Event Shapes at NLLA+NNLO and a New Measurement of α_s

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LC09 Workshop

Universität Zürich

Outline

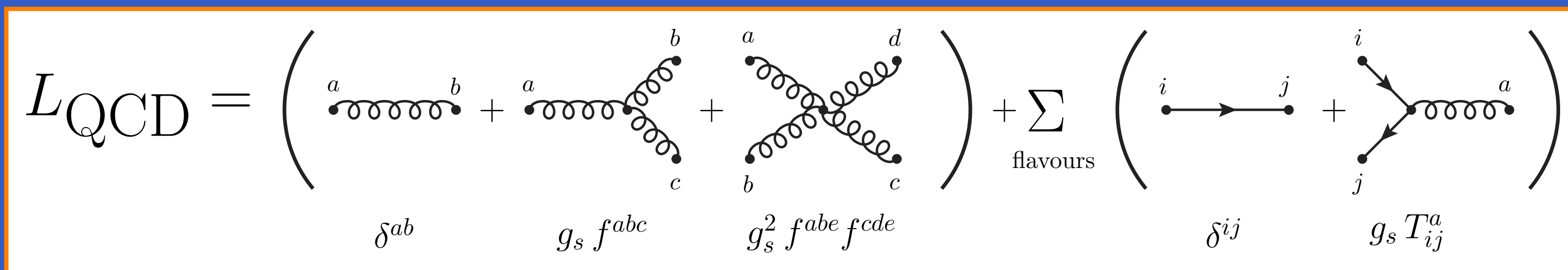
- Motivation
- Event Shapes Observables
- Cross Section Calculations
  - Fixed Order Calculations
  - Resummed Calculations
  - Matching
- Determination of α_S
  - Uncertainties
  - NNLO vs NLLA+NNLO Fits
  - Hadronization Issues
- Conclusions and Outlook

Motivation

- LHC era is around the corner
  - it will be a discovery machine, however...
  - ... need excellent understanding of background,
  - ... and precise prediction of signals.
- Challenge for perturbative QCD:
  can we achieve a high enough precision in perturbative calculations?

Motivation

- Apart the quark masses, there is only one free parameter in the QCD lagrangian,



$$\alpha_s = g_s^2 / (4\pi)$$

- Can be extracted with good accuracy from e^+e^- data, however
- the value of α_s from LEP data suffers mainly from theoretical scale uncertainty:

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0013(\text{had}) \pm 0.0047(\text{scale})$$

[LEPQCDWG]

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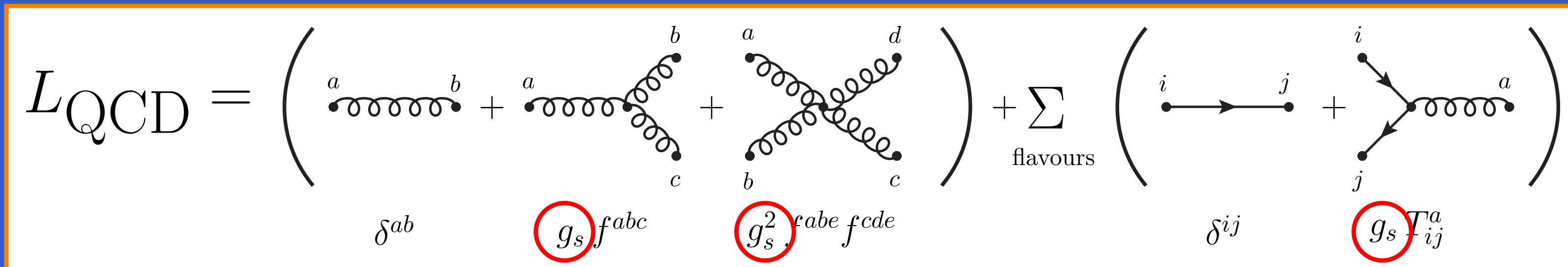

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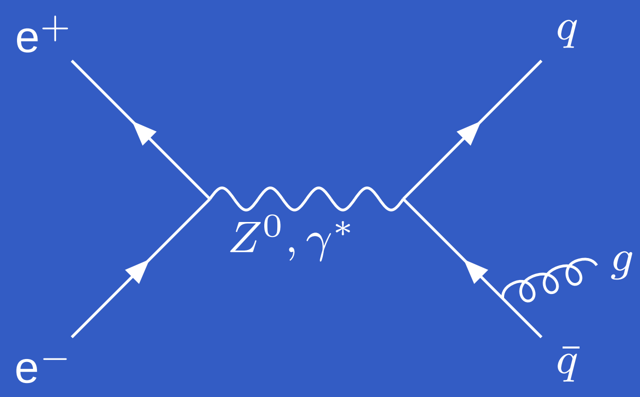
Event Shape Observable

- $e^+e^- \rightarrow 3$ jets at leading order:



$$\frac{d\sigma}{dE_g\, d\cos\theta \bar{q}g} \propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g(1-\cos\theta_{\bar{q}g})}$$

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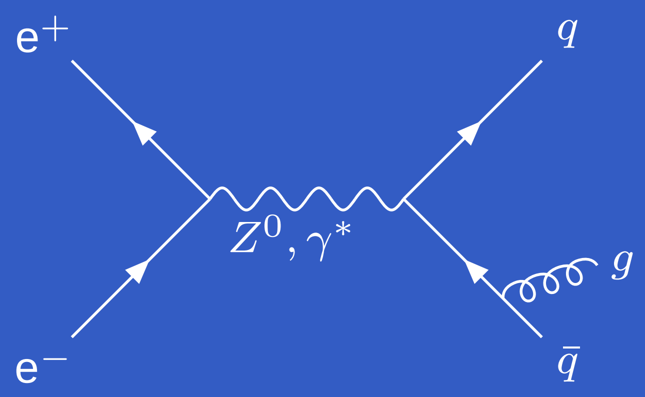

Born cross section for $Z, \gamma \rightarrow q\bar{q}$

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Bremsstrahlung:
enhancement for $E_g \rightarrow 0$
and for $\theta_{\bar{q}g} \rightarrow 0$

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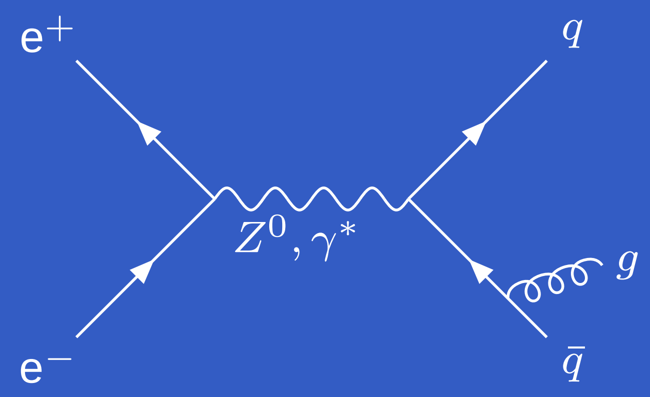

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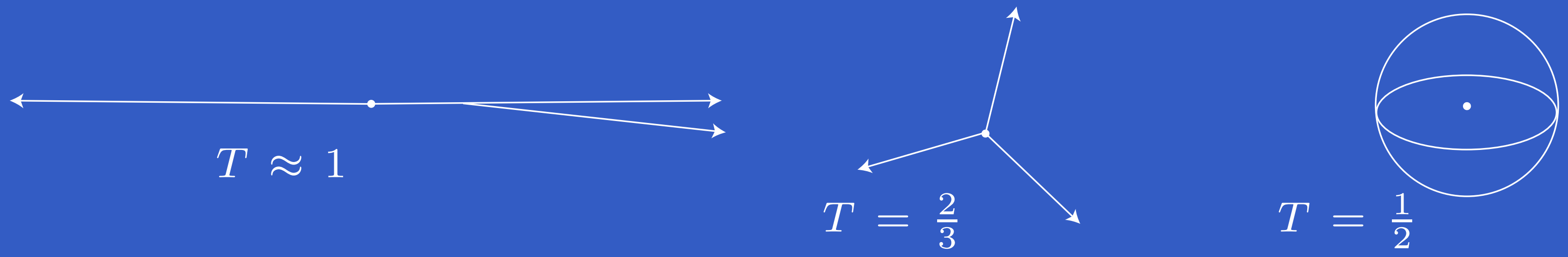
- Experimental observable:
  $\longrightarrow$ jet rates (number of jets),
  $\longrightarrow$ event shape observables.

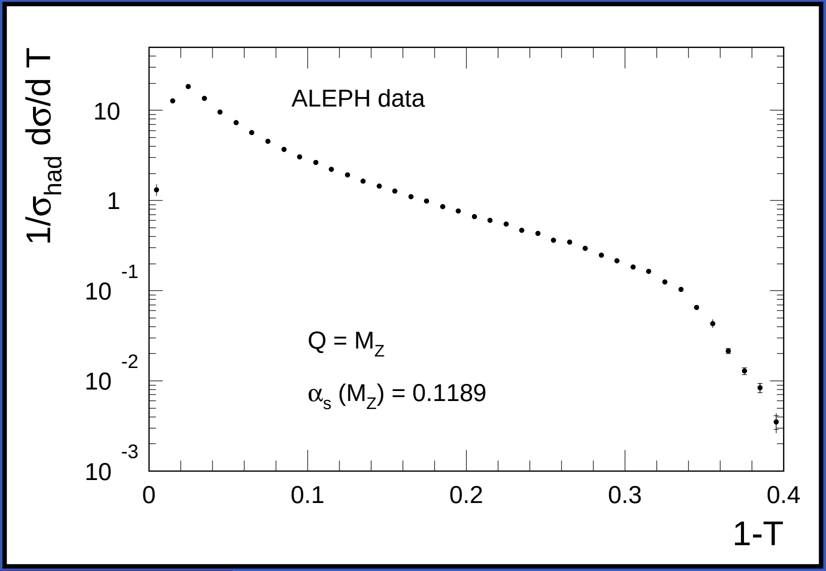
- Well suited also for theoretical pQCD predictions since many are IR and collinear safe.

Event shape observables

- Parametrize geometrical properties of energy-momentum flow,
- canonical example: Thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$





Event shape observables

Progress in theoretical predictions:

- State-of-the-art up to recently:
  - NLO calculations, [Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour.]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]

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  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]
- Very important progress in the last two years
  - NNLO calculations and matching with NLLA of the LEP standard set of event shape observables, [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich; Weinzierl; Gehrmann, G.L., Stenzel]
  - N^3LL resummation in SCET and matching with NNLO for T, [Schwartz; Becher, Schwartz]
  - Non-perturbative $1/Q$ corrections to NLLA+NNLO for T, [Davison, Webber]

Fixed Order Calculations

- For an observable y the differential cross section at NNLO is given by $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}\ , x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

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LO	$\gamma^* \to q\bar{q}g$	tree level	NNLO	$\gamma^* \to q\bar{q}g$	two loop
				$\gamma^* \to q\bar{q}gg$	one loop
NLO	$\gamma^* \to q\bar{q}g$	one loop		$\gamma^* \to q\bar{q}q\bar{q}$	one loop
	$\gamma^* \to q\bar{q}gg$	tree level		$\gamma^* \to q\bar{q}q\bar{q}g$	tree level
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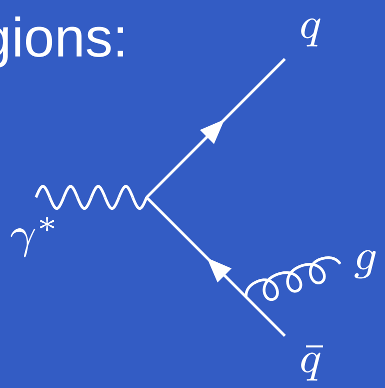
	$\gamma^* \to q\bar{q}gg$	tree level		$\gamma^* \to q\bar{q}q\bar{q}g$	tree level
	$\gamma^* \to q\bar{q}q\bar{q}$	tree level		$\gamma^* \to q\bar{q}ggg$	tree level

- Coefficient functions $\frac{dA}{dy}$, $\frac{dB}{dy}$, $\frac{dC}{dy}$ are functions of $L \equiv \ln\frac{1}{y}$,

Fixed Order Calculations

- Logarithms are originated from integration over soft and collinear regions:



$$\propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g(1-\cos\theta_{\bar{q}g})}$$

- Integrating over the phase space:

$$\frac{d\sigma}{dy} \propto \int \frac{dE_g}{E_g} \frac{d\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}} \delta\left(y - y\left(E_g, \theta_{\bar{q}g}\right)\right)$$
$$\propto \frac{1}{y} \log\left(\frac{1}{y}\right)$$

- They describe the enhancement due to soft and collinear emissions.

Fixed Order Calculations

- Consider cumulative cross section $R(y,Q,\mu) \equiv \frac{1}{\sigma_{\text{had}}}\int_0^y \frac{d\sigma(x,Q,\mu)}{dx}dx,$

$$R(y,Q,\mu) = 1 + \mathcal{A}(y)\,\bar{\alpha}_s(\mu) + \mathcal{B}(y,x_\mu)\,\bar{\alpha}_s^2(\mu) + \mathcal{C}(y,x_\mu)\,\bar{\alpha}_s^3(\mu).$$

$\bar{\alpha}_s\mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
$\bar{\alpha}_s^2\mathcal{B}(y,x_\mu)$	$\bar{\alpha}_s^2 L$	$\bar{\alpha}_s^2 L^2$	$\bar{\alpha}_s^2 L^3$	$\bar{\alpha}_s^2 L^4$		
$\bar{\alpha}_s^3\mathcal{C}(y,x_\mu)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	$\bar{\alpha}_s^3 L^3$	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

Contribution becomes smaller ↓

- If L is NOT large, contributions become smaller line-by-line.
- In phase space region where $y \to 0$, $L \to \infty$:

$\longrightarrow$ coefficient functions become large spoiling the convergence of the series expansion.

- Main contribution comes from highest power of the logarithms.

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Need RESUMMATION!

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Resummed Calculations

- Idea: resum the highest powers of the logarithms to all orders in perturbation theory

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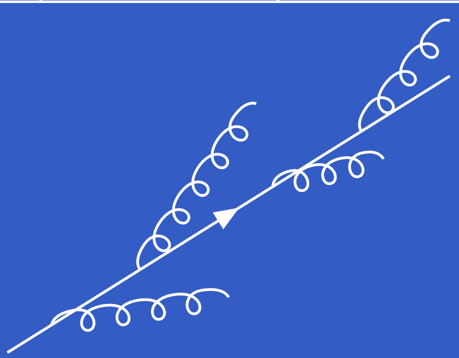
- Leading logarithms
- Next-to-Leading logarithms
- From trivial exponentiation



Resummed Calculations

- For suitable observables, resummation of logarithms leads to exponentiation

$$\Sigma(y) = e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \ldots}$$

with

$$L\, g_1(\alpha_s L) = G_{12} L^2 \bar{\alpha}_s + G_{23} L^3 \bar{\alpha}_s^2 + G_{34} L^4 \bar{\alpha}_s^3 + \ldots \text{ (LL)}$$

$$g_2(\alpha_s L) = G_{11} L \bar{\alpha}_s + G_{22} L^2 \bar{\alpha}_s^2 + G_{33} L^3 \bar{\alpha}_s^3 + \ldots \text{ (NLL)}$$

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- Integrated cross section at NLLA to be matched with NNLO:

$$R(y) = \left(1 + C_1\alpha_s + C_2\alpha_s^2 + C_3\alpha_s^3\right) \times e^{L\,g_1(\alpha_s L) + g_2(\alpha_s L) + \bar{\alpha}_s^2 G_{21} L + \bar{\alpha}_s^3 G_{32} L^2 + \bar{\alpha}_s^3 G_{31} L} + D(y)$$

$$\Rightarrow R(y) = \underbrace{C(\alpha_s)\,\Sigma(y)}_{\text{logarithmic part}} + \underbrace{D(y)}_{\text{remainder function: } \longrightarrow 0 \text{ as } y \longrightarrow 0}$$

C_1, C_2, C_3, G_{21}, G_{32}, G_{31}, $D(y)$: to be determined by matching with fixed order.

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Matching

- Different matching schemes
  - R-matching scheme:
    - Two predictions for $R(y)$ are matched and double-counting terms are subtracted.
    - Unknown matching coefficients $C_1, C_2, C_3, G_{21}, G_{32}, G_{31}$ numerically determined from fixed order result.

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  - Log(R)-matching scheme:
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Log-R matching scheme

- To NLLA + NNLO the integrated cross section in the Log-R matching scheme is given by

$$
\begin{aligned}
\ln\left(R\left(y,\alpha_S\right)\right) &= L\,g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right) \\
&+ \bar{\alpha}_S\left(\mathcal{A}\left(y\right) - G_{11}L - G_{12}L^2\right) + \\
&+ \bar{\alpha}_S^2\left(\mathcal{B}\left(y\right) - \frac{1}{2}\mathcal{A}^2\left(y\right) - G_{22}L^2 - G_{23}L^3\right) \\
&+ \bar{\alpha}_S^3\left(\mathcal{C}\left(y\right) - \mathcal{A}\left(y\right)\mathcal{B}\left(y\right) + \frac{1}{3}\mathcal{A}^3\left(y\right) - G_{33}L^3 - G_{34}L^4\right) .
\end{aligned}
$$

- fixed order
- resummation

Log-R matching scheme

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• fixed order • resummation

- To ensure the vanishing of the matched expression at the kinematical boundary $y_{\max}$

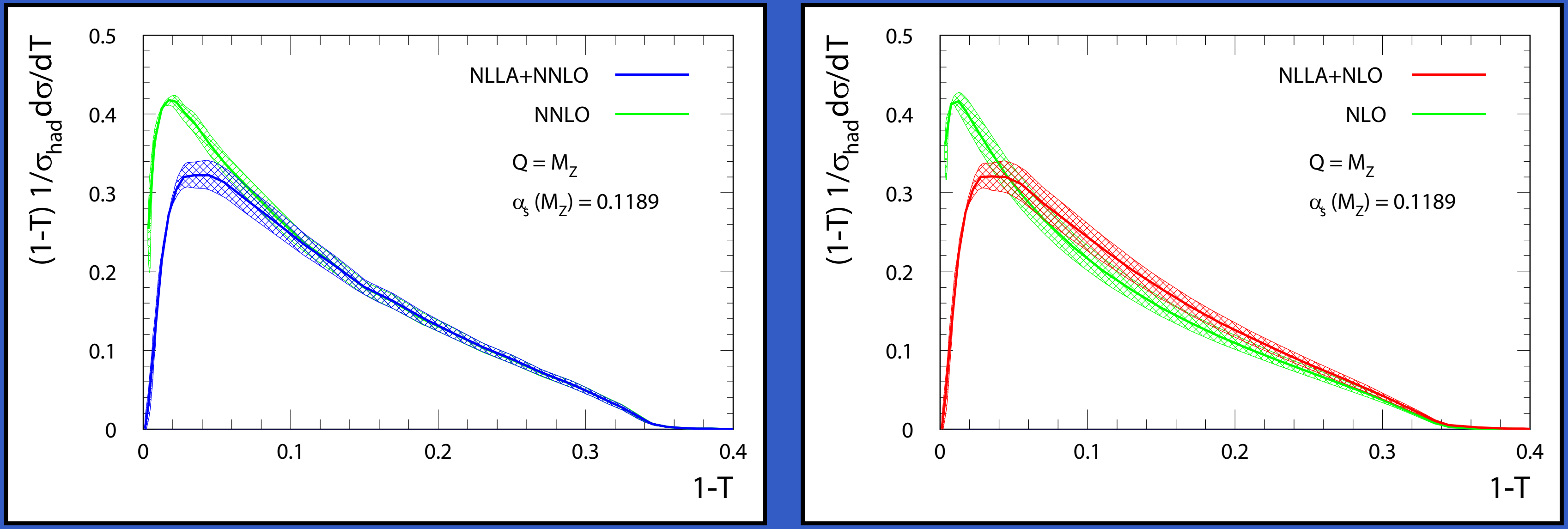
$$L \longrightarrow \tilde{L} = \frac{1}{p}\ln\left(\left(\frac{y_0}{y\,x_L}\right)^p - \left(\frac{y_0}{y_{\max}\,x_L}\right)^p + 1\right),$$

with $y_0 = 6$ for $y = C$ and $y_0 = 1$ otherwise, ($x_L = p = 1$).

[Ford, Jones, Salam, Stenzel, Wicke.]

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$



- Difference between NLLA+NNLO and NNLO restricted to the two-jet region, whereas NLLA+NLO differ in normalisation throughout the full kinematical range.

Determination of α_S

- Recent works:
  - α_S fit using only theoretical NNLO predictions and ALEPH data,
    [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Stenzel.]
  - α_S fit using theoretical N^{3}LLA predictions and ALEPH data,
    [Becher, Schwartz]
  - α_S fit using theoretical NNLO and NLLA+NNLO predictions and JADE data,
    [Bethke, Kluth, Pahl, Schieck and JADE Collaboration.]
  - α_S fit using the the matched NLLA+NNLO predictions and ALEPH data.
    [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, G. L., Stenzel.]

Determination of α_S: uncertainties

- Experimental uncertainties:
  - track reconstr., event selection, detector corrections: cut variations or MC
  - background and ISR (LEP2),

  $\sim 1\%$

- Hadronization uncertainties:
  - difference between various models for hadronization: $\sim 0.7 - 1.5\%$

    Pythia (String frag.), Herwig (Cluster frag.), Ariadne (Dipole + String frag.),

- Theoretical uncertainties (pQCD and resummation):
  - variation of x_μ, x_L, y_{max}, p and matching scheme,
  - uncertainty for b-quark mass correction.

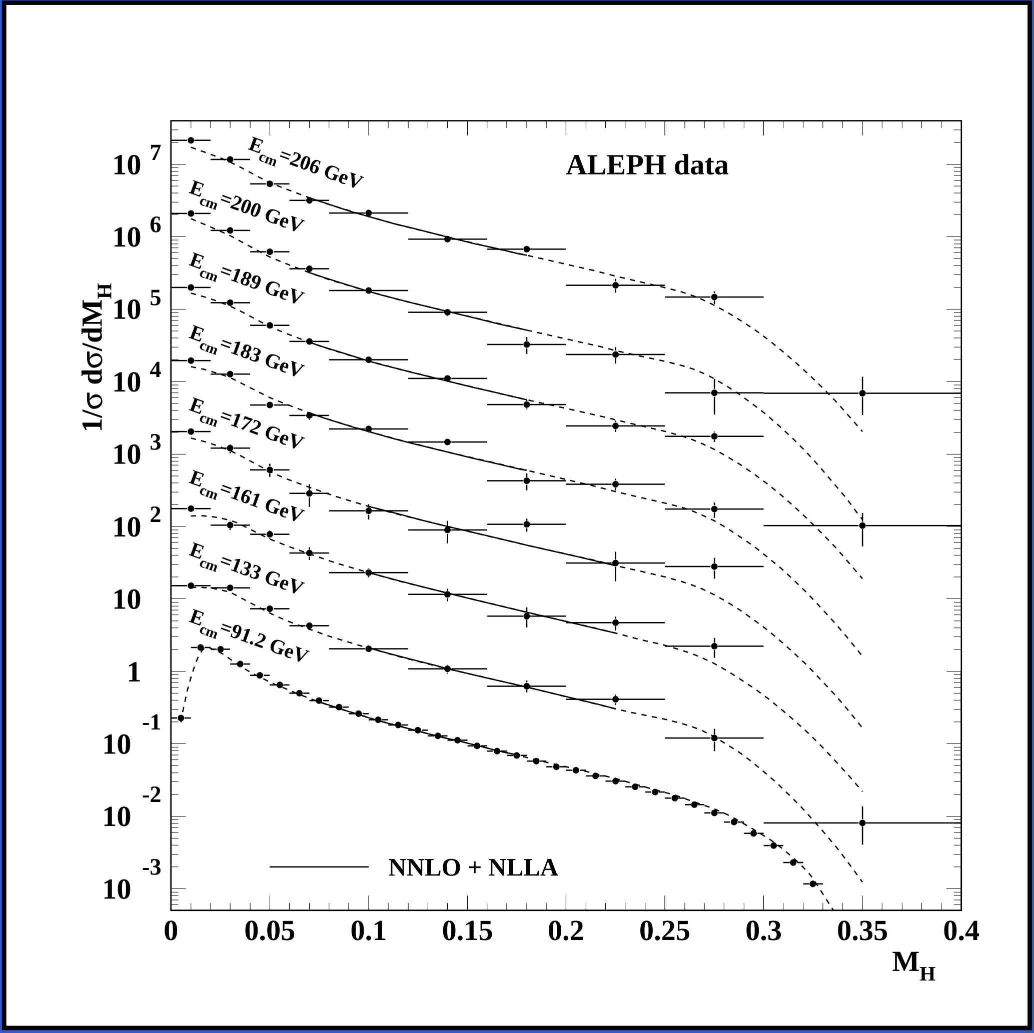
  $\sim 3.5 - 5\%$

- Uncertainty band method to estimate missing higher orders

  [Ford, Jones, Salam, Stenzel, Wicke.]

Determination of α_S: NLLA+NNLO fits

- data are fit in the central part of the event shape distribution,
- only statistical uncertainties are included in the χ^2.
- good fit quality (but includes still large statistical uncertainties of C-coefficient)



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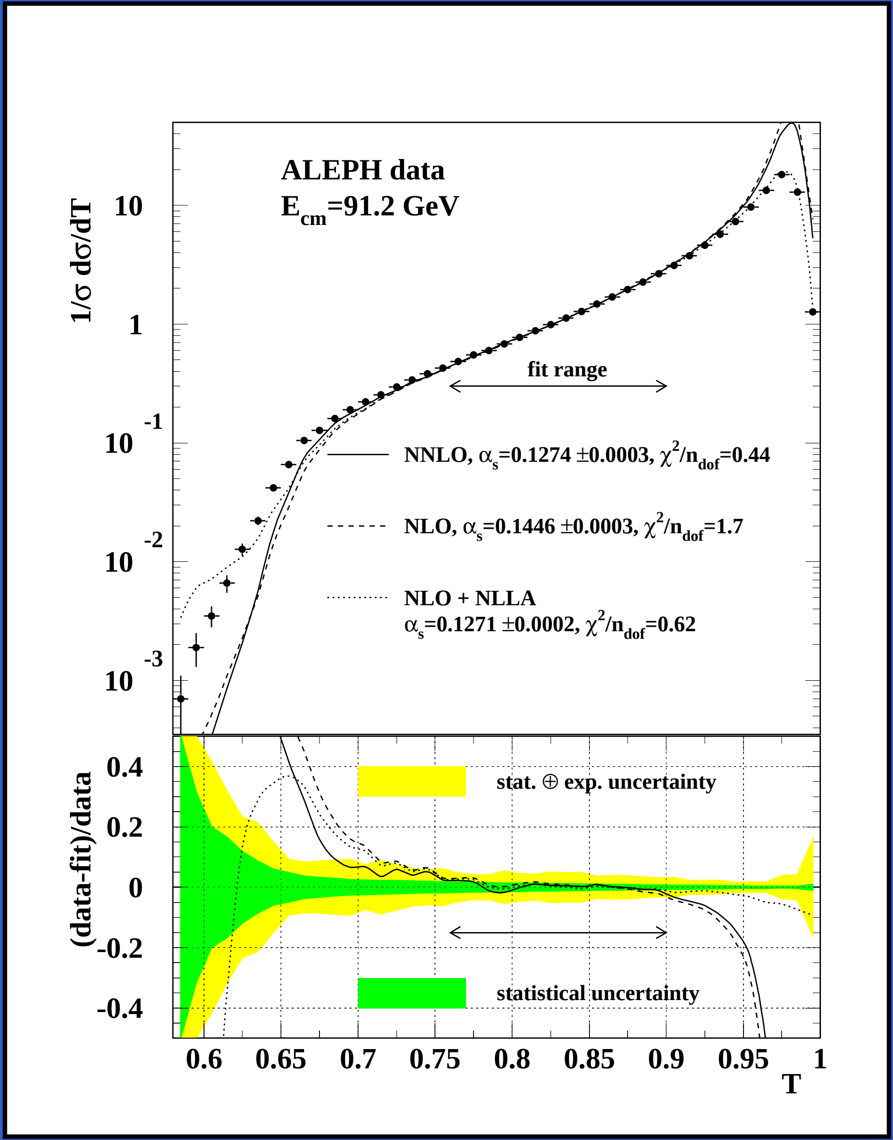
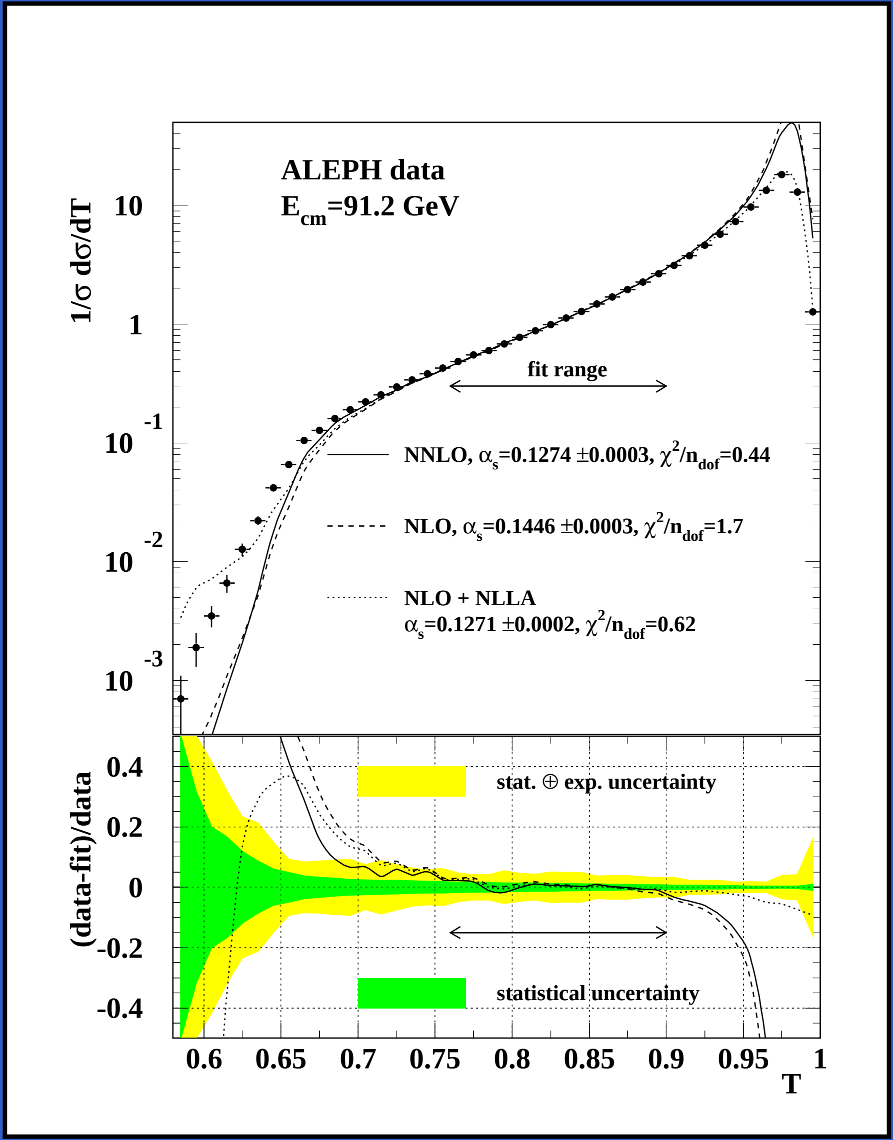
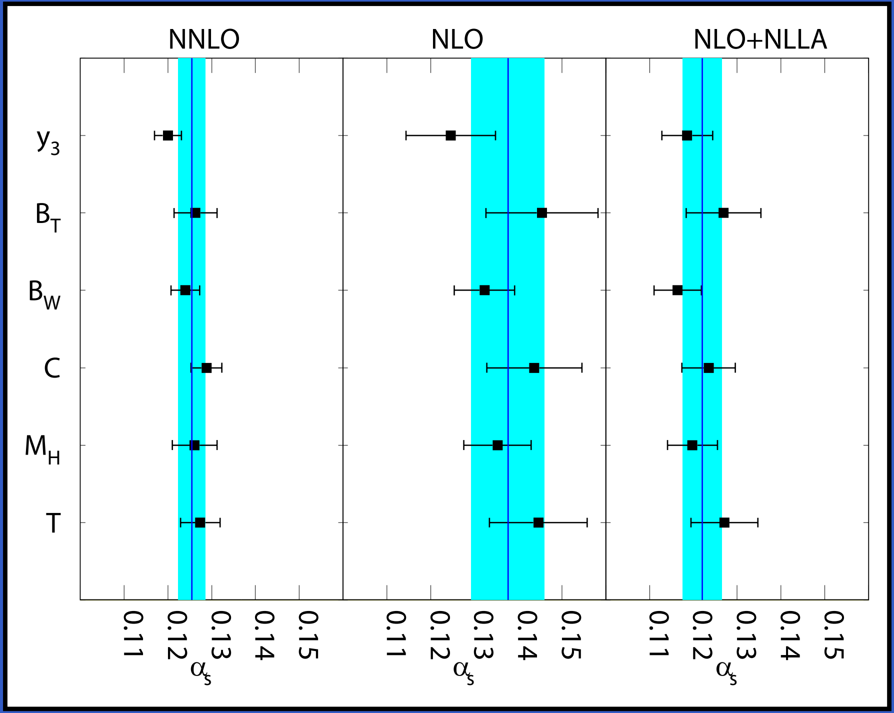


- fit to fixed order calculations gives higher values for α_S,
- tendency to decrease from NLO to NNLO.

Determination of α_S: NNLO fits





- much less scatter at NNLO
- reduced perturbative uncertainty: 0.003

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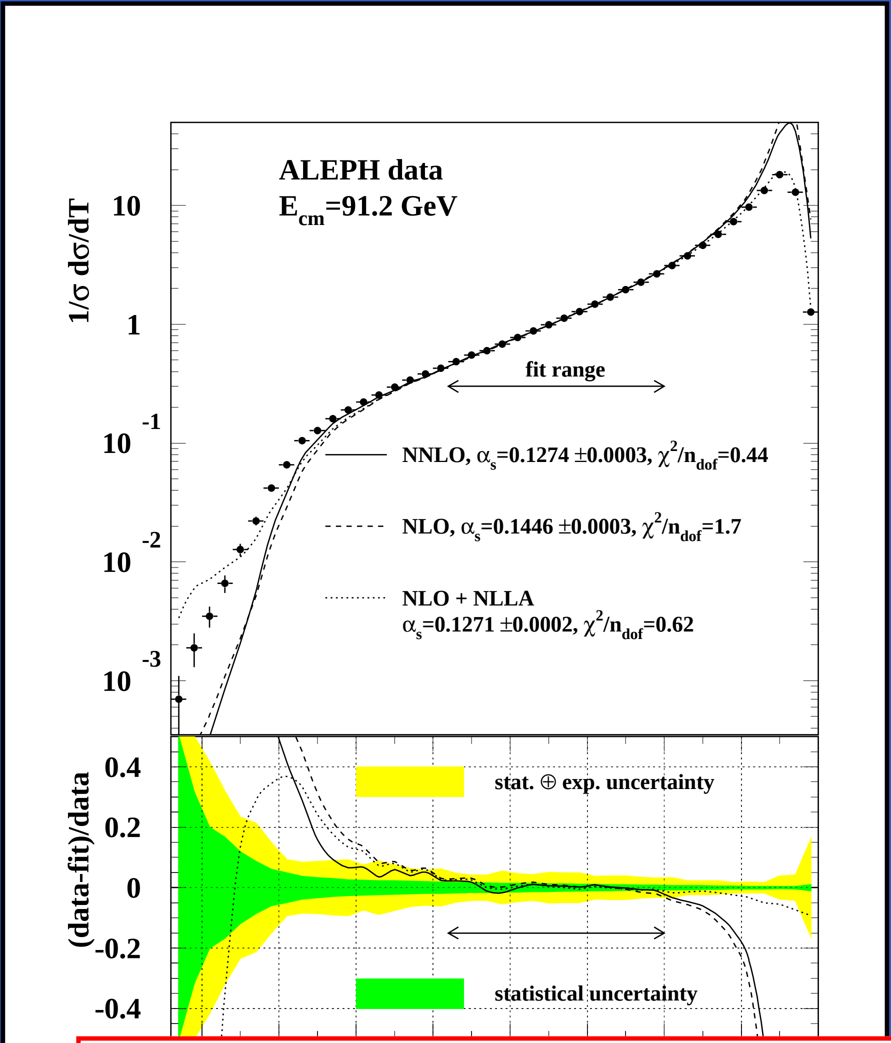

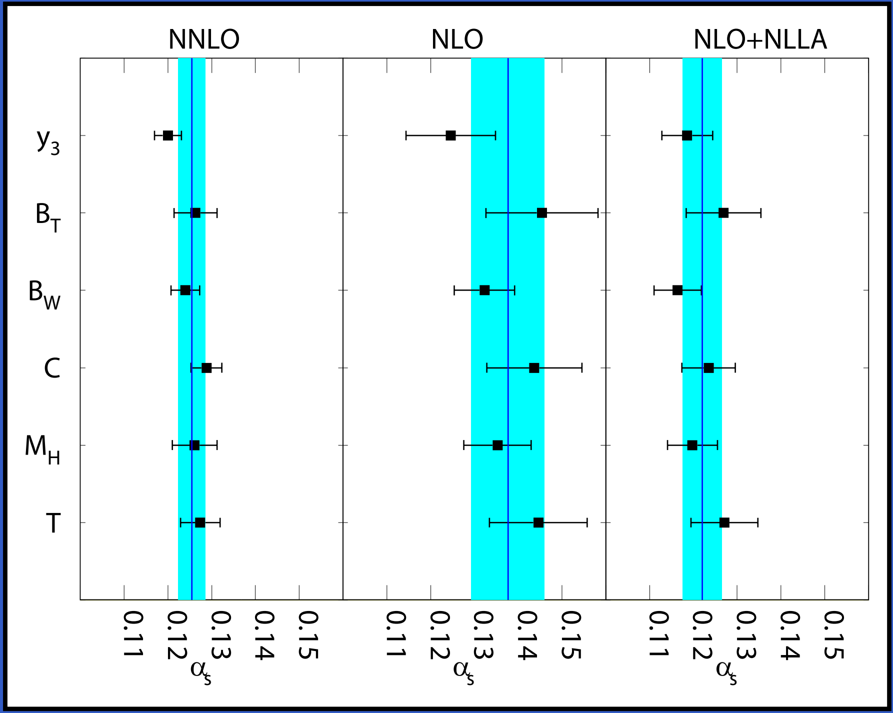

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$$\alpha_s(M_Z) = 0.1240 \pm 0.0008(\text{stat.}) \pm 0.0010(\text{exp.}) \pm 0.0011(\text{had.}) \pm 0.0029(\text{theo.})$$

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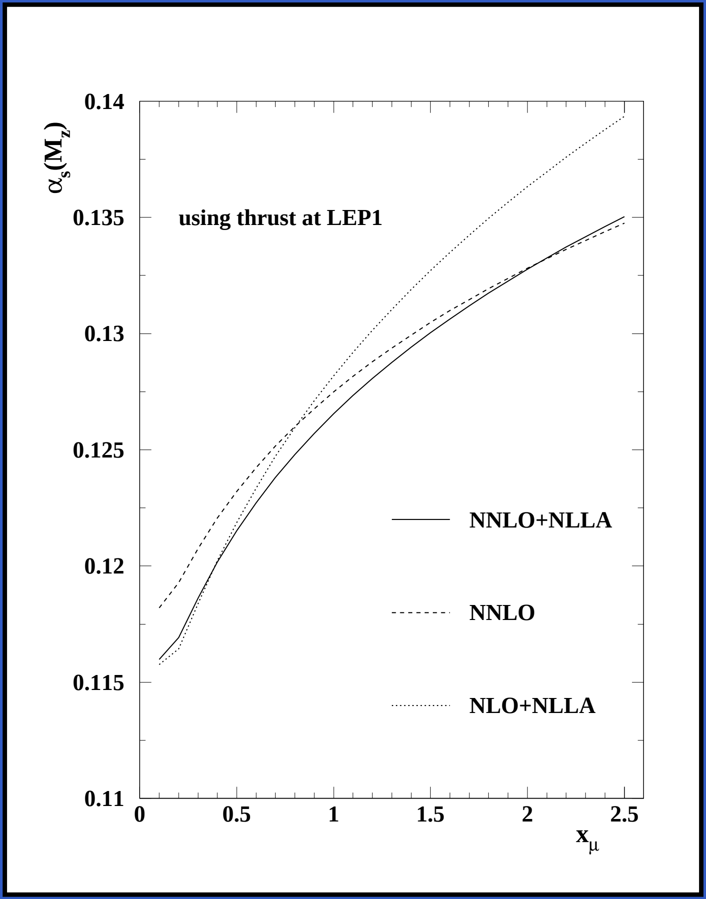
Determination of α_S: NNLO+NLLA fits

- Beware: consistent matching would require full NNLLA results (at present known only for T).
- a slight increase of the scale uncertainty is observed: two loop running terms not compensated in NLLA.

data set	LEP1 + LEP2	LEP2
$\alpha_s(M_Z)$	0.1224	0.1224
stat. error	0.0009	0.0011
exp. error	0.0009	0.0010
pert. error	0.0035	0.0034
hadr. error	0.0012	0.0011
total error	0.0039	0.0039



Determination of α_S: perturbative uncertainty

- The $\log$ R(μ)- matching scheme:
  - compute the two-loop terms proportional to the renormalization scale in resummation and matching functions,
  - central values of individual fits are not affected...
  - ... but theoretical uncertainty is much reduced.

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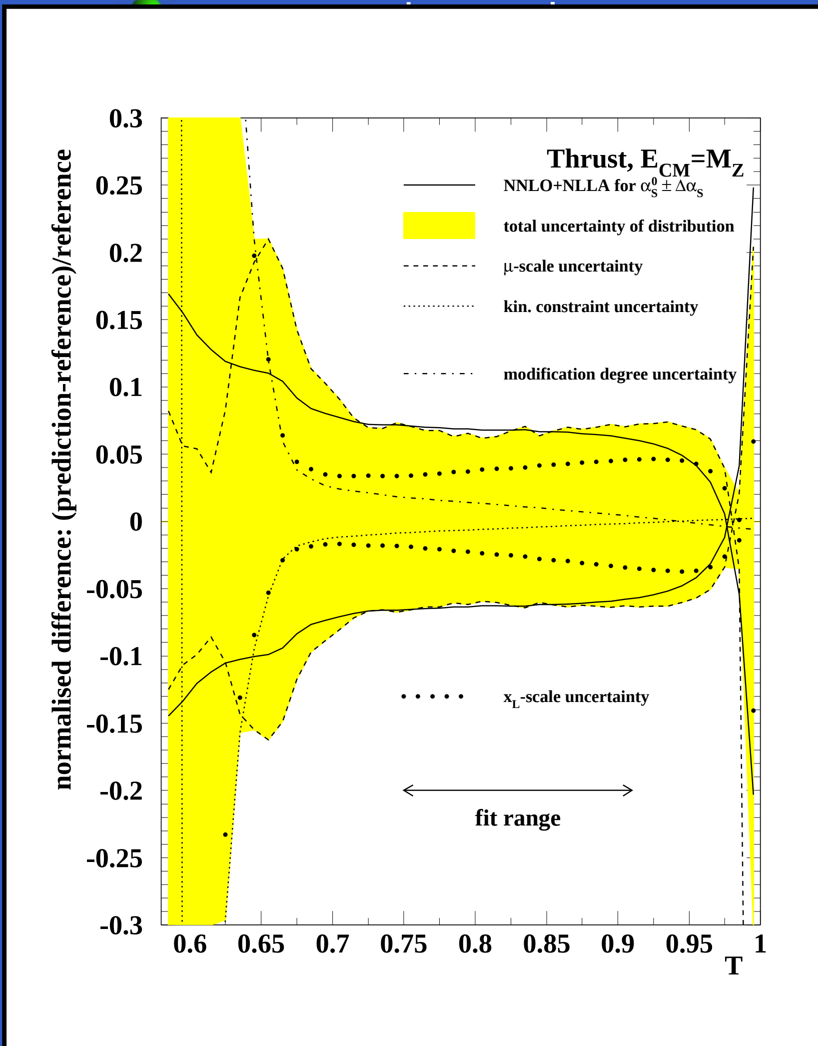

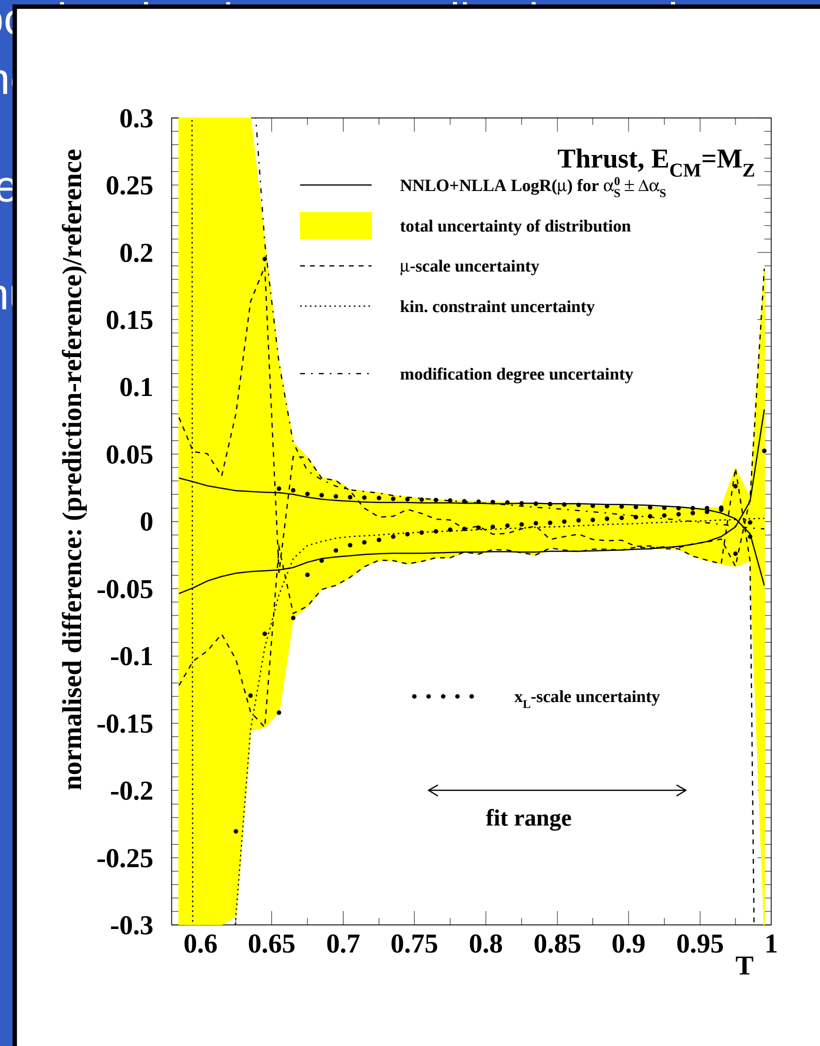

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$\log$ R

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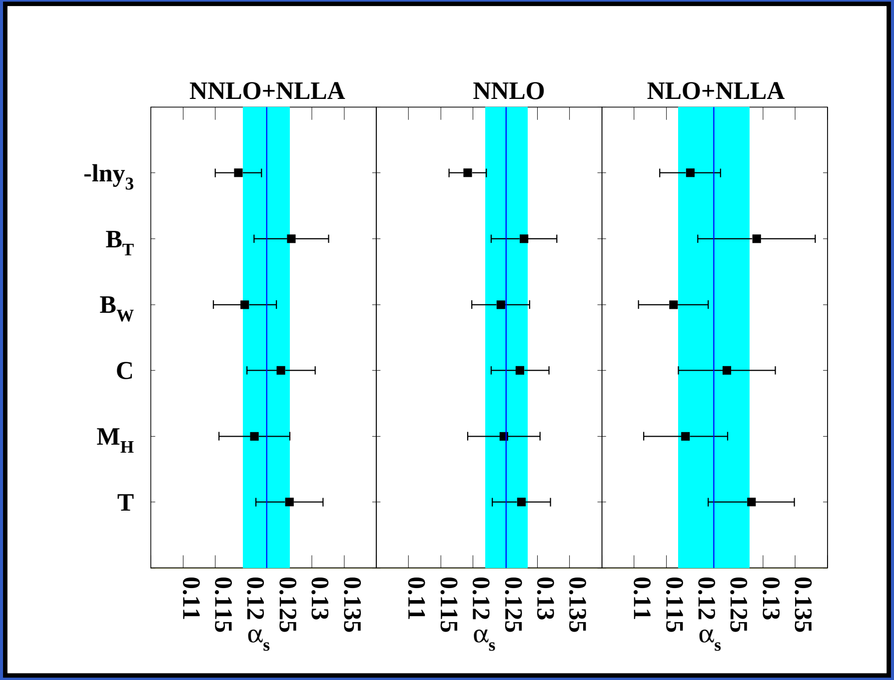
$\log$ R(μ)

data set	LEP1 + LEP2	LEP2
$\alpha_s(M_Z)$	0.1227	0.1226
stat. error	0.0008	0.0010
exp. error	0.0009	0.0010
pert. error	0.0022	0.0021
hadr. error	0.0012	0.0011
total error	0.0028	0.0028

- cancelations are probably overestimated,
- conservative result is more reliable.

Determination of α_s: NNLO+NLLA results



- Two class of observables:
  - T, C-par., B_{tot}: higher fit result $\rightarrow$ sizeable missing higher order
  - -$\log y_3$, B_w, M_H: lower fit result, $\rightarrow$ good convergence of pert. expansion

$\longrightarrow$ also seen by NNLO study of moments [Gehrmann-deRidder, Gehrmann, Glover, Heinrich]

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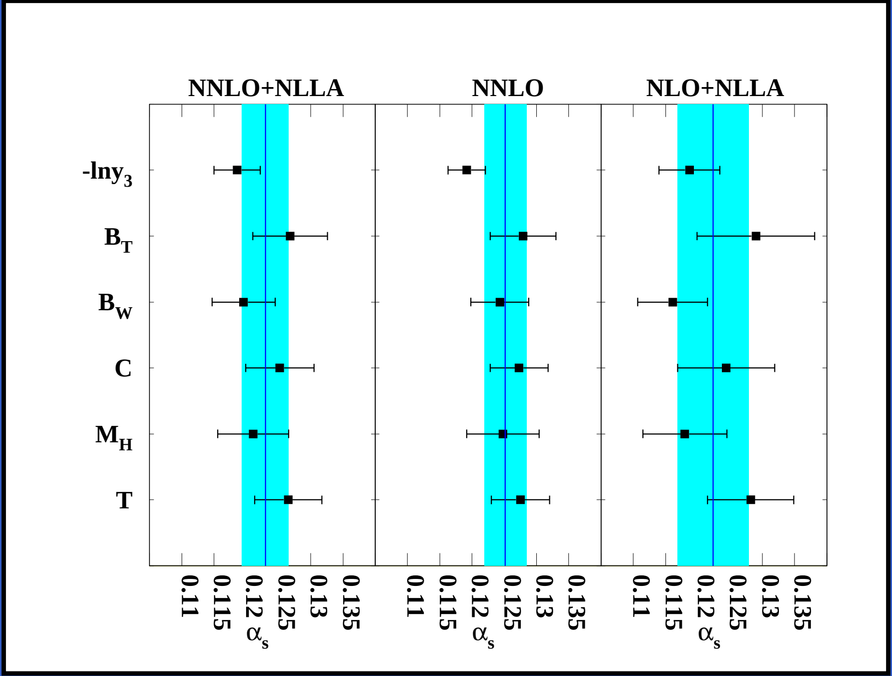

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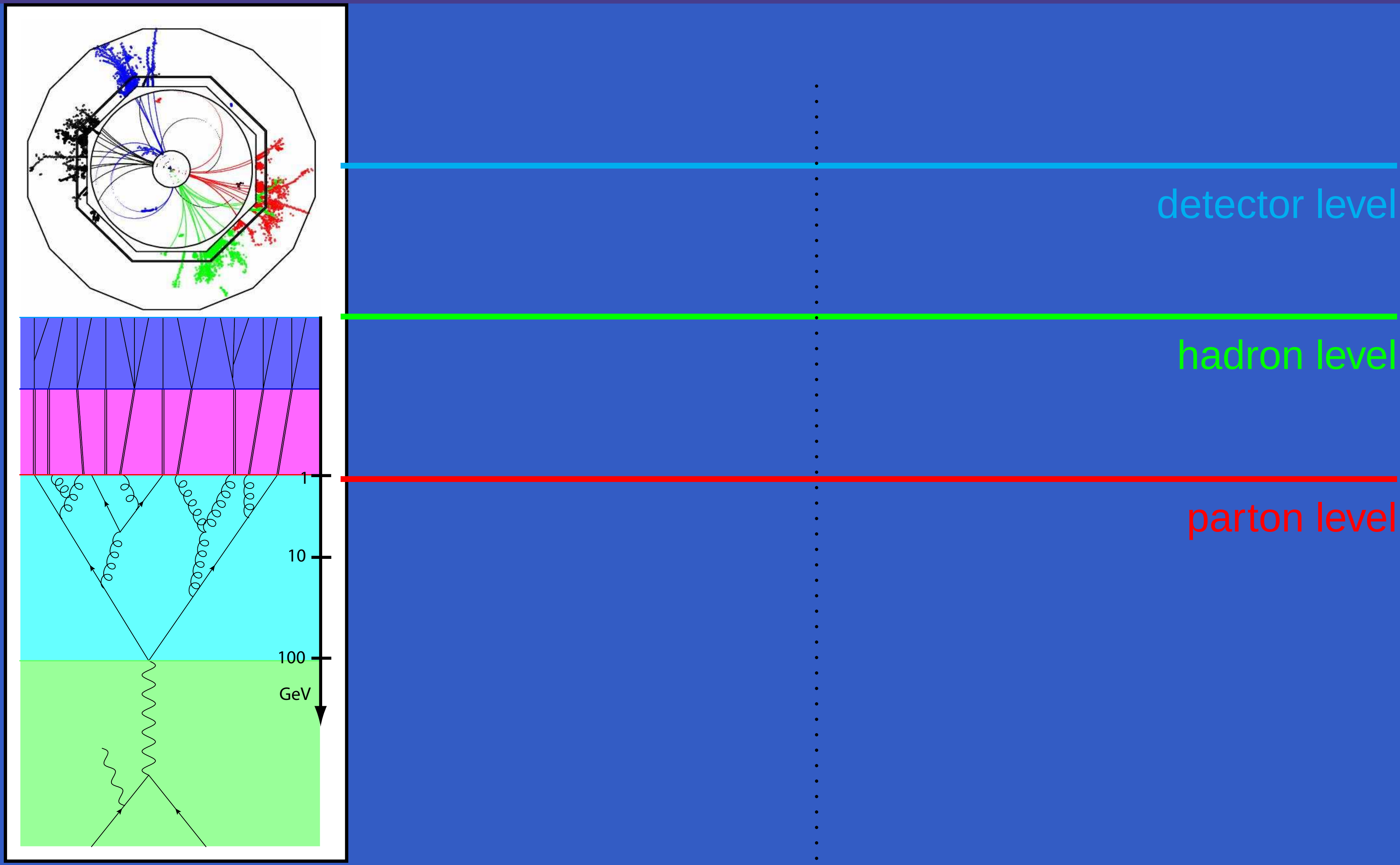
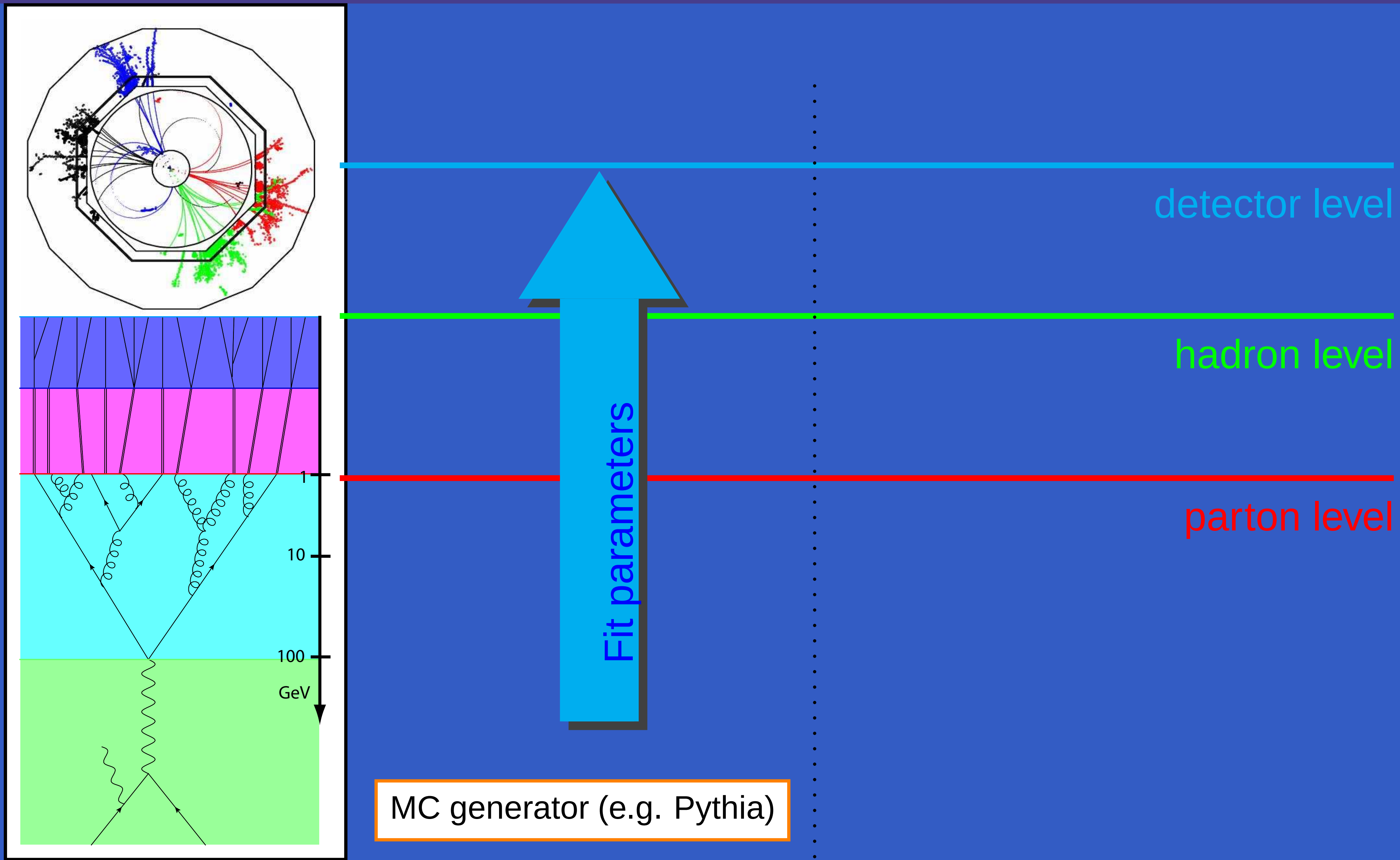
What about hadronization corrections?

Determination of α_s: Hadronization



Determination of α_s: Hadronization



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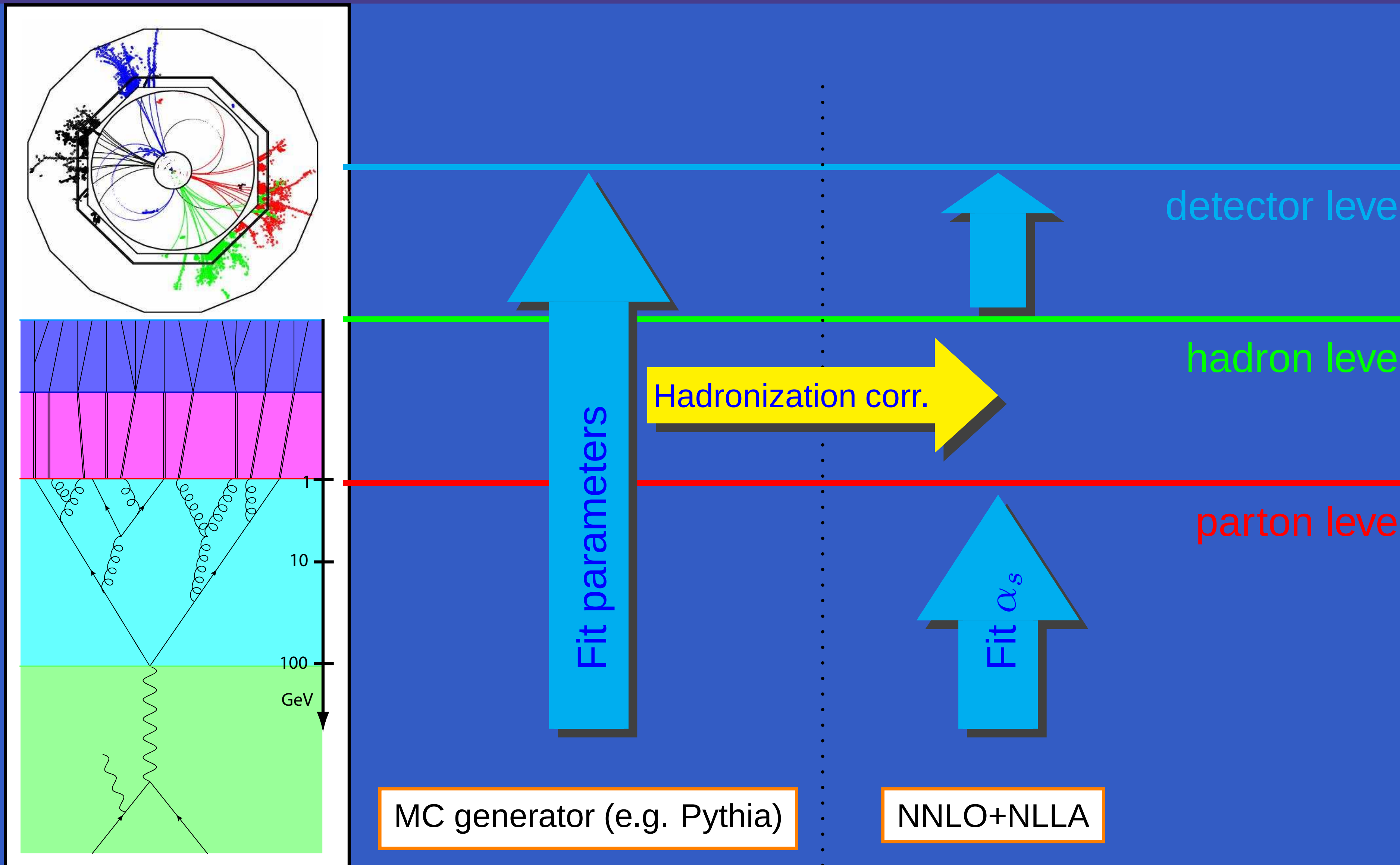

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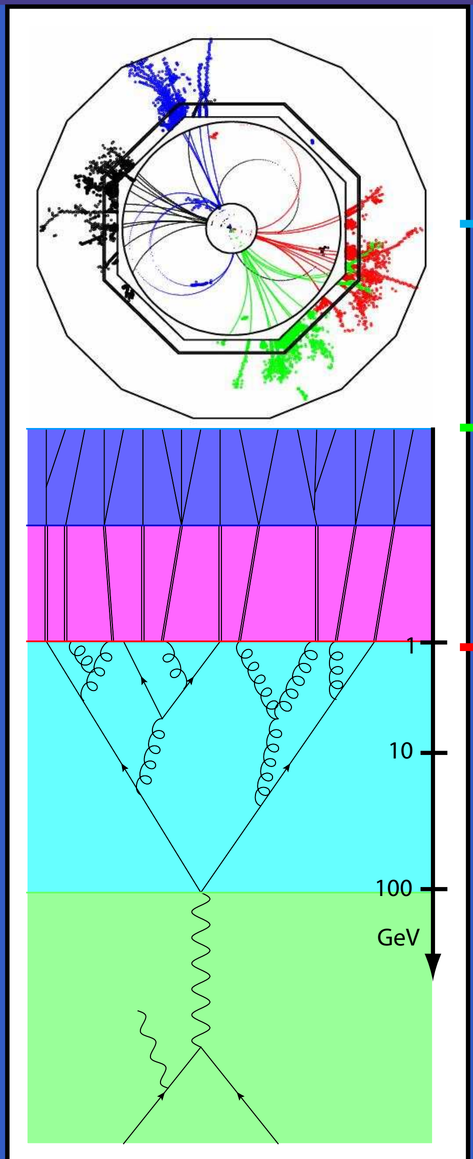
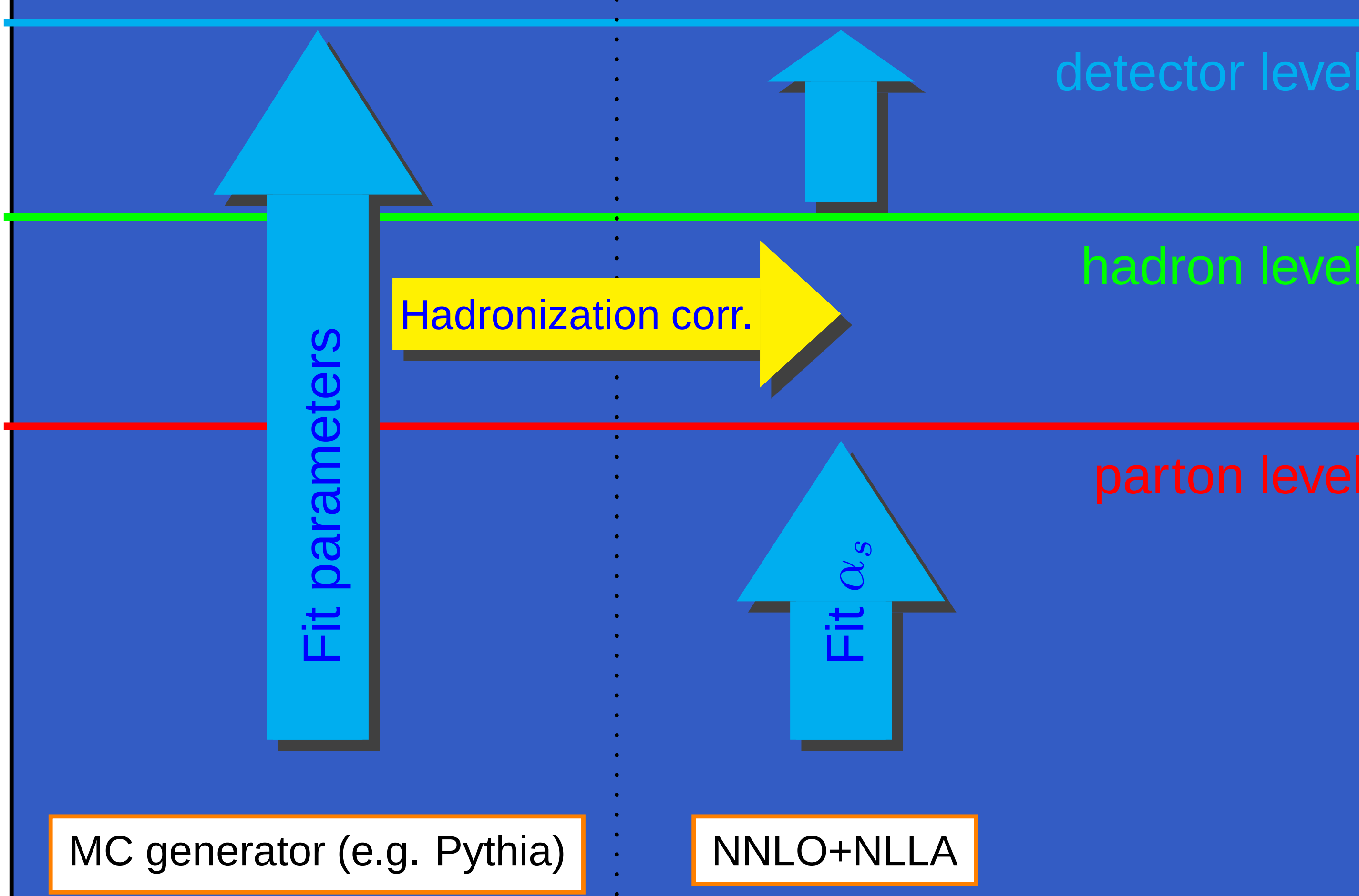
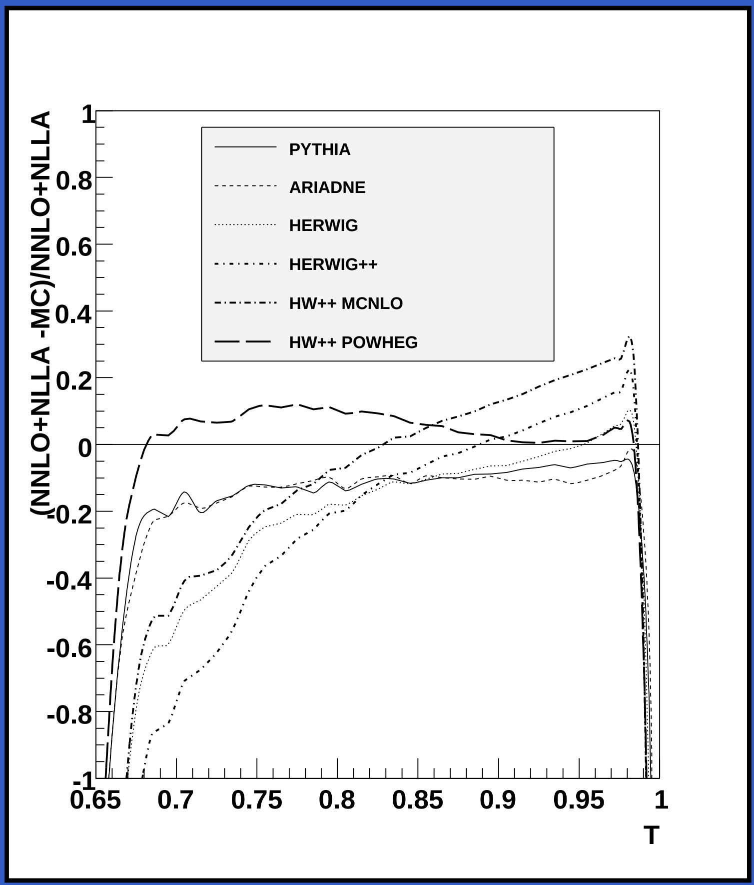
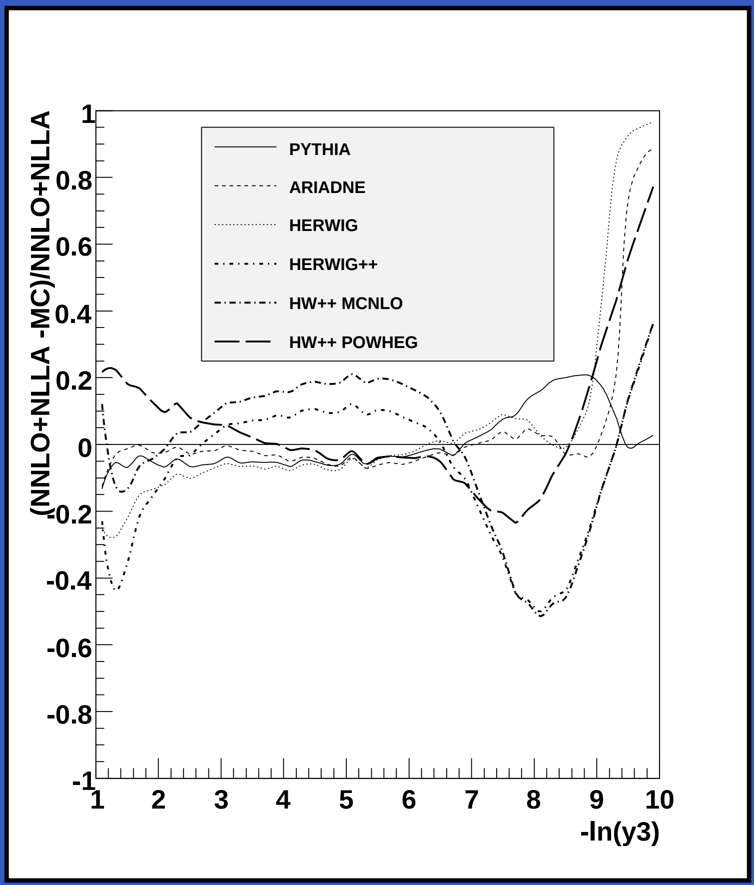


Problematic if parton level in MC and pQCD predictions are very different



Determination of α_s: Hadronization





- Thrust: parton level higher than in NNLO+NLLA

→ Pythia parameters tuned such that missing HO terms are (over-)compensated and hadronization corrections are effectively too small

Determination of α_s: Hadronization

$\alpha_s\,(M_z)$	T	C	M_H	B_W	B_T	$-\ln y_3$
PYTHIA	0.1266	0.1252	0.1211	0.1196	0.1268	0.1186
χ^2/N_{dof}	0.16	0.47	4.4	4.4	0.84	1.89
ARIADNE	0.1285	0.1268	0.1234	0.1212	0.1258	0.1202
χ^2/N_{dof}	0.96	0.52	2.5	3.1	2.15	1.41
HERWIG	0.1256	0.1242	0.1253	0.1203	0.1258	0.1203
χ^2/N_{dof}	0.5	0.65	4.4	2.0	2.15	0.8
HW++	0.1242	0.1228	0.1299	0.1212	0.1238	0.1168
χ^2/N_{dof}	6.6	3.2	3.3	1.33	2.65	0.56
HW++ MCNLO	0.1234	0.1220	0.1292	0.1220	0.1232	0.1175
χ^2/N_{dof}	10.7	4.2	2.2	1.1	5.7	0.69
HW++ POWHEG	0.1189	0.1179	0.1236	0.1169	0.1224	0.1142
χ^2/N_{dof}	1.46	2.55	3.8	3.9	1.54	0.56

Conclusions and outlook

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$$\alpha_s\,(M_Z) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

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  - reduced scatter and scale dependence,
  - indication of still missing higher order corrections
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- Further steps and improvements:
  - EW-corrections computed recently, [Denner, Dittmaier, Gehrmann, Kurtz; Carloni-Calame, Moretti, Piccini, Ross]
  - resummation of subleading logarithms (for all observables), [Becher, Schwartz],
  - further hadronization and power correction studies. [Davison, Webber]

Backup Slides

Fixed Order Calculations

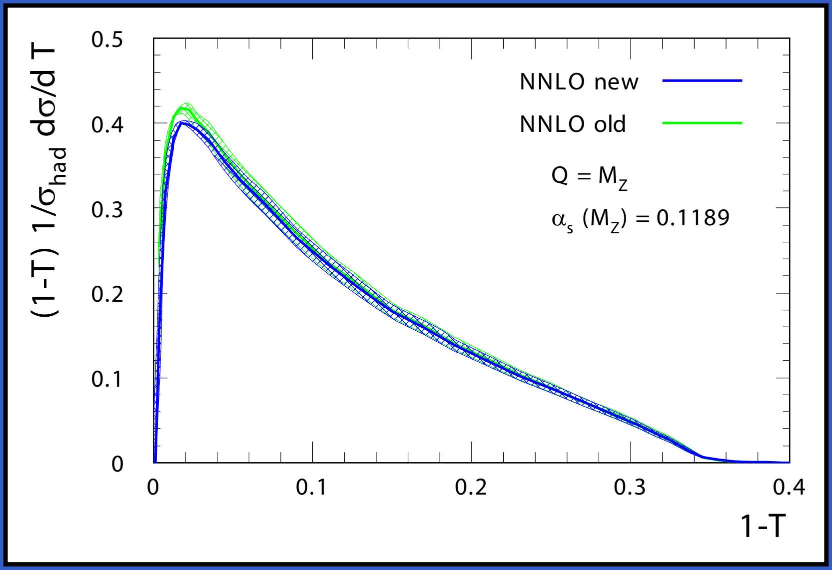
- NLO and NNLO calculations: [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich]
  - careful subtraction of real and virtual divergencies using antenna method:

$$
\begin{aligned}
d\sigma_{\mathrm{NLO}} &= \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NLO}} \right) + \left[\int_{d\Phi_{m+1}} d\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \int_{d\Phi_m} d\sigma^{V}_{\mathrm{NLO}} \right] \\
d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_{m+2}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+2}} d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \\
&+ \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}} - d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+1}} d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \\
&+ \int_{d\Phi_m} d\sigma^{\mathrm{V},2}_{\mathrm{NNLO}} .
\end{aligned}
$$

  - Implemented in the EERAD3 integration programme.

Fixed Order Calculations

- Inconsistency in the treatment of large-angle radiation, [Weinzierl]
  - inconsistency was corrected and cross-checks are in progress,
  - numerically minor changes in the kinematical region of interest for phenomenology.



Fixed Order Calculations

- Theoretical NNLO prediction $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}\ , x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_0}\,\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

However:

$$\frac{1}{\sigma_{\mathrm{had}}}\,\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \frac{\sigma_0}{\sigma_{\mathrm{had}}\,(Q,\mu)}\,\frac{1}{\sigma_0}\,\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu)$$

Measured

Theory prediction

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Measured

Theory prediction

- use simple expansion in α_s, or "exact" ratio up to calculated order
- issues:
  - mass effects
  - EWK effects (factorization)
  - $\longrightarrow$ Effects below per-cent range

Renormalization scale dependence

- The full renormalization scale dependence is given by making the following replacements,

$$\alpha_s \longrightarrow \alpha_s(\mu) \;,$$

$$\mathcal{B}(y) \longrightarrow \mathcal{B}(y,\mu) = 2\,\beta_0\;\ln x_\mu\,\mathcal{A}(y) + \mathcal{B}(y) \;,$$

$$\mathcal{C}(y) \longrightarrow \mathcal{C}(y,\mu) = (2\,\beta_0\;\ln x_\mu)^2\,\mathcal{A}(y) + 2\;\ln x_\mu\;[2\,\beta_0\mathcal{B}(y) + 2\,\beta_1\,\mathcal{A}(y)] + \mathcal{C}(y) \;,$$

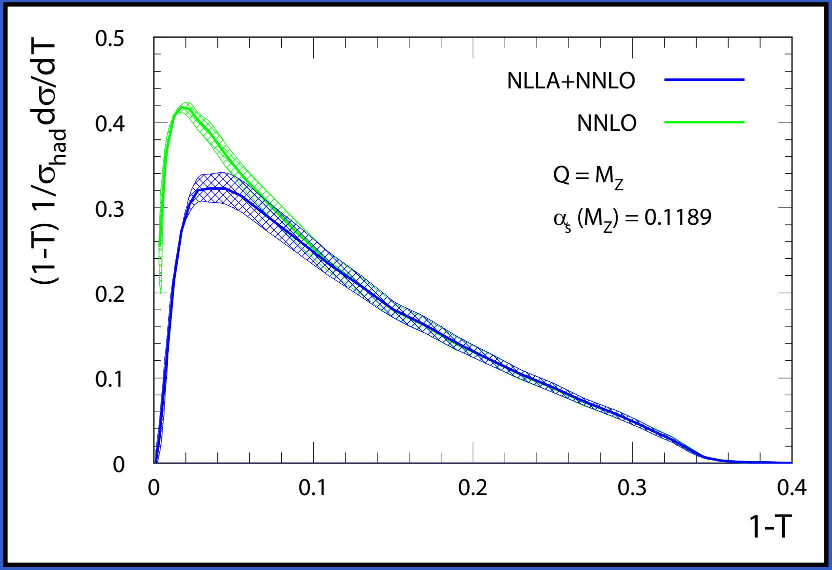
$$g_2(\alpha_S L) \longrightarrow g_2\left(\alpha_S L,\mu^2\right) = g_2(\alpha_S L) + \frac{\beta_0}{\pi}(\alpha_S L)^2\; g_1'(\alpha_S L)\;\ln x_\mu \;,$$

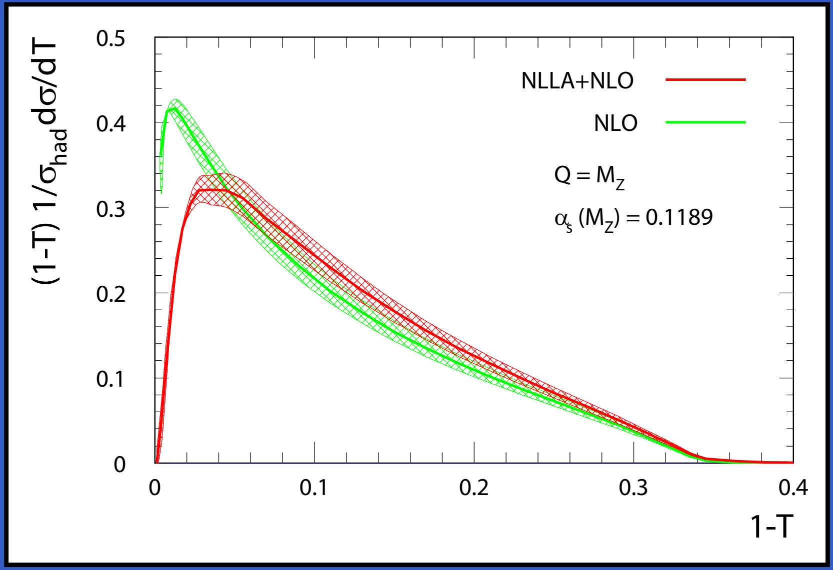
$$G_{22} \longrightarrow G_{22}(\mu) = G_{22} + 2\beta_0 G_{12}\ln x_\mu \;,$$

$$G_{33} \longrightarrow G_{33}(\mu) = G_{33} + 4\beta_0 G_{23}\ln x_\mu \;.$$

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$





- Difference between NLLA+NNLO and NNLO restricted to the two-jet region, whereas NLLA+NLO differ in normalisation throughout the full kinematical range.

Results: renormalization scale dependence

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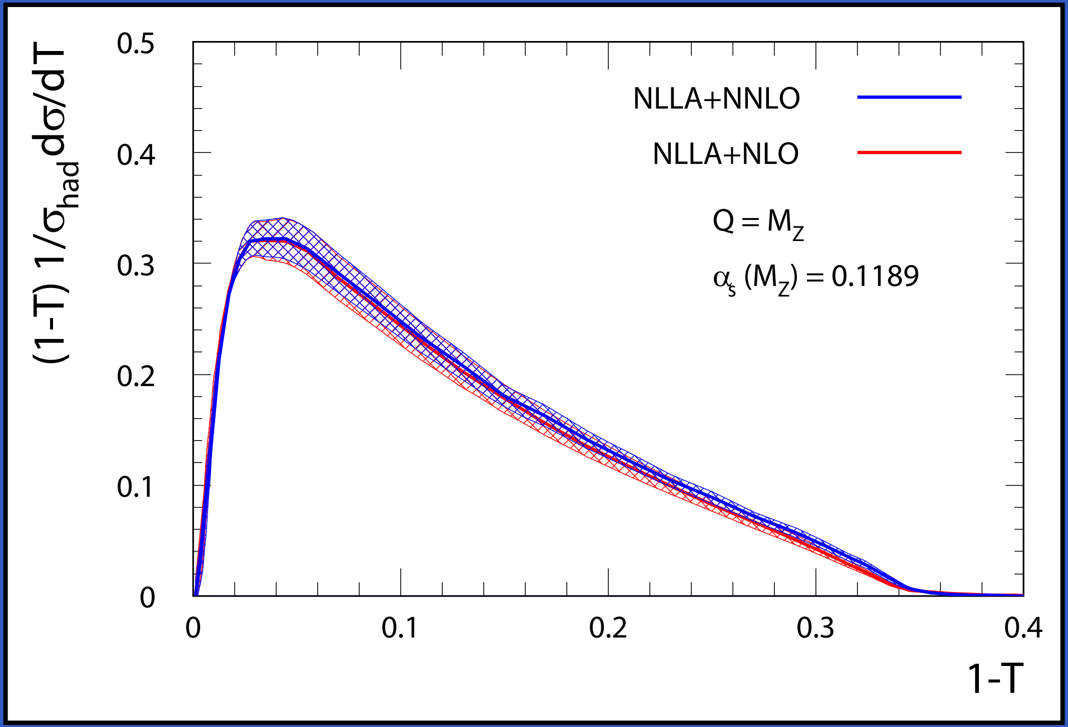

- Difference between NLLA+NNLO and NLLA+NLO moderate in the three-jet region.
- Renormalization scale dependence reduced in three-jet region.

Results: renormalization scale dependence

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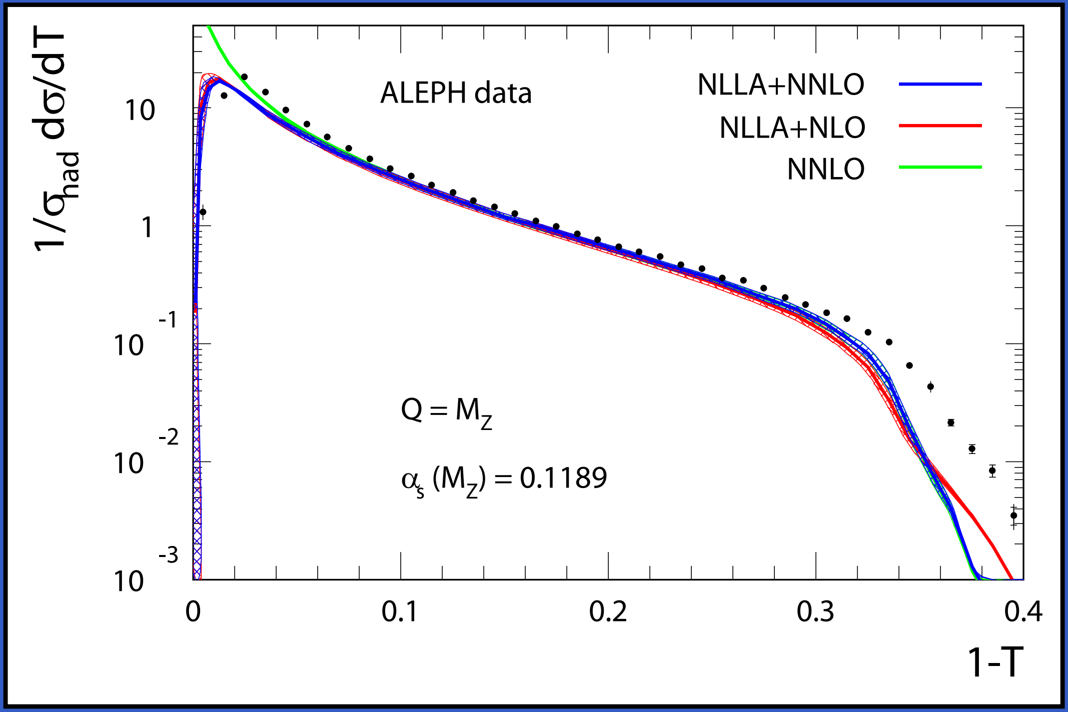
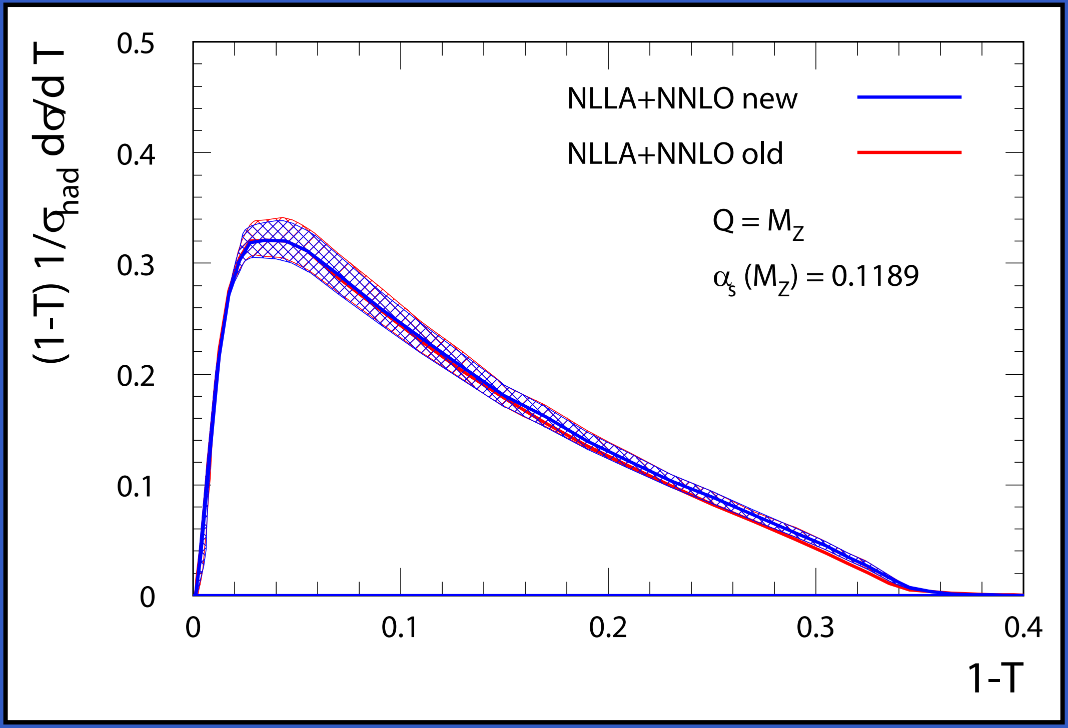


- Description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region.

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$



- Comparison between matched results using old and corrected new histograms: the small difference in the IR region disappears, resummation becomes dominant.

Results

- Full set of event shape variables:
  - Heavy jet mass: $\rho = \frac{M_H^2}{s} = \max_i \frac{1}{E_{\text{vis}}} \left(\sum_{k \in H_i} p_k\right)^2$
  - C-parameter: $\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p}_k|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p}_k|}$,

$$C = 3\left(\Theta^{11}\Theta^{22} + \Theta^{22}\Theta^{33} + \Theta^{33}\Theta^{22} - \Theta^{12}\Theta^{12} - \Theta^{23}\Theta^{23} - \Theta^{31}\Theta^{31}\right)$$

  - Total jet broadening: $B_T = B_1 + B_2$
  - Wide jet broadening: $B_W = \max(B_1, B_2)$,

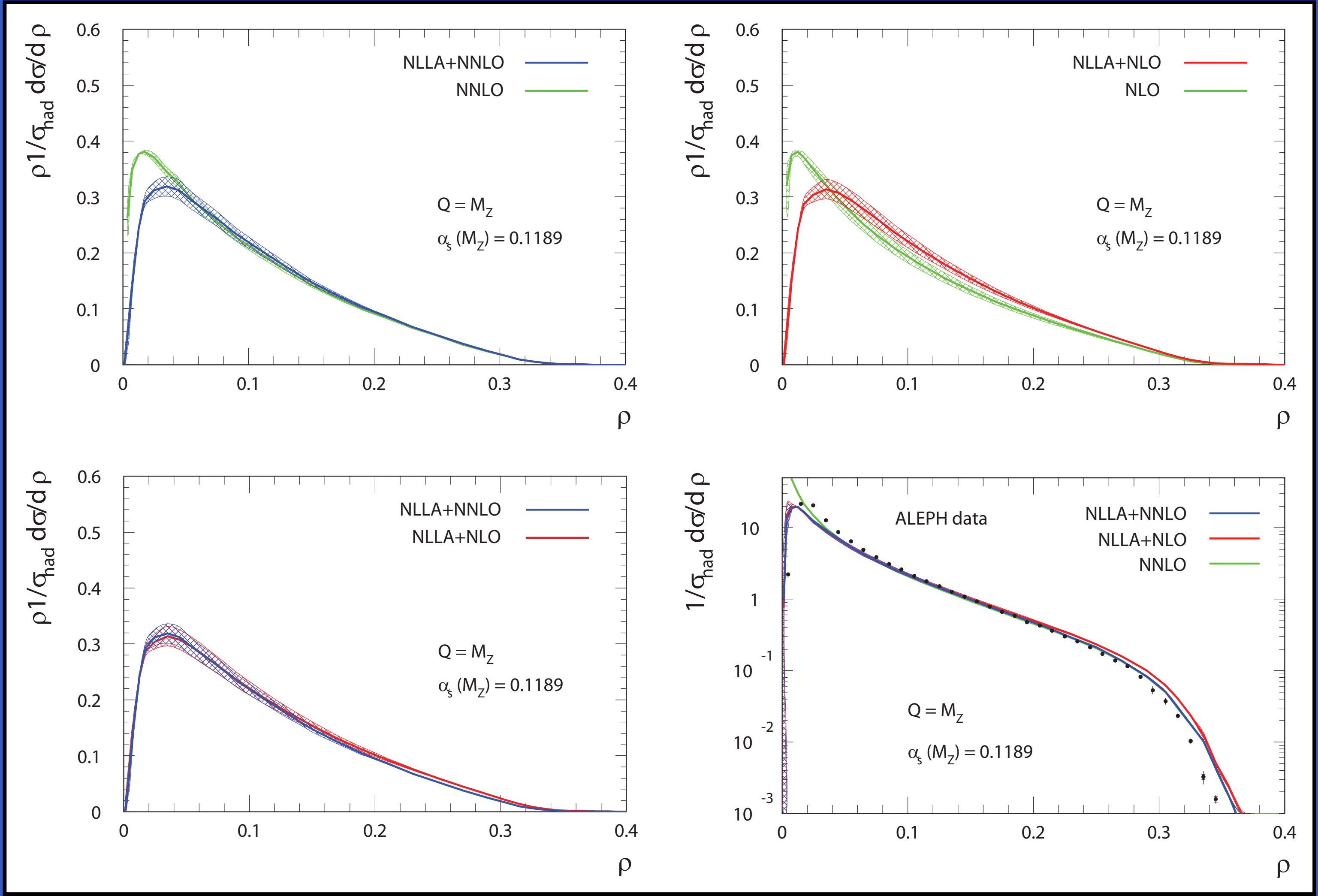
$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_k |\vec{p}_k|}$$

  - Two-to-three jet parameter for Durham jet algorithm:

$$y_{ij,D} = \frac{2 \min\left(E_i^2, E_j^2\right)(1 - \cos\theta_{ij})}{E_{\text{vis}}^2}$$

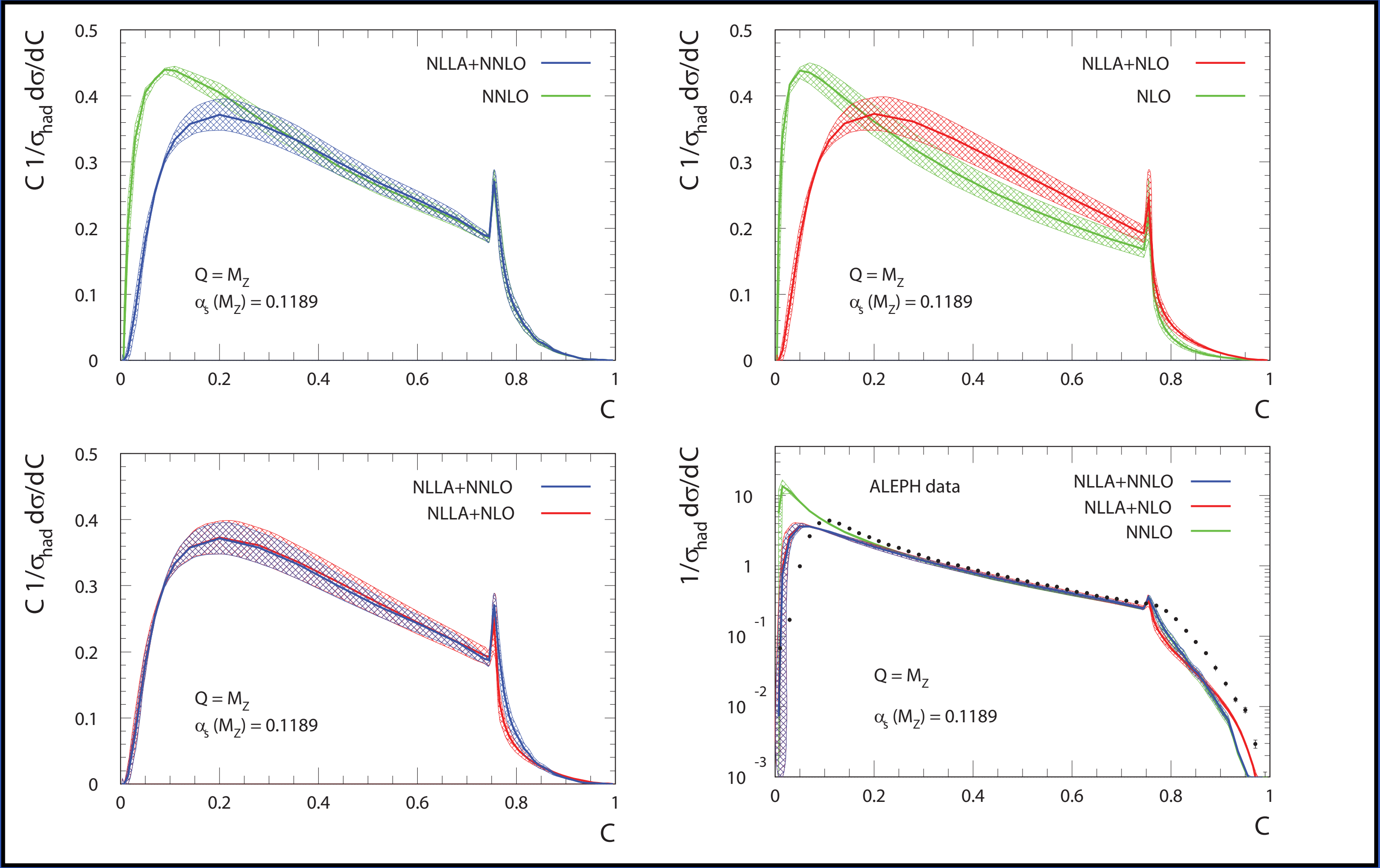
Results

- Heavy Jet Mass ρ:



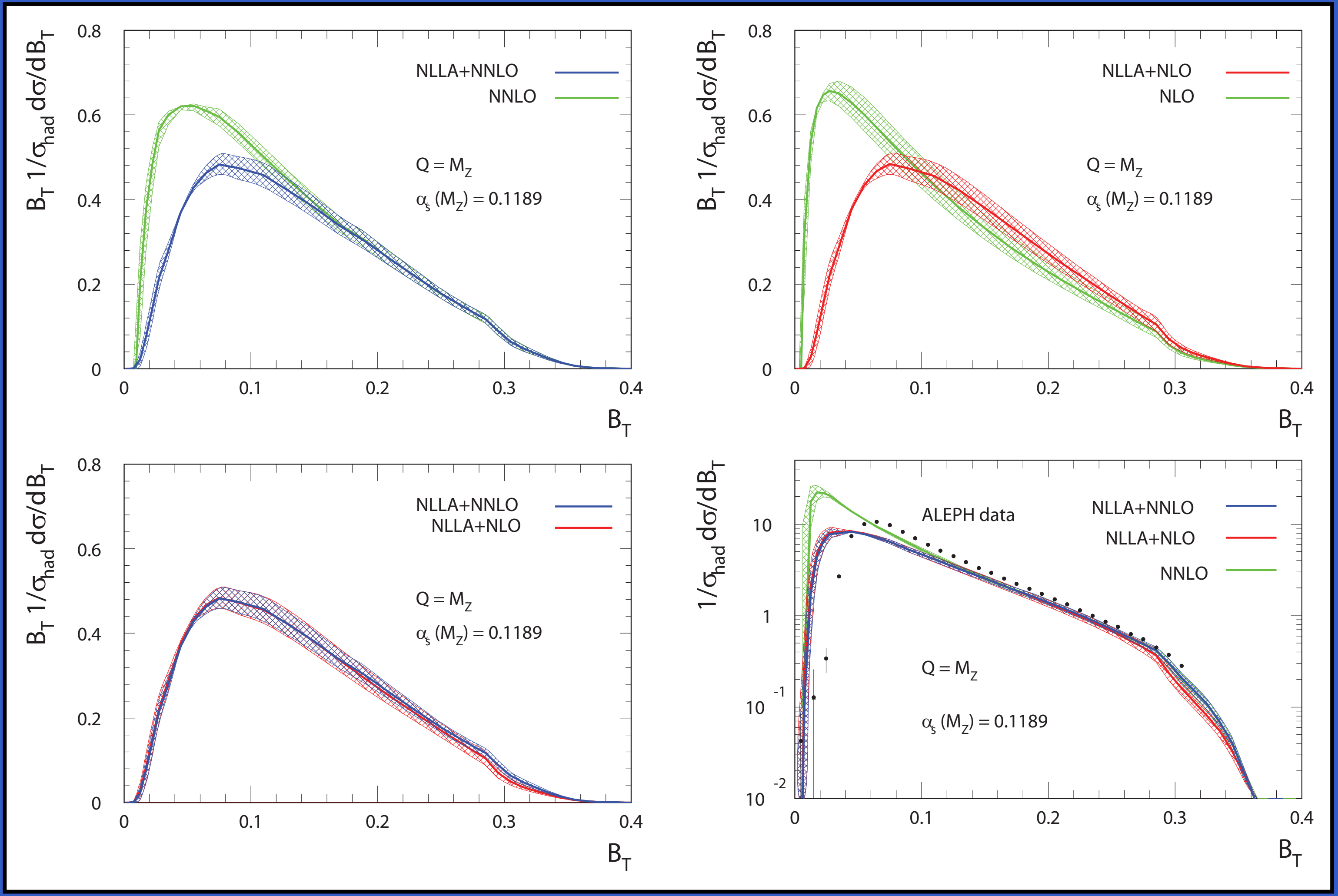
Results

- C-parameter C:



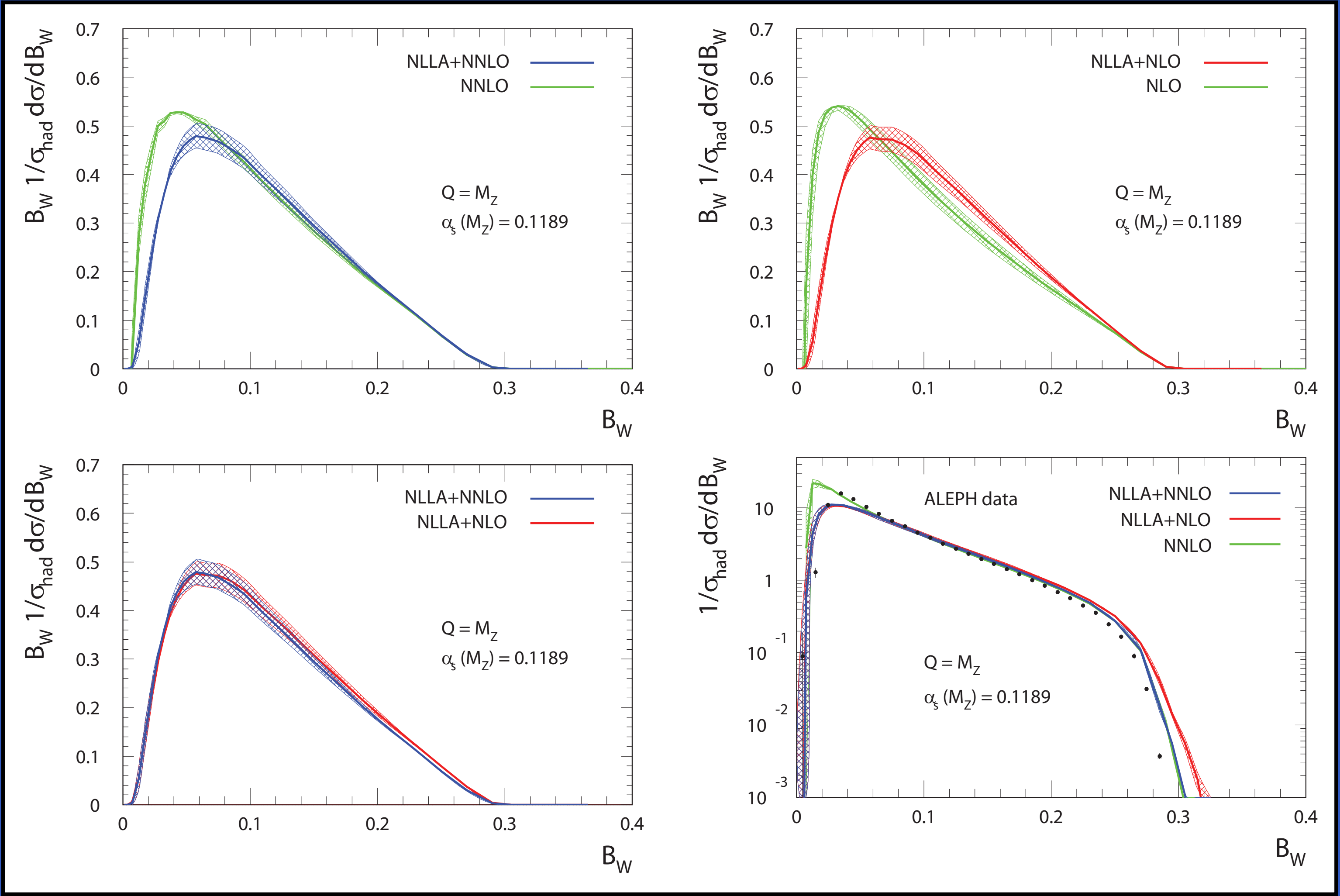
Results

- Total jet broadening B_T:



Results

- Wide jet broadening B_W:



Results

- Two-to-three jet parameter for Durham algorithm Y_3: