Higher orders and resummations for precision physics

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Next challenges at colliders

- Precision QCD
  - H,W,Z and heavy quark hadroproduction
    $\longrightarrow$ measured with high experimental accuracy
  - Multijet final states
    $\longrightarrow$ background to SUSY, UED, ...
    $\longrightarrow$ measurement of couplings ($e^+e^- \to t\bar{t}H$, $e^+e^- \to HHZ$)
  - Precision measurement of α_S from event shapes

- LO is not enough
  - Large renormalization scale uncertainty (α_S scale not defined)
  - Large factorization scale uncertainty
  - Large corrections from higher orders
  - Jet structure appears only beyond LO

  $\longrightarrow$ Reliable predictions only at NLO
  $\longrightarrow$ Reliable estimate of errors only at NNLO
  $\longrightarrow$ Resummation necessary in some region of the phase space

State of the Art - at a glance

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3	NNNLO	NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- LO Automated and under control, even for multiparticle final states
- NLO Well understood for $2 \to 1$ and $2 \to 2$ in SM and beyond
- NLO Many new $2 \to 3$ calculations from Les Houches wish list since 2007
- NLO Very first $2 \to 4$ LHC cross section in 2008 $q\bar{q} \to t\bar{t}b\bar{b}$
- NLO Important developments in automation, $W + 3$ jets (2009)
- NNLO Inclusive and exclusive Drell-Yan and Higgs cross sections
- NNLO $e^+e^- \to 3$ jets, but still waiting for $pp \to \text{jets},\ W + \text{jet},\ t\bar{t}, VV$
- NNNLO F_2, F_3 and form-factors

NLO Automation

- Combination of infrared divergent parts (dipole subtraction) has become standard and automated

  [Gleisberg,Krauss(SHERPA);Frederix,Gehrmann,Greiner(MadGraph) Seymour,Tevlin(TevJet)Hasegawa,Moch,Uwer]

- One-loop matrix elements: major breakthroughs

Unitarity Methods

Use unitarity cuts on loop diagrams to compute tensor coefficients as products of tree amplitudes

[Bern,Dixon,Dunbar,Kosower(94); Britto,Cachazo,Feng(04); Berger,Bern,Dixon,Forde,Kosower(06); Giele,Kunzst,Melnikov(08)]

OPP Method

New reduction formalism for tensor integrals: reduce 1-loop amplitudes to scalar integrals at the integrand level

[Ossola,Papadopoulos,Pittau(06)]

implemented in **BlackHat, Helac/CutTools, Rucola**

Available codes

- **Rocket** [Giele,Zanderighi(08)]
  - up to 1-loop 20 gluon amplitudes! [Giele,Zanderighi(08)]
  - 1-loop W+5j amplitudes [Ellis,Giele,Kunzst,Melnikov,Zanderighi(08)]
  - NLO W+3j cross section [Ellis,Melnikov,Zanderighi(08)]
- **BlackHat** [Berger et al.(08)]
  - 1-loop 8 gluon amplitudes
  - 1-loop W+5j amplitudes (2008)
  - NLO W+3j cross section (2009)
- **Helac/CutTools** [Cafarella et al.(09)]
  - 1-loop amplitudes for $q\bar{q}, gg \to t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg, Wggg, Zggg$
  - NLO $pp \to t\bar{t}b\bar{b}$ cross section

[Bevilacqua,Czakon,Papadopoulos,Pittau,Worek(09)]
[see also Bredenstein,Denner,Dittmaier,Pozzorini(09)]

- Goal at NLO: all $2 \to 4(5,6)$ processes with Unitarity/OPP methods

NLO Matching

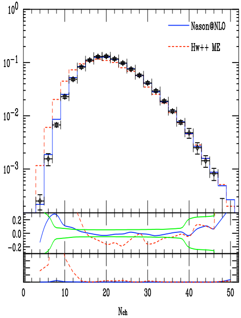
Parton Shower Generator	**Matrix Element Generator**
Resums leading logs to all orders	Only go up to NLO
High multiplicity *hadrons* in final state	Low multiplicity *partons* in final state
Good for regions of low relative p_T	Good for regions of high relative p_T
Total rate accurate to LO	Total rate accurate to NLO

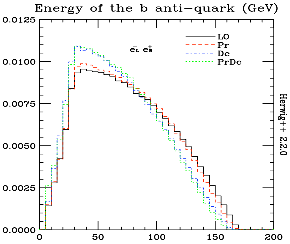
The perfect matching

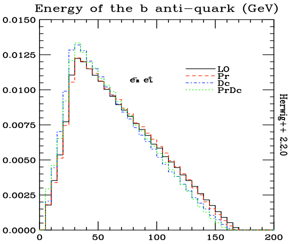
- generates total rates accurate at NLO
- treats hard emission as in Matrix Element Generators
- treats soft/collinear emission as in Parton Shower Generators
- generates a set of fully exclusive events which can be interfaced with a hadronization model

NLO Matching

- **MC@NLO** [Frixione,Webber(02)]
  - Add difference between exact(ME) NLO and approx.(PS) NLO
  - → dependent on the shower details
  - → difference may be negative
- **POWHEG** [Nason(04)]
  - Generate the hardest emission at NLO accuracy (mod. Sudakov)
  - Angular-ordered showers: add truncated shower from hard scale
  - → always positive weights







Ingredients for NNLO

- For a general $2 \to n$ process we need
  - Two-loop amplitude for $2 \to n$
  - One-loop amplitude for $2 \to n+1$
  - Tree-level amplitude for $2 \to n+2$
- Each term has its own singularities
  - Ultraviolet (removed by renormalization)
  - Infrared (have to cancel among each other)

$\longrightarrow$ Much more difficult than NLO cancellation!

Cancellation of singularities

- Fully inclusive quantities
  - analytical computation of contributions is possible
  - explicit cancellation of singularities

  $\longrightarrow$ DIS [Zijlstra,van Neerven(92)]

  $\longrightarrow$ Single Hadron [Rijken,vanNeerven(97);Mitov,Moch(06)]

  $\longrightarrow$ DY [Hamberg,van Neerven,Matsuura(91)]

  $\longrightarrow$ H [Harlander,Kilgore(02);Anastasiou,Melnikov(02);Ravindran,Smith,van Neerven(03)]
- Fully exclusive quantities (real world!)
  - IR singularity structure at NNLO understood [Catani,Grazzini;Campbell,Glover;Bern,DelDuca,Kilgore,Schmidt; Kosower,Uwer;Sterman,Tejeda-Yeomans]
  - numerical integration still very difficult

  $\longrightarrow$ **Sector Decomposition**

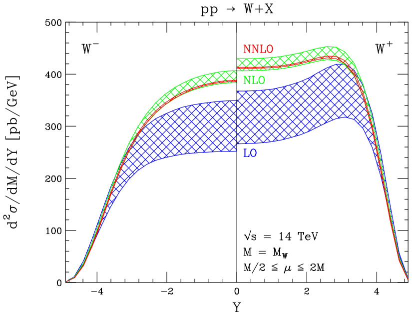
  $\longrightarrow$ **Subtraction Method**

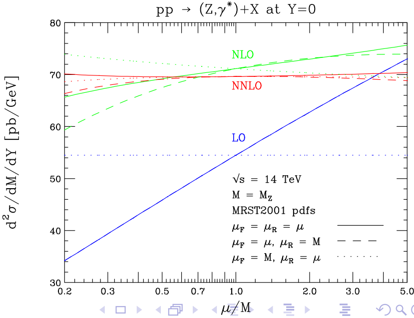
"Split the integration region into sectors, each containing a single singularity, and explicit the pole by expanding it into distributions"

Binoth,Heinrich[00,04];Anastasiou,Melnikov,Petriello[04]

AMP developed a fully automated procedure to compute pole coefficients and finite terms and applied it to

$$\text{H/W/Z(04), QED } \mu\text{-decay(05), } b \to c l \bar{\nu}_l \text{(08)}$$





Subtraction Method

"Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton"

NLO:Ellis,Ross,Terrano[81];Frixione,Kunzst,Signer[95];Catani,Seymour[96]
(NNLO):Kosower[03,05];Weinzierl[03];Frixione,Grazzini[04]
Gehrmann,Glover[05];Somogyi,Trocsanyi,DelDuca[05,07]

$$
\begin{aligned}
d\sigma &= \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n \\
d\sigma &= \int_{n+1} (r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1}) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n
\end{aligned}
$$

The *Antenna Subtraction Method* developed by A and T. Gehrmann and Glover has been used for the NNLO QCD calculation of

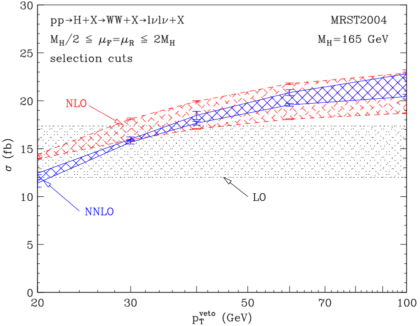
$$e^+e^- \to 3 \text{ jets } \textit{(next talk)}$$

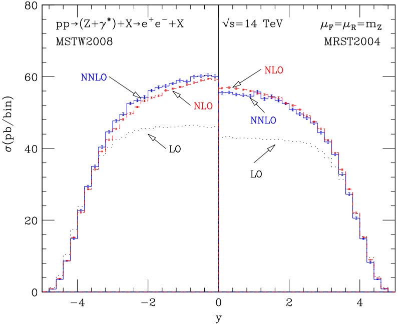
A.Gehrmann,T.Gehrmann,Glover,Heinrich[07]

Subtraction Method

NNLO subtraction has been applied also to Higgs and Vector Boson production at the LHC

H:Catani,Grazzini[07];W,Z:Catani,Cieri,DeFlorian,Ferrera,Grazzini[09]





Cuts greatly affect the impact of NNLO corrections in the Higgs case!

Resummation: well-known examples

- $\log(Q/Q_0)$
  - evolution of pdfs from input scale Q_0 to hard scale Q
  - collinear radiation from colliding partons: single logs
  - systematically resummed by DGLAP equation
- $\log(Q/\sqrt{S})$
  - hadronic c.m. energy $\sqrt{S}$ much larger than hard scale Q
  - multiple radiation over wide rapidity range: single logs
  - systematically resummed by BFKL equation
- $\log(Q^2/q_T^2)$
  - systems with invariant-mass $Q \gg q_T$
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation
- $\log(1-T)$ *(next talk)*
  - when the event is *pencil-like*, i.e. T→1
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation

Resummation: the main idea

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\dots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\dots$	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
LL	NLL	NNLL	$\dots$	$\dots$	

- Ratio of two successive rows: $\mathcal{O}(\alpha_s L^2)$
- improved expansion
  - *reorganization* of the terms into *towers of logs*
  - *all-order summation* of the terms in each class
- key-point: *exponentiation*

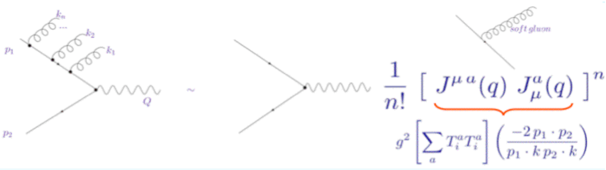
$$\sigma^{res} \sim \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right]$$

- Ratio of two successive columns: $\mathcal{O}(1/L)$

Exponentiation

The observable must fulfill factorization properties both for

- dynamics (matrix element)
  - → in the soft limit, multigluon amplitudes fulfill *generalized factorization formulae* given in terms of *single gluon emission probability*



- kinematics (phase space)
  - → usually factorizable working in *conjugate space*

$$\delta^{(2)}(q_T - q_{T1} - \cdots - q_{Tn}) \quad = \quad \int d^2b\, e^{ib\cdot q_T}\, \Pi_i\, e^{ib\cdot q_T}$$

$$\log(Q^2/q_T^2) \quad \to \quad \log(Q^2 b^2)$$

→ generalized exponentiation of single gluon emission

[Collins,Soper,Sterman(1985)]

Matching with fixed-order

The resummed result has to be properly matched with the fixed-order calculation to avoid double counting

$$\sigma = \sigma^{res} + \sigma^{fix} - \sigma^{asym}$$

where σ^{asym} = expansion of resummed result to same order

- $q_T \ll Q$: $\sigma^{fix} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{res}$
- $q_T > Q$: $\sigma^{res} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{fix}$
- intermediate q_T: matching $\rightarrow \sigma$

The all-orders crew

- Higgs

  $\longrightarrow$ Bozzi, Catani, deFlorian, Grazzini, Nason, Moch, Vogt, Laenen, Magnea, Idilbi, Ji, Ma, Yuan, Kulesza, Sterman, Vogelsang, ..., SEGMENTATION FAULT

- Drell-Yan

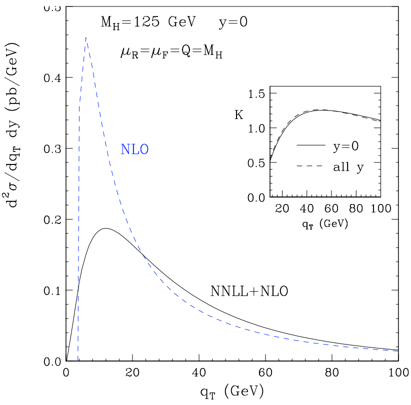
  $\longrightarrow$ Balazs, Yuan, Ellis, Ross, Veseli, Kulesza, Stirling, Sterman, Vogelsang, Bozzi, Catani, DeFlorian, Ferrera, Grazzini, Qiu, Zhang, Vogt, ..., SEGMENTATION FAULT

- Event Shapes

  $\longrightarrow$ Catani, Trentadue, Turnock, Webber, Andersen, Gardi, Rathsman, Banfi, Salam, Zanderighi, Gehrmann, Luisoni, Stenzel, Berger, Sterman, ..., SEGMENTATION FAULT

- SUSY

  $\longrightarrow$ Bozzi, Fuks, Klasen, Debove, Morel, Li, Ledroit, ..., SEGMENTATION FAULT



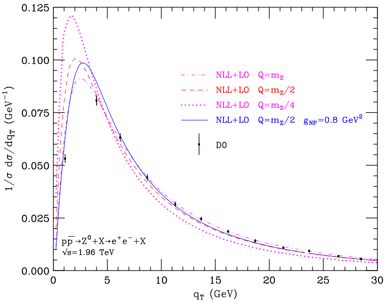
- NLO
  - divergent
  - unphysical peak
- NNLL+NLO
  - well-behaved
  - physical peak
  - converges to NLO at high q_T
- q_T-dependent K-factor

$$K(q_T, y) = \frac{d\sigma_{NNLL+NLO}/(dq_T\, dy)}{d\sigma_{NLO}/(dq_T\, dy)}$$

$\rightarrow$ mild rapidity dependence

$\rightarrow$ resummation relevant both at small and intermediate q_T

Drell-Yan at NLL+LO [Bozzi,Catani,deFlorian,Ferrera,Grazzini (08)]



- Normalized q_T distribution
- Scales fixed to Z mass

$\longrightarrow$ Uncertainty dominated by Q variation

$\longrightarrow$ Good agreement with Run II D0 data

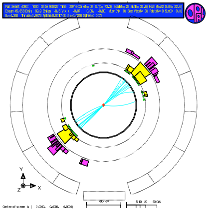
Experimental errors are smaller than theoretical uncertainty
$\rightarrow$ more accurate perturbative predictions (NNLL+NLO)

Event shapes

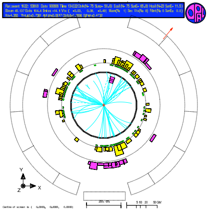
- Event-shape variables $V(p_1,\ldots,p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.
- Thrust: longitudinal particle alignment

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \qquad \Sigma(t) = \mathrm{Prob}(1 - T < t)$$

Pencil-like event: $t \gtrsim 0$



Planar event: $t \simeq 1/3$



Summary

- ILC is useful for QCD (α_S, jets, event shapes, heavy quarks)
- Precision QCD is mandatory for ILC (high experimental accuracy)
- More and more sofisticated tools are becoming available
- Need to continue the effort on the theoretical side

Thanks for your attention!