuracy.

The experimental total decay width of  $\pi_2(1670)$  reads  $\Gamma_{\pi_2(1670)}^{\text{exp,tot}} = (260 \pm 9) \text{ MeV}$ . The value in our model is  $\approx 238.8 \text{ MeV}$ . The reason why the sum of the theoretical and experimental decay modes in our model is slightly smaller than the experimental total width is due to the fact that the model does not include the decay  $\pi_2(1670) \rightarrow f_0(500) \pi$ , which is  $\approx 26 \text{ MeV}$ .

Experimentally, the decay channel  $K_2(1770) \rightarrow \bar{K}_2^*(1430) \pi$  is dominant. The total decay width of  $K_2(1770)$  is  $\Gamma_{K_2(1770)}^{\text{exp,tot}} = (186 \pm 14) \text{ MeV}$ . Theoretically, the  $K_2(1770) \rightarrow \bar{K}_2^*(1430) \pi$  decay mode is the dominant one ( $84.5 \pm 5.6 \text{ MeV}$ ).

The full theoretical decay width of  $K_2(1770)$  amounts to  $(162.0 \pm 15.4) \text{ MeV}$  which is compatible with the experimental value.

## Comments/2



Most of the decays which are predicted by our model to vanish were consistently not seen in experiments. Yet, the decays  $\pi_2(1670) \rightarrow a_2(1230)\eta$  and  $K_2(1270) \rightarrow K^*(892)\eta$  are expected to be small but not zero. They were not yet measured, hence they represent a test of our approach as soon as new experimental data will be available.

Various branching ratios of  $K_2(1770)$  can be calculated from Tab. At present, experimental results are missing (no average or fit is quoted by PDG ). In this respect, our approach makes predictions for new future experimental measurements. In this context, it will also be possible to determine the mixing angle of the bare  $1^1D_2$  and  $1^3D_2$  configurations into the physical  $K_2(1770)$  and  $K_2(1820)$  states [so far,  $K_2(1770)$  is dominated by  $1^1D_2$ ].



## Isoscalar sector: mixing

$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos \beta_{pt} & \sin \beta_{pt} \\ -\sin \beta_{pt} & \cos \beta_{pt} \end{pmatrix} \begin{pmatrix} \eta_{2,N} \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} \equiv \bar{s}s \end{pmatrix}$$

Let us allow for a large mixing angle!

# Isoscalar sector: naive results

Decay process	Theory (MeV) ( $\beta_{pt} = 14.8^\circ$ )	Theory (MeV) ( $\beta_{pt} = 0.0^\circ$ )	Experiment (MeV)
$\eta_2(1645) \rightarrow \bar{K}^*(892) K + c.c.$	$3.2 \pm 0.4$	$8.6 \pm 1.1$	seen
$\eta_2(1645) \rightarrow a_2(1320) \pi$	$315.6 \pm 21.2$	$337.8 \pm 22.6$	
$\eta_2(1645) \rightarrow \bar{K}_2^*(1430) K + c.c.$	0	0	
$\eta_2(1645) \rightarrow f_2(1270) \eta$	0	0	not seen
$\eta_2(1645) \rightarrow f_2(1270) \eta'(958)$	0	0	
$\eta_2(1645) \rightarrow f_2'(1525) \eta$	0	0	
$\eta_2(1645) \rightarrow f_2'(1525) \eta'(958)$	0	0	
$\eta_2(1870) \rightarrow \bar{K}^*(892) K + c.c.$	$60.1 \pm 8.0$	$45.6 \pm 6.1$	
$\eta_2(1870) \rightarrow a_2(1320) \pi$	$32.2 \pm 2.1$	0	
$\eta_2(1870) \rightarrow \bar{K}_2^*(1430) K + c.c.$	0	0	
$\eta_2(1870) \rightarrow f_2(1270) \eta$	$2.5 \pm 0.2$	$0.1 \pm 0.1$	
$\eta_2(1870) \rightarrow f_2(1270) \eta'(958)$	0	0	
$\eta_2(1870) \rightarrow f_2'(1525) \eta$	0	0	
$\eta_2(1870) \rightarrow f_2'(1525) \eta'(958)$	0	0	

Table 5: Decays of  $I = 0$  pseudo-tensor states. The total decay widths are  $\Gamma_{\eta_2(1645)}^{\text{tot}} = (181 \pm 11)$  MeV and  $\Gamma_{\eta_2(1870)}^{\text{tot}} = (225 \pm 14)$  MeV.

ArXiv: 1608.08777

# Only a large and negative mixing angle works

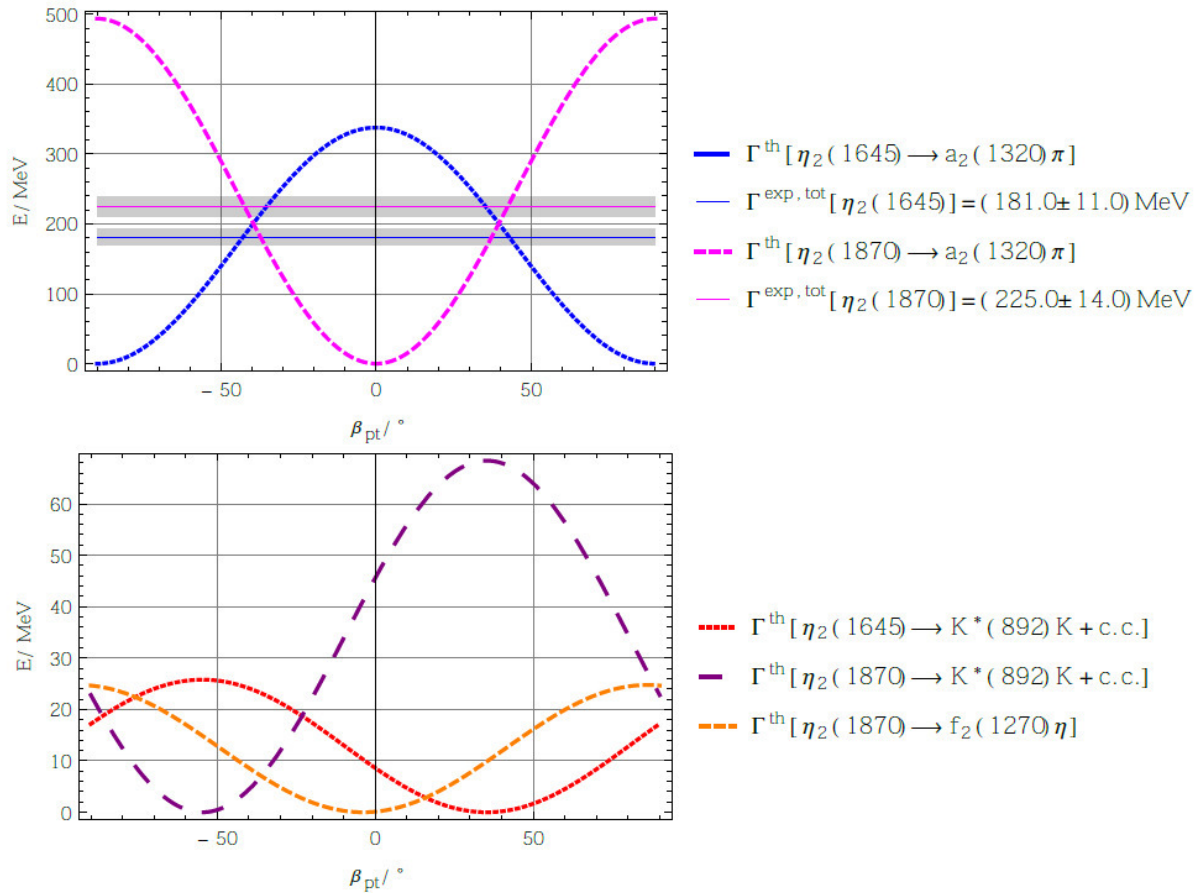


Figure 1: Upper panel:  $\eta_2(1645) \rightarrow a_2(1320)\pi$  and  $\eta_2(1870) \rightarrow a_2(1320)\pi$  as function of  $\beta_{pt}$ . The bands correspond to the total decay widths of  $\eta_2(1645)$  and  $\eta_2(1870)$ . Lower panel:  $\eta_2(1645) \rightarrow K^*(892)K$ ,  $\eta_2(1870) \rightarrow K^*(892)K$ , and  $\eta_2(1870) \rightarrow f_2(1270)\eta$  as function of  $\beta_{pt}$ . Only for  $\beta_{pt} \approx -40^\circ$  the decay  $\eta_2(1870) \rightarrow K^*(892)K$  is suppressed. For further details, see discussion in the main text.

# Results in the isoscalar sector for a large mixing

Decay process	Theory (MeV) ( $\beta_{pt} = -42^\circ$ )	Experiment (MeV)
$\eta_2(1645) \rightarrow \bar{K}^*(892) K + c.c.$	24.7	seen
$\eta_2(1645) \rightarrow a_2(1320) \pi$	186.5	
$\eta_2(1645) \rightarrow \bar{K}_2^*(1430) K + c.c.$	0	
$\eta_2(1645) \rightarrow f_2(1270) \eta$	0	not seen
$\eta_2(1645) \rightarrow f_2(1270) \eta'(958)$	0	
$\eta_2(1645) \rightarrow f_2'(1525) \eta$	0	
$\eta_2(1645) \rightarrow f_2'(1525) \eta'(958)$	0	
$\eta_2(1870) \rightarrow \bar{K}^*(892) K + c.c.$	3.3	
$\eta_2(1870) \rightarrow a_2(1320) \pi$	221.0	
$\eta_2(1870) \rightarrow \bar{K}_2^*(1430) K + c.c.$	0	
$\eta_2(1870) \rightarrow f_2(1270) \eta$	9.4	
$\eta_2(1870) \rightarrow f_2(1270) \eta'(958)$	0	
$\eta_2(1870) \rightarrow f_2'(1525) \eta$	0	
$\eta_2(1870) \rightarrow f_2'(1525) \eta'(958)$	0	

Table 6: Decays of  $I = 0$  pseudotensor states. The total decay widths are  $\Gamma_{\eta_2(1645)}^{\text{tot}} = (181 \pm 11)$  MeV and  $\Gamma_{\eta_2(1870)}^{\text{tot}} = (225 \pm 14)$  MeV.

ArXiv: 1608.08777

# Comments



The experimental total width of  $\eta_2(1645)$  is  $(181 \pm 11)$  MeV, while the theoretical result for  $\beta_{pt} = -42^\circ$  reads 211 MeV

The experimental total width of  $\eta_2(1870)$  is  $(225 \pm 14)$  MeV; the corresponding theoretical width for  $\beta_{pt} = -42^\circ$  reads 233.7 MeV.

The suppressed decay of  $\eta_2(1870)$  into  $K^*(892) K$  is the result of a destructive interference due to a large and negative strange-nonstrange mixing angle (similar to the one in the pseudoscalar sector).

In conclusion, it is possible to interpret the resonances [ $\pi_2(1670)$ ,  $K_2(1770)$ ,  $\eta_2(1645)$ ,  $\eta_2(1870)$ ] as the ground-state  $\bar{q}q$  pseudotensor mesons nonet if a large and negative mixing angle of about  $-42^\circ$  is considered. In this respect, there is – at the stage of the present experimental knowledge – no need to include further additional fields, such as an hybrid pseudotensor state, in the model.

# Branching ratios of $\eta_2(1870) / 1$

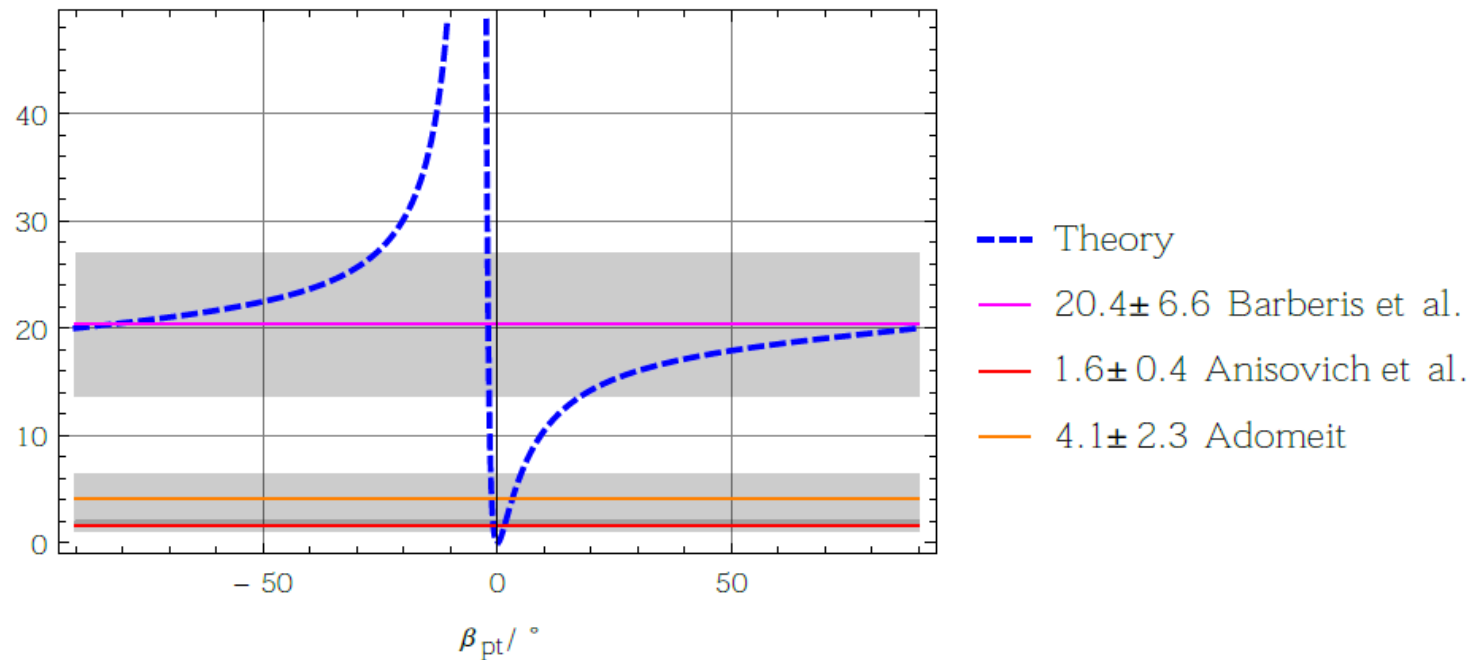


$\eta_2(1870)$			
Branching ratio	Theory	Experiment	Collaboration
$\Gamma^{\text{exp}}(a_2(1320) \pi) / \Gamma^{\text{exp}}(f_2(1270) \eta)$	$\approx 23.5$	<b><math>1.7 \pm 0.4</math></b>	average by [1]
		$1.60 \pm 0.4$	Anisovich et al. [32]
		$20.4 \pm 6.6$	Barberis et al. [33]
		$4.1 \pm 2.3$	Adomeit [34]
$\Gamma^{\text{exp}}(a_2(1320) \pi) / \Gamma^{\text{exp}}(a_0(980) \pi)$		<b><math>32.6 \pm 12.6</math></b>	Barberis et al. [33]
$\Gamma^{\text{exp}}(a_0(980) \pi) / \Gamma^{\text{exp}}(f_2(1270) \eta)$		<b><math>0.48 \pm 0.45</math></b>	Barberis et al. [33]

Table 7: Theoretical and experimental branching ratios for  $\eta_2(1870)$ . The mixing angle  $\beta_{pt} \approx -42^\circ$  is used for theoretical predictions. Bold numbers are also bold in PDG [1].

Large conflict for the first entry! Which experiment is right? Can we get new data?

# Branching ratios of $\eta_2(1870)$ /2



ArXiv: 1608.08777

# Considerations

If new experimental data **confirms** our results,

- we have good candidates for a ground-state pseudotensor meson nonet.
- the large mixing angle  $\beta_{pT} \approx -40^\circ$  would be a mystery which deserves a detailed study.
- the current phenomenological study should be redone, including higher order corrections.

If new experimental data **is at odd** with our results,

- an understanding of the lowlying pseudotensor states as a standard quark-antiquark nonet would be hard.
- $\eta_2(1870)$  could be wrongly assigned as a  $\bar{q}q$ -state.
- possible further mixings with (hybrid) states could be included in the model.



# Large mixing angle: where does it come from?



PHYSICAL REVIEW D **97**, 091901(R) (2018)

Rapid Communications

## How the axial anomaly controls flavor mixing among mesons

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$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos \theta_P & \sin \theta_P \\ -\sin \theta_P & \cos \theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

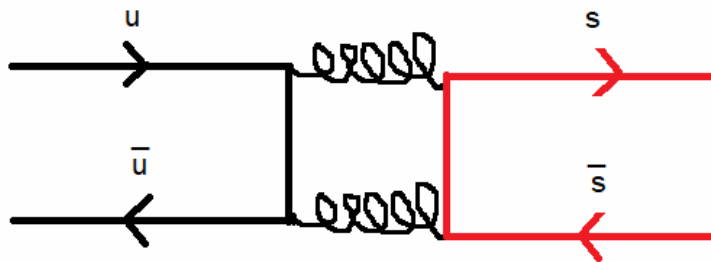
$$\theta_P \simeq -42^\circ$$

$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos \beta_{pt} & \sin \beta_{pt} \\ -\sin \beta_{pt} & \cos \beta_{pt} \end{pmatrix} \begin{pmatrix} \eta_{2,N} \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} \equiv \bar{s}s \end{pmatrix}$$

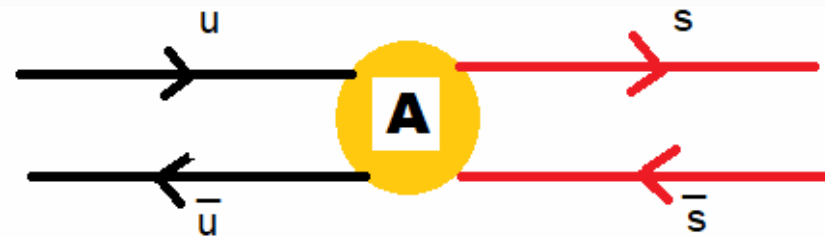
$$\beta_{pt} = -42^\circ$$

# Large mixing angle: where does it come from? /2

Such a mixing is suppressed...



But this can be large



- For pseudoscalar mesons:  $\eta(547)$  and  $\eta'(958)$ .  $\Theta_{\text{mix}} = -42^\circ$  Large mixing caused by the axial anomaly.
- For vector mesons:  $\omega(782)$  and  $\phi(1020)$ .  $\Theta_{\text{mix}} = -3^\circ$  Very small mixing.
- For tensor mesons:  $f_2(1270)$  and  $f_2'(1525)$ .  $\Theta_{\text{mix}} = 3^\circ$  Also very small mixing. Why?
- Pseudotensor mesons: also large, but confirmation is needed.

Details in: **1709.07454**

# Excited vector mesons

Strong and radiative decays of excited vector mesons and  
predictions for a new  $\phi(1930)$  resonance

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**Phys.Rev. D96 (2017) no.5, 054033, arXiv: 1708.02593**


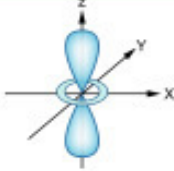
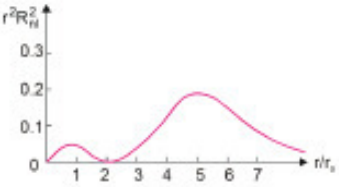
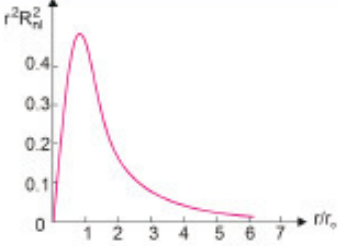
# Excited vector mesons

$n^{2s+1}\ell_J$	$J^{PC}$	$l = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$l = \frac{1}{2}$ $u\bar{s}, \bar{d}s; \bar{d}s, -\bar{u}s$	$l = 0$ $f'$	$l = 0$ $f$	$\theta_{\text{quad}}$ [°]	$\theta_{\text{lin}}$ [°]
$1^1S_0$	$0^{-+}$	$\pi$	$K$	$\eta$	$\eta'(958)$	-11.4	-24.5
$1^3S_1$	$1^{--}$	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.1	36.4
$1^1P_1$	$1^{+-}$	$b_1(1235)$	$K_{1B}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1^3P_0$	$0^{++}$	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1^3P_1$	$1^{++}$	$a_1(1260)$	$K_{1A}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1^3P_2$	$2^{++}$	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	32.1	30.5
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1^3D_1$	$1^{--}$	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1^3D_2$	$2^{--}$		$K_2(1820)$				
$1^3D_3$	$3^{--}$	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1^3F_4$	$4^{++}$	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1^3G_5$	$5^{--}$	$\rho_5(2350)$	$K_5^*(2380)$				
$1^3H_6$	$6^{++}$	$a_6(2450)$			$f_6(2510)$		
$2^1S_0$	$0^{-+}$	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2^3S_1$	$1^{--}$	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

L=2, S = 1 coupled to: J = 1

L=0, S = 1 (but n = 2): J = 1

# Excited vector mesons: properties

Type of excitation	Radially excited vector mesons	Angular momentum excited vector mesons
Quantum numbers	$n \ 2^{S+1}L_J = 2^3S_1$	$n \ 2^{S+1}L_J = 1^3D_1$
Notation	$V_E$	$V_D$
S	1 $\uparrow\uparrow$	1 $\uparrow\uparrow$
n	2	1
L	0	2
orbital		
Radial function		
Associated states	$\rho(1450), K^*(1410), \phi(1680), \omega(1420)$	$\rho(1700), K^*(1680), \phi_P, \omega(1650)$
Decay types	$V_E \rightarrow PP$ $V_E \rightarrow VP$ $V_E \rightarrow \gamma P$	$V_D \rightarrow PP$ $V_D \rightarrow VP$ $V_D \rightarrow \gamma P$

# Matrices of fields

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\eta_{N^+} + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_{N^-} - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix}$$

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega^\mu + \rho^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} & K_i^{\mu^{*+}} \\ \rho^{\mu-} & \frac{\omega^\mu - \rho^{\mu 0}}{\sqrt{2}} & K^{\mu^{*0}} \\ K^{\mu^{*-}} & K^{\mu^{*0}} & \phi^\mu \end{pmatrix}$$

$$V_E^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_E^\mu + \rho_E^{\mu 0}}{\sqrt{2}} & \rho_E^{\mu+} & K_E^{\mu^{*+}} \\ \rho_E^{\mu-} & \frac{\omega_E^\mu - \rho_E^{\mu 0}}{\sqrt{2}} & K_E^{\mu^{*0}} \\ K_E^{\mu^{*-}} & K_E^{\mu^{*0}} & \phi_E^\mu \end{pmatrix}$$

$$V_D^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_D^\mu + \rho_D^{\mu 0}}{\sqrt{2}} & \rho_D^{\mu+} & K_D^{\mu^{*+}} \\ \rho_D^{\mu-} & \frac{\omega_D^\mu - \rho_D^{\mu 0}}{\sqrt{2}} & K_D^{\mu^{*0}} \\ K_D^{\mu^{*-}} & K_D^{\mu^{*0}} & \phi_D^\mu \end{pmatrix}$$

- $P = \{\pi, K, \eta, \eta'\}$
- $V = \{\rho(770), K^*(892), \phi(1020), \omega(782)\}$
- $V_E = \{\rho(1450), K^*(1410), \phi(1680), \omega(1420)\}$
- $V_D = \{\rho(1700), K^*(1680), \phi_p, \omega(1650)\}$

# Lagrangians

The Lagrangian of the model is:

$$\mathcal{L} = \mathcal{L}_{1,E} + \mathcal{L}_{1,D} + \mathcal{L}_{2,E} + \mathcal{L}_{2,D},$$

where:

$$\mathcal{L}_{1,E} = ia_E Tr[\partial^\mu P, V_{E,\mu}]P \quad \mathcal{L}_{1,D} = ia_D Tr[\partial^\mu P, V_{D,\mu}]P$$

$$\mathcal{L}_{2,E} = b_E Tr[\tilde{V}_E^{\mu\nu} \{V_{\mu\nu}, P\}] \quad \mathcal{L}_{2,D} = b_D Tr[\tilde{V}_D^{\mu\nu} \{V_{\mu\nu}, P\}]$$

$a_E, a_D, b_E, b_D$  – coupling constants of the different decay types.

- $R \rightarrow \gamma P$  through „vector meson dominance”

$$V_{\mu\nu} \rightarrow V_{\mu\nu} + \frac{e_0}{g_\rho} Q F_{\mu\nu}$$

$F_{\mu\nu}$  – field strength tensor for photons

$$e_0 = \sqrt{4\pi\alpha} \quad \alpha \approx 1/137 \quad g_\rho \approx 5.5 \pm 0.5 \quad Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$$



# Strong and radiative decay widths

## TYPE OF DECAY

- $R \rightarrow PP$

$$\Gamma_{R \rightarrow PP} = S \frac{|\vec{k}|^3}{6\pi m_R^2} \left[ \frac{a_i}{2} \lambda_{RPP} \right]^2$$

- $R \rightarrow VP, R \rightarrow \gamma P$

$$\Gamma_{R \rightarrow VP} = S \frac{|\vec{k}|^3}{12\pi} \left[ \frac{b_i}{2} \lambda_{RVP} \right]^2$$

## EXAMPLES

- $K^*(1410) \rightarrow K\eta$

$$\Gamma_{K^*(1410) \rightarrow K\eta} = \frac{|\vec{k}|^3}{6\pi m_{K^*(1410)}^2} \left[ \frac{a_E}{2} \frac{1}{2} (\cos\theta_p - \sqrt{2}\sin\theta_p) \right]^2$$

- $\phi(1680) \rightarrow \phi(1020)\eta$

$$\Gamma_{\phi(1680) \rightarrow \phi(1020)\eta} = \frac{|\vec{k}|^3}{12\pi} \left[ \frac{b_E}{2} \frac{\sin\theta_p}{\sqrt{2}} \right]^2$$

where:

$$|\vec{k}| = \frac{\sqrt{m_R^2 + (m_a^2 - m_b^2)^2 - 2(m_a^2 + m_b^2)m_R^2}}{2m_R};$$

$m_R$  – mass of the decaying resonance;

$a_i, b_i$  – coupling constants ( $i = E, D$ );

$m_a, m_b$  – masses of decay products;

$S$  – symmetry factor;

# Results: radially excited vector mesons/1



TABLE VII. Decays widths of (predominantly) radially excited vector mesons into a pseudoscalar meson and a ground-state vector meson ( $V_E \rightarrow VP$ ).

Decay process $V_E \rightarrow VP$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \rightarrow \omega\pi$	$74.7 \pm 31.0$	$\sim 84 \pm 13$ seen by Clegg 94 [47]
$\rho(1450) \rightarrow K^*(892)K$	$6.7 \pm 2.8$	Possibly seen by Coan 04 [62]
$\rho(1450) \rightarrow \rho(770)\eta$	$9.3 \pm 3.9$	$< 16.0 \pm 2.4$ by Donnachie 91 [49]
$\rho(1450) \rightarrow \rho(770)\eta'$	$\approx 0$	Not listed in PDG
$K^*(1410) \rightarrow K\rho$	$12.0 \pm 5.0$	$< 16.2 \pm 1.5$ by PDG
$K^*(1410) \rightarrow K\phi$	$\approx 0$	Not listed in PDG
$K^*(1410) \rightarrow K\omega$	$3.7 \pm 1.5$	Not listed in PDG
$K^*(1410) \rightarrow K^*(892)\pi$	$28.8 \pm 12.0$	$> 93 \pm 8$ by PDG
$K^*(1410) \rightarrow K^*(892)\eta$	$\approx 0$	Not listed in PDG
$K^*(1410) \rightarrow K^*(892)\eta'$	$\approx 0$	Not listed in PDG
$\omega(1420) \rightarrow \rho\pi$	$196 \pm 81$	Dominant, $\Gamma_{\text{tot}} = (180 - 250)$ by PDG
$\omega(1420) \rightarrow K^*(892)K$	$2.3 \pm 1.0$	Not listed in PDG
$\omega(1420) \rightarrow \omega(782)\eta$	$4.9 \pm 2.0$	Not listed in PDG
$\omega(1420) \rightarrow \omega(782)\eta'$	$\approx 0$	Not listed in PDG
$\phi(1680) \rightarrow K\bar{K}^*$	$110 \pm 46$	Dominant, $\Gamma_{\text{tot}} = 150 \pm 50$ by PDG
$\phi(1680) \rightarrow \phi(1020)\eta$	$12.2 \pm 5.1$	Seen by Achasov 14 [63]
$\phi(1680) \rightarrow \phi(1020)\eta'$	$\approx 0$	Not listed in PDG

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# Results: radially excited vector mesons/2



TABLE VI. Decays widths of (predominantly) radially excited vector mesons into two pseudoscalar mesons ( $V_E \rightarrow PP$ ).

Decay process $V_E \rightarrow PP$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \rightarrow \bar{K}K$	$6.6 \pm 1.4$	$< 6.7 \pm 1.0$ by Donnachie 91 [49]
$\rho(1450) \rightarrow \pi\pi$	$30.8 \pm 6.7$	$\sim 27 \pm 4$ , seen by Clegg 94 [47]
$K^*(1410) \rightarrow K\pi$	$15.3 \pm 3.3$	$15.3 \pm 3.3$ by PDG
$K^*(1410) \rightarrow K\eta$	$6.9 \pm 1.5$	Not listed in PDG
$K^*(1410) \rightarrow K\eta'$	$\approx 0$	Not listed in PDG
$\omega(1420) \rightarrow \bar{K}K$	$5.9 \pm 1.3$	Not listed in PDG
$\phi(1680) \rightarrow \bar{K}K$	$19.8 \pm 4.3$	Seen by Buon 82 [54]

TABLE VIII. Decay widths of (predominantly) radially excited vector mesons into a photon and a pseudoscalar meson ( $V_E \rightarrow \gamma P$ ).

Decay process $V_E \rightarrow \gamma P$	Theory (MeV)	Experiment (MeV)
$\rho(1450) \rightarrow \gamma\pi$	$0.072 \pm 0.042$	Not listed
$\rho(1450) \rightarrow \gamma\eta$	$0.23 \pm 0.14$	$\sim 0.2-1.5$ (see text)
$\rho(1450) \rightarrow \gamma\eta'$	$0.056 \pm 0.033$	Not listed
$K^*(1410) \rightarrow \gamma K$	$0.18 \pm 0.11$	$< 0.0529$ MeV seen by PDG and Alavi-Harati 02B [64]
$\omega(1420) \rightarrow \gamma\pi$	$0.60 \pm 0.36$	$1.90 \pm 0.75$ (see text)
$\omega(1420) \rightarrow \gamma\eta$	$0.023 \pm 0.014$	Not listed
$\omega(1420) \rightarrow \gamma\eta'$	$0.0050 \pm 0.0030$	Not listed
$\phi(1680) \rightarrow \gamma\eta$	$0.14 \pm 0.09$	Seen
$\phi(1680) \rightarrow \gamma\eta'$	$0.076 \pm 0.045$	Not listed

arXiv: 1708.02593

## Which mass for the missing state?

TABLE I. Mass differences between the members of the two nonets of excited vector mesons.

$V_E$	$\rho(1450)$	$K^*(1410)$	$\omega(1420)$	$\phi(1680)$
$V_D$	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$	$\phi(???)$
Difference	250 MeV	270 MeV	230 MeV	?

Hence, we can estimate the mass of  $\phi(???)$  as

$$m_{\phi(???)} \simeq (m_{\phi(1680)} + 250 \pm 20) \text{ MeV} = 1930 \pm 20 \text{ MeV}.$$

From now on we shall call this hypothetical state

$$\phi(???) \equiv \phi(1930).$$

# Orbitally excited/1

TABLE X. Decays widths of (predominantly) orbitally excited vector mesons into a pseudoscalar meson and a ground-state vector meson ( $V_D \rightarrow VP$ ).

Decay process $V_D \rightarrow VP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \rightarrow \omega\pi$	$140 \pm 59$	Seen (see text)
$\rho(1700) \rightarrow K^*(892)K$	$56 \pm 23$	$83 \pm 66$ MeV (see text)
$\rho(1700) \rightarrow \rho\eta$	$41 \pm 17$	$68 \pm 42$ MeV (see text)
$\rho(1700) \rightarrow \rho\eta'$	$\approx 0$	Not listed in PDG
$K^*(1680) \rightarrow K\rho$	$64 \pm 27$	$101 \pm 35$ by PDG
$K^*(1680) \rightarrow K\phi$	$13 \pm 6$	Not listed in PDG
$K^*(1680) \rightarrow K\omega$	$21 \pm 9$	Not listed in PDG
$K^*(1680) \rightarrow K^*(892)\pi$	$81 \pm 34$	$96 \pm 33$ by PDG
$K^*(1680) \rightarrow K^*(892)\eta$	$0.5 \pm 0.2$	Not listed in PDG
$K^*(1680) \rightarrow K^*(892)\eta'$	$\approx 0$	Not listed in PDG
$\omega(1650) \rightarrow \rho\pi$	$370 \pm 156$	$\sim 205, 154 \pm 44, \sim 273, 120 \pm 18$ (see text)
$\omega(1650) \rightarrow K^*(892)K$	$42 \pm 18$	Not listed in PDG
$\omega(1650) \rightarrow \omega(782)\eta$	$32 \pm 13$	$\sim 100, 56 \pm 30$ (see text)
$\omega(1650) \rightarrow \omega(782)\eta'$	$\approx 0$	Not listed in PDG
$\phi(1930) \rightarrow K\bar{K}^*$	$260 \pm 109$	Resonance not yet known
$\phi(1930) \rightarrow \phi(1020)\eta$	$67 \pm 28$	Resonance not yet known
$\phi(1930) \rightarrow \phi(1020)\eta'$	$\approx 0$	Resonance not yet known

arXiv: 1708.02593

## Orbitally excited/2

TABLE IX. Decay widths of (predominantly) orbitally excited vector mesons into two pseudoscalar mesons ( $V_D \rightarrow PP$ ).

Decay process $V_D \rightarrow PP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \rightarrow \bar{K}K$	$40 \pm 11$	$8.3^{+10}_{-8.3}$ MeV (see text)
$\rho(1700) \rightarrow \pi\pi$	$140 \pm 37$	$75 \pm 30$ by Becker 79 [65]
$K^*(1680) \rightarrow K\pi$	$82 \pm 22$	$125 \pm 43$ by PDG
$K^*(1680) \rightarrow K\eta$	$52 \pm 14$	Not listed in PDG
$K^*(1680) \rightarrow K\eta'$	$0.72 \pm 0.02$	Not listed in PDG
$\omega(1650) \rightarrow \bar{K}K$	$37 \pm 10$	Not listed in PDG
$\phi(1930) \rightarrow \bar{K}K$	$104 \pm 28$	Resonance not yet known

TABLE XI. Decay widths of (predominantly) orbitally excited vector mesons into a photon and a pseudoscalar meson ( $V_D \rightarrow \gamma P$ ).

Decay process $V_D \rightarrow \gamma P$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \rightarrow \gamma\pi$	$0.095 \pm 0.058$	Not listed
$\rho(1700) \rightarrow \gamma\eta$	$0.35 \pm 0.21$	Not listed
$\rho(1700) \rightarrow \gamma\eta'$	$0.13 \pm 0.08$	Not listed
$K^*(1680) \rightarrow \gamma K$	$0.30 \pm 0.18$	Not listed
$\omega(1650) \rightarrow \gamma\pi$	$0.78 \pm 0.47$	Not listed
$\omega(1650) \rightarrow \gamma\eta$	$0.035 \pm 0.021$	Not listed
$\omega(1650) \rightarrow \gamma\eta'$	$0.012 \pm 0.007$	Not listed
$\phi(1930) \rightarrow \gamma\eta$	$0.19 \pm 0.12$	Resonance not yet known
$\phi(1930) \rightarrow \gamma\eta'$	$0.13 \pm 0.08$	Resonance not yet known

# Prediction for $\phi(1930)$

Can one find this state?

TABLE XII. Summary table for the putative state  $\phi(1930)$ .

Meson $\phi(1930)$	
Quark composition	$\approx s\bar{s}$
Old spectroscopy notation	(Predom.) $n^{2S+1}L_J = 1^3D_1$
$n$	(Predom.) 1
$S$	(Predom.) $1\uparrow\uparrow$
$L$	(Predom.) 2
$J^{PC}$	$1^{--}$
Mass	$\approx 1930 \pm 40$ MeV
Decays	
Decay channel	Decay width (MeV)
$\phi(1930) \rightarrow \bar{K}K$	$104 \pm 28$
$\phi(1930) \rightarrow K\bar{K}^*$	$260 \pm 109$
$\phi(1930) \rightarrow \Phi(1020)\eta$	$67 \pm 28$
$\phi(1930) \rightarrow \Phi(1020)\eta'$	$\approx 0$
$\phi(1930) \rightarrow \gamma\eta$	$0.19 \pm 0.12$
$\phi(1930) \rightarrow \gamma\eta'$	$0.13 \pm 0.08$

arXiv: 1708.02593

## Conclusions



- There are open issues in the realm of conventional mesons below 2 GeV
- $a_1(1260)$ : decay into  $\rho\pi$  and  $\gamma\pi$
- Pseudoscalar mesons: decays of  $\eta_2(1870)$
- Excited vector mesons: search for  $\phi(1930)$



Thanks

Back-up slides

# Results of the eLSM (11 parameters, 21 exp. quantities)

Error from PDG or 5% of exp.  
Scalar-isoscalar sector not  
included.

$$\chi_{red}^2 = 1.2$$

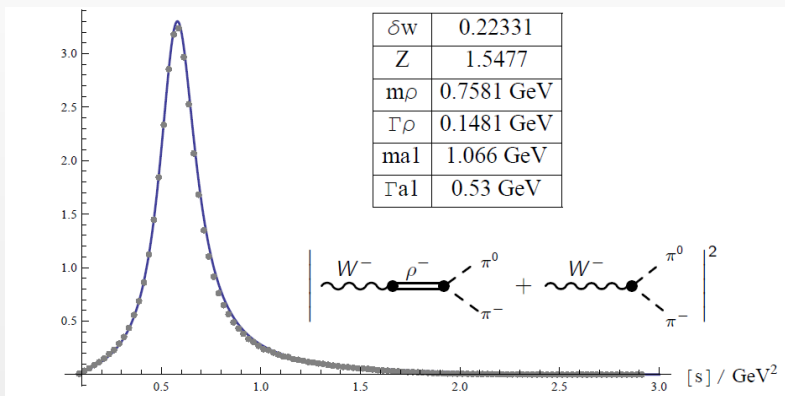
Observable	Fit [MeV]	Experiment [MeV]
$f_\pi$	$96.3 \pm 0.7$	$92.2 \pm 4.6$
$f_K$	$106.9 \pm 0.6$	$110.4 \pm 5.5$
$m_\pi$	$141.0 \pm 5.8$	$137.3 \pm 6.9$
$m_K$	$485.6 \pm 3.0$	$495.6 \pm 24.8$
$m_\eta$	$509.4 \pm 3.0$	$547.9 \pm 27.4$
$m_{\eta'}$	$962.5 \pm 5.6$	$957.8 \pm 47.9$
$m_\rho$	$783.1 \pm 7.0$	$775.5 \pm 38.8$
$m_{K^*}$	$885.1 \pm 6.3$	$893.8 \pm 44.7$
$m_\phi$	$975.1 \pm 6.4$	$1019.5 \pm 51.0$
$m_{a_1}$	$1186 \pm 6$	$1230 \pm 62$
$m_{f_1(1420)}$	$1372.5 \pm 5.3$	$1426.4 \pm 71.3$
$m_{a_0}$	$1363 \pm 1$	$1474 \pm 74$
$m_{K_0^*}$	$1450 \pm 1$	$1425 \pm 71$
$\Gamma_{\rho \rightarrow \pi\pi}$	$160.9 \pm 4.4$	$149.1 \pm 7.4$
$\Gamma_{K^* \rightarrow K\pi}$	$44.6 \pm 1.9$	$46.2 \pm 2.3$
$\Gamma_{\phi \rightarrow \bar{K}K}$	$3.34 \pm 0.14$	$3.54 \pm 0.18$
$\Gamma_{a_1 \rightarrow \rho\pi}$	$549 \pm 43$	$425 \pm 175$
$\Gamma_{a_1 \rightarrow \pi\gamma}$	$0.66 \pm 0.01$	$0.64 \pm 0.25$
$\Gamma_{f_1(1420) \rightarrow K^*K}$	$44.6 \pm 39.9$	$43.9 \pm 2.2$
$\Gamma_{a_0}$	$266 \pm 12$	$265 \pm 13$
$\Gamma_{K_0^* \rightarrow K\pi}$	$285 \pm 12$	$270 \pm 80$

arXiv:1208.0585

# The weak tau-decay into mesons

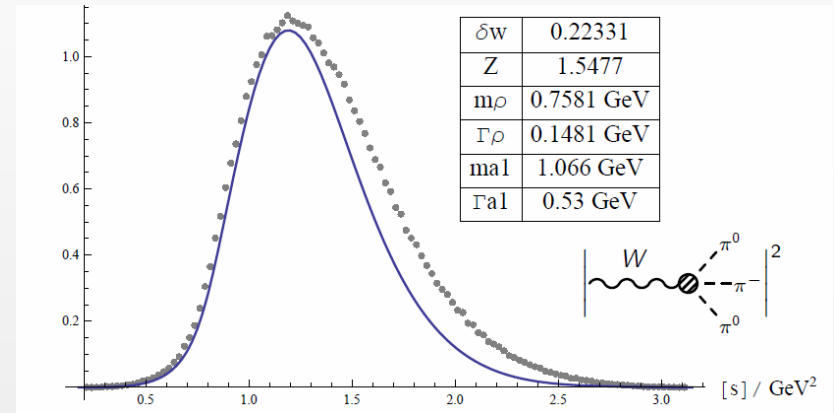
$$\tau \rightarrow W^- \nu_\tau \rightarrow \pi\pi\nu_\tau$$

$$\frac{1}{N_\nu} \frac{dN_\nu}{ds}$$



$$\tau \rightarrow W^- \nu_\tau \rightarrow \pi\pi\pi\nu_\tau$$

$$\frac{1}{N_A} \frac{dN_A}{ds}$$



A. Habersetzer and F. G., *J.Phys.Conf.Ser.* **599** (2015) 012011

$$\begin{aligned}
 & \left| W^- \rightarrow \pi^- \pi^0 \right|^2 \\
 &= \frac{1}{2} \left| 2 W^- \rightarrow \pi^- \pi^0 + W^- \rightarrow \pi^- \pi^0(k_1) \pi^0(k_2) + W^- \rightarrow \pi^- \pi^0(k_3) \pi^0(k_1) \right. \\
 & \left. + 2 W^- \rightarrow \pi^- \pi^0 + W^- \rightarrow \pi^- \pi^0(k_1) \pi^0(k_2) + W^- \rightarrow \pi^- \pi^0(k_3) \pi^0(k_1) \right|^2
 \end{aligned}$$

# Quark-antiquark currents

Meson	$n^{2S+1}L_J$	$J^{PC}$	$S$	$L$	Hermitian quark current operators
pseudoscalar	$1^1S_0$	$0^{-+}$	0	0	$P_{ij} = \bar{q}_j i\gamma^5 q_i$
vector	$1^3S_1$	$1^{--}$	1		$V_{ij}^\mu = \bar{q}_j \gamma^\mu q_i$
pseudovector	$1^1P_1$	$1^{+-}$	0	1	$P_{ij}^\mu = \bar{q}_j \gamma^5 \overleftrightarrow{\partial}^\mu q_i$
scalar	$1^3P_0$	$0^{++}$	1		$S_{ij} = \bar{q}_j q_i$
axial vector	$1^3P_1$	$1^{++}$	1		$A_{ij}^\mu = \bar{q}_j \gamma^5 \gamma^\mu q_i$
tensor	$1^3P_2$	$2^{++}$	1		$X_{ij}^{\mu\nu} = \bar{q}_j i \left[ \gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3} \tilde{G}^{\mu\nu} \overleftrightarrow{\not{\partial}} \right] q_i$
pseudotensor	$1^1D_2$	$2^{-+}$	0	2	$T_{ij}^{\mu\nu} = \bar{q}_j i \left[ \gamma^5 \overleftrightarrow{\partial}^\mu \overleftrightarrow{\partial}^\nu - \frac{2}{3} \tilde{G}^{\mu\nu} \overleftrightarrow{\partial}_\alpha \overleftrightarrow{\partial}^\alpha \right] q_i$
excited vector	$1^3D_1$	$1^{--}$	1		$S_{ij}^\mu = \bar{q}_j \overleftrightarrow{\partial}^\mu q_i$
axial tensor	$1^3D_2$	$2^{--}$	1		$B_{ij}^{\mu\nu} = \bar{q}_j i \left[ \gamma^5 \gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^5 \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3} \tilde{G}^{\mu\nu} \gamma^5 \overleftrightarrow{\not{\partial}} \right] q_i$
spin-3 tensor	$1^3D_3$	$3^{--}$	1		...

# Pseudotensor glueball: predictions

“ $G_2(3040)$ ”	
Branching ratio	Theory
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(K_2^*(1430) K + c.c.)$	0.9
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2(1270) \eta)$	6.0
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2(1270) \eta'(958))$	8.5
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2'(1525) \eta)$	9.0
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2'(1525) \eta'(958))$	11.0

# Other considerations on pseudotensor mesons



## Our model

- couples pseudotensor mesons to pseudoscalar, vector and tensor mesons.
- reproduces present experimental data for  $\pi_2(1670)$  and  $K_2(1770)$ .
- identifies  $\eta_2(1870)$  and  $\eta_2(1645)$  with the  $\bar{q}q$  pseudotensor meson nonet, if non-strange-strange mixing is large.
- predicts a large non-strange-strange mixing angle  $\beta_{pT} \approx -40^\circ$  in the isoscalar sector.
- contributes to the discussion on conflicting experimental results for the branching ratios of  $\eta_2(1870)$ .

# Useful ratios from PDG

Branching ratios from PDG for  $\eta_2(1645)$ :

$$\Gamma^{\text{exp}}(\bar{K} K \pi) / \Gamma^{\text{exp}}(a_2(1320) \pi) = 0.07 \pm 0.03,$$

$$\Gamma^{\text{exp}}(a_2(1320) \pi) / \Gamma^{\text{exp}}(a_0(980) \pi) = 13.1 \pm 2.3.$$

Branching ratio from PDG for  $\eta_2(1870)$ :

$$\Gamma^{\text{exp}}(a_2(1320) \pi) / \Gamma^{\text{exp}}(f_2(1270) \eta) \neq 0.$$